SHAHEED BHAGAT SINGH STATE TECHNICAL CAMPUS, FEROZEPUR

ROLL No: | | | | | | | | | | | | | **Total number of pages:[ ]**

Total number of questions:06

B.Tech. 1st Sem

Engg. Mathematics-I

Subject Code:BTAM-101A

Paper ID:

Time allowed: 3 Hrs **Max Marks: 60**

Important Instructions:

- **All questions are compulsory**
- **Assume any missing data**
- **Additional instructions, if any**

PART A (2×10)

All COs

Q. 1. Short-Answer Questions:

(a) If $\vec{a} = x^2yz\,\hat{i} - 2xz^3\,\hat{j} + xz^2\hat{k}$ and $\vec{b} = 2z\,\hat{i} + y\,\hat{j} - x^2\hat{k}$ Find $\frac{\partial^2}{\partial x\,\partial y}(\vec{a} \times \vec{b})$ at $(1, 0, -2)$

(b) Find the points on the surface $z^2 = xy + 1$ nearest to the origin .

(c) If $u = f(r, s)$, $r = x + y$ and $s = x - y$ Then show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2\frac{\partial u}{\partial r}$$

(d) Write all the salient features of curve tracing.

(e) Find the radius of curvature of the circle $x^2 + y^2 = a^2$ at any point.

(f) If $x = uv, y = \frac{u}{v}$, *prove that* $JJ' = 1$

(g) Evaluate $\int_1^2 \int_0^x \frac{1}{x^2+y^2} dydx$

(h) Find the C.G. of the area of the parabola $y^2 = 4ax$ bounded by the x-axis and the latus rectum.

(i) Prove that $div(\emptyset\vec{f}) = \emptyset div\vec{f} + \vec{f}.grad\emptyset$

(j) Prove that $\vec{A} = 3y^4z^2\,\hat{i} + 4x^3z^2\,\hat{j} - 3x^2y^2\hat{k}$ is solenoidal .

PART B (8×5)

Q. 2. Trace the curve $x(x^2 + y^2) = a(x^2 - y^2)$. COa

OR

Find the radius of curvature to the curve $x = 3acost - acos3t$, $y = 3asint - asin3t$ at any point. COa

Q. 3. Find the M.I about the x-axis of the lamina of the part of parabola $y = \sqrt{x}$ lying between (0,0) and (4,2) COb

OR

Find the entire length of the cardioid $r = (1 + cos\theta)$ and show that the upper COb

half of the curve is bisected by the line $\theta = \frac{\pi}{3}$ COc

Q. 4. State Euler's theorem and prove that if $u = \tan^{-1}\frac{x^3+y^3}{x+y}$

then $x2\ \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = sin4u - sin2u$

OR

find the minimum value of the function $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = a + b + c$ COc COd

Q. 5. Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = 4 - 4x$

OR COd

Evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x + y)dxdy$ by changing the order of integration

COe

Q. 6. Verify Stoke's theorem for $\vec{F} = (y - z + 2)\hat{\imath} + (yz + 4)\hat{\jmath} - xz\hat{k}$ over the surface of a cube x = 0 , y = 0 , z = 0 , x =2 , y = 2 , z = 2 above XOY plane .

OR

A vector field is given by $\overrightarrow{F} = (\sin y)\hat{\imath} + x(1+\cos y)\hat{\jmath}$ Evaluate the line integral over a circular path given by equation $x^2 + y^2 = a^2$, z = 0. COe