⟨Elimination⟩

$$\begin{aligned} x+2y+z &= 2 \\ 3x+8y+z &= 12 \\ 4y+z &= 2 \end{aligned} \Rightarrow \boxed{Ax=b}$$

augmented column

First pivot

second pivot

$$\begin{bmatrix} \boxed{1} & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} \begin{matrix} 2 \\ 12 \\ 2 \end{matrix} \Rightarrow \begin{bmatrix} \boxed{1} & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & 4 & 1 \end{bmatrix} \begin{matrix} 2 \\ 6 \\ 2 \end{matrix}$$

"A"

*pivots can't be '0'

$$\Rightarrow \begin{bmatrix} \boxed{1} & 2 & 1 \\ 0 & \boxed{2} & -2 \\ 0 & 0 & \boxed{5} \end{bmatrix} \begin{matrix} 2 \\ 6 \\ -10 \end{matrix} \Rightarrow \begin{bmatrix} x+2y+z=2 \\ 2y-2z=6 \\ 5z=-10 \end{bmatrix}$$

"C"

third pivot

"U"

(upper-triangular)

$z=-2$

$y=1$

$x=2$

back substitution

$$\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = 3\cdot \text{col}1 + 4\,\text{col}2 + 5\,\text{col}3$$

matrix x column = column

$$\begin{bmatrix} 1 & 2 & 7 \end{bmatrix}\begin{bmatrix} - & - & - \\ - & - & - \\ - & - & - \end{bmatrix} = 1\cdot \text{row}1 + 2\,\text{row}2 + 7\,\text{row}3$$

row x matrix = row

using the idea above...

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} \Rightarrow$$ what subtracts $3\cdot$row1 from row2 ?

$$\begin{bmatrix} & ? & \end{bmatrix}\begin{bmatrix} 1 & 2 & 1 \\ 3 & 8 & 1 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix}$$

$\downarrow$

$\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ → take 1 of row 1, none of other rows

$$\begin{bmatrix} -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E_{21}

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} \Rightarrow \text{row3} - 2 \cdot \text{row2}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}}_{E_{32}} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & -2 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\boxed{E_{32}(E_{21}A) = U}$$

$$\Rightarrow (E_{32} \cdot E_{21})A = U$$

Associative Law

* Permutation (exchange rows)

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

"P"

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} b & a \\ d & c \end{bmatrix}$$

"P"

* Inverses

add subtract

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

"E" "I"

matrix that un-does what "E" does $\Rightarrow E^{-1}$