1

Estimate mean and standard deviations for both distributions using maximum likelihood. Use sum and sum of squares from question.

Mu1 = sum1 / n = 201.8

Mu2 = sum2 / n = 253.7

$$std1 = \sqrt{\frac{ssq_1}{n} - mu_1^2} = 30.23$$

$$std2 = 46.56$$

$$p(\omega_1) = 0.9$$

$$p(\omega_2) = 0.1$$

$$p(x|\omega_1) = \frac{1}{\sqrt{2\pi}\sigma 1}\exp(-\left(\frac{x-\mu 1}{\sigma 1}\right)^2/2) = 0.00594$$

$$p(x|\omega_2) = 0.00821$$

$$p(x) = p(x|\omega_1)p(\omega_1) + p(x|\omega_2)p(\omega_2) = 0.00617$$

So:

$$p(\omega_2|x) = \frac{p(x|\omega_2)p(\omega_2)}{p(x)} = \mathbf{0.1331}$$

2

(a) Linear regression – put into standard simultaneous equations.

m = 3

sum x = 5

sum x^2 = 9

sum y = 6

sum yx = 1 + 4 + 6 = 11

So equations are:

$3\beta_1 + 5\beta_2 = 6$ (1)

$5\beta_1 + 9\beta_2 = 11$ (2)

(1)/3 x 5: $5\beta_1 + \frac{25}{3}\beta_2 = 10$

subtract (2): $\left(\frac{25}{3} - 9\right)\beta_2 = 1$

So $\beta_2 = \frac{3}{2} = 1.5$

$\beta_1 = 2 - \frac{5}{3}\beta_2 = -0.5$

So equation is y = 1.5x - 0.5

(b)

PCA:

Get rid of mean initially:

$$\bar{x} = \frac{5}{3}$$

$$\bar{y} = 2$$

$$\begin{pmatrix} 2 & 3 \\ 2 & 2 \\ 1 & 1 \end{pmatrix} - \boldsymbol{\mu} = \begin{pmatrix} \frac{1}{3} & 1 \\ \frac{1}{3} & 0 \\ -\frac{2}{3} & -1 \end{pmatrix}$$

Calculate covariance matrix:

$$C = \begin{pmatrix} \frac{2}{3} & 1 \\ 1 & 2 \end{pmatrix}$$

Get eigenvectors – large one is:

$$v = \begin{pmatrix} -2 + \sqrt{13} \\ 3 \end{pmatrix}$$

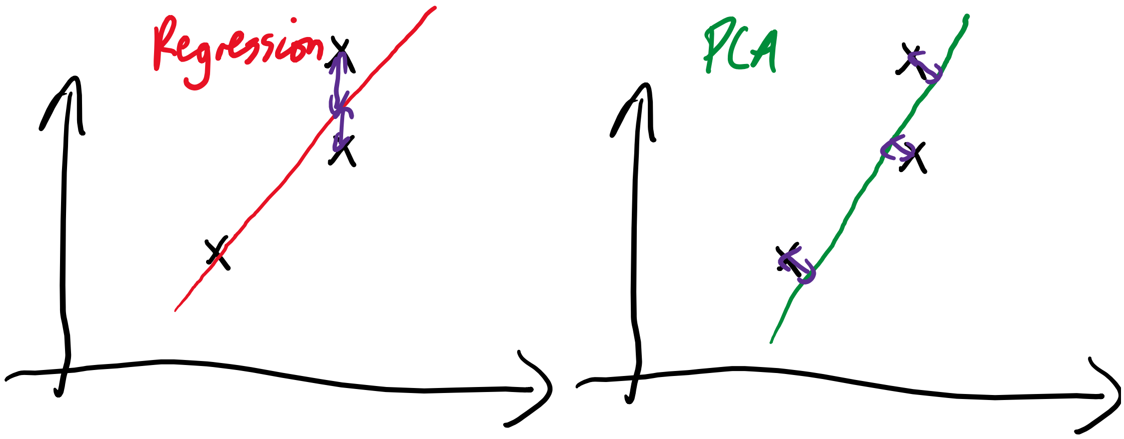
Put into an equation – subtract off mean terms from x and y and make gradient between them along the eigenvector:

$$y - 2 = (x - 5/3)\frac{3}{-2 + \sqrt{13}}$$

so

$$y = 1.87x - 1.12$$

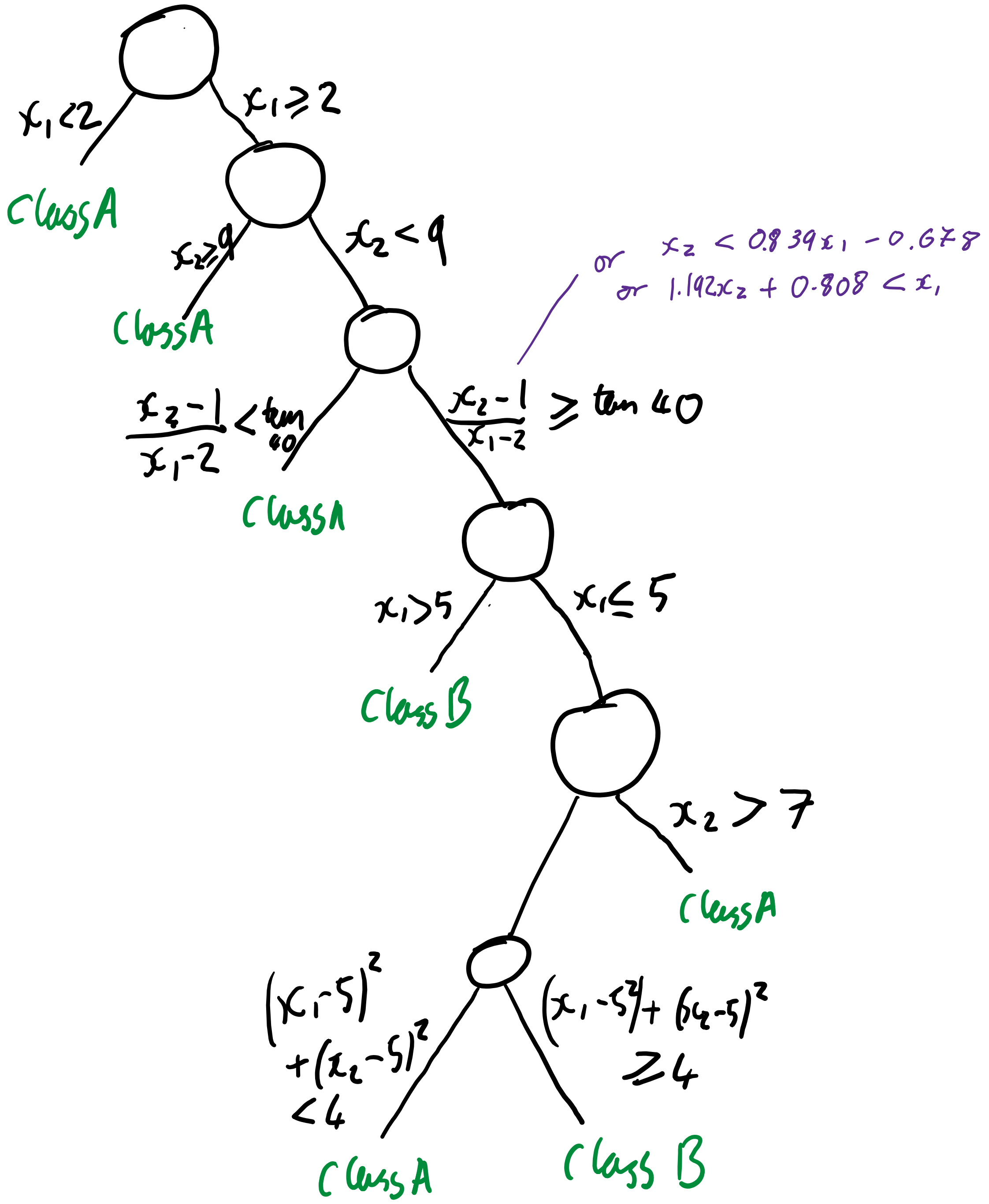
(c)



Regression minimises the square of the y distances. Principal component analysis instead minimises the square of the perpendicular distance between the line and the points.

PCA makes no distinction between the two parameters, x and y – both are fitted equally - which fits with it being an unsupervised method. However, regression minimises the error in the y parameter, the output under a supervised method, which is to be fitted.

3


x1<2
x1≥2
Class A
x2≥9
x2<9
Class A
or x2 < 0.839x1 − 0.678
or 1.192x2 + 0.808 < x1
(x2−1)/(x1−2) < tan 40
(x2−1)/(x1−2) ≥ tan 40
Class A
x1>5
x1≤5
Class B
x2>7
Class A
(x1−5)² + (x2−5)² < 4
(x1−5)² + (x2−5)² ≥ 4
Class A
Class B

4

Define $X = 3$, $Y = 2$, as horizontal and vertical distances respectively between the two given points. Take $x1 = cx$, $y1 = cy$ as the horizontal and vertical distances from point A and the equidistant point, and $x2$, $y2$ as the same from point B. $x1 + x2 = X$, $y1+y2 = Y$.

L1 dist:

Sum of absolute components

$|x1| + |y1| = |x2| + |y2|$

$|cx| + |cy| = |X - cx| + |Y - cy|$

$cx + cy = X – cx – Y + cy$

(since $Y < cy$)

$cx = (X – Y)/2 = 0.5$

Linf dist:

Maximum of absolute components

$Max(|x1|,|y1|) = Max(|x2|,|y2|)$

$Max(|cx|,|cy|) =$ cy since $cx = 0.5$ from above and cy must be > 2 from question

$Max(|x2|,|y2|) = Max(|X-cx|,|cy-Y|) = Max(|2.5|,|cy-2|)$

$cy = cy-2$ is incompatible (and also unlikely that $cy-2 > 2.5$) so take $cy = 2.5$

Point C is at (0.5, 2.5)

(b)

L2 dist is sum of the squares (rooted)

L2 dist (squared) from (0, 0): $0.5 ^2 + 2.5 ^2 = 6.5$

$x2 = 3 – 0.5 = 2.5$

$y2 = 2 – 2.5 = -0.5$

L2 dist (squared) from (3, 2): $2.5 ^2 + (-0.5) ^2 = 6.5$

Same dist.

5

(a) Direction must be between the two points, so

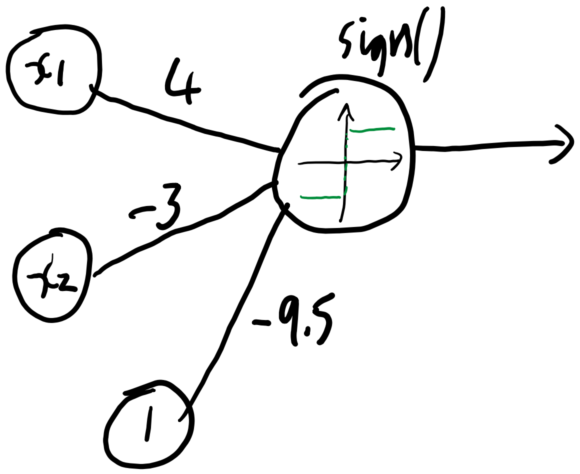
$$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} 7-3 \\ 2-5 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

Bias value must be defined such that get zero out at the centre (5, 3.5):

$$\begin{pmatrix} 4 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 3.5 \end{pmatrix} + w_0 = 0$$

$$20 - 10.5 + w_0 = 0$$

$$w_0 = -9.5$$



[Give 1 mark if just a 'step function' is listed rather than specifically the sign()]

(b) The line equidistant from the two points under the L2 norm is straight, so a linear separation is sufficient. The addition of more nodes and/or layers is unnecessary since the single node can fully capture this linear problem.

6.

3 parameter space, therefore 3 principal components. Need to find the third one.

Must be orthogonal to the other two – do a cross product to get this

$$\frac{1}{9}\begin{pmatrix}2\\1\\2\end{pmatrix} \times \begin{pmatrix}1\\2\\-2\end{pmatrix} = \frac{1}{9}\begin{pmatrix}-2-4\\2+4\\4-1\end{pmatrix} = \begin{pmatrix}-2/3\\2/3\\1/3\end{pmatrix}$$

Don't need to worry about magnitude – both original vectors are unit vectors, so cross product must have unit magnitude.