Math for Taehoon

Sunghee Yun

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- What is the smallest possible value of n that makes $7! \times n$ a perfect square?

  Answer: 35

  Proof: Since

  $$7! = 2 \times 3 \times (2 \times 2) \times 5 \times (2 \times 3) \times 7 = 2^4 \times 3^2 \times 5 \times 7 = (2^2 \times 3)^2 \times 5 \times 7, \quad (1)$$

  we need at least one 5 and one 7 to make $7! \times n$ a perfect square. Therefore the smallest possible value of n that makes $7! \times n$ a perfect square is $5 \times 7 = 35$. Note that $7! \times 35 = (2^2 \times 3 \times 5 \times 7)^2$.

- If there are 18 dragons with either 2 or 3 heads, and the total number of heads is 42, how many 2-headed dragons are there?

  Answer: 12

  Proof: Let x be the number of 2-headed dragons and y be the number of 3-headed dragons. Since there are 18 dragons, we have the following equation:

  $$x + y = 18 \quad (2)$$

  Also, the total number of heads is 42, thus we have

  $$2 \times x + 3 \times y = 42 \quad (3)$$

  Now if we multiply 3 to both the left-hand-side (LHS) and right-hand-side (RHS) of (2), we obtain the following two equations:

  $$\begin{aligned} 3 \times x + 3 \times y &= 3 \times (x + y) = 3 \times 18 \quad (4) \\ 2 \times x + 3 \times y &= 42 \quad (5) \end{aligned}$$

  Now we subtract (5) from (4), we get

  $$x = (3 \times x + 3 \times y) - (2 \times x + 3\times) = 3 \times 18 - 42 = 54 - 42 = 12. \quad (6)$$

  Since we initially assumed that x is the number of 2-headed dragons, the answer is 12.

- Brenda and Sally run in opposite directions on a circular track, starting at diametrically opposite points. They first meet after Brenda has run 100 meters. They next meet after Sally has run 150 meters past their first meeting point. Each girl runs at a constant speed. What is the length of the track in meters?

  Answer: 350 meters.

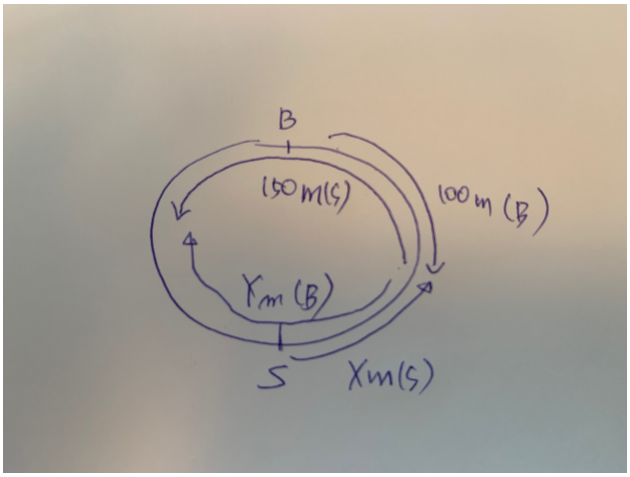

Figure 1: Race of Brenda and Sally. B and S stand for Brenda and Sally respectively.

Proof: Figure 1 shows the track Brenda and Sally run. X refers to the distance that Sally runs until she meets Brenda and Y refers to the distance that Brenda runs past the first meeting point until she meets Sally again. Now since they run at constant speed, the ratio of 100 to X and that of Y to 150 are the same, *i.e.*,

$$100 : X = Y : 150 \Leftrightarrow \frac{100}{X} = \frac{Y}{150} \Leftrightarrow X \times Y = 100 \times 150. \tag{7}$$

Now we know that the sum of the distance that Brenda runs and the distance that Sally runs until they first meet is half the circumference of the track. We also know that the sum of the distance that Brenda runs and the distance that Sally runs until they meets second time is the circumference of the track. If we express this as an equation, we have

$$Y + 150 = 2 \times (X + 100). \tag{8}$$

Now if we multiply 2 to both side of this equation and use (7), we have

$$\begin{aligned} & X \times Y + 150 \times X = 2 \times (X \times X + 100 \times X) \\ \Leftrightarrow \quad & 100 \times 150 + 150 \times X = 2 \times (X \times X + 100 \times X) \\ \Leftrightarrow \quad & X \times X + 25 \times X - 100 \times 75 = 0 \\ \Leftrightarrow \quad & (X - 75) \times (X + 100) = 0. \end{aligned}$$

Therefore $X = 75$ and the circumference (or the length) of the track is $2 \times (X + 100) = 350$.