Math Solution for Hyunjin

Sunghee Yun

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Let x and y be the amount of job Nikki and Kelly can write for one hour respectively. In other words, assume that x and y are the rates of Nikki and Kelly respectively. If Nikki work alone, it will take $1/x$ to do the job, and if Kelly work alone, it will take $1/y$. Thus, we have

$$1/x = 1/y - 4 \text{ and } 1/(x+y) = 20, \tag{1}$$

which are equivalent to

$$4xy + y - x = 0 \text{ and } 20(x+y) = 1. \tag{2}$$

Since $y = 1/20 - x$, we plug this in the first equation to obtain

$$(4x+1)(1/20 - x) - x = 0 \Leftrightarrow (4x+1)(20x-1) + 20x = 0 \Leftrightarrow 80x^2 + 36x - 1 = 0. \tag{3}$$

The computer calculation yields (*e.g.*, using `numpy.roots` with Python)

$$x = -0.47624689 \text{ or } 0.02624689. \tag{4}$$

Since x is a positive real number,

$$x = 0.02624689 \text{ and } y = 1/20 - x = 0.0237531094, \tag{5}$$

and it'd take Nikki and Kelly

$$1/x = 38.0998 \text{ hours} \tag{6}$$

and

$$1/y = 42.0998 \text{ hours} \tag{7}$$

respectively (to do the job).

Now let us do the calculation using the quadratic formula. The roots of $80x^2 + 36x - 1 = 0$ are

$$x = \frac{-18 \pm \sqrt{18^2 + 80}}{80} = \frac{-9 \pm \sqrt{9^2 + 20}}{40} = \frac{-9 \pm \sqrt{101}}{40} \tag{8}$$

Thus if we take the positive number, we have

$$x = \frac{-9 + \sqrt{101}}{40} \text{ and } y = 1/20 - x = \frac{11 - \sqrt{101}}{40}, \tag{9}$$

and it'd take Nikki and Kelly

$$1/x = \frac{40}{-9 + \sqrt{101}} = \frac{40}{-9 + \sqrt{101}} \cdot \frac{9 + \sqrt{101}}{9 + \sqrt{101}} = 2 \cdot (9 + \sqrt{101}) \text{ hours} \tag{10}$$

and

$$1/y = \frac{40}{11 - \sqrt{101}} = \frac{40}{11 - \sqrt{101}} \cdot \frac{11 + \sqrt{101}}{11 + \sqrt{101}} = 2 \cdot (11 + \sqrt{101}) \text{ hours} \tag{11}$$

respectively.