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MATHEMATICAL MODELING (CO2011)

Assignment

”The SIR Model in COVID-19 prediction”

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Contents

1 Exercise 2 - The RK4 method in solving the SIR system

1.1 Preliminary

The most widely known member of the Runge–Kutta family is generally referred to as "RK4", the "classic Runge–Kutta method" or simply as "the Runge–Kutta method".RK4 is one of the classic methods for numerical integration of ODE models.
Consider the following initial value problem of ODE

$$\begin{aligned} \frac{dy}{dt} &= f(t, y) \\ y(t_0) &= y_0 \end{aligned} \tag{1}$$

where y(t) is the unknown function (scalar or vector) which I would like to approximate.
The iterative formula of RK4 method for solving ODE (1) is as follows

$$\begin{aligned} y_{n+1} &= y_n + \frac{\Delta t}{6}(k_1 + 2k_2 + 2k_3 + k4) \\ k_1 &= f(t_n, y_n) \\ k_2 &= f(t_n + \frac{\Delta t}{2}, y_n + \frac{k_1 \Delta t}{2}) \\ k_3 &= f(t_n + \frac{\Delta t}{2}, y_n + \frac{k_2 \Delta t}{2}) \\ k_4 &= f(t_n + \Delta t, y_n + k_3 \Delta t) \\ t_{n+1} &= t_n + \Delta t \\ n &= 0, 1, 2, 3, ... \end{aligned} \tag{2}$$

The SIR model is defined as (**??**), (**??**), (**??**). where S(t) is the number of susceptible people in the population at time t, I(t) is the number of infectious people at time t, R(t) is the number of recovered people at time t, β is the transmission rate, γ represents the recovery rate, and N=S(t)+I(t)+R(t) is the fixed population.

According to the general iterative formula (2), the iterative formulas for S(t), I(t) and R(t) of SIR model can be written out.

$$\begin{aligned} S_{n+1} &= S_n + \frac{\Delta t}{6}(k_1^S + 2k_2^S + 2k_3^S + k4^S) \\ k_1^S &= f(t_n, S_n, I_n) = -\frac{\beta S_n I_n}{N} \\ k_2^S &= f(t_n + \frac{\Delta t}{2}, S_n + \frac{k_1^S \Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}) = -\frac{\beta}{N}(S_n + \frac{k_1^S \Delta t}{2})(I_n + \frac{k_1^I \Delta t}{2}) \\ k_3^S &= f(t_n + \frac{\Delta t}{2}, S_n + \frac{k_2^S \Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}) = -\frac{\beta}{N}(S_n + \frac{k_2^S \Delta t}{2})(I_n + \frac{k_2^I \Delta t}{2}) \\ k_4^S &= f(t_n + \Delta t, S_n + k_3^S \Delta t, I_n + k_3^I \Delta t) = -\frac{\beta}{N}(S_n + k_3^S \Delta t)(I_n + k_3^I \Delta t) \end{aligned} \tag{3}$$

$$
\begin{aligned}
I_{n+1} &= I_n + \frac{\Delta t}{6}(k_1^I + 2k_2^I + 2k_3^I + k4^I) \\
k_1^I &= f(t_n, S_n, I_n) = \frac{\beta S_n I_n}{N} - \gamma I_n \\
k_2^I &= f(t_n + \frac{\Delta t}{2}, S_n + \frac{k_1^S \Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}) = \frac{\beta}{N}(S_n + \frac{k_1^S \Delta t}{2})(I_n + \frac{k_1^I \Delta t}{2}) - \gamma(I_n + k_1^I \Delta t) \\
k_3^I &= f(t_n + \frac{\Delta t}{2}, S_n + \frac{k_2^S \Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}) = \frac{\beta}{N}(S_n + \frac{k_2^S \Delta t}{2})(I_n + \frac{k_2^I \Delta t}{2}) - \gamma(I_n + k_2^I \Delta t) \\
k_4^I &= f(t_n + \Delta t, S_n + k_3^S \Delta t, I_n + k_3^I \Delta t) = \frac{\beta}{N}(S_n + k_3^S \Delta t)(I_n + k_3^I \Delta t) - \gamma(I_n + k_3^I \Delta t)
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
R_{n+1} &= R_n + \frac{\Delta t}{6}(k_1^R + 2k_2^R + 2k_3^R + k_4^R) \\
k_1^R &= f(t_n, I_n) = \gamma I_n \\
k_2^R &= f(t_n + \frac{\Delta t}{2}, I_n + \frac{k_1^I \Delta t}{2}) = \gamma(I_n + k_1^I \Delta t) \\
k_3^R &= f(t_n + \frac{\Delta t}{2}, I_n + \frac{k_2^I \Delta t}{2}) = \gamma(I_n + k_2^I \Delta t) \\
k_4^R &= f(t_n + \Delta t, I_n + k_3^I \Delta t) = \gamma(I_n + k_3^I \Delta t)
\end{aligned}
\tag{5}
$$

Note that since the population N = S(t) + I(t) + R(t) is constant, there will have $\frac{dS}{dt} + \frac{dI}{dt} + \frac{dr}{dt} = 0$. Therefore, only two of the three ODEs are independent and sufficient to solve the ODEs. Here, only iterative formulas for S(t) and I(t) are used and R(t) is calculated by S(t)=N - I(t) - R(t).