Math Kangaroo Solutions: Grade 5-6, 2013

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January 18, 2020

1. (E)
2. (C)
3. (C)
4. (B)
5. (E)
6. (B)
7. (B)
8. (E)
9. (C)
10. (D)
11. (C)
12. (C)
13. (D)
14. (B)
15. (A)

    Solution: 13 (1-by-1) + 4 (2-by-2) + 5 (3-by-3) + 4 (4-by-4) + 1 (5-by-5) = 24
16. (D)
17. (D)
18. (A)

    Solution: (10, 60), (11, 61), ..., (49, 99)
19. (C)
20. (D)
21. (A)
22. (D)
23. (B)

Solution:

$$\begin{array}{cccc} A & B & A & B \\ & & & | \\ A & B & B & A \\ & & | & \\ A & A & A & B \\ & | & & \\ B & B & B & B \end{array}$$

24. (A)

25. (B)

26. (D)

    Solution: The three digits numbers which satisfy the property have one of these forms:

    $$9\square 6,\ 8\square 5,\ 7\square 4,\ 6\square 3,\ 5\square 2,\ 4\square 1$$

    There are 10 digits for each of these forms, hence there are 60 3 digit numbers (which satisfy the given property).
    Another way to see this is, if we let abc a three digit number, the property holds for that number if and only if

    $$100 \times a + 10 \times b + 1 \times c - 297 = 1 \times a + 10 \times b + 100 \times c \Leftrightarrow 99 \times (a - c) = 297 \Leftrightarrow a - c = 3.$$

    Therefore the possible tuples for (a, b, c) are

    $$(4, b, 1),\ (5, b, 2),\ (6, b, 3),\ (7, b, 4),\ (8, b, 5),\ (9, b, 6),$$

    which is basically a strict mathematical proof for the first argument.

27. (B)

28. (E)

    Solution: One possibility is all of them were liars. Another possibility is that 1006 of them were liars and 1007 of them were knights. Therefore there are at least two possibilities.

29. (D)

    Solution:

    $$(a, b, c) \rightarrow (b + c, a + c, a + b)$$

    The maximum difference from $(b + c, a + c, a + b)$ is $\max\{|(b + c) - (a + c)|, |(b + c) - (a + b)|, |(a + c) - (a + b)|\} = \max\{|b - a|, |c - a|, |c - b|\}$, which is the maximum difference from (a, b, c). Therefore the "changesum" procedure does not change the maximum difference.

30. (B)

    Solution:

    $$4 \times (1 + 2 + 3 + 4 + 5 + 6) - 2 \times (1 + 1 + 3 + 3) = 4 \times 21 - 2 \times 8 = 68$$