# Delegated cheap talk: a theory of investment banking

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## 1 Brief Introduction

We study a sender-receiver game of cheap messages in which the incentive of the sender (a seller) is orthoganal to that of the receiver (a buyer). Any direct talk from the seller to the buyer is useless. However, the seller can usefully inform the buyer by delegating the talk to an intermediary. He does so by offering a joint contract that aligns the intermediary's incentives partially to the seller and partially to the buyer.

## 2 Model

There are three players: Seller (S), Intermediary (I), and Buyer (B). The seller is an entrepreneur who owns a firm with an uncertain state $\omega \in \Omega = \{0, 1\}$ of either good ($\omega = 1$) or bad ($\omega = 0$). There is a publicly known, objective prior probability $p \in (0, 1)$ on the good state. The intermediary is an investment bank that can acquire private information about the firm's state and send a cheap message to the buyer. The buyer is a consortium of investors whose final action is $a \in A = \{0, 1\}$ for *investing* (or *buying*, $a = 1$) and *not investing* (or *not buying*, $a = 0$) in the seller's firm.

In Stage 1 (the "contracting stage"), the seller makes a joint take-it-or-leave-it offer of a *contract* $t = (t_0, t_1) \in \mathbb{R}^2$ to the intermediary and the buyer, where $t_\omega$ is the amount of transfer from the buyer to the intermediary when the buyer invests ($a = 1$) and the realized state is $\omega$. If either the intermediary or the buyer rejects the offer, the game ends, and all players get zero payoffs. If both accept the offer, the game moves to Stage 2.

In Stage 2 (the "due diligence stage"), the intermediary decides and publicly announces an *information structure* or a *due diligence structure*, a pair $(q, r) \in \mathcal{E} = \{(q, r) : 0 \leq q \leq p \leq r \leq 0\}$. The information structure determines a Bayes-plausible distribution of the

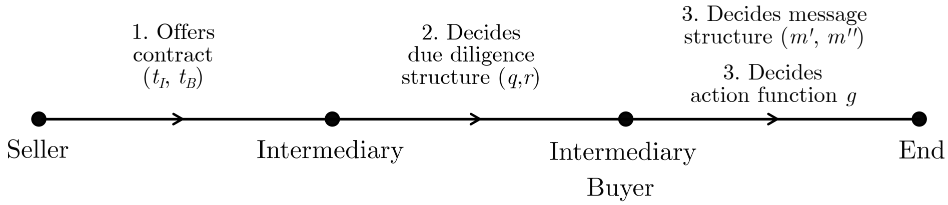


Figure 1: Summary of timing

intermediary's *type* or *posterior belief* $\widetilde{p}$ revealed privately to the intermediary at the game's end. If $q = p = r$, then $\widetilde{p} = p$. If $q < r$, then

$$\widetilde{p} = \begin{cases} q, & \text{with probability } 1 - f(q,r) \text{ and} \\ r, & \text{with probability } f(q,r), \end{cases} \tag{1}$$

where we define $f(q,r) = \frac{p-q}{r-q}$. We say that a due diligence structure $(q,r)$ is *uninformative* if $q = p = r$. Otherwise, it is *informative*. The interpretation of this stage is that the intermediary inspects the firm's business and operations before talking to the buyer.

Lastly, in Stage 3 (the "talking stage"), the intermediary and the buyer make a simultaneous move. The intermediary decides a *message structure* $(m', m'')$, where $m'$ and $m''$ are elements of a set $M$ of finitely many possible messages. The messages $m'$ and $m''$ correspond to those sent when the realized posterior belief is $q$ and $r$, respectively. It is required that if $q = r$, then $m' = m''$. Meanwhile, the buyer decides an *action function* $g : M \longrightarrow A$. It means that the buyer's final action is $a = g(m)$ when she receives a message $m$. The interpretation of this stage is that the intermediary's talk is *cheap*: it is costless, nonbinding, and unverifiable.

As the game ends, Nature draws the intermediary's type $\widetilde{p}$ from the probability distribution (1). If $\widetilde{p} = q$, the intermediary's message and the buyer's action are determined as $m'$ and $a = g(m')$. If $\widetilde{p} = r$, they are $m''$ and $a = g(m'')$. Finally, Nature draws the firm's state $\omega \in \Omega = \{good, bad\}$ with probabilty $\Pr(\omega = good) = \widetilde{p}$. Figure 1 shows a summary of the game's timing.

The seller's ex-post payoff is

$$v(a, \omega) = a. \tag{2}$$

The intermediary's ex-post payoff is

$$\pi_t(a, \omega, q, r) = a t_\omega - c(q, r), \tag{3}$$

where $c$ is the *informational cost function*. It is the expected reduction in entropy scaled by a constant $\lambda > 0$:

$$c(q,r) = \lambda h(p) - \lambda\Big[\big(1 - f(q,r)\big)h(q) + f(q,r)h(q)\Big], \tag{4}$$

where $h$ is the Shannon entropy function defined as

$$h(z) = \lim_{x \to z}\left[-(1-x)\log(1-x) - x\log x\right], \quad \text{for all } z \in [0,1]. \tag{5}$$

The buyer's ex-post payoff is

$$u_t(a,\omega) = \left[(1-\omega)\cdot(-1) + \omega \cdot R - t_\omega\right] a, \tag{6}$$

where $R \in (0, \frac{1}{p} - 1)$ is a constant interpreted as the rate of return to the buyer's investment. The requirement that $R < \frac{1}{p} - 1$, or equivalently $p < \frac{1}{1+R}$, means that the prior expected rate of return is negative, so the buyer is unwilling to invest *ex ante*.

The seller's strategy is a contract $t$. The intermediary's strategy is a pair of mappings: (i) one that assigns a due diligence structure $(q,r)$ to every contract $t \in \mathbb{R}^2$ and (ii) one that assigns a message structure $(m', m'')$ to every $\big(t,(q,r)\big) \in \mathbb{R}^2 \times \mathcal{E}$. The buyer's strategy is a mapping that assigns an action function $g$ to every $\big(t,(q,r)\big) \in \mathbb{R}^2 \times \mathcal{E}$. We analyze the pure-strategy subgame perfect Nash equilibria of this game.

# 3 Optimal design of investment banking contracts

## 3.1 Equilibrium in talking

Let a feasible contract $t$ and a due diligence structure $(q,r)$ be given, and consider the subgame in the talking stage (Stage 3). Let $T(s)$ denote the expected transfer from the buyer to the intermediary given the buyer's action $a = 1$ and the probability $s$ on the good state:

$$T(s) = (1-s)\cdot t_0 + s \cdot t_1. \tag{7}$$

Let $U(s)$ denote the expected payoff for the buyer given her action $a = 1$ and the probability $s$ on the good state. That is,

$$U(s) = (1-s)\cdot(-1) + s\cdot R - T(s). \tag{8}$$

It is straightforward to characterize the equilibria when the due diligence structure is uninformative. Suppose $q = p = r$. If $U(p) < 0$, then the only Nash equilibria of the subgame are $\big((m^*, m^*), g^*\big)$ where $m^*$ is any message in $M$ and $g^*(m) = 0$ for all $m \in M$. Conversely, if $U(p) > 0$, then the only Nash equilibria of the subgame are $\big((m^*, m^*), g^*\big)$ where $m^*$ is any element of $M$ and $g^*(m) = 1$ for all $m \in M$. If $U(p) = 0$, then a strategy profile $\big((m^*, m^*), g^*\big)$ is an equilibrium if and only if $g^*(m^*)T(p) \geq g^*(m)T(p)$ for all messages $m \in M$.

The equilibria are more interesting when the due diligence structure is informative, when $q < r$. They depend on how the interemdiary's incentives relate to the buyer's.

**Definition 1.** The intermediary's incentives are *aligned* with the buyer's if, for all $s \in \{q, r\}$,

1. $T(s) \leq 0$ and $U(s) \leq 0$, or
2. $T(s) \geq 0$ and $U(s) \geq 0$.

The intermediary's incentives are *biased against investing* if $T(s) \leq 0$ for all $s \in \{q, r\}$ with strict inequality for some $s \in \{q, r\}$. They are *biased toward investing* if $T(s) \geq 0$ for all $s \in \{q, r\}$ with strict inequality for some $s \in \{q, r\}$. They are *biased* if they are either biased against investing or biased toward investing. They are *balanced* if they are not biased.

The following lemma characterizes the Nash equilibria of the talking-stage subgame when $q < r$.

**Lemma 1.** *Suppose $q < r$, and let a strategy profile $\big((m^q, m^r), g\big)$ be given. Let $M_0 = \{m \in M : g(m) = 0\}$ and $M_1 = \{m \in M : g(m) = 1\}$.*

1. *(Pooled actions) Suppose either $M_0$ or $M_1$ is empty. Then $\big((m^q, m^r), g\big)$ is an equilibrium if and only if it satisfies one of (a)–(d):*
   - *(a) $m^q = m^r$, $M_0 = M$ and $U(p) \leq 0$,*
   - *(b) $m^q = m^r$, $M_1 = M$ and $U(p) \geq 0$,*
   - *(c) $m^q \neq m^r$, $M_0 = M$ and $U(s) \leq 0$ for all $s \in \{q, r\}$, or*
   - *(d) $m^q \neq m^r$, $M_1 = M$ and $U(s) \geq 0$ for all $s \in \{q, r\}$.*
2. *(Separated actions) Suppose both $M_0$ and $M_1$ are nonempty. Then $\big((m^q, m^r), g\big)$ is an equilibrium if and only if it satisfies one of (a)–(c):*
   - *(a) The intermediary's incentives are biased against investing. For all $s \in \{q, r\}$, $m^s \in M_0$ and $U(s) \leq 0$.*
   - *(b) The intermediary's incentives are biased toward investing. For all $s \in \{q, r\}$, $m^s \in M_1$ and $U(s) \geq 0$.*

(c) *The intermediary's incentives are balanced and aligned with the buyer's. For all $s \in \{q, r\}$, $m^s \in M_0$ if $U(s) \leq 0$ and $m^s \in M_1$ if $U(s) \geq 0$.*

**Proof of Lemma 1.** [to be completed] ■

Lemma 1 shows that, while there always exists an equilibrium with meaningless talk (or *babbling*), talk can sometimes be meaningful and even useful in equilibrium. The cases 1(a) and 1(b) of Lemma 1 show *meaningless* talk: the equilibria consist of the intermediary sending the same message regardless of his private information and the buyer not listening. In the cases 1(c), 1(d), 2(a), and 2(b), the talk can be *meaningful* as the intermediary's message possibly reveals his private information. Still, the talk is *uninfluential* because the buyer does not act differently based on the information. Only in the case 2(c)—when the agents' incentives are balanced and aligned—the talk is meaningful and influential. The following definition and corollary summarize this result.

**Definition 2.** Consider an equilibrium $((m', m''), g)$ of the talking-stage subgame. Its message structure $(m', m'')$ is *meaningful* if $m' \neq m''$ and *meaningless* otherwise. It is *influential* if $g(m') \neq g(m'')$ and *uninfluential* otherwise.

**Corollary 1.** *There exists an equilibrium with influential talk in the talking-stage subgame if and only if the intermediary's incentives are balanced and aligned with the buyer's.*

**Proof of Corollary 1.** [to be completed] ■

Our next result says that all equilibria with useful talk lead to the same *outcome* (that is, the pair of buyer's actions when the realized posterior belief $\widetilde{p}$ is $q$ and $r$, respectively). Similarly, all equilibria with useless talk lead to the same outcome.

**Corollary 2.** *Let two equilibria $e_1 = ((m_1', m_1''), g_1)$ and $e_2 = ((m_2', m_2''), g_2)$ of the talking-stage subgame be given. If $(m_1', m_1'')$ and $(m_2', m_2'')$ are both useful or both useless, then $g(m_1') = g(m_2')$ and $g(m_1'') = g(m_2'')$.*

**Proof of Corollary 2.** [to be completed] ■

In conclusion, we see that in any talking-stage subgame given a contract $t$ and a due diligence structure $(q, r)$, there are at least one distinct equilibrium outcome (uninfluential talk) and at most two distinct equilibrium outcomes (uninfluential talk and influential talk). It is easy to see that the outcome with influential talk weakly Pareto-superior to the outcome with uninfluential talk. In the following sections, therefore, we focus on an equilibrium of this subgame with a Pareto-efficient talk: one with influential talk when it exists and one with uninfluential talk if otherwise.

## 3.2 Optimal information acquisition

Let a feasible contract $t$ be given. Define $\big((m'(q,r), m''(q,r)), g^*(q,r)\big)$ as a Pareto-efficient talking-stage equilibrium for the contract $t$ when the information structure is $(q,r)$. Let $a'(q,r) = g^*(q,r)(m'(q,r))$ and $a''(q,r) = g^*(q,r)(m''(q,r))$. As before, define $T(s) = (1-s)t_0 + st_1$.

**Definition 3.** The intermediary's *expected profit* given a due diligence structure $(q,r)$ is

$$\Pi(q,r) = (1-f(q,r))a'(q,r)T(q) + f(q,r)a''(q,r)T(r) - c(q,r). \tag{9}$$

A due diligence structure $(q^*, r^*)$ is *optimal* if it maximizes $\Pi(q,r)$ subject to the *incentive-alignment constraint*: the investor's incentives are balanced and aligned with the buyer's whenever $q < r$.

**Lemma 2.** *There exists a unique optimal due diligence structure $(q,r)$ for every contract $t \in \mathbb{R}^2$. If $q < r$,*

$$\lambda\left[\frac{dh(q)}{dq} - \frac{h(r)-h(q)}{r-q}\right] \geq \frac{(1-r)t_0 + rt_1}{r-q} \tag{10}$$

$$\lambda\left[\frac{h(r)-h(q)}{r-q} - \frac{dh(r)}{dr}\right] \geq (t_1 - t_0) - \frac{(1-r)t_0 + rt_1}{r-q}, \tag{11}$$

$$q \leq \min\left\{\frac{1+t_0}{1+R-t_1+t_0}, \frac{-t_0}{t_1-t_0}\right\}, \text{ and} \tag{12}$$

$$r \geq \max\left\{\frac{1+t_0}{1+R-t_1+t_0}, \frac{-t_0}{t_1-t_0}\right\}, \tag{13}$$

*where one of (10) and (12) hold with equality and one of (11) and (13) hold with equality.*

**Proof of Lemma 2.** [to be completed]

## 3.3 Optimal contracts

Equations (10) and (11) are the *incentive-compatibility constraints* for the intermediary's choice of the due diligence structure. Equations (12) and (13) are the *incentive-alignment* *constraints* for the talking-stage subgame. The *intermediary's participation consraint* is

$$\Pi_t(q,r) \geq 0. \tag{14}$$

The *buyer's participation consraint* is

$$(1 - f(q,r))\, U_t(q) + f(q,r) U_t(r) \geq 0 \tag{15}$$

**Definition 4.** A contract $t^*$ is *(seller-)optimal* if there exists a due diligence structure $(q^*, r^*)$ such that

$$(t^*, (q^*, r^*)) \in \underset{(t,(q,r))}{\operatorname{argmax}}\, f(q,r) \tag{16}$$

subject to the incentive-compatibility constraints (10)–(11), the incentive-alignment constraints (12)–(13), and the participation constraints (14)–(15).

**Lemma 3.** *There exists a unique* $(q^*, r^*)$ *such that*

$$(q^*, r^*) = \underset{(q,r)\in\mathcal{E}}{\operatorname{argmax}}\, f(q,r), \tag{17}$$

*subject to* $r \geq \frac{1}{1+R}$ *and* $\lambda\left[(r-q)h'(q) - h(r) + h(q)\right] = (1-r)(-1) + rR.$

**Proof of Lemma 3.** [to be completed]

**Theorem 1.** *Let* $(q^*, r^*)$ *be the unique solution of (17). There exists* $\tau \in (0, R)$ *such that a contract* $t = (t_0, t_1)$ *is optimal if and only if it satisfies* $t_0 \in [-1, 0)$, $t_1 \in (0, \tau]$, *and*

$$t_1 = R - \left(\frac{1}{r^*} - 1\right)(1 + t_0)\,. \tag{18}$$

**Proof of Theorem 1.** [to be completed]

Theorem 1 means that any optimal contract has a due diligence structure with a binding incentive-alignment constraint, specifically, on the high posterior belief. The interpretation of this result is that a seller-optimal contract distorts the intermediary's incentives in favor of the seller to a maximal degree while (1) maintaining the intermediary's talk influential to the buyer and (2) respecting the intermediary's incentive compatibility.

## Notes

$$f(q,r) = \frac{p-q}{r-q} \tag{19}$$

$$1 - f(q,r) = \frac{r-p}{r-q} \tag{20}$$

$$f_q(q,r) = -\frac{r-p}{(r-q)^2} \tag{21}$$

$$f_r(q,r) = -\frac{p-q}{(r-q)^2} \tag{22}$$

$$c(q,r) = \lambda h(p) - \lambda\big(1 - f(q,r))h(q) - \lambda f(q,r)h(r) \tag{23}$$

$$c_q(q,r) = \lambda\left[f_q(q,r)\big(h(q) - h(r)\big) - \big(1 - f(q,r)\big)h'(q)\right] \tag{24}$$

$$c_r(q,r) = \lambda\left[f_r(q,r)\big(h(q) - h(r)\big) - f(q,r)h'(r)\right] \tag{25}$$

$$H(q,r) = \frac{h(r) - h(q)}{r-q} \tag{26}$$

$$H_q(q,r) = \frac{-h'(q)}{(r-q)^2} \tag{27}$$

$$H_r(q,r) = \frac{-h'(r)}{(r-q)^2} \tag{28}$$

$$h'(x) = \log(1-x) - \log x \tag{29}$$

$$h''(x) = -\frac{1}{1-x} - \frac{1}{x} \tag{30}$$