Dynamics and Cognitive Models **(Due: 04/09/19)**

Assignment #7

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Problem 1: Solve the following differential equations using integrating factors

(1) $tx' + 2x = t^2 - t + 1$ where $x(1) = 1/2$

(2) $x' - (1/t)x = t$ where $x(1) = 1/2$

Solution

(1) The given equation can be written as follows:

$$x^{'} + \frac{2}{t}x = 1 - t + \frac{1}{t}$$

Then, the integrating factor is $\exp(\int \frac{2}{t}dt) = t^2$. Multiply both sides by the factor.

$$t^2\frac{dx}{dt} + 2tx = t^3 - t^2 + t$$

Integrate both sides.

$$\int \left(t^2\frac{dx}{dt} + 2tx\right) dt = t^2x + C = t^3 - t^2 + t$$

where C is a constant of integration. Since $x(1) = 1/2$, $C = \frac{1}{2}$. Therefore,

$$x(t) = t - 1 + t^{-1} - \frac{1}{2}t^{-2}$$

(2) The integrating factor of the given equation is $\exp(\int -\frac{1}{t}dt) = \frac{1}{|t|}$. Multiply both sides by the factor. Then, for both $t > 0$ and $t < 0$, we get

$$\frac{1}{t} - \frac{1}{t^2}x = 1$$

Integrate both sides.

$$\int \left(\frac{1}{t} - \frac{1}{t^2}x\right) dt = \frac{x}{t} = t + C$$

where C is a constant of integration. Since $x(1) = 1/2$, $C = -\frac{1}{2}$. Therefore,

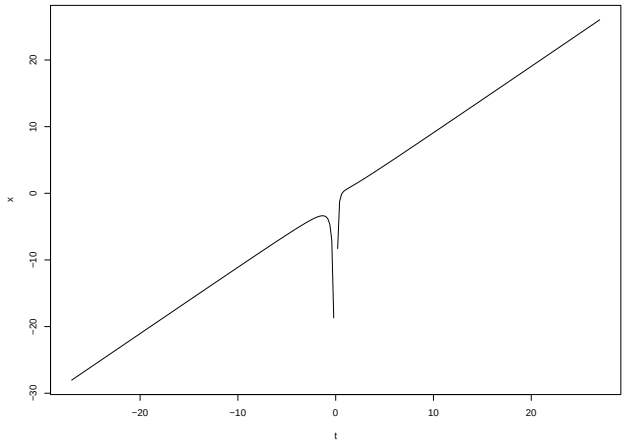
$$x(t) = t^2 - \frac{1}{2}t$$

Problem 2: Draw a graph that shows the relationship between x and t

Solution

(1)

$$x(t) = t - 1 + t^{-1} - \frac{1}{2}t^{-2}$$



(2)

$$x(t) = t^2 - \frac{1}{2}t$$