

Assignment #6

Instructor: Sungryong Koh*Name:* Sungjae Cho, *ID:* 2017-28413**Problem 1: Solve the following differential equation.**

(1) $x' = -x$ where $x(0) = 1$

Solution

(1) The given equation can be written as follows:

$$\begin{aligned}\frac{dx}{dt} &= -x \\ -\frac{1}{x}dx &= dt\end{aligned}$$

Integrate both sides.

$$\int -\frac{1}{x}dx = \int dt$$

$$-\ln|x| = t + C$$

$$|x| = Ae^{-t}$$

where C is a constant of integration and $A > 0$. Since $x(0) = 1$, $A = 1$. Therefore,

$$x = e^{-t} \text{ or } x = -e^{-t}$$

□

Problem 2: Estimate $x(1)$ using the Euler method when Δt is 1, 0.1, and 0.01.

Solution

The Euler method estimates $x'(1)$ as

$$\frac{x(1 + \Delta t) - x(1)}{\Delta t}$$

Then, if $x = e^{-t}$, $x'(1)$ is approximated as follows for each Δt :

$$x'(1) \approx \begin{cases} \frac{e^{-2} - e^{-1}}{1} = -0.2325, & \text{if } \Delta t = 1 \\ \frac{e^{-1.1} - e^{-1}}{0.1} = -0.3501, & \text{if } \Delta t = 0.1 \\ \frac{e^{-1.01} - e^{-1}}{0.01} = -0.3660, & \text{if } \Delta t = 0.01 \end{cases}$$

Then, if $x = -e^{-t}$, $x'(1)$ is approximated as follows for each Δt :

$$x'(1) \approx \begin{cases} \frac{-e^{-2} + e^{-1}}{1} = 0.2325, & \text{if } \Delta t = 1 \\ \frac{-e^{-1.1} + e^{-1}}{0.1} = 0.3501, & \text{if } \Delta t = 0.1 \\ \frac{-e^{-1.01} + e^{-1}}{0.01} = 0.3660, & \text{if } \Delta t = 0.01 \end{cases}$$

□