Dynamics and Cognitive Models **(Due: 04/02/19)**

Assignment #6

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Problem 1: Solve the following differential equation.

(1) $x' = -x$ where $x(0) = 1$

Solution

(1) The given equation can be written as follows:

$$\frac{dx}{dt} = -x$$
$$-\frac{1}{x}dx = dt$$

Integrate both sides.

$$\int -\frac{1}{x}dx = \int dt$$

$$-\ln|x| = t + C$$

$$|x| = Ae^{-t}$$

where C is a constant of integration and $A > 0$. Since $x(0) = 1$, $A = 1$. Therefore,

$$x = e^{-t} \text{ or } x = -e^{-t}$$

□

Problem 2: Estimate $x(1)$ using the Euler method when Δt is 1, 0.1, and 0.01.

Solution

We know $x(1) = -x'(1)$. The Euler method estimates $x'(1)$ as

$$\frac{x(1+\Delta t) - x(1)}{\Delta t}$$

Then, if $x = e^{-t}$, $x(1)$ is approximated as follows for each Δt:

$$x(1) = -x'(1) \approx \begin{cases} \frac{-e^{-2}+e^{-1}}{1} = 0.2325, & \text{if } \Delta t = 1 \\ \frac{-e^{-1.1}+e^{-1}}{0.1} = 0.3501, & \text{if } \Delta t = 0.1 \\ \frac{-e^{-1.01}+e^{-1}}{0.01} = 0.3660, & \text{if } \Delta t = 0.01 \end{cases}$$

The analytic solution is

$$x(1) = e^{-1} = 0.3679$$

If $x = -e^{-t}$, $x(1)$ is approximated as follows for each Δt:

$$x(1) = -x'(1) \approx \begin{cases} \frac{e^{-2}-e^{-1}}{1} = -0.2325, & \text{if } \Delta t = 1 \\ \frac{e^{-1.1}-e^{-1}}{0.1} = -0.3501, & \text{if } \Delta t = 0.1 \\ \frac{e^{-1.01}-e^{-1}}{0.01} = -0.3660, & \text{if } \Delta t = 0.01 \end{cases}$$

The analytic solution is

$$x(1) = -e^{-1} = -0.3679$$

Therefore, the absolute errors between each approximate solution and analytic solution are:

$$\begin{cases} |0.3679 - 0.2325| = |(-0.3679) - (-0.2325)| = 0.1354, & \text{if } \Delta t = 1 \\ |0.3679 - 0.3501| = |(-0.3679) - (-0.3501)| = 0.0178, & \text{if } \Delta t = 0.1 \\ |0.3679 - 0.3660| = |(-0.3679) - (-0.3501)| = 0.0019, & \text{if } \Delta t = 0.01 \end{cases}$$

$\square$