

## Assignment #7

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## Problem 1: Solve the following differential equations using integrating factors

(1)  $tx' + 2x = t^2 - t + 1$  where  $x(1) = 1/2$

(2)  $x' - (1/t)x = t$  where  $x(1) = 1/2$

## Solution

(1) The given equation can be written as follows:

$$x' + \frac{2}{t}x = 1 - t + \frac{1}{t}$$

Then, the integrating factor is  $\exp(\int \frac{2}{t} dt) = t^2$ . Multiply both sides by the factor.

$$t^2 \frac{dx}{dt} + 2tx = t^3 - t^2 + t$$

Integrate both sides.

$$\int \left( t^2 \frac{dx}{dt} + 2tx \right) dt = t^2 x + C = t^3 - t^2 + t$$

where  $C$  is a constant of integration. Since  $x(1) = 1/2$ ,  $C = \frac{1}{2}$ . Therefore,

$$x(t) = t - 1 + t^{-1} - \frac{1}{2}t^{-2}$$

(2) The integrating factor of the given equation is  $\exp(\int -\frac{1}{t} dt) = \frac{1}{|t|}$ . Multiply both sides by the factor. Then, for both  $t > 0$  and  $t < 0$ , we get

$$\frac{1}{t} - \frac{1}{t^2}x = 1$$

Integrate both sides.

$$\int \left( \frac{1}{t} - \frac{1}{t^2}x \right) dt = \frac{x}{t} = t + C$$

where  $C$  is a constant of integration. Since  $x(1) = 1/2$ ,  $C = -\frac{1}{2}$ . Therefore,

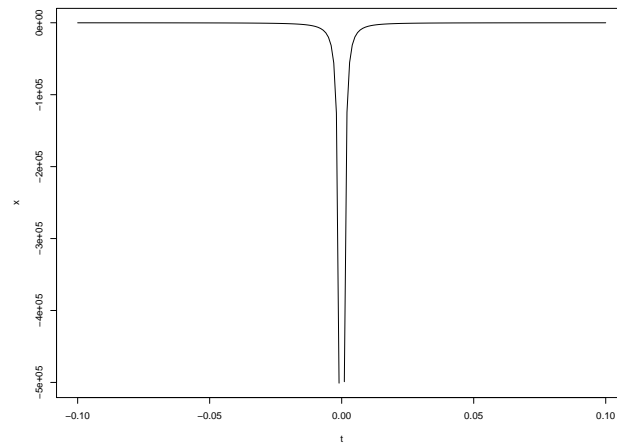
$$x(t) = t^2 - \frac{1}{2}t$$

**Problem 2:** Draw a graph that shows the relationship between  $x$  and  $t$

**Solution**

(1)

$$x(t) = t - 1 + t^{-1} - \frac{1}{2}t^{-2}$$



(2)

$$x(t) = t^2 - \frac{1}{2}t$$

