

# Math Solution for Hyunjin

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Let  $x$  and  $y$  be the amount of job Nikki and Kelly can write for one hour respectively. In other words, assume that  $x$  and  $y$  are the rates of Nikki and Kelly respectively. If Nikki work alone, it will take  $1/x$  to do the job, and if Kelly work alone, it will take  $1/y$ . Thus, we have

$$1/x = 1/y - 4 \text{ and } 1/(x + y) = 20, \quad (1)$$

which are equivalent to

$$4xy + y - x = 0 \text{ and } 20(x + y) = 1. \quad (2)$$

Since  $y = 1/20 - x$ , we plug this in the first equation to obtain

$$(4x + 1)(1/20 - x) - x = 0 \Leftrightarrow (4x + 1)(20x - 1) + 20x = 0 \Leftrightarrow 80x^2 + 36x - 1 = 0. \quad (3)$$

The computer calculation yields (*e.g.*, using `numpy.roots` with Python)

$$x = -0.47624689 \text{ or } 0.02624689. \quad (4)$$

Since  $x$  is a positive real number,

$$x = 0.02624689 \text{ and } y = 1/20 - x = 0.0237531094, \quad (5)$$

and it'd take Nikki and Kelly

$$1/x = 38.0998 \text{ hours} \quad (6)$$

and

$$1/y = 42.0998 \text{ hours} \quad (7)$$

respectively (to do the job).

Now let us do the calculation using the quadratic formula. The roots of  $80x^2 + 36x - 1 = 0$  are

$$x = \frac{-18 \pm \sqrt{18^2 + 80}}{80} = \frac{-9 \pm \sqrt{9^2 + 20}}{40} = \frac{-9 \pm \sqrt{101}}{40} \quad (8)$$

Thus if we take the positive number, we have

$$x = \frac{-9 + \sqrt{101}}{40} \text{ and } y = 1/20 - x = \frac{11 - \sqrt{101}}{40}, \quad (9)$$

and it'd take Nikki and Kelly

$$1/x = \frac{40}{-9 + \sqrt{101}} = \frac{40}{-9 + \sqrt{101}} \cdot \frac{9 + \sqrt{101}}{9 + \sqrt{101}} = 2 \cdot (9 + \sqrt{101}) \text{ hours} \quad (10)$$

and

$$1/y = \frac{40}{11 - \sqrt{101}} = \frac{40}{11 - \sqrt{101}} \cdot \frac{11 + \sqrt{101}}{11 + \sqrt{101}} = 2 \cdot (11 + \sqrt{101}) \text{ hours} \quad (11)$$

respectively.