Yun Family Math Problems

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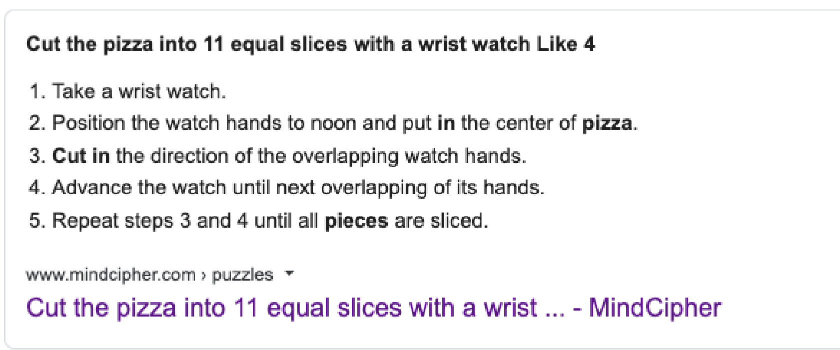
1 Jisun's pizze problem (21-MAR-2020)

While we were chatting over Zoom on 21-MAR-2020, Jisun brought up the below problem:

> Suppose that you have a round pizza and an analog watch or clock. How can you divide it into 11 (exactly) equal pieces?

Though this problem looked really interesting, I myself could not figure it out. Taehoon said you can erase the digit 12, which was a very creative idea. However, nobody could give the correct question to it.

On 23-MAR-2020, Jisun asked again, "Has anybody solved the pizza problem?" over KakaoTalk. I googled it and found the following answer.



Cut the pizza into 11 equal slices with a wrist watch Like 4

1. Take a wrist watch.
2. Position the watch hands to noon and put **in** the center of **pizza**.
3. **Cut in** the direction of the overlapping watch hands.
4. Advance the watch until next overlapping of its hands.
5. Repeat steps 3 and 4 until all **pieces** are sliced.

www.mindcipher.com › puzzles

Cut the pizza into 11 equal slices with a wrist ... - MindCipher

And Jisun confirmed that that is the right answer.

Then my tendency to want to prove things rigorously made me have to spend my time on writing the below. However, I'm pretty sure that this will be helpful for my nephews (and niece), so this would definitely not waste of time. :)

Here I want to write two different ways to see this, the first of which is kind of repeat what is written above.

Proof:

1. One way to prove this can go as follows.

   If you position the hour hand (HH) and the minute hand (MH) at noon and let the watch operator as it usually does, then the two hands do not meet until 1PM (the left clock in Figure 1) because the MH goes faster than the HH. Now if the time passes a bit more, there comes a moment that the two hands meets exactly (the right clock in Figure 1). Now mark this position (or angle) on the pizza. Then wait until the two hands meet again and mark that position, too. You repeat this until the two meets again at midnight.

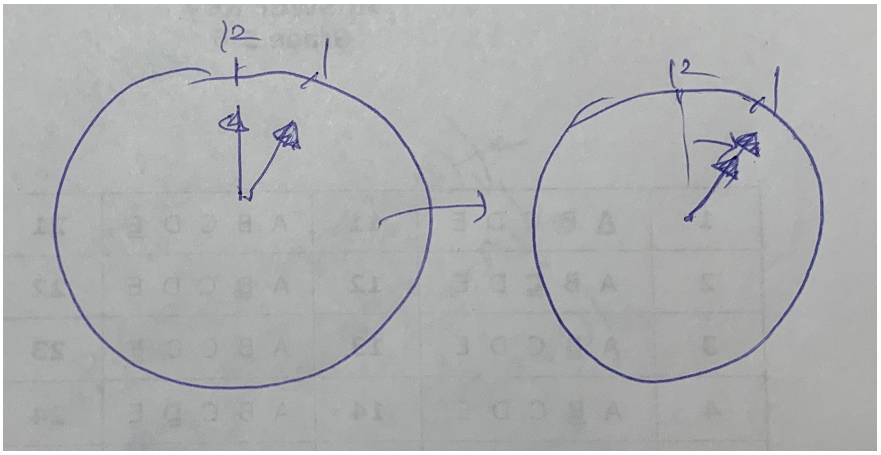


Figure 1: The locations of hour and minute hands at 1PM (left) and those when it meets the next time (right).

Now ask how many times the two hands have met after noon, *i.e.*, exclude noon. Since they met a bit after 1PM, then a more bit after 2PM, . . . , and after 11PM. When they met after 11PM is at midnight. Thus, *they have met 11 times.*

Because you can insist that the time between two consecutive *rendezvious* of the two hands is the same for every time, this divides 360 degrees into 11 equal angles.

2. Now let us examine this quantitatively. Let x be the angle between HH at midnight and HH when it meets MH next time as shown in Figure 2. Because HH goes to the position of 1 when MH does one rotation, *i.e.*, the speed of MH is 12 times faster then HH, the angle y in Figure 2 is 12 times smaller than x, or equivalently, x is 12 times larger than y. Then we have

$$x = 12 \times y. \tag{1}$$

Now we have two variables, x and y, but only one equation. In this case, (in general) we cannot decide unique values for the variables. However, we have one more equation since x is equal to y plus 360/12. Therefore we have

$$x = y + \frac{360}{12} \Leftrightarrow 12 \times x = 12 \times y + 360 \tag{2}$$

We obtained the right side equality by multiplying 12 on both sides of the left equality in (2). Now if you replace $12 \times y$ with x in (1), we obtain

$$12 \times x = x + 360 \Leftrightarrow 11 \times x = 360 \Leftrightarrow x = 360/11. \tag{3}$$

Therefore our method *indeed* divides the pizza into 11 equal-sized pieces. ■

An interesting observation is that this number 11 comes from the calculation of $12 - 1$. Thus, if half day were 13 hours, the technique would give us 12 pizza pieces, and, *e.g.*, if half day were 18 hours, the technique would give us 17 pizza pieces. In general, if half day were n hours, the technique would give us $n - 1$ pizze pieces! This type of generalization is a good practice to improve your math skills (*I* believe).

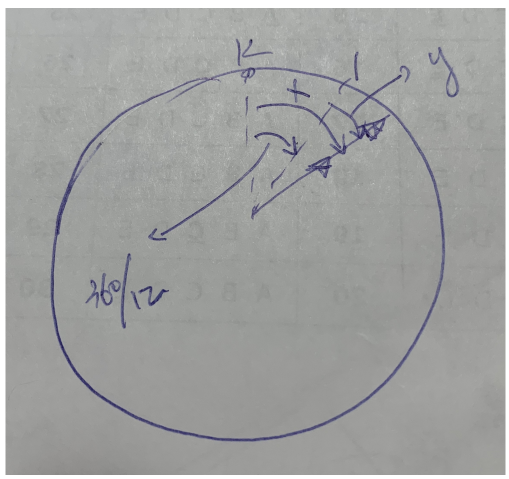


Figure 2: Pizza problem: x refers to the angle between 12 o'clock and where the minute hand meets the hour hand, and y refers to the angle between 1 o'clock and where the minute hand meets the hour hand.