Factorization into $\boxed{A = LU}$

$$\underbrace{AB}_{I}\underline{(B^{-1}A^{-1})} = I \quad \Rightarrow (AB)^{-1} = B^{-1}A^{-1}$$

$$\Downarrow$$

$$A\,I\,A^{-1} = I$$

$$AA^{-1} = I \longrightarrow \underbrace{(A^{-1})^T}_{} A^T = I$$

inverse of A^T : $(A^T)^{-1}$

$$\begin{array}{ccc} E_{21} & A & U \\ \begin{bmatrix} 1 & 0 \\ -4 & 1 \end{bmatrix} & \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = & \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \end{array}$$

-1

$$\begin{array}{ccc} A & L & U \\ \begin{bmatrix} 2 & 1 \\ 8 & 7 \end{bmatrix} = & \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} & \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \end{array}$$

Lower T. Upper T

$$= \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1/2 \\ 0 & 1 \end{bmatrix}$$

$$E_{32}E_{31}E_{21}A = U$$

$$A = (E_{32}E_{31}E_{21})^{-1}U$$

$$= (E_{21})^{-1} \cdot (E_{31})^{-1} \cdot (E_{32})^{-1} U = LU$$

$$\overset{E_{32}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -5 & 1 \end{bmatrix}} \overset{E_{21}}{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \overset{E}{\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 10 & -5 & 1 \end{bmatrix}}$$

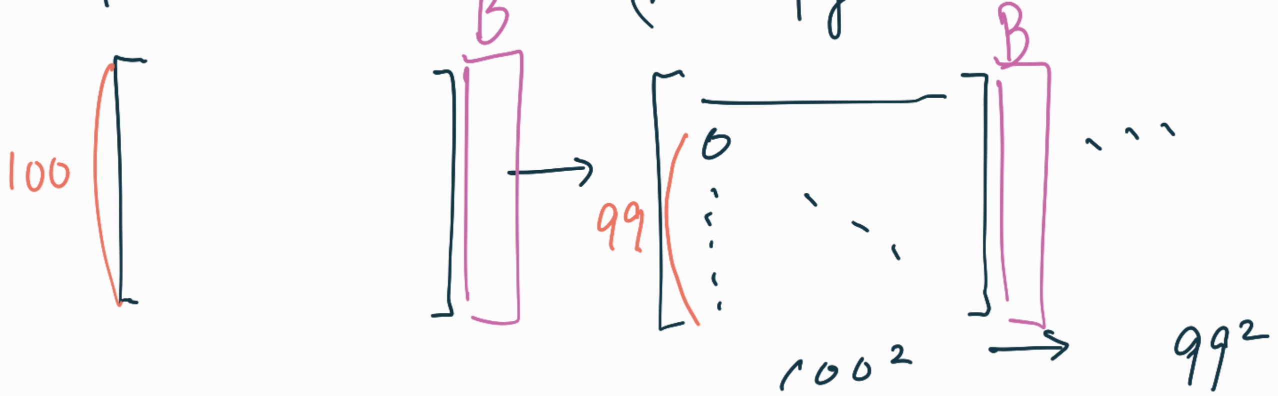
$$\overset{(E_{21})^{-1}}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{(E_{32})^{-1}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 0 \end{bmatrix}} = \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}}$$

$EA = U$ / $\underline{A = LU}$

if no row exchanges,
the multipliers go directly
into L

How many operations on $(n \times n)$ matrix A ?

$n = 100$ (multiply + sub)



$$\Rightarrow n^2+(n-1)^2+(n-2)^2+\cdots+2^2+1^2$$

$$\approx \boxed{\frac{1}{3}n^3} \text{ on } A \quad \boxed{n^2} \text{ on } B$$

Permutations $3\times 3 \Rightarrow 6$ ps

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$$

$$P_{12}=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{13}=\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\boxed{P^{-1}=P^T}$$

$$P_{23}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$+ \quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$