Data Abstraction

Announcements

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All Programmers

Great Programmers

Rational Numbers

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$$\frac{\text{numerator}}{\text{denominator}}$$

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Exact representation of fractions

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Exact representation of fractions

A pair of integers

Rational Numbers

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As soon as division occurs, the exact representation may be lost! (Demo)

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Assume we can compose and decompose rational numbers:

- `rational(n, d)` returns a rational number `x`

Rational Numbers

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Exact representation of fractions

A pair of integers

As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

- `rational(n, d)` returns a rational number `x`
- `numer(x)` returns the numerator of `x`

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Exact representation of fractions

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Assume we can compose and decompose rational numbers:

- rational(n, d) returns a rational number x
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Rational Numbers

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Exact representation of fractions

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Assume we can compose and decompose rational numbers:

Constructor → `rational(n, d)` returns a rational number x

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As soon as division occurs, the exact representation may be lost! (Demo)

Assume we can compose and decompose rational numbers:

Constructor

- `rational(n, d)` returns a rational number x

Selectors

- `numer(x)` returns the numerator of x
- `denom(x)` returns the denominator of x

Rational Number Arithmetic

Example

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5}$$

Example

General Form

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$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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$$\frac{nx}{dx} * \frac{ny}{dy}$$

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{nx}{dx} * \frac{ny}{dy} \qquad \frac{nx*ny}{dx*dy}$$

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

$$\frac{3}{2} + \frac{3}{5}$$

Example

$$\frac{nx}{dx} * \frac{ny}{dy} = \frac{nx*ny}{dx*dy}$$

General Form

Rational Number Arithmetic

$$\frac{3}{2} * \frac{3}{5} = \frac{9}{10}$$

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General Form

Rational Number Arithmetic Implementation

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- `rational(n, d)` returns a rational number x
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```
def mul_rational(x, y):
    return rational(numer(x) * numer(y),
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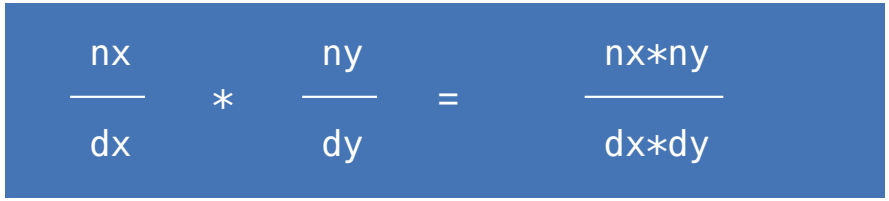

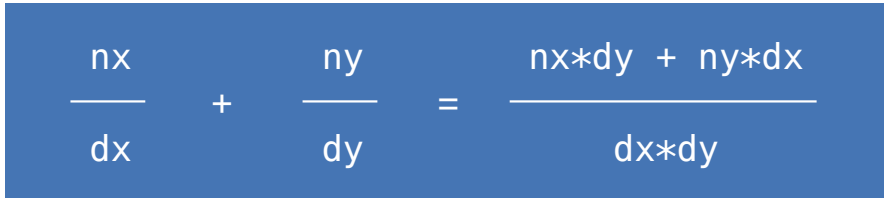

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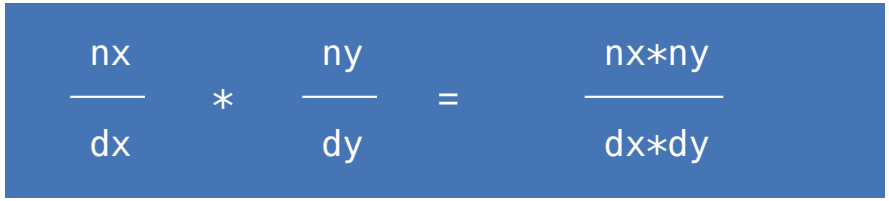

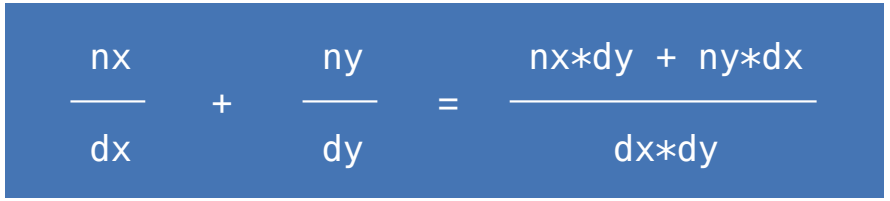

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These functions implement an abstract representation for rational numbers

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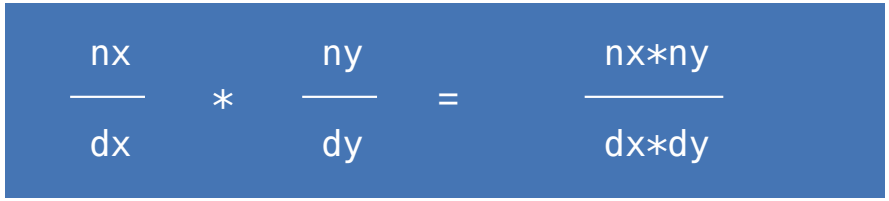
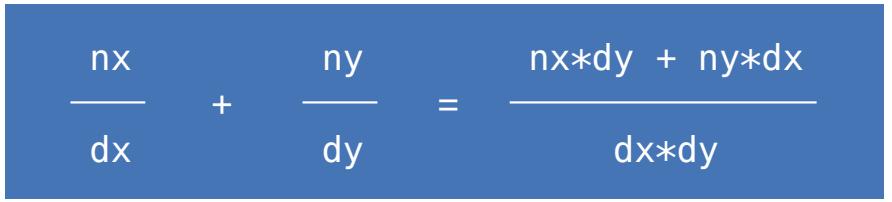



```
def add_rational(x, y):
    nx, dx = numer(x), denom(x)
    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)
```



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def print_rational(x):
    print(numer(x), '/', denom(x))
```

Constructor

Selectors

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    ny, dy = numer(y), denom(y)
    return rational(nx * dy + ny * dx, dx * dy)

def print_rational(x):
    print(numer(x), '/', denom(x))

def rationals_are_equal(x, y):
    return numer(x) * denom(y) == numer(y) * denom(x)
```

$$\frac{nx}{dx} + \frac{ny}{dy} = \frac{nx*dy + ny*dx}{dx*dy}$$

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Representing Rational Numbers

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def rational(n, d):
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def numer(x):
    """Return the numerator of rational number X."""
    return x[0]
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Select item from a list

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Select item from a list

(Demo)

Reducing to Lowest Terms

Example:

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Reducing to Lowest Terms

Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2} \qquad \frac{2}{5} + \frac{1}{10}$$

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Reducing to Lowest Terms

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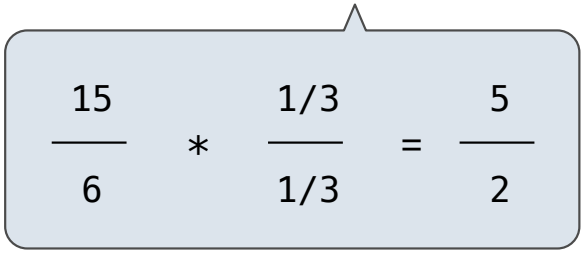

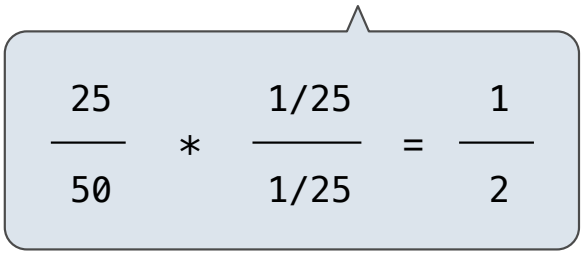


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from math import gcd
```

Reducing to Lowest Terms

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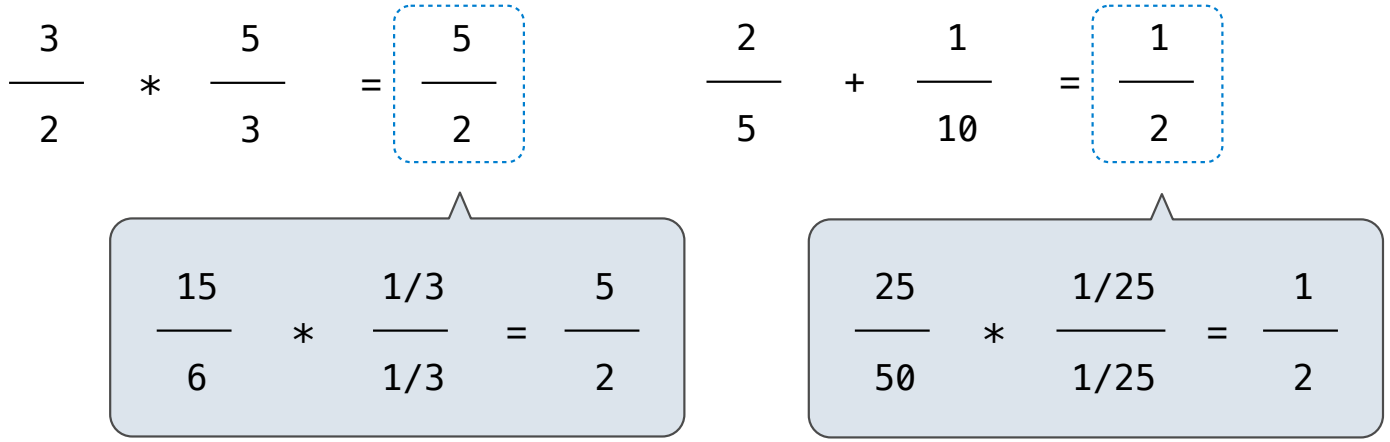


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Reducing to Lowest Terms

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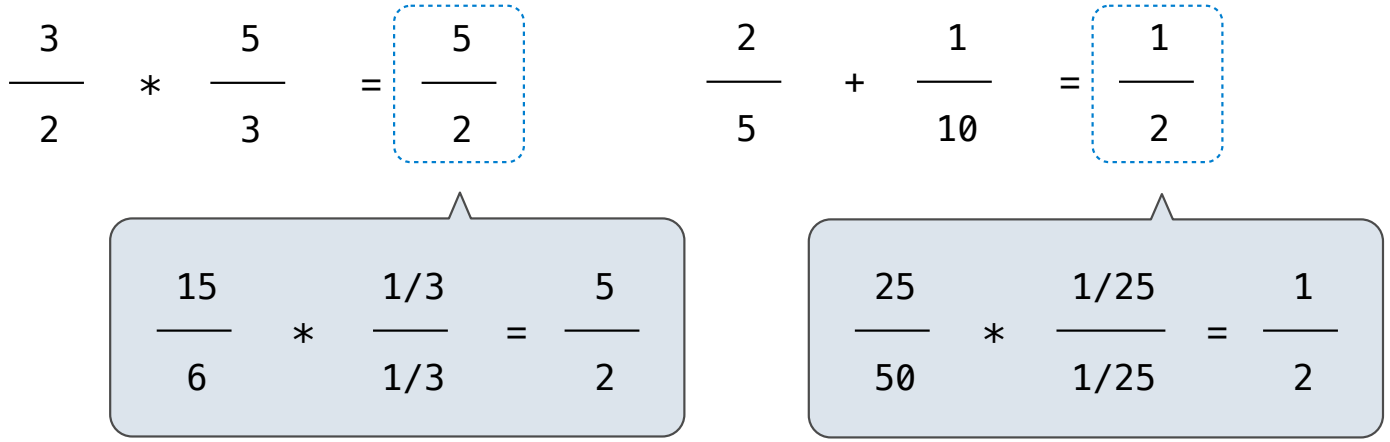


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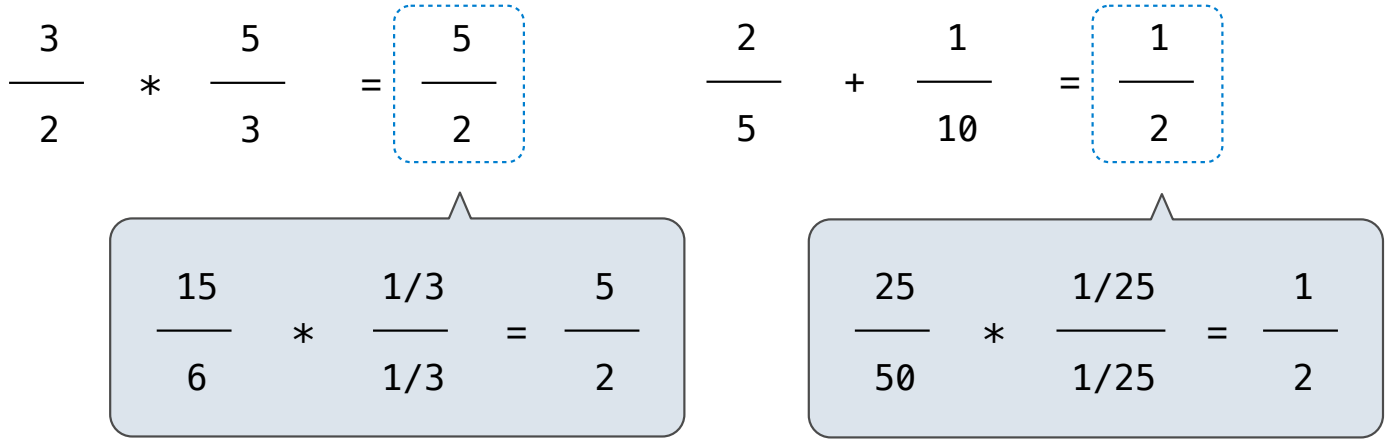


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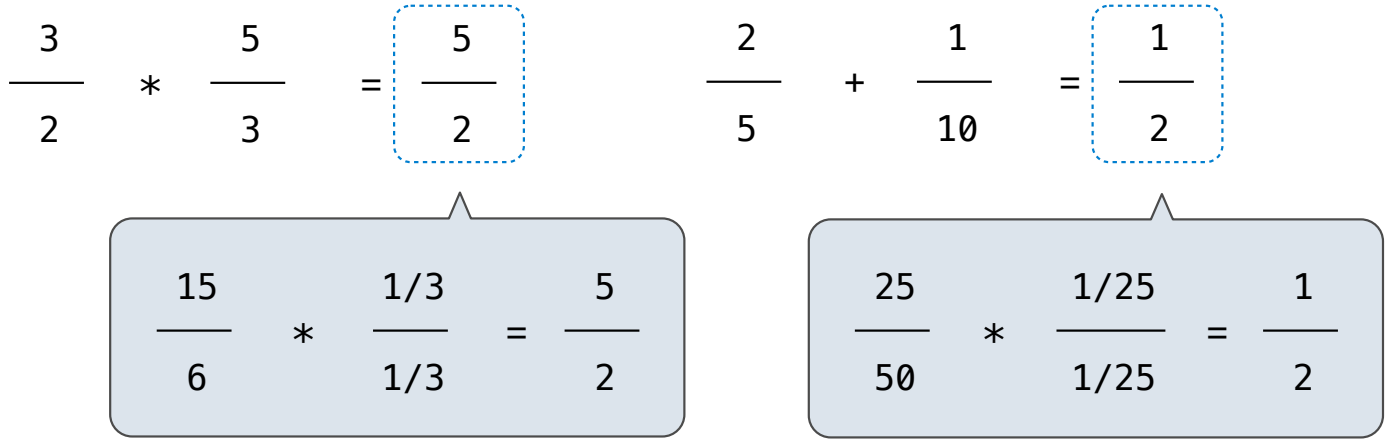


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from math import gcd

def rational(n, d):
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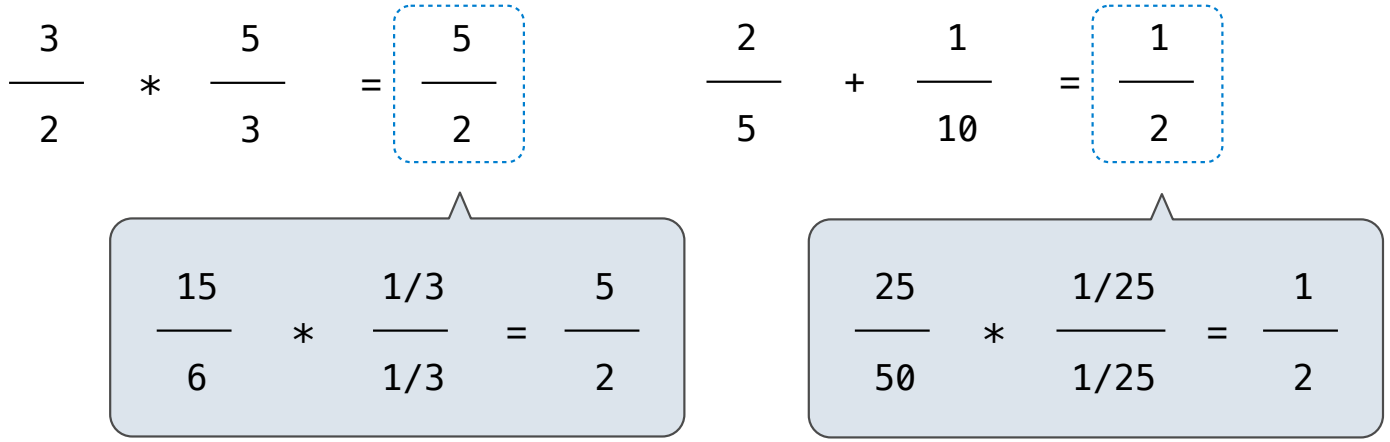


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Greatest common divisor

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Example:

$$\frac{3}{2} * \frac{5}{3} = \frac{5}{2} \qquad \frac{2}{5} + \frac{1}{10} = \frac{1}{2}$$

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(Demo)

Abstraction Barriers

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation		

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
Use rational numbers to perform computation	whole data values	

Abstraction Barriers

Parts of the program that...	Treat rationals as...	Using...
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Implementation of lists

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Implementation of lists

Violating Abstraction Barriers

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add_rational( [1, 2], [1, 4] )

def divide_rational(x, y):
    return [ x[0] * y[1], x[1] * y[0] ]
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Violating Abstraction Barriers

Does not use constructors

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And no constructor!

Violating Abstraction Barriers

Data Representations

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(Demo)

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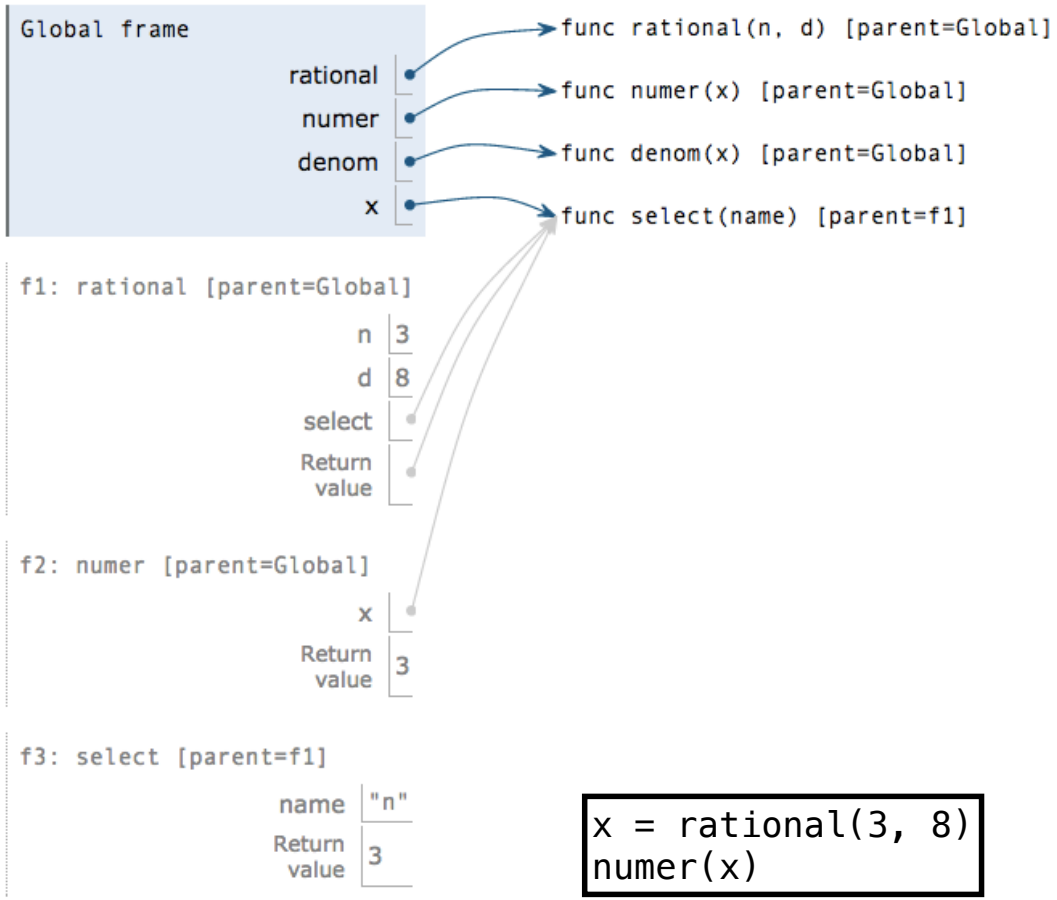


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