Math for Taehoon

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- What is the smallest possible value of n that makes $7! \times n$ a perfect square?

  Answer: 35

  Proof: Since

  $$7! = 2 \times 3 \times (2 \times 2) \times 5 \times (2 \times 3) \times 7 = 2^4 \times 3^2 \times 5 \times 7 = (2^2 \times 3)^2 \times 5 \times 7, \quad (1)$$

  we need at least one 5 and one 7 to make $7! \times n$ a perfect square. Therefore the smallest possible value of n that makes $7! \times n$ a perfect square is $5 \times 7 = 35$. Note that $7! \times 35 = (2^2 \times 3 \times 5 \times 7)^2$.

- If there are 18 dragons with either 2 or 3 heads, and the total number of heads is 42, how many 2-headed dragons are there?

  Answer: 12

  Proof: Let x be the number of 2-headed dragons and y be the number of 3-headed dragons. Since there are 18 dragons, we have the following equation:

  $$x + y = 18 \quad (2)$$

  Also, the total number of heads is 42, thus we have

  $$2 \times x + 3 \times y = 42 \quad (3)$$

  Now if we multiply 3 to both the left-hand-side (LHS) and right-hand-side (RHS) of (2), we obtain the following two equations:

  $$\begin{aligned} 3 \times x + 3 \times y &= 3 \times (x + y) = 3 \times 18 & (4) \\ 2 \times x + 3 \times y &= 42 & (5) \end{aligned}$$

  Now we subtract (5) from (4), we get

  $$x = (3 \times x + 3 \times y) - (2 \times x + 3 \times) = 3 \times 18 - 42 = 54 - 42 = 12. \quad (6)$$

  Since we initially assumed that x is the number of 2-headed dragons, the answer is 12.

- Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

  Answer: 350 meters.

  Proof: Figure 1 shows the track Brenda and Sally run. X refers to the distance that Sally runs until she meets Brenda and Y refers to the distance that Brenda runs past the first meeting point until she meets Sally again. Now since they run at constant speed, the ratio of 100 to X and that of Y to 150 are the same, *i.e.*,

  $$100 : X = Y : 150 \Leftrightarrow \frac{100}{X} = \frac{Y}{150} \Leftrightarrow X \times Y = 100 \times 150. \quad (7)$$

  Now we know that the sum of the distance that Brenda runs and the distance that Sally runs until they first meet is half the circumference of the track. We also know that the sum

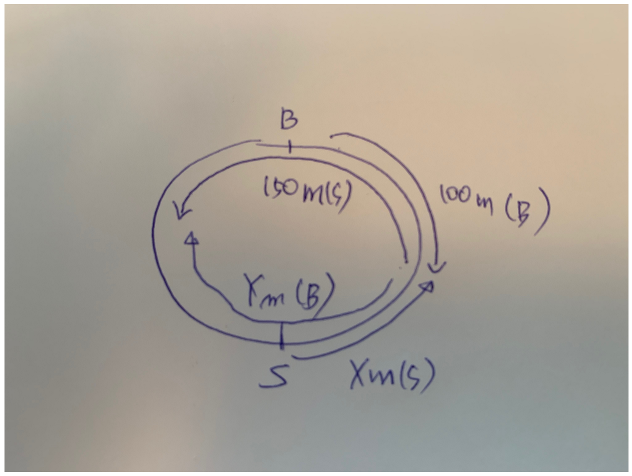

Figure 1: Race of Brenda and Sally. B and S stand for Brenda and Sally respectively.

of the distance that Brenda runs and the distance that Sally runs until they meets second time is the circumference of the track. If we express this as an equation, we have

$$Y + 150 = 2 \times (X + 100). \tag{8}$$

Now if we multiply 2 to both side of this equation and use (7), we have

$$\begin{aligned} & X \times Y + 150 \times X = 2 \times (X \times X + 100 \times X) \\ \Leftrightarrow\ & 100 \times 150 + 150 \times X = 2 \times (X \times X + 100 \times X) \\ \Leftrightarrow\ & X \times X + 25 \times X - 100 \times 75 = 0 \\ \Leftrightarrow\ & (X - 75) \times (X + 100) = 0. \end{aligned}$$

Therefore $X = 75$ and the circumference (or the length) of the track is $2 \times (X + 100) = 350$.

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1. In this addition problme, ...

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

Solution:

Since O is an even number, the previous addition, *i.e.*, $W + W$ should not generate overflow. Therefore, W should be less than 5. Then $O = 4$ and $F = 1$ since $7 + 7 = 14$. Since $O = 4$, $R = 8$. Now the above equation becomes

$$\begin{array}{cccc} & 7 & W & 4 \\ + & 7 & W & 4 \\ \hline 1 & 4 & U & 8 \end{array}$$

Now since $O = 4$ and $F = 1$, the only possible values for W are 2 and 3. But if $W = 2$, then $U = 4$ and $U = O$. Therefore $W = 3$. In summary,

$$
\begin{array}{cccc}
 & 7 & 3 & 4 \\
+ & 7 & 3 & 4 \\
\hline
1 & 4 & 6 & 8
\end{array}
$$

2. Mr. Harman needs ...

   Answer: $(122 + 125 + 127)/2 = 187$.

   Solution: Let X, Y, and Z be the weights of the three boxes. Then we have

$$
\begin{array}{rcl}
X + Y & = & 122 \\
Y + Z & = & 125 \\
X + Z & = & 127
\end{array}
$$

   Since the sum of all three quantities in left-hand-side (LHS) is the same as the sum of all three quantities in right-hand-side (RHS), we have

$$2 \times (X + Y + Z) = 122 + 125 + 127, \tag{9}$$

   thus

$$X + Y + Z = (122 + 125 + 127)/2 = 187. \tag{10}$$

3. The students in Mrs. Reed's English class are reading ...

   Answer: $760 \times \frac{1.5}{1+1.5} = 760 \times \frac{3}{5} = 456$.

   Solution: Note that Chandra's reading speed is 1.5 times bigger than that of Bob. If we let x be the number of pages that Bob should read, then $1.5 \times x$ is the number of pages that Chandra should read. Thus we have $x + 1.5 \times x = 2.5 \times x = 760$ and

$$x = \frac{760}{2.5} = 760 \times \frac{2}{5}. \tag{11}$$

   Thus the number of pages Chandra should read is

$$x \times 1.5 = 760 \times \frac{\cancel{2}}{5} \times \frac{3}{\cancel{2}} = 152 \times 3 = 456. \tag{12}$$

4. Landy drove ...

   Answer:

$$\frac{10 + 20 + 30}{10/30 + 20/20 + 30/10} = 13.8 \text{ miles/hour} \tag{13}$$

5. The Incredible ...

   Answer: The 11th jump!

Solution:

$$\begin{aligned}
\text{1st jump} &= 1 \\
\text{2nd jump} &= 2 \\
\text{3rd jump} &= 4 \\
\text{4th jump} &= 8 \\
\text{5th jump} &= 16 \\
\text{6th jump} &= 32 \\
\text{7th jump} &= 64 \\
\text{8th jump} &= 128 \\
\text{9th jump} &= 256 \\
\text{10th jump} &= 512 \\
\text{11th jump} &= 1024
\end{aligned}$$

6. The Amaco ...

Answer: $15 - 11 = 4$

Solution: Let X and Y be the number of seventh graders and that of sixth graders respectively who bought pencils. Let P be the price of one pencil in cents. Then we have

$$\begin{aligned}
X \times P &= 143 = 11 \times 13 \\
Y \times P &= 195 = 15 \times 13
\end{aligned}$$

Thus P is a common divisor of 143 and 195. Thus P is 1 or 13. However, if $P = 1$, then $Y = 195$. But $Y \leq 30$, hence $P = 13$. Therefore, $X = 11$ and $Y = 15$. Thus $Y - X = 4$.

Q1 $88 + 86 + 91 + 92 + 87 + 90 + 89 + 93 + 92 + 88$?

Answer:

$$\begin{aligned}
&88 + 86 + 91 + 92 + 87 + 90 + 89 + 93 + 92 + 88 \\
&= (90 - 2) + (90 - 4) + (90 + 1) + (90 + 2) + (90 - 3) \\
&\quad +(90 + 0) + (90 - 1) + (90 + 3) + (90 + 2) + (90 - 2) \\
&= 90 \times 10 + (-2 - 4 + 1 + 2 - 3 + 0 - 1 + 3 + 2 - 2) \\
&= 90 \times 10 - 4 = 896.
\end{aligned}$$

Q2 **Answer:** 90 inches.

Q3 **Answer:** 4.

Q4 **Answer:** $3 \times 8 \times 2 = 48$.

Q5 **Answer:** 1249

Q6 **Answer:** 56.25

$$x \times \frac{4}{5} \times \frac{4}{5} = 36 \Rightarrow x = 36 \times \frac{5}{4} \times \frac{5}{4} = 56.25 \tag{14}$$

Q7 **Answer:** $2 \times 2/2 = 2$

Q8 Answer: Captain Hook found 30 diamonds.

$$\begin{aligned} H + P &= 80 \\ P + S &= 70 \\ H + S &= 50 \end{aligned}$$

thus

$$2 \times H = (H + P) + (H + S) - (P + S) = 80 + 50 - 70 = 60 \Rightarrow H = 30.$$

Q9 Answer: $15 + 10 \times 2 = 35$.

Q10 Answer: 25 hours.

Since the 1st pipe can fill the swimming pool 1.5 times faster than the 2nd pipe, using both pipes will be 2.5 ($= 1 + 1.5$) times fater than using the 2nd pipe alone. Thus using only 2nd pipe will be 2.5 slower than using both pipes, hence it'd take $10 \times 2.5 = 25$ hours to fill the swimming pool.

Q11 Answer: $12 \times 12 - 4 \times 4 - 4 \times (4 \times 4)/2 = 96$.

Q12 Answer: $(30 + 90 + 50)/(1 + 2 + 1) = 42.5$ miles/hour.

Q13 Answer: 4 months

$$1 \ \cancel{2} \ \cancel{3} \ \cancel{4} \ 5 \ \cancel{6} \ 7 \ \cancel{8} \ \cancel{9} \ \cancel{10} \ 11 \ \cancel{12}$$

Q17 Answer: 12 kilograms

Assume that x kilograms of raisins and y kilograms of nuts were used for making the snakcs. Then

$$\begin{aligned} x + y &= 20 \\ 3.5 \times x + 4.75 \times y &= 20 \times 4 \end{aligned}$$

If we multiply 4.75 to both sides of the first equation, we have

$$\begin{aligned} 4.75 \times x + 4.75 \times y &= 20 \times 4.75 \\ 3.5 \times x + 4.75 \times y &= 20 \times 4 \end{aligned}$$

If we subtract the second equation from the first one, we have

$$\begin{aligned} & (4.75 - 3.5) \times x = 20 \times (4.75 - 4) \\ \Leftrightarrow \quad & 1.25 \times x = 20 \times 0.75 \\ \Leftrightarrow \quad & x = 20 \times 0.75/1.25 = 20 \times \frac{3}{5} = 12 \end{aligned}$$