Presentation at FINX LAB

The Log-Normal Prior Tangency Portfolio with S&P500

- April 2024 Financial Research Letters Paper by Olha Bodnar, Taras Bodnar, and Vihelm Niklasson: "Constructing Bayesian Tangency Portfolios under Short-Selling Restrictions"

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FINANCIAL INNOVATION
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CONTENTS

X.1 Background and Development of the 'Log-Normal Prior Tangency Portfolio'

What background leads to the development of the “Log-Normal Prior Tangency Portfolio”?

- **Markowitz's Ideal and Its Limitation**:
  - Markowitz's framework is foundational in finance, but a key challenge in applying his mean-variance portfolio theory in practice is the inherent uncertainty in estimating the mean vector and covariance of asset returns. To address this, both frequentist and Bayesian approaches have been explored in academic research.

- **Frequentist vs. Bayesian Approaches**:
  - An example of a frequentist approach is robust optimization, which aims to improve portfolio performance by addressing the issue of parameter uncertainty in asset returns. This is achieved by imposing constraints inspired by robust optimization to limit the elements of the covariance matrix. In contrast, the Bayesian approach seeks to enhance portfolio performance by addressing the same issue through the integration of prior information and the consideration of both parameter and model uncertainty.

- **Limitations of Traditional Bayesian Approaches**:
  - In Bayesian approaches, portfolios are constructed by incorporating investors' beliefs into the prior. However, traditional Bayesian methods have their limitations, especially when investors prefer portfolios that are predominantly long.

- **New Bayesian Approach:**
  - The novel Bayesian approach in this paper is designed specifically for portfolios that accept only long positions through a specialized prior.
  - The 'Log-Normal Prior Tangency Portfolio': This new approach exclusively allows long positions through a custom prior.

Bayesian Tangency Portfolio

$\tilde{x}_1, \ldots, \tilde{x}_n$: the p-dimensional vectors of asset returns for p assets

$r_{f,i}$: the return of the risk-free asset at time i

$\mathbf{1}$: a p-dimensional vector of ones

$$x_1 = \tilde{x}_1 - r_{f,1}\mathbf{1},$$
$$\vdots$$
$$x_n = \tilde{x}_n - r_{f,n}\mathbf{1}$$

(the vectors of excess returns)

$x_{1:n} = (x_1, \ldots, x_n)$: the matrix consisting of n realizations of p-dimensional vector of the excess stock returns

The vector μ: the expected returns of the excess returns

The vector Σ: the covariance matrix associated with the excess returns

Assumptions:

μ and Σ are independent and follow a multivariate normal distribution.

Given these assumptions:

The joint distribution of the asset returns, conditionally on μ and Σ, belongs to the exponential family.

The canonical parameters:

$$\mathbf{v} = \Sigma^{-1}\mu \quad \text{and} \quad \Omega = \Sigma^{-1}$$

The canonical statistic $(\mathbf{t}, -T/2)$:

$$\mathbf{t} = \sum_{i=1}^{n} \mathbf{x}_i \quad \text{and} \quad T = \sum_{i=1}^{n} \mathbf{x}_i \mathbf{x}_i^\top.$$

The likelihood function:

$$f(\mathbf{x}_{1:n}|\mu, \Sigma) \propto \det(\Sigma)^{-n/2} \exp\left(-\frac{1}{2}\sum_{i=1}^{n}(\mathbf{x}_i - \mu)^\top \Sigma^{-1}(\mathbf{x}_i - \mu)\right)$$

$$= \det(\Omega)^{n/2} \exp\left(-\frac{n}{2}\mathbf{v}^\top \Omega^{-1}\mathbf{v}\right) \exp\left[\left(\sum_{i=1}^{n}\mathbf{x}_i\right)^\top \mathbf{v} + \mathrm{tr}\left(\Omega\left(-\frac{1}{2}\sum_{i=1}^{n}\mathbf{x}_i\mathbf{x}_i^\top\right)\right)\right]$$

Bayesian Tangency Portfolio

Given the canonical representation of the likelihood function, the optimal portfolio weights are found by solving

$$\max_{\mathbf{w}} \left\{ \mathbf{w}^\top \mu - \frac{\gamma}{2} \mathbf{w}^\top \Sigma \mathbf{w} \right\},$$

where $\mathbf{w}$ represents the portfolio weights and γ is the risk aversion coefficient.

The optimal weights are

$$\mathbf{w} = \gamma^{-1} \Sigma^{-1} \mu = \gamma^{-1} \mathbf{v},$$

indicating that the solution is proportional to the unscaled tangency portfolio weights.

- When there are no short-selling restrictions, the optimal weights in mean-variance optimization can be expressed as shown on the left.
- This formulation facilitates direct modeling of the tangency portfolio weights and allows for the seamless incorporation of an investor's beliefs into the prior distribution.
- This approach accommodates both long and short positions.

Let's consider a case when portfolios consisting solely of long positions is preferred

- To account for this investor preference (or when the short-selling is restricted), it is necessary to create a specialized prior that accommodates only long positions.

Prior Based on New Bayesian Methodology

- We identified the need for a specialized prior that supports only long positions. Moving forward, this paper developed a prior distribution that allows only positive values, ensuring that the tangency portfolio weights remain positive.
  - How? Since the support of the prior sets the maximum limit for the support of the posterior, modeling the logarithm of $\boldsymbol{\nu}$ as a multivariate normal distribution can be an effective approach.

Priors:

$$\log(\mathbf{v}) \sim \mathcal{MN}(\log(\mathbf{w}_0), \kappa\mathbf{I})$$

$$\sigma_i \sim C^+(\beta) \quad \text{for } i = 1, 2, \dots, p$$

$$\mathbf{R} \sim \mathrm{LKJ}(\eta)$$

$$\Omega = \mathrm{diag}(\sigma)\mathbf{R}\mathrm{diag}(\sigma)$$

Notations:

Matrix Ω: the inverse of a covariance matrix

$\mathcal{MN}$: the multivariate normal distribution

C^+: the Half-Cauchy distribution

LKJ: the Lewandowski-Kurowicka-Joe (LKJ) distribution

$\mathbf{I}$: the identity matrix

κ, β, and η: the constants determining the scales and shapes of the prior distributions

$\mathbf{w}_0$: the prior belief about the optimal portfolio weights

Posterior Based on New Bayesian Methodology

The posterior for $\mathbf{v}$ and Ω when $\pi(\mathbf{v}, \Omega)$ is the prior we mentioned before:

$$\pi(\mathbf{v}, \Omega | \mathbf{x}_{1:n}) \propto \det(\Omega)^{n/2} \exp\left(-\frac{n}{2}\mathbf{v}^\top \Omega^{-1} \mathbf{v}\right) \exp\left(\mathbf{t}^\top \mathbf{v} + \mathrm{tr}\left(-\frac{1}{2}\Omega \mathbf{T}\right)\right) \pi(\mathbf{v}, \Omega)$$

- As the sample size increases, the influence of the prior decreases, in accordance with the Bernstein–von Mises theorem.
- Even though the impact of the prior lose its power, the prior plays an important role under short-selling restrictions, which means it guarantees that a posterior distribution of portfolio weights is positive. In a same vein, Bayesian estimators derived from this posterior will produce only positive tangency portfolio weights.

Issue with Bayesian Inference based on The 'Log-Normal Prior Tangency Portfolio'

- The complexity of the posterior distribution in a new Bayesian approach makes sampling for portfolio weight estimators difficult. To resolve this issue, the Markov Chain Monte Carlo (MCMC) methodology is utilized while this research applies this new method.

X.2 Methods Used to Ensure Robustness

Which approaches does the novel method in the paper take to ensure robustness?

- Using the Probabilistic Sharpe Ratio with a Zero Benchmark:
  - To enhance the reliability of Sharpe ratio comparisons, a key metric in the analysis, the probabilistic Sharpe ratio with a zero benchmark was employed, as recommended by Bailey and Lopez de Prado (2012). This method outperforms traditional hypothesis testing, providing a more robust approach to addressing the inherent uncertainty in Sharpe ratio estimation.
- Conducting Sensitivity Analysis:
  - Sensitivity analysis was conducted by comparing the outcomes when κ is 1 and when κ is 2. This comparison demonstrates how closely the proposed method aligns with the theoretical results. In this context, a lower Kappa implies a greater prior weight, indicating a stronger belief in the results.
- Testing During Extreme Market Conditions:
  - Backtesting was conducted during periods of extreme market volatility, such as the 2008 financial crisis and the 2020 COVID-19 pandemic, to validate that the proposed method yields consistent results even under such conditions.

X.3 Implementation and Comparison with Original Results

Implementation

- Monthly portfolio rebalancing was conducted using a 60-month rolling window, and the S&P 500 index was used.
- The analysis specifically utilized S&P 500 data from January 2006 to June 2023, sourced from AlphaVantage and FinancialModelingPrep. This required a dataset consisting of 1,035 stock prices and market capitalizations.
- https://github.com/sungbinnpark/FINXLAB_ResearchProject.git

Comparison and Analysis of Results: Original Study vs. Implementation

- **[Conclusion]**: Both in the original study and during the implementation, the Log-Normal Prior method, with κ set to 1 and 2, generally demonstrated superior performance across various metrics for different strategies.
  - With a portfolio size of 10, the Log-Normal Prior method showed strong performance across metrics such as cumulative return, CAGR, Sharpe Ratio, Calmar Ratio, and others.
  - In the original study using the Log-Normal Prior method with a portfolio size of 10, a cumulative return of 539.170% was reported when $\kappa = 2$, along with the highest Sharpe Ratio and Sortino Ratio. In the implementation, a cumulative return of 540.446% was achieved when $\kappa = 1$, with the highest Sharpe Ratio and a Sortino Ratio that was very close to the peak.
    - » Since 'a lower value of κ signifies a slightly greater influence in the prior weights,' this suggests that the increased influence of prior weights when $\kappa = 1$ may have contributed to the improved portfolio performance (Bodnar, Olha et al. 3).
  - In the original study, the Value-weighted and Black-Litterman portfolios also demonstrated strong performance on certain metrics. Similarly, the implementation similarly showed strong performance on certain metrics for both the Value-weighted and Black-Litterman portfolios.
  - In the original study, the Equally-weighted, Jorion Hyperparameter, and Shrinkage methodologies demonstrated suboptimal performance. The implementation yielded similarly subpar results for these approaches.

Original Results with a Portfolio Size of 10

Table 1
Performance metrics for various strategies with a portfolio size of 10, between January 2006 and June 2023. The best and worst values in each row are highlighted in green and red, respectively.

	Value-Weighted	Equally-Weighted	Log-Normal $\kappa = 1$	Log-Normal $\kappa = 2$	Black–Litterman	Jorion Hyperpar.	Shrinkage	Min. Variance
Cum. return	470.277%	323.815%	510.298%	539.170%	437.268%	405.787%	478.609%	243.692%
CAGR	10.463%	8.604%	10.892%	11.185%	10.087%	9.707%	10.554%	7.311%
Sharpe ratio	0.504	0.452	0.521	0.534	0.503	0.478	0.529	0.436
Prob. Sharpe ratio	98.266%	97.039%	98.500%	98.695%	98.239%	97.668%	98.619%	96.602%
Sortino ratio	0.726	0.638	0.738	0.757	0.723	0.673	0.747	0.624
Calmar ratio	0.205	0.167	0.227	0.246	0.205	0.185	0.211	0.180
Max. DD	−51.090%	−51.628%	−48.068%	−45.436%	−49.228%	−52.476%	−50.040%	−40.660%
Avg. loss	−0.934%	−0.858%	−0.953%	−0.917%	−0.890%	−0.918%	−0.860%	−0.691%
Avg. return	0.049%	0.041%	0.051%	0.052%	0.047%	0.046%	0.048%	0.033%
Avg. win	0.874%	0.800%	0.891%	0.865%	0.831%	0.872%	0.815%	0.676%
Best day	11.897%	10.879%	13.498%	16.664%	11.308%	12.454%	13.708%	10.739%
Worst day	−10.946%	−10.135%	−13.319%	−16.152%	−10.379%	−12.948%	−12.415%	−8.575%
Ann. Vol.	22.258%	20.011%	22.361%	22.228%	21.170%	21.917%	20.801%	16.565%
Daily VaR	−2.257%	−2.033%	−2.266%	−2.251%	−2.147%	−2.225%	−2.107%	−1.683%
Avg. turnover	4.411%	7.573%	6.619%	9.794%	4.314%	21.248%	16.945%	14.707%

Implementation with a Portfolio Size of 10

	Value-Weighted	Equally-Weighted	Log-Normal κ =1	Log-Normal κ =2	Black-Litterman	Jorion Hyperpar.	Shrinkage	Min. Variance
Cum. return	508.51%	382.77%	540.45%	439.62%	471.80%	278.75%	313.33%	257.43%
CAGR	10.87%	9.42%	11.20%	10.11%	10.48%	7.91%	8.45%	7.55%
Sharpe Ratio	0.538	0.488	0.539	0.495	0.538	0.4	0.459	0.449
Prob. Sharpe Ratio	98.763%	97.88%	98.76%	98.03%	98.76%	95.21%	97.21%	96.98%
Sortino Ratio	0.762	0.685	0.761	0.695	0.761	0.557	0.644	0.64
Calmar Ratio	0.216	0.175	0.224	0.181	0.216	0.118	0.153	0.183
Max. DD	-50.26%	-53.89%	-50.04%	-55.92%	-48.41%	-66.87%	-55.26%	-41.25%
Avg. Loss	-0.91%	-0.87%	-0.94%	-0.91%	-0.87%	-0.93%	-0.78%	-0.70%
Avg. Return	0.05%	0.04%	0.05%	0.05%	0.05%	0.04%	0.04%	0.03%
Avg. Win	0.86%	0.81%	0.88%	0.86%	0.82%	0.87%	0.72%	0.68%
Best Day	10.78%	10.62%	12.59%	14.54%	10.13%	12.46%	13.03%	10.26%
Worst Day	-11.24%	-10.78%	-13.02%	-14.78%	-10.66%	-12.95%	-12.09%	-8.94%
Ann. Vol.	21.04%	20.16%	21.93%	21.85%	20.00%	22.15%	18.98%	16.60%
Daily VaR	-2.13%	-2.05%	-2.22%	-2.22%	-2.02%	-2.26%	-1.93%	-1.69%
Avg. Turnover	3.43%	6.79%	5.64%	9.12%	3.38%	21.97%	16.47%	15.06%

Comparison of Metrics

Original Results with a Portfolio Size of 20

Table 2
Performance metrics for various strategies with a portfolio size of 20, between January 2006 and June 2023. The best and worst values in each row are highlighted in green and red, respectively.

	Value-Weighted	Equally-Weighted	Log-Normal $\kappa = 1$	Log-Normal $\kappa = 2$	Black–Litterman	Jorion Hyperpar.	Shrinkage	Min. Variance
Cum. return	413.721%	315.687%	432.677%	439.548%	386.048%	325.988%	273.239%	279.066%
CAGR	9.805%	8.484%	10.033%	10.113%	9.458%	8.636%	7.818%	7.914%
Sharpe ratio	0.492	0.454	0.503	0.507	0.491	0.440	0.407	0.485
Prob. Sharpe ratio	98.008%	97.076%	98.200%	98.259%	97.986%	96.663%	95.497%	97.850%
Sortino ratio	0.703	0.640	0.712	0.715	0.701	0.616	0.568	0.688
Calmar ratio	0.185	0.158	0.198	0.205	0.185	0.138	0.124	0.196
Max. DD	−52.973%	−53.682%	−50.621%	−49.344%	−51.074%	−62.491%	−62.853%	−40.322%
Avg. loss	−0.892%	−0.818%	−0.901%	−0.887%	−0.849%	−0.899%	−0.870%	−0.664%
Avg. return	0.046%	0.040%	0.047%	0.047%	0.044%	0.042%	0.039%	0.035%
Avg. win	0.823%	0.773%	0.825%	0.815%	0.783%	0.859%	0.815%	0.627%
Best day	11.030%	11.018%	12.025%	13.705%	10.357%	13.398%	13.283%	10.276%
Worst day	−11.222%	−10.922%	−12.661%	−14.168%	−10.638%	−13.602%	−13.068%	−9.211%
Ann. Vol.	21.109%	19.467%	21.003%	20.973%	20.071%	21.212%	20.927%	15.800%
Daily VaR	−2.141%	−1.977%	−2.129%	−2.126%	−2.036%	−2.156%	−2.130%	−1.602%
Avg. turnover	3.236%	6.430%	4.669%	6.912%	3.188%	25.564%	20.272%	17.799%

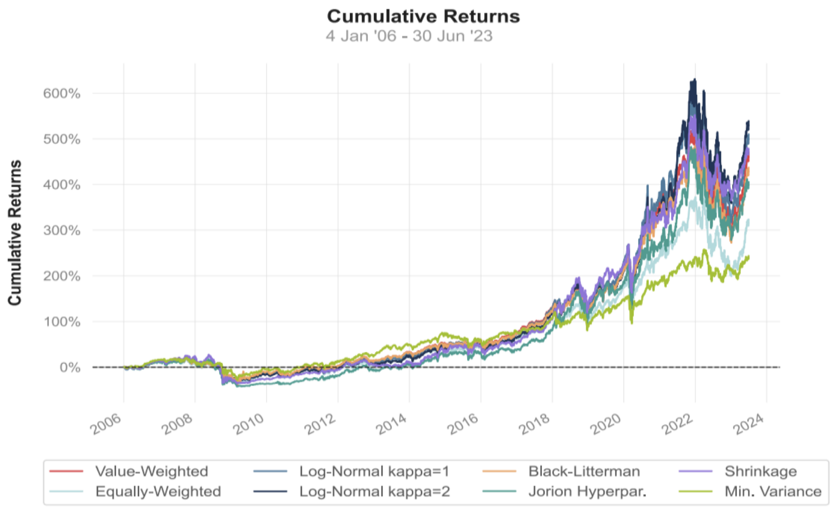
Implementation with a Portfolio Size of 20

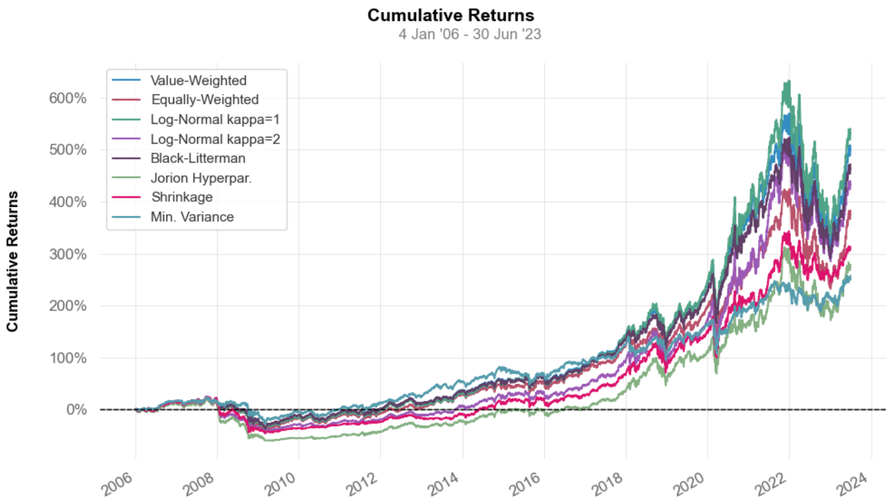
	Value-Weighted	Equally-Weighted	Log-Normal κ =1	Log-Normal κ =2	Black-Litterman	Jorion Hyperpar.	Shrinkage	Min. Variance
Cum. return	461.90%	369.16%	490.99%	474.94%	429.55%	425.03%	406.71%	313.64%
CAGR	10.37%	9.24%	10.69%	10.51%	10.00%	9.94%	9.72%	8.45%
Sharpe Ratio	0.527	0.487	0.535	0.527	0.527	0.493	0.489	0.514
Prob. Sharpe Ratio	98.60%	97.88%	98.71%	98.58%	98.59%	98.00%	97.91%	98.41%
Sortino Ratio	0.745	0.688	0.755	0.742	0.744	0.692	0.685	0.73
Calmar Ratio	0.209	0.182	0.223	0.22	0.21	0.183	0.178	0.216
Max. DD	-49.52%	-50.63%	-47.98%	-47.86%	-47.66%	-54.29%	-54.48%	-39.22%
Avg. Loss	-0.88%	-0.82%	-0.90%	-0.88%	-0.84%	-0.91%	-0.87%	-0.67%
Avg. Return	0.05%	0.04%	0.05%	0.05%	0.05%	0.05%	0.05%	0.04%
Avg. Win	0.81%	0.78%	0.82%	0.81%	0.77%	0.87%	0.82%	0.63%
Best Day	10.93%	10.81%	11.62%	13.55%	10.27%	12.98%	12.72%	10.26%
Worst Day	-11.27%	-11.04%	-12.29%	-14.08%	-10.68%	-13.60%	-13.11%	-9.27%
Ann. Vol.	20.38%	19.62%	20.77%	20.82%	19.37%	21.45%	21.03%	15.89%
Daily VaR	-2.06%	-1.99%	-2.10%	-2.11%	-1.96%	-2.18%	-2.13%	-1.61%
Avg. Turnover	2.74%	6.34%	4.21%	6.32%	2.72%	25.72%	19.28%	17.38%

Comparison of Metrics

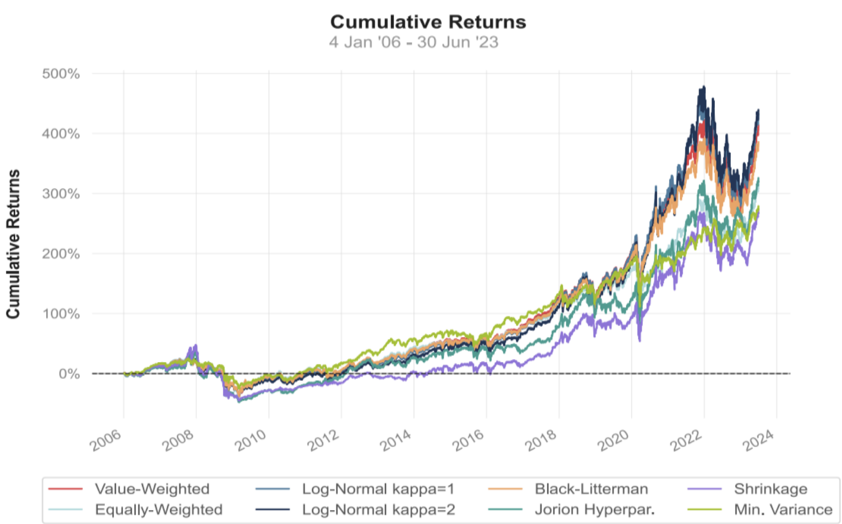
Original Result | **Implementation**

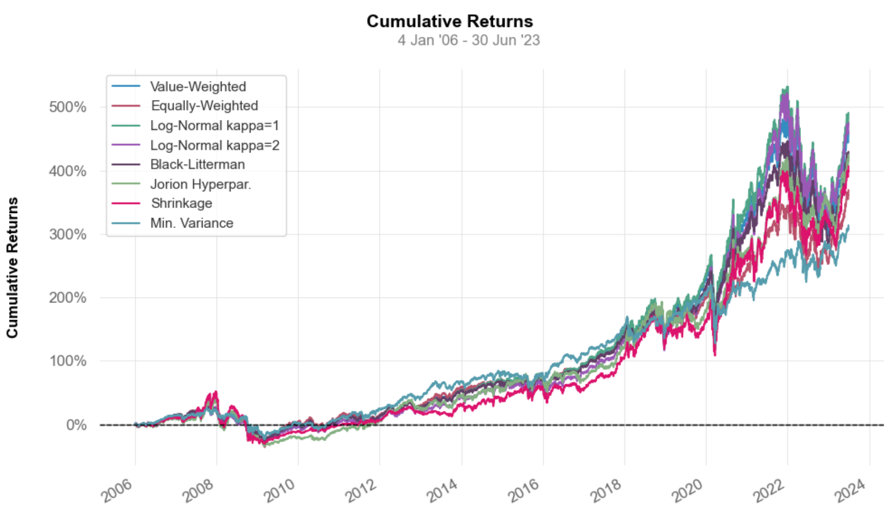
Portfolio Size of 10





Portfolio Size of 20

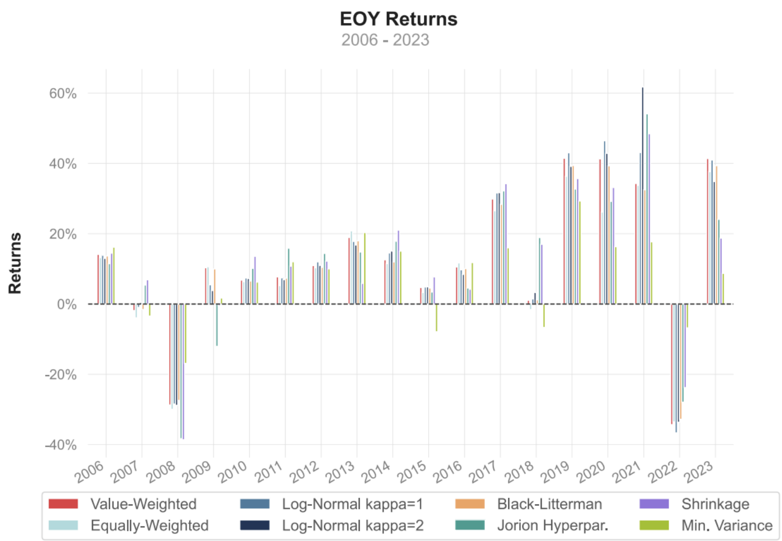


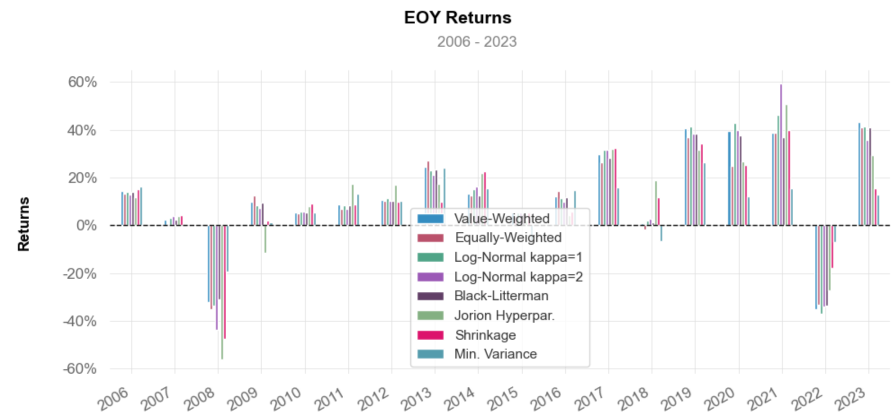


Comparison of Metrics

Original Result | **Implementation**

Portfolio Size of 10





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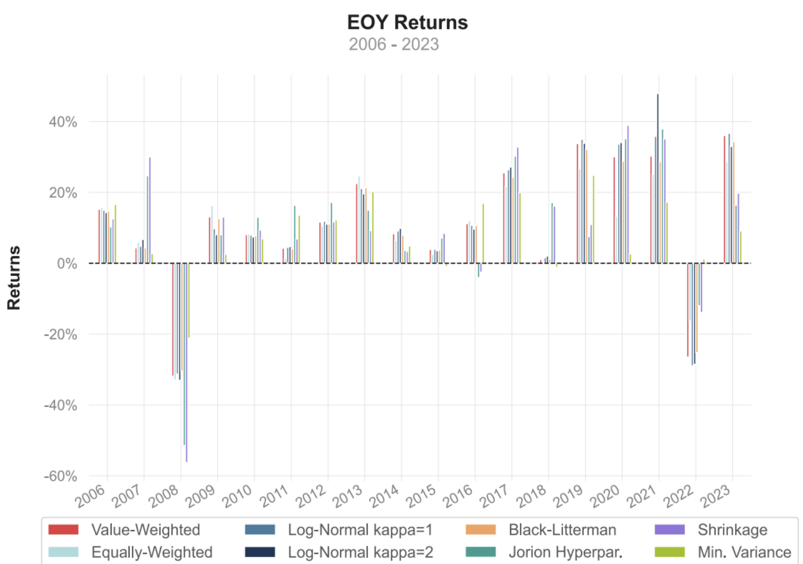
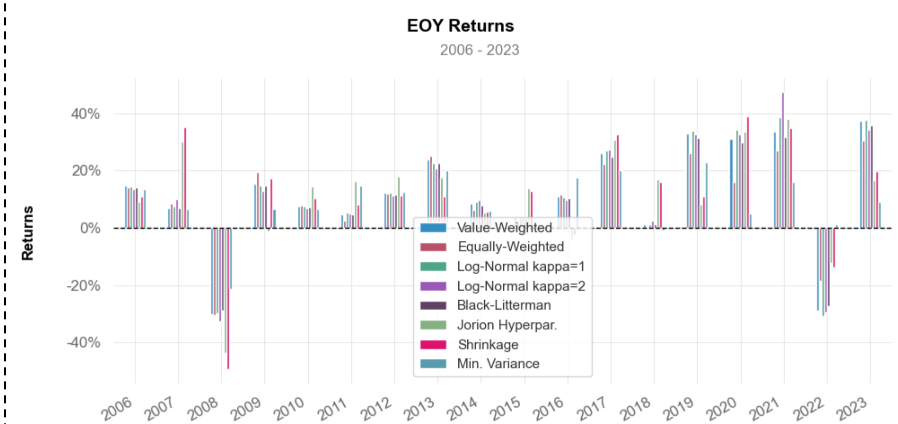
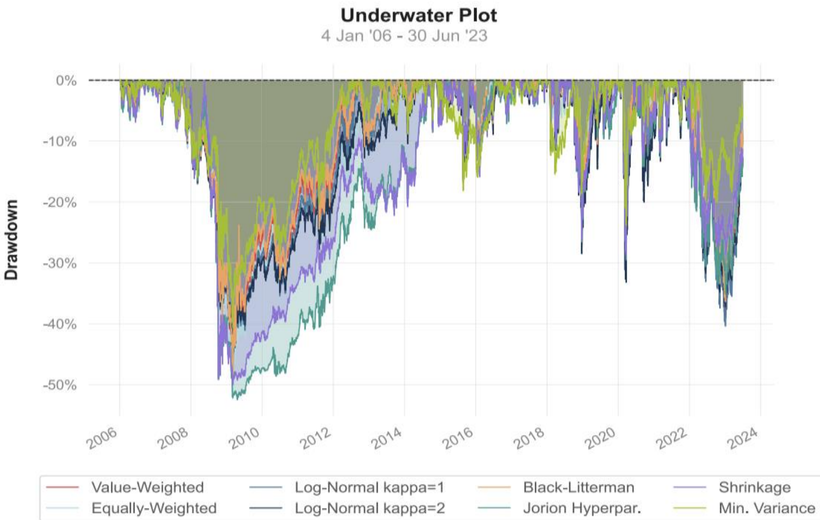


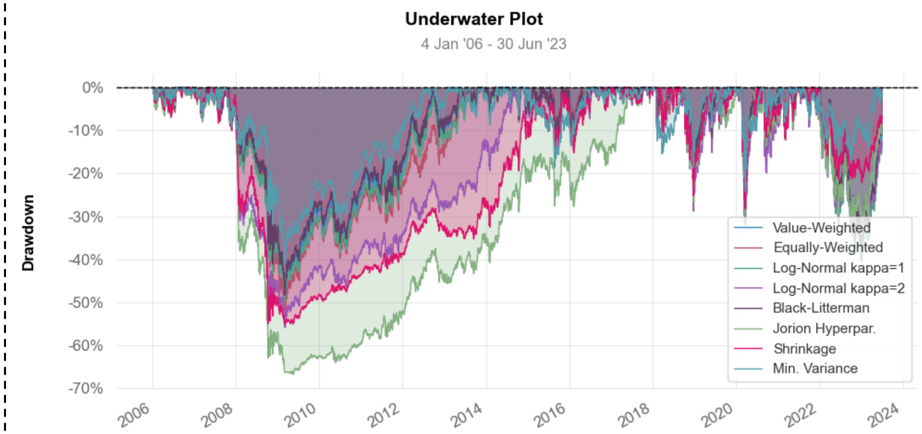


Comparison of Metrics

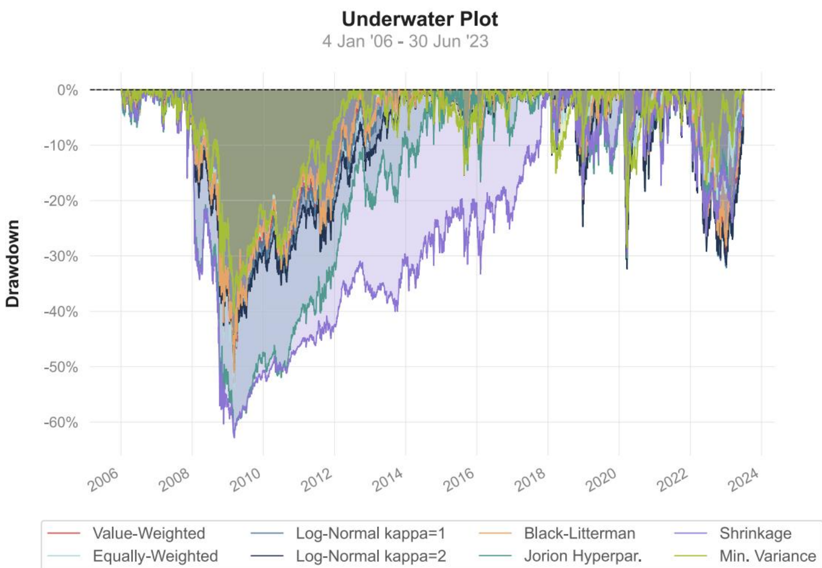
Original Result | **Implementation**

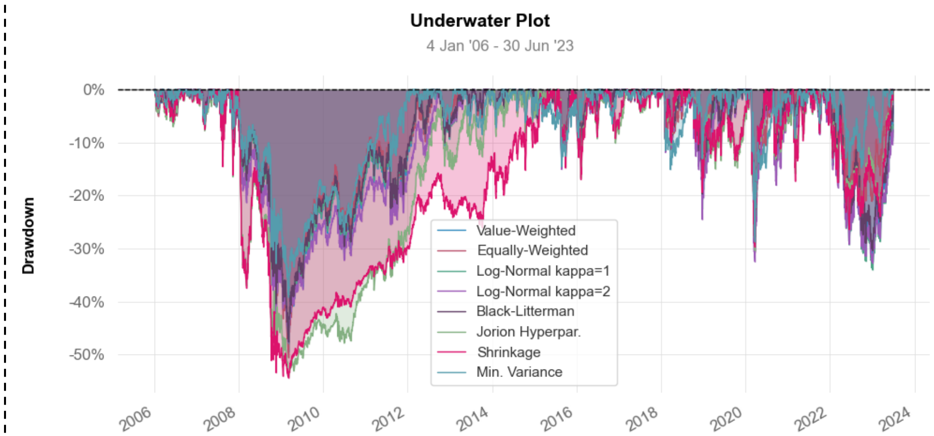
Portfolio Size of 10





Portfolio Size of 20

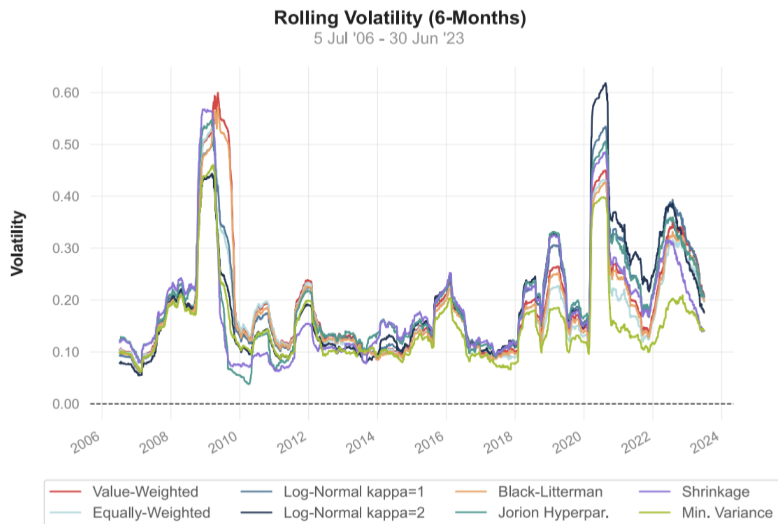


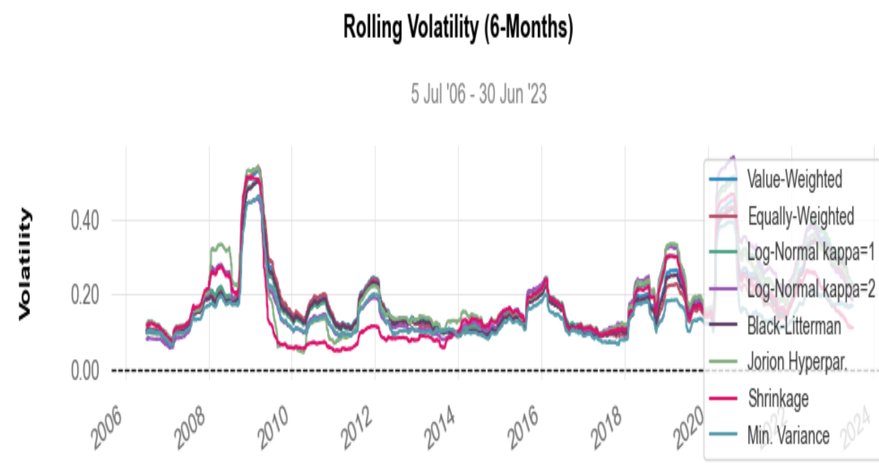


Comparison of Metrics

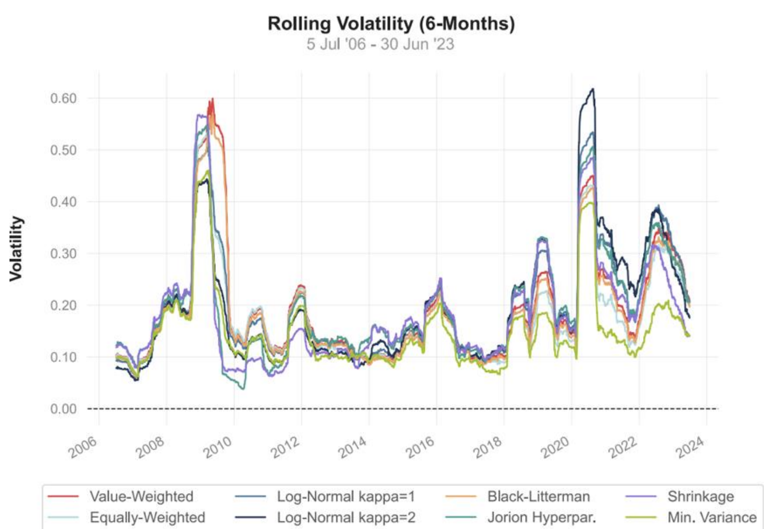
Original Result | **Implementation**

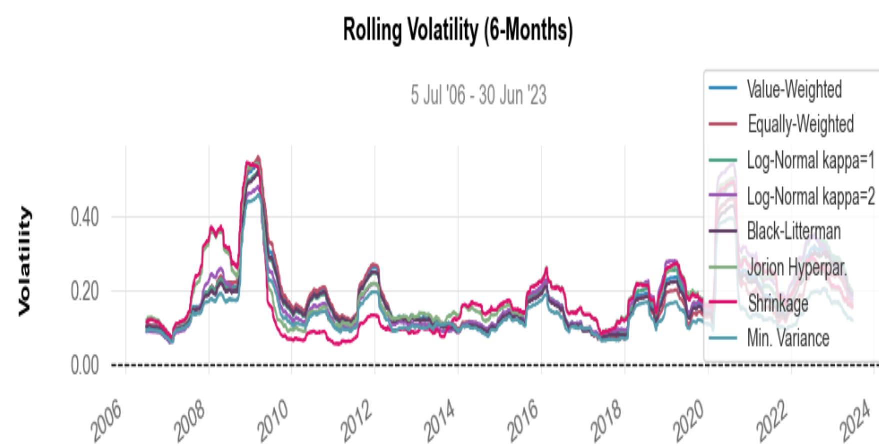
Portfolio Size of 10





Portfolio Size of 20



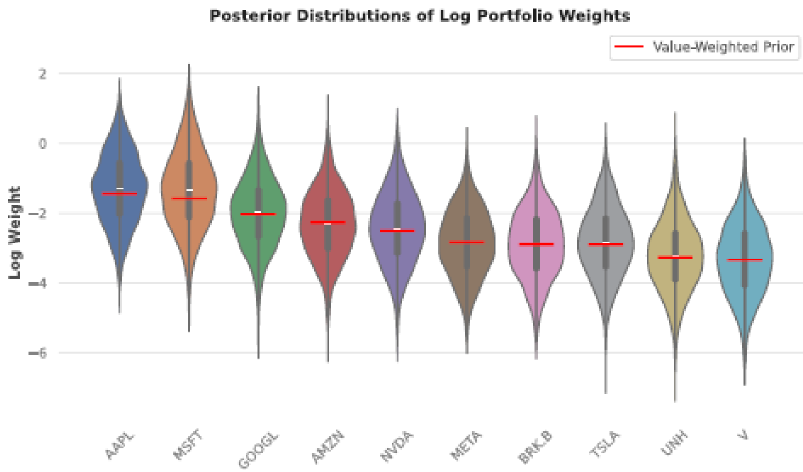


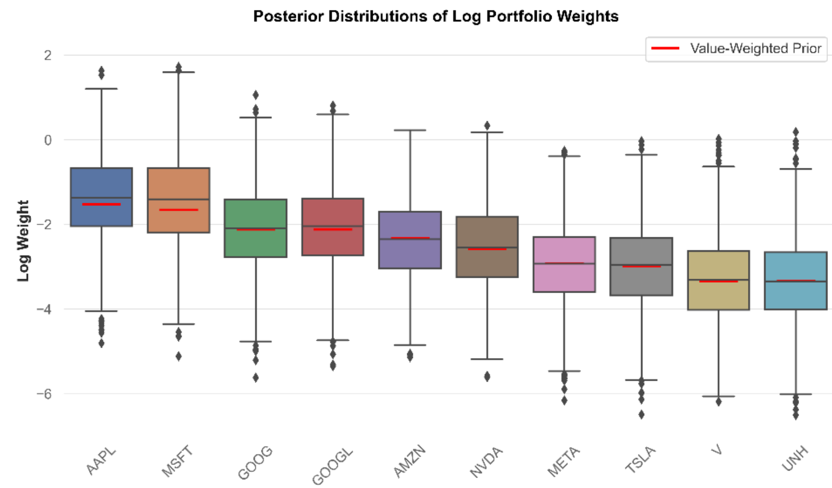
Comparison of MCMC Results

Original Result (Portfolio Size of 10)

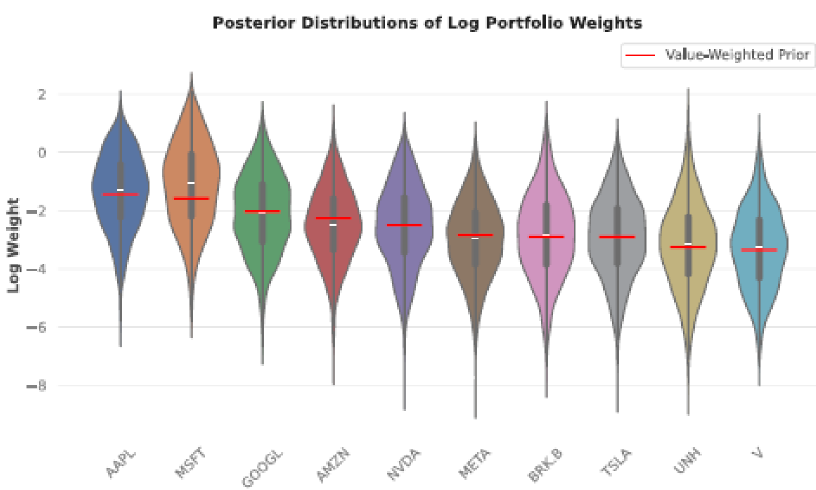
Implementation (Portfolio Size of 10)

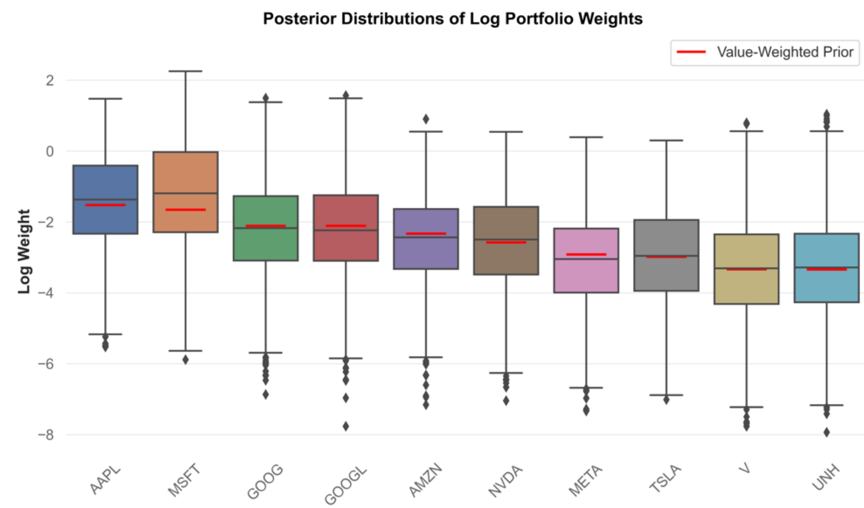
Log Normal $\kappa = 1$





Log Normal $\kappa = 2$



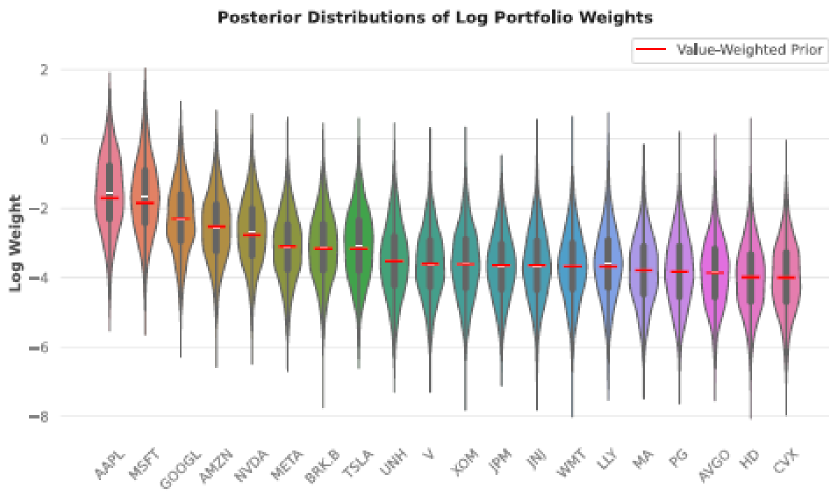


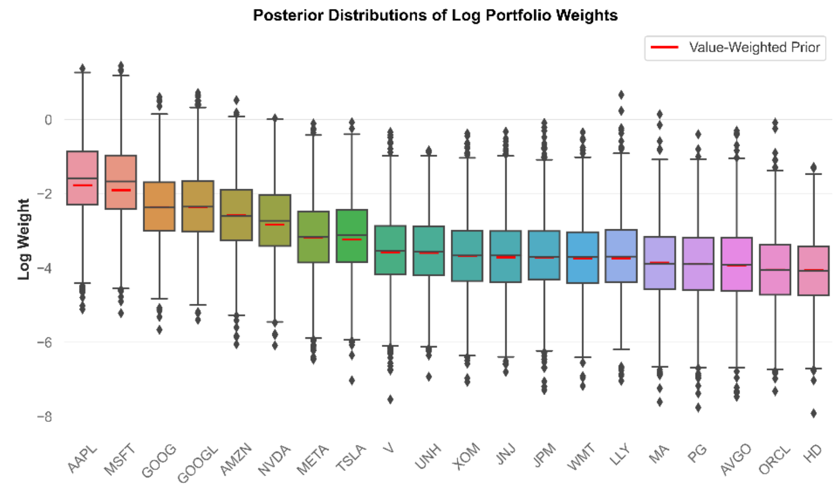
Comparison of MCMC Results

Original Result (Portfolio Size of 20)

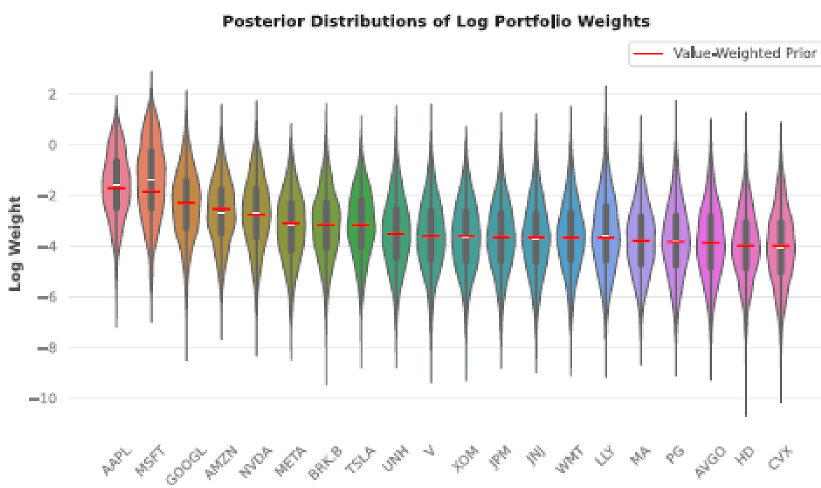
Implementation (Portfolio Size of 20)

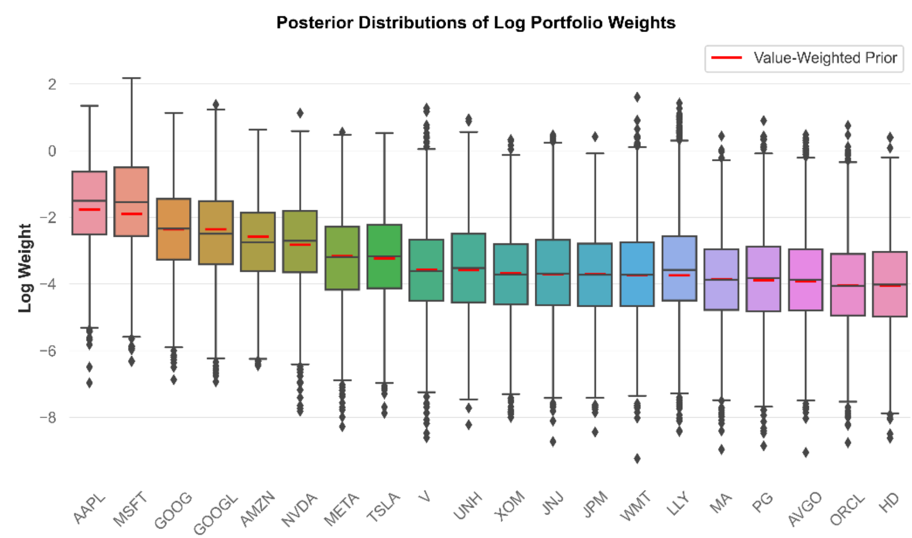
Log Normal κ =1





Log Normal κ =2





X.4 Explanation of the Backtesting Methodology

What is QuantStats Python Package?

- “QuantStats Python library that performs portfolio profiling, allowing quants and portfolio managers to understand their performance better by providing them with in-depth analytics and risk metrics.” (ranaroussi)

How does QuantStats actually work in implementation?

```
def plot_performance(portfolio_setups_simple_returns_df,
                     portfolio_setups_excess_simple_returns_df,
                     potfolio_size):

    # Plot daily returns
    qs.plots.returns(portfolio_setups_simple_returns_df,
                          savefig = f"./results/returns_{potfolio_size}")

    # Plot yearly returns
    qs.plots.yearly_returns(portfolio_setups_simple_returns_df,
                          savefig = f"./results/yearly_returns_{potfolio_size}")

    # Plot rolling Sharpe
    qs.plots.rolling_sharpe(portfolio_setups_excess_simple_returns_df,
                          savefig = f"./results/rolling_sharpe_{potfolio_size}")

    # Plot rolling Sortino
    qs.plots.rolling_sortino(portfolio_setups_excess_simple_returns_df,
                          savefig = f"./results/rolling_sortino_{potfolio_size}")

    # Plot rolling volatility
    qs.plots.rolling_volatility(portfolio_setups_simple_returns_df,
                          savefig = f"./results/rolling_volatility_{potfolio_size}")

    # Plot drawdown
    qs.plots.drawdown(portfolio_setups_simple_returns_df,
                     savefig=f"./results/drawdown_{potfolio_size}")
```

- As shown in the image on the left, running the 'portfolio_evaluation.py' file initiates the execution of the QuantStats Python package. After that, it generates plots for daily returns, yearly returns, rolling Sharpe ratio, rolling volatility, and drawdown.

- In a similar vein, other functionalities of QuantStats are also executed during the implementation process.

X.5 Challenges and Insights

Which challenges did we face?

- Delayed Processing Time Due to Code Redundancy and Inefficiencies
  - **Processing Time**: Running main.py on the lab computer took a total of three days.
  - **MCMC Simulation Error**: An error occurred while running a Markov Chain Monte Carlo (MCMC) simulation due to compatibility issues with the pymc library.
    - » Our team resolved this issue by upgrading the pymc library to the latest version using the command pip install --upgrade pymc cloudpickle.

- API Malfunction and Data Inconsistencies
  - **API Malfunction**: The paper stated that the API could retrieve data from January 1, 2000, to the present, but in reality, only five years of data from the start date were accessible.
    - » Our team solved this issue by ensuring complete data retrieval through five separate API calls per ticker.
  - **Inaccessible Stock Data**: Certain stock price data could not be retrieved. This issue is presumed to be due to the exclusion of delisted stocks from the historical S&P 500 list.
    - » Our team identified the potential for discrepancies in the paper's results due to missing data on delisted stocks, which also sparked our interest in exploring data imputation methods to address this issue.

Further Exploration

- Challenges with Missing Data During Data Processing:
  - Our team encountered issues with missing data during the processing stage. To address this, we explored various data imputation techniques. Following feedback from our advisor at the lab meeting, we reviewed relevant studies from the ACM International Conference on AI in Finance (ICAIF) on this topic.
    - One example we came across was the paper titled "A Fast Non-Linear Coupled Tensor Completion Algorithm for Financial Data Integration and Imputation.“

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- Quastions about Hyperparameters and Robustness
  - Our team wonders whether the performance of the new Bayesian method presented in the paper may be highly sensitive to specific hyperparameter settings, raising concerns about overfitting rather than genuine robustness.
  - Through further study and reflection, we have come to understand that a truly robust model should maintain consistent performance across a variety of hyperparameter settings and data conditions. In other words, a robust model should not experience significant performance degradation with slight changes in hyperparameters or when new data is introduced but should instead retain a stable level of performance.

Further Exploration

- Markov Chain Monte Carlo (MCMC):
  - The 'Log-Normal Prior Tangency Portfolio' method described in this paper faces challenges with direct Bayesian inference due to the complexity of the posterior distribution, which makes sampling difficult. To tackle this issue, MCMC was used for sampling. Despite the high computational cost, MCMC remains a widely recognized method for recovering distributions in fields such as statistics, and it is therefore commonly used, even though it may be inefficient.
  - Our team discovered that there are computational methods, such as Gibbs Sampling, aimed at mitigating the high computational cost associated with MCMC. We also realized that MCMC has evolved into various forms tailored to specific domains. For instance, in finance, MCMC is often applied to stochastic properties like factorial distributions, autocorrelation functions, and volatility clustering.

- Low-Frequency Returns and Conditional Normality:
  - Our team struggled to understand the concept of monthly portfolio rebalancing using a rolling window of n = 60 months, particularly when it came to the discussion of low-frequency returns and conditional normality. It wasn't until we came across Eugene Fama's research that we understood the emphasis on using low-frequency returns like monthly data. Fama's study suggests that employing low-frequency returns is more likely to satisfy the assumption of conditional normality compared to using high-frequency returns.

X.6 Reference

Reference

Acknowledgment

- This presentation is based entirely on the content of 'Constructing Bayesian Tangency Portfolios under Short-Selling Restrictions' by Olha Bodnar, Taras Bodnar, and Vilhelm Niklasson. Our team would like to express our deep gratitude to the authors, Olha Bodnar, Taras Bodnar, and Vilhelm Niklasson, for their valuable research that served as the foundation for this presentation.
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Reference

- Bodnar, Olha, et al. “Constructing Bayesian tangency portfolios under short-selling restrictions.” *Finance Research Letters*, vol. 62, Apr. 2024, p. 105065, https://doi.org/10.1016/j.frl.2024.105065.

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