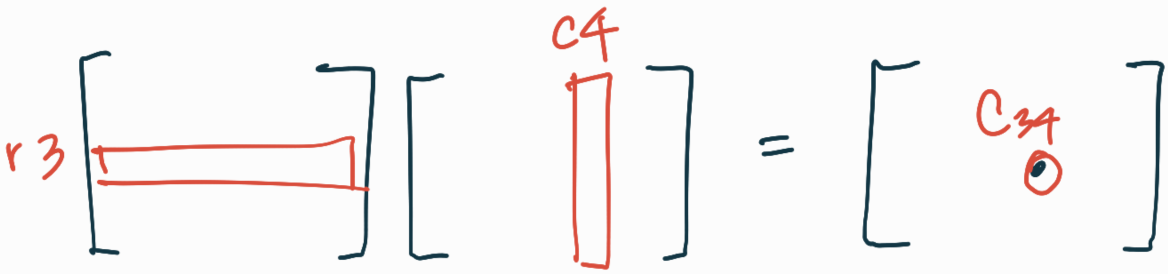
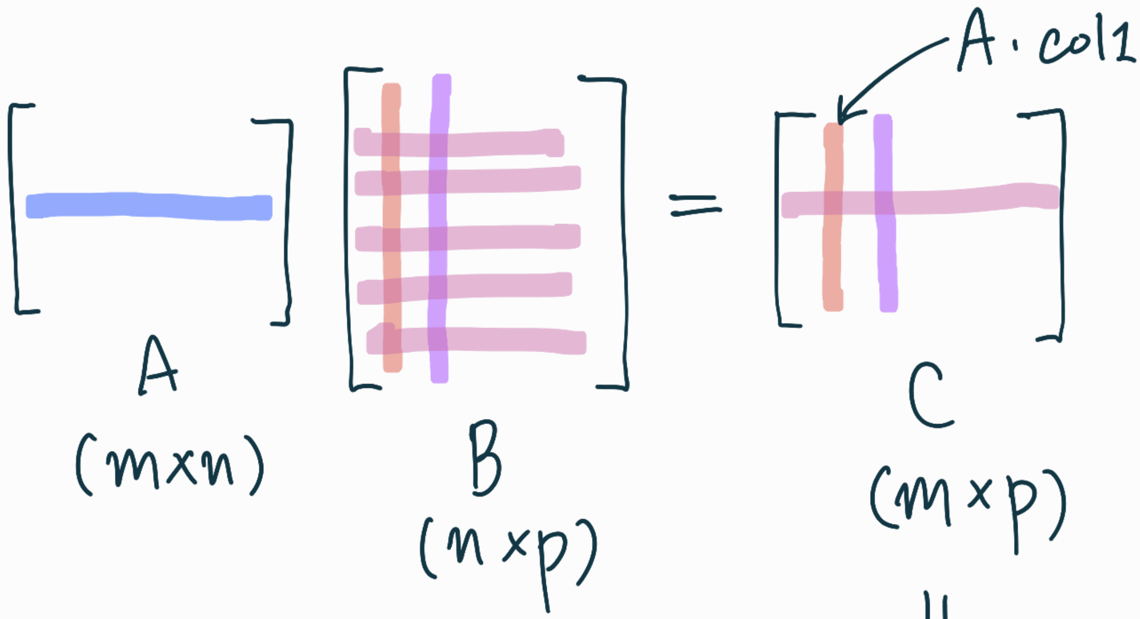


$A\ (m \times n)$ $\quad B\ (n \times p)$ $\quad C = AB\ (m \times p)$

$$C_{34} = (\text{row3 of } A) \times (\text{col4 of } B)$$

$$= a_{31} \cdot b_{14} + a_{32} \cdot b_{24} + \cdots = \sum_{k=1}^{n} a_{3k} b_{k4}$$



$\Downarrow$

* columns of C are combinations of columns of A

* rows of C are combinations of rows of B

column of A × row of B
$(m \times 1)$ $(1 \times p)$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 4 & 8 \\ 6 & 12 \end{bmatrix}$$

$*AB = \text{sum of } \left\{ (\text{Cols of } A) \times (\text{rows of } B) \right\}$

$$\begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \end{bmatrix} + \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} \begin{bmatrix} 3 & 4 \end{bmatrix}$$

Block Multiplication

$$\underset{A}{\left[\begin{array}{c|c} A_1 & A_2 \\ \hline A_3 & A_4 \end{array}\right]} \underset{B}{\left[\begin{array}{c|c} B_1 & B_2 \\ \hline B_3 & B_4 \end{array}\right]} = \left[\begin{array}{c|c} & \\ \hline & \end{array}\right]$$

$A_1B_1 + A_2B_3$

INVERSES (square)

$$A^{-1} A = I = AA^{-1}$$

if this exists... → $\boxed{\text{invertible, non-singular}}$

[singular / no inverse]

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$$

I can find a vector $x \neq 0$ with $Ax = 0$

$$\underset{A}{\begin{bmatrix} & \\ & \end{bmatrix}} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

[non-singular]

$$\underset{A}{\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}} \underset{A^{-1}}{\begin{bmatrix} a & c \\ b & d \end{bmatrix}} = \underset{I}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

$A \times$ column j of A^{-1} = column j of I

[Gauss-Jordan]

solve two equation at once

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\rightarrow \left[\begin{array}{cc:cc} 1 & 3 & 1 & 0 \\ 2 & 7 & 0 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{cc:cc} 1 & 3 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{array}\right]$$

$$\qquad A \qquad\quad I$$

$$\longrightarrow \left[\begin{array}{cc:cc} \boxed{\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}} & & \boxed{\begin{matrix} 7 & -3 \\ -2 & 1 \end{matrix}} \end{array}\right]$$

$$\qquad I \qquad\quad A^{-1}$$

$E's\,[A \;\; I] =$

$[I \,, \, ?]$

$$\begin{bmatrix} EA = I & E = A^{-1} \\ EI = ? & ? = E = A^{-1} \end{bmatrix}$$