몰빵이 왜 좋은 방법이 아니냐면

Reviewing

Portfolio Selection

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Abstract

"투자 할 때 기대이익이 최대가 되도록 하는 것 말고,
기대이익과 **분산을** 고려하여 분산 투자하는 것이 좋다."

"분산을 고려했을 때, **몰빵이 optimal이 아님**을 보임."

Discussion:

시계열 데이터에서 어떻게 평균과 분산 개념을 근사시켜 사용할 수 있을까?

시계열 데이터에서 평균과 분산과 같은 분포 개념을 적용하는 것이 괜찮을까?

Introduction

The process of selecting a portfolio

1. Observation and experience

2. Choice of portfolio.

Introduction

The process of selecting a portfolio

1. Observation and experience

2. ***Choice of portfolio.***

Introduction: Discounted Return in Discounted-Flow

Suppose there are N securities;
let r_{it} be the anticipated return at time t per dollar invested in security i;
let d_{it} be the rate at which the return on the i^{th} security at time t is discounted back to the present;
let X_i be the relative amount invested in security i
$\sum X_i = 1, \forall X_i \geq 0.$

$$R = \sum_{i=1}^{N} X_i (\sum_{t=1}^{\infty} d_{it} r_{it})$$

$R_i = \sum_{t=1}^{\infty} d_{it} r_{it}$ is the discounted return of the i^{th} security.
$R = \sum X_i R_i$ where R_i is independent of X_i, which is a weighted average of R_i with the X_i as non-negative weights.

Introduction: Discounted Return in Discounted-Flow

To maximize R, we let $X_i = 1$ for i with maximum R_i.
If several $R_{a_a}, a = 1, \ldots, K$ are maximum then any allocation with

$$\sum_{a=1}^{K} Xa_a = 1$$

maximizes R.
It results **no diversified portfolio**,
confronting the maxim that an investor **should diversify** portfolio.

Introduction

Two maxims contradicts in discounted-flow point of view.

- The investor should diversify.

- The investor should maximize expected return.

Introduction

We saw that **the expected returns rule** is inadequate.

We propose **expected returns-variance of returns (E-V) rule**.

Preliminary

Let Y be a random variable that can take on a finite number of values $y_1, Y_2, \ldots, y_N$. Let the probability that $Y = y_1$ be p_1; $Y = y_2$ be p_2 etc. The expected value of Y is

$$E = p_1 y_1 + p_2 y_2 + \ldots + p_N y_N$$

The variance of Y is defined to be

$$V = p_1(y_1 - E)^2 + p_2(y_2 - E)^2 + \ldots + p_N(y_N - E)^2.$$

Preliminary

Suppose we have a number of random variables: $R_1, \ldots, R_n$. If R is a weighted sum of the R_i

$$R = a_1 R_1 + a_2 R_2 + \ldots + a_n R_n$$

then R is also a random variable.
The expected value of a weighted sum is the weighted sum of the expected values. I.e. $E(R) = a_1 E(R_1) + a_2 E(R_2) + \ldots + a_n E(R_n)$. Plus, we must define "covariance." The covariance of R_i and R_j is

$$\sigma_{ij} = E\{[R_i - E(R_i)][R_j - E(R_j)]\}$$

σ_{ij} can be expressed in terms of the familiar correlation coefficient (ρ_{ij})

$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$$

Preliminary

The variance of a weighted sum is

$$V(R) = \sum_{i=1}^{N} a_i^2 V(X_i) + 2 \sum_{i=1}^{N} \sum_{j>1}^{N} a_i a_j \rho_{ij}$$

Using the fact that $\sigma_i = \sigma_{ii}$

$$V(R) = \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j \rho_{ij}$$

E-V Rule

The yield (R) on the portfolio is

$$R = \sum R_i X_i.$$

The expectation and the variance are

$$E = \sum_{i=1}^{N} X_i \mu_i$$

$$V = \sum_i \sum_j X_i X_j \sigma_i \sigma_j \rho_{ij}$$

E-V Rule

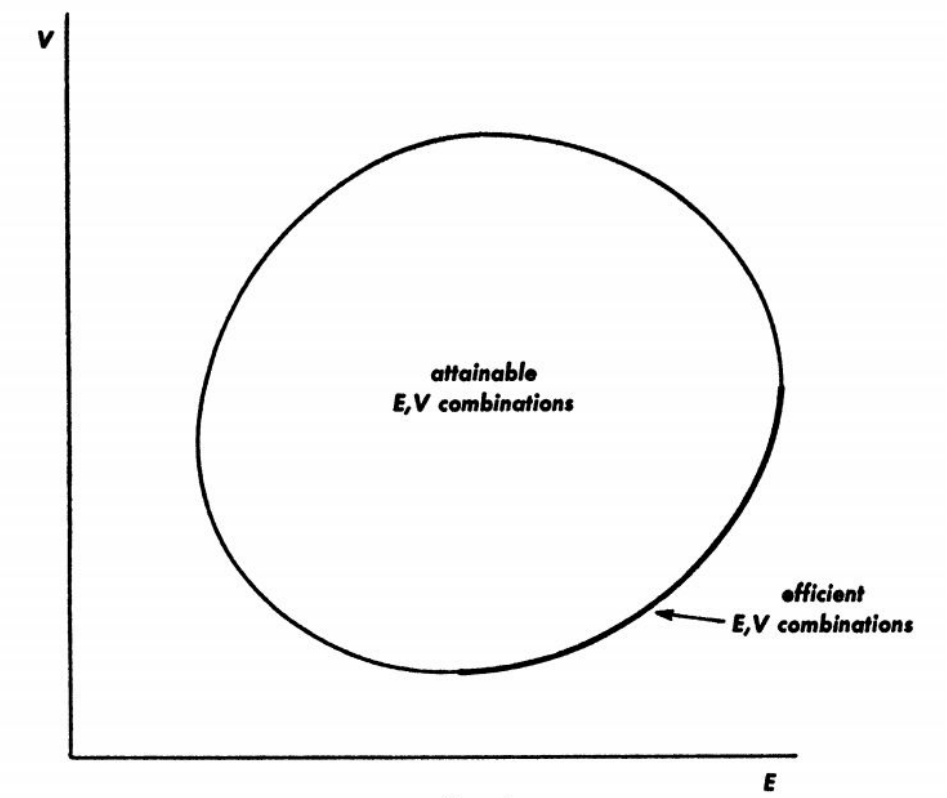

FIG. 1

Example

Let us consider the case of three securities. In the three security case our model reduces to

1. $E = \sum_{i=1}^{3} X_i \mu_i$
2. $V = \sum_{i=1}^{3} \sum_{j=1}^{3} X_i X_j \sigma_{ij}$
3. $\sum_{i=1}^{3} X_i = 1$
4. $X_i \geq 0$ for $i = 1, 2, 3$.

From 3. we get

$$3`. X_3 = 1 - X_1 - X_2$$

We can simply write

$$E = E(X_1, X_2)$$

$$V = V(X_1, X_2)$$

$$X_1 \geq 0, X_2 \geq 0, 1 - X_1 - X_2 \geq 0$$

Example

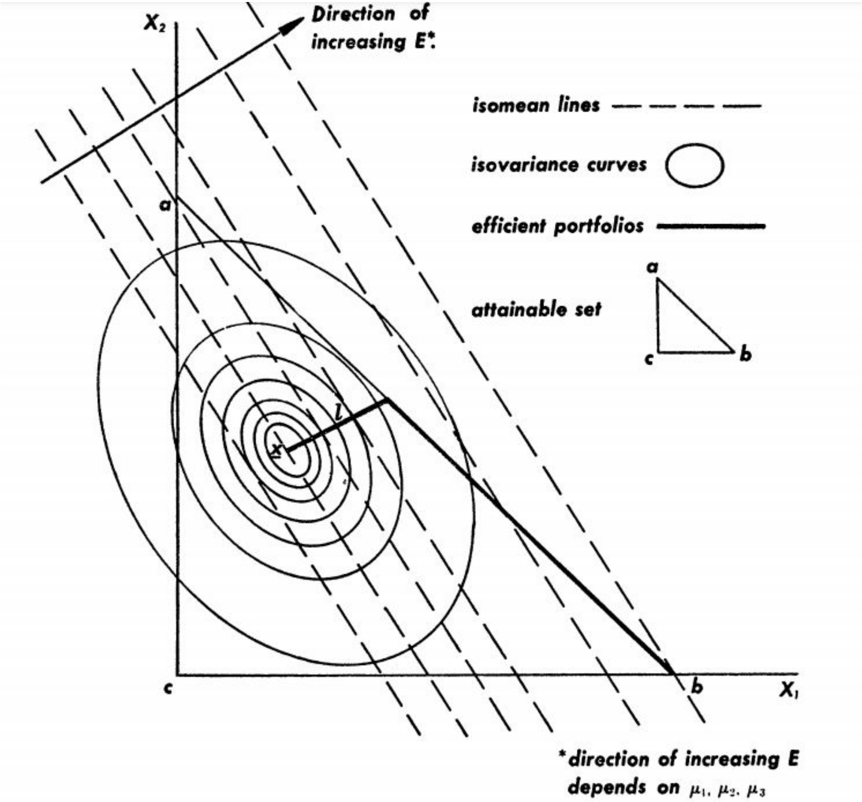

FIG. 2

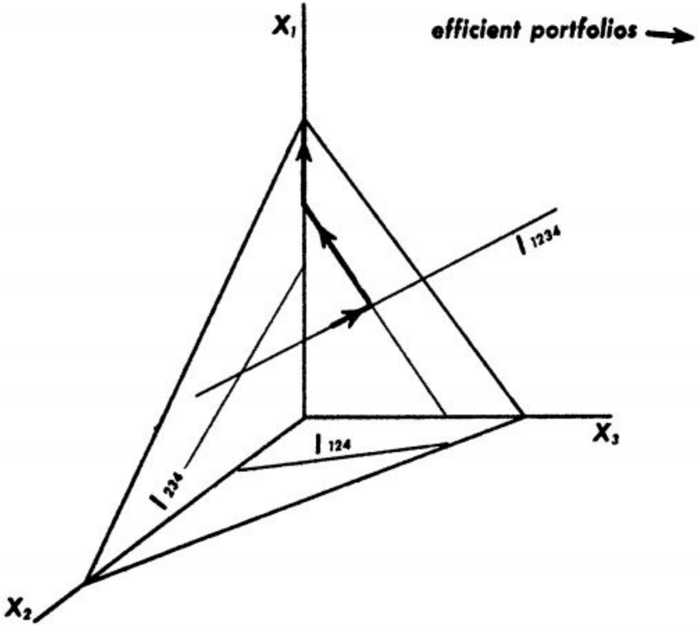

FIG. 4

Example

The paper contains more insights and examples.

Conclusion

If we can model securities in random variable form,

E-V rule, considering variance of returns, suggests diversified portfolio curve.

Discussion

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