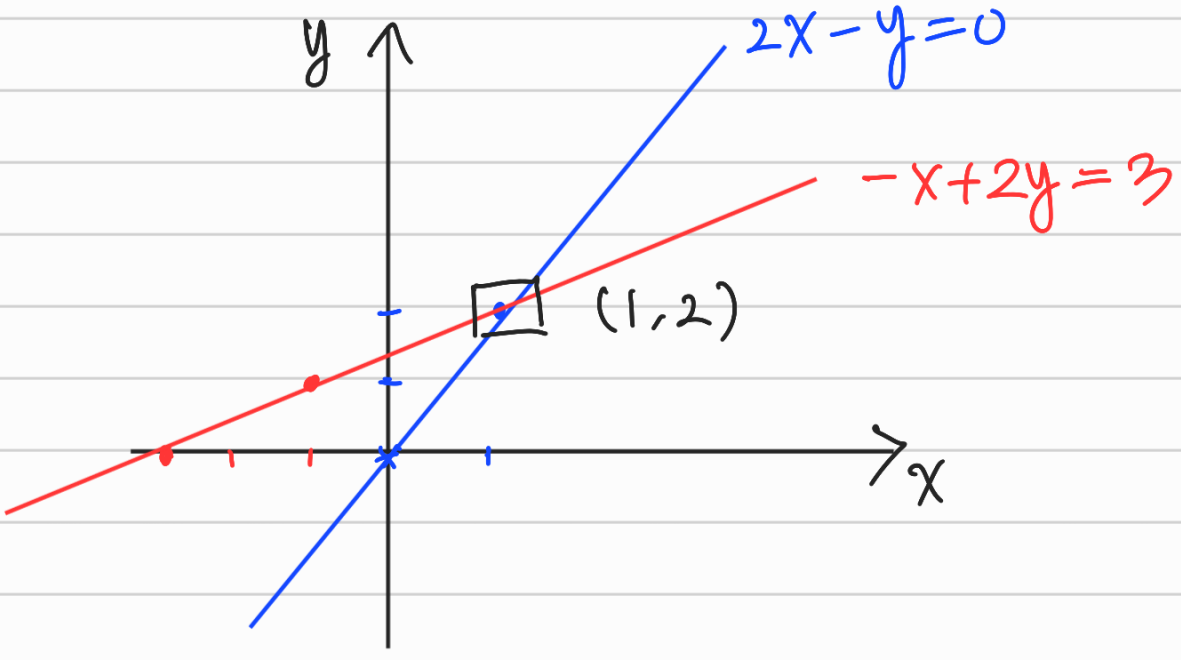
〈n equation . n unknowns〉

$$\begin{aligned} 2x - y &= 0 \\ -x + 2y &= 3 \end{aligned} \Rightarrow \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$A \cdot X = b$$

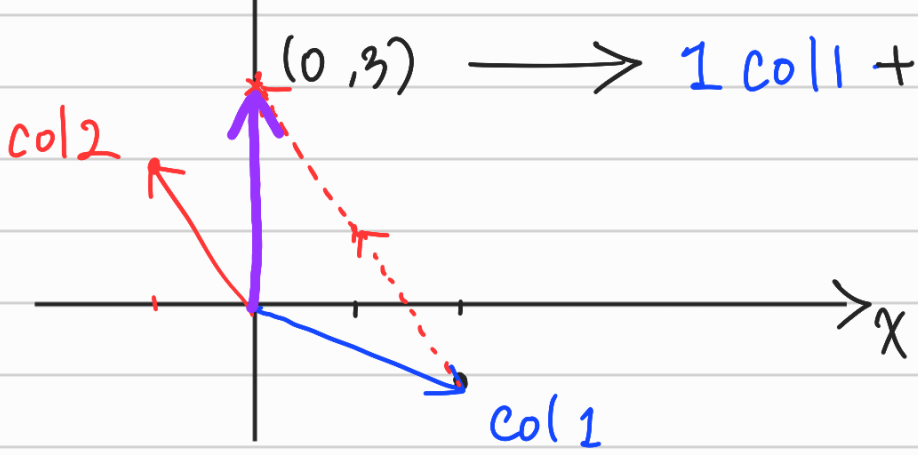
① Row picture



② Column picture

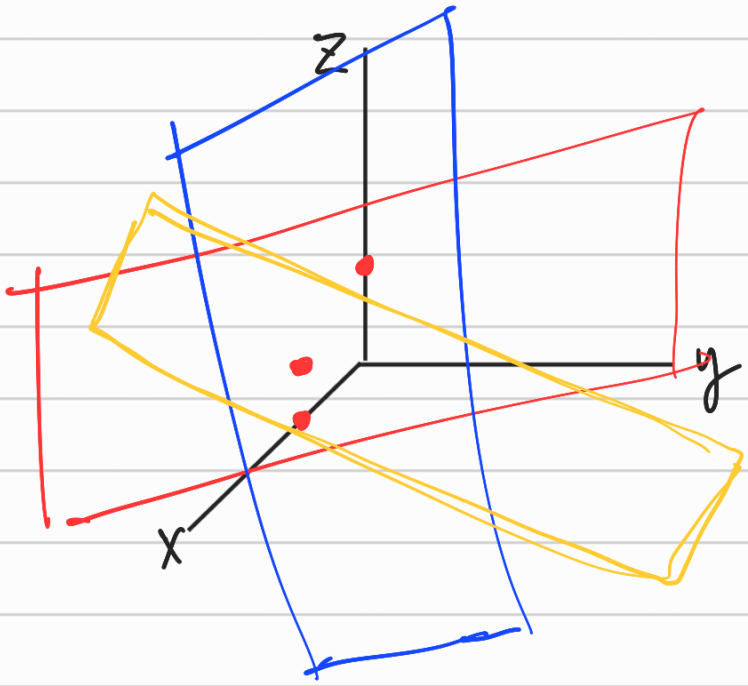
$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

⇒ linear combination of columns



$$
\begin{aligned}
2x - y &= 0 \\
-x + 2y - z &= -1 \\
-3y + 4z &= 4
\end{aligned}
$$

$$
A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \qquad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}
$$

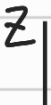


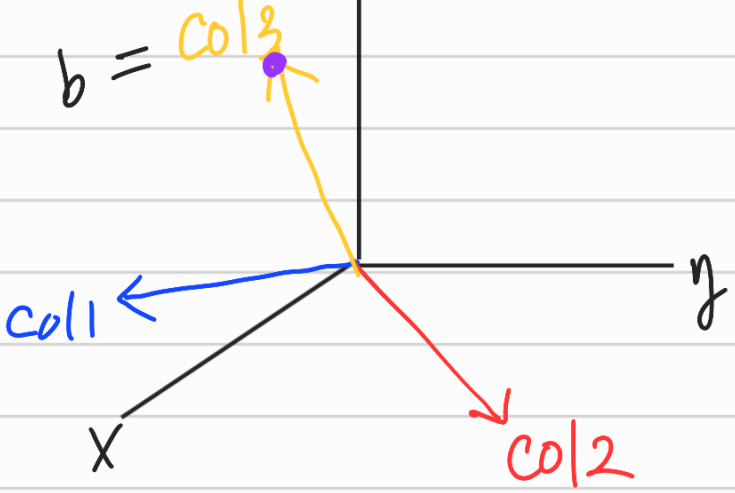
(Row picture)

→ 3 plains meet on one point

(Column picture)

$$
x \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} + z \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}
$$

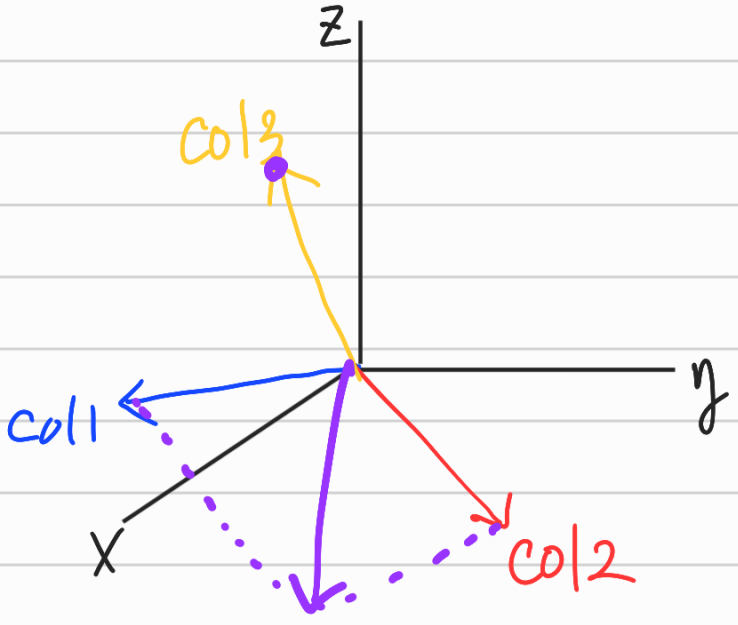




$$[x=0 \quad y=0 \quad z=1]$$

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}1\\1\\-3\end{bmatrix}$$

$$[x=1 \quad y=1 \quad z=0]$$



⟨ Can I solve $Ax=b$ for every b ? ⟩

⇓

Do the linear combinations of the cols

fill 3-D space?

* When could it go wrong?

↳ if the columns of the matrice lie in the same plane

⇓

all the combinations lie in that plane

⇒ Singular. not invertible

* Matrice multiplication in column picture

$$Ax = b$$

$$\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

↳ combination of columns in A