Delegated Cheap Talk: A Theory of Investment Banking

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Abstract

We study the role of investment banking as delegated cheap talk. Two agents in our model have conflicting interests: a seller simply wants to sell his business, whereas a buyer wants to invest in a good business. We find that no direct talk between the two is informative. However, the seller can influence the buyer by contracting with an intermediary and delegating the communication. Any successful contract requires the intermediary to share the risk of loss with the buyer. A seller-optimal contract maximizes the intermediary's bias in favor of the seller while maintaining minimal alignment with the buyer's incentives.

Keywords: Investment banking, Cheap talk, Costly information acquisition, Conflict of interest
JEL Codes: D83, G24

1 Introduction

1.1 Motivation and results

When Mark Zuckerberg announced his plan to sell the shares of Facebook to the public in 2012, he chose Morgan Stanley as the company's investment bank. That is, he chose the bank as the lead underwriter that would help manage Facebook's initial public offering (IPO). Among the underwriter's most crucial responsibilities were (a) *due diligence*—inspecting the company's business and finances, and (b) *roadshow*—meeting large institutional investors to

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convince them to buy the shares. For these services, Morgan Stanley earned 1.1 percent of the total 16 billion US dollars raised. More typical, moderate-sized IPOs in the United States usually pay investment banks even more hefty fraction, of 7 percent (Chen and Ritter, 2000; Ritter, 2023). The standard explanation behind such large compensation is that investment banks reduce the information asymmetry between the seller and the buyer through expertise (Baron and Holmström, 1980; Baron, 1982; Ramakrishnan and Thakor, 1984; Biais, Bossaerts, and Rochet, 2002) or reputation (Beaty and Ritter, 1986; Booth and Smith II, 1986; Carter and Manaster, 1990; Chemmanur and Fulghieri, 1994).

Yet this view of investment banks as reputable experts seem largely at odds with what the general public thinks, what the popular media portray, and even what the investment bankers say about themselves. For example, the protesters of the left-wing populist movement in 2011, Occupy Wall Street, cried "We are the 99%," referring to investment banks as the undeservingly rich one percent that caused the 2008 financial crisis. Although with much exaggeration, movies such as *Wall Street* (1987), *Margin Call* (2011), and *The Wolf of Wall Street* (2013) depict investment bankers as sleazy, smooth-talking salesmen. In their memoir *Monkey Business: Swinging through the Wall Street jungle*, the former investment bankers Rolfe and Troob (2009) write:

> [Investment bankers] only want to say good things. The better they can make the company sound, the easier it will be for them to sell the securities. The easier it is for them to sell the securities, the more certain they'll be that the clients will be happy. That means fees. Fees are important.

Motivated by the stark contrast between the academic and popular views, this paper studies whether investment banks can be valuable even when they have neither expertise nor reputation, by being *delegated talkers*. Our model has one contract designer and two players: a seller (entrepreneur) owning a company with an uncertain future value, an intermediary (investment bank), and a buyer (investor) initially unwilling to invest in the firm. It captures the key stages of a typical IPO, in which the investment bank conducts due diligence, generates private information about the firm, and communicates with potential investors. First, in the *contracting stage*, the seller designs a contract that specifies state-contingent transfers from the buyer to the intermediary. Next, in the *due diligence stage*, the intermediary publicly announces its due diligence structure, namely, the structure of its costly information acquisition on the company's uncertain state. She then observes a private signal generated from that information structure. Finally, in the *roadshow stage*, the intermediary talks to the buyer who then decides whether to invest in the company or not. Importantly, the talk is *cheap*; that is, the intermediary's message is costless, non-binding, and unverifiable. The players' preferences are such that the seller simply wants to sell his firm, the intermediary wants to maximize her profit, and the buyer wants to invest only if his expected return is large enough.

In this setting, we show that a contract that implements a positive success rate necessarily requires that the intermediary shares the risk of loss with the buyer (Theorem 1). Namely, when the buyer experiences losses, the intermediary also incurs losses; when the buyer benefits from gains, the intermediary also benefits. The intuition is simple. For the intermediary's communication to be credible to the buyer, the incentives of the intermediary and the buyer must be at least somewhat aligned. The result is similar to one in the canonical cheap talk game of Crawford and Sobel (1982), in which the conflict of interest between the sender and the receiver must be small enough for their communication to be informative. Our prediction is also consistent with a well-known observed phenomenon: IPOs in the United States are predominantly under "firm commitment" contracts—in which the investment bank takes on the risk of buying and reselling all of the shares—rather than "best-efforts" contracts, in which the investment bank receives a fixed fee and does not face such a risk.[1]

Building on the previous observation, we characterize the seller-optimal contract: an investment banking contract is seller-optimal if and only if it is incentive-compatible with a due diligence structure that maximizes the success rate subject to a binding *incentive alignment* constraint (Theorem 2). The incentive alignment constraint is a requirement that the intermediary and the buyer agree on the preferred action in all realizations of the intermediary's due diligence. In short, it requires that the intermediary has no incentive to deceive the buyer. Therefore, this characterization means that a seller-optimal contract maximizes the intermediary's bias in favor of the seller while maintaining minimal alignment with the buyer's incentives. Intuitively, if the intermediary's incentives are too distorted in favor of the seller, the buyer would stop believing in the intermediary's message. This result potentially explains why entrepreneurs (sellers) are often willing to pay substantial fees—about 7 percent of the capital raised—to investment banks: by closely aligning the banks' incentives with their own, they increase the likelihood of a successful sale.

Taken together, our results imply that the delegated communication can make the seller strictly better off relative to two benchmarks: (1) when the seller himself takes the investment bank's role and (2) when the buyer assumes that role. First, when the seller serves as his own investment bank, his incentives are to recommend the buyer to invest regardless of his private information. Consequently, even if the seller has greater expertise about his own firm than anyone else, his talk cannot influence the seller. An important precondition for this result, however, is that the seller has limited liability, meaning that the seller cannot commit

[1]For further information on the different types of IPO contracts and the prevalence of firm commitment contracts, see Ritter (2003) and Eckbo, Masulis, and Norli (2007).

to making state-contingent transfers. Second, when the buyer serves as the investment bank, he acquires the information about the firm more objectively than desired by the seller. In contrast to these benchmarks, using the intermediary allows the seller to design the intermediary's incentives to maximize the success rate while keeping the intermediary's talk credible and informative.

1.2 Related literature

To the best of our knowledge, our paper is the first study to systematically investigate the role of investment banks as delegated cheap talkers and derive the conditions under which their communication is informative and optimal. It bridges two distinct branches of existing literature, on financial intermediation and strategic communication. First, our results contribute to the theory of financial intermediation by isolating the value of investment banks as delegated talkers. Second, we extend the theory of strategic communication by endogenizing the conflict of interest between the sender and the receiver.

Investment banking and financial intermediation. Our paper revisits the mature literature on the role of investment banks as information producers that reduce the information asymmetry between the seller and the buyer. A common assumption in the literature is that investment banks have *expertise*, acquiring information more easily than others. For example, in Baron (1982) and Biais, Bossaerts, and Rochet (2002), the investment bank is *ex ante* better informed about the firm's market value than the seller. In Baron and Holmström (1980), the seller and the intermediary have the same prior information about the firm but only the latter observes an additional signal. In Ramakrishnan and Thakor (1984), the intermediary collects information at a lower cost than the buyer by forming a coalition.

Another frequent assumption in the literature is that investment banks have *reputation*—loosely defined—which makes them credible sources of information. For example, Beaty and Ritter (1986) and Booth and Smith II (1986) argue that the intermediary facing a repeated game maintains its reputation of being truthful and thus can influence the buyer. Carter and Manaster (1990) directly assume that more reputable intermediary is better able to assess the seller's firm. In Chemmanur and Fulghieri (1994), the intermediary has private information on its cost of information acquisition, inducing it to evaluate the seller's firm sufficiently accurately to gain the reputation of credibility. Empirically, in their survey on the IPO literature, Ritter and Welch (2002) argue that neither expertise nor reputation of investment banks are primary driver of their observed phenomena, although Fang (2005) and Brau and Fawcett (2006) argue otherwise. More recent works continue to broadly conform to either views (Eckbo, Masulis, and Norli, 2007; Ljungqvist, 2007; Ragupathy, 2011; Lee and Masulis, 2011; Katti and Phani, 2016; Lowry et al., 2017). In response to this literature, our work appeals neither to these aspects yet show that the intermediary can emerge as an information producer through an optimal contract.

More broadly, our paper contributes to the theory of financial intermediation, on the principal-agent relationship between financial intermediaries (agents) and their clients (principals). A canonical view in the literature[2] is that of Diamond (1984). He argues that financial intermediaries serve as *delegated monitors*, pooling deposits from many customers and making loans to entrepreneurs as it is too costly for individual depositors to monitor the loans. This view thus focuses on the relationship between the intermediary and the investor (depositor). In contrast, we focus on the relationship between the intermediary and the seller (entrepreneur), and argue that, to entrepreneurs, intermediaries serve as *delegated talkers*.

Strategic communication. Our paper augments the existing theories of strategic communication in Sender-Receiver games by endogenizing both the information acquisition and the conflict of interest. In the classic paper by Crawford and Sobel (1982), the sender and the receiver have partially aligned preferences over the receiver's action, which enables the sender's cheap talk to be informative. In their work, however, the sender's incentives and information are exogenously given.

Many subsequent studies thus add endogenous information acquisition to the classic cheap talk game. For example, in Austen-Smith (1994), the receiver knows neither the sender's cost of information nor his binary decision of acquiring information. The paper shows that this uncertainty improves the seller's credibility. Pei (2015) is another example. Using a more general set of information structures and a monotonic information cost function, he finds that the sender always fully reveals the acquired information. In Argenziano, Severinov, and Squintani (2016), the sender collects costly information through multiple Bernoulli trials before sending a cheap message to the receiver. They find that the sender collects more information than the receiver himself would, to avoid being ignored. In Deimen and Szalay (2019), the sender collects information about the ideal actions of both the sender and the receiver before communicating with the receiver. They show that the sender chooses to collect less information about his own state to make his talk more credible to the receiver. Most recently, Lyu and Suen (2022) and Kreutzkamp (2022) let the sender collects information before talking to the receiver, and study the seller's optimal experiment from the space of general information structures.

[2]Bhattacharya and Thakor (1993) and Thakor (2020) provide comprehensive surveys of the existing theories.

In our knowledge, only one existing paper goes a step further and lets both the sender's information acquisition and his conflict of interest be determined in the model.[3] In Ivanov (2010), there are (a) a privately informed, biased expert, (b) an intermediary (sender), and (c) an unbiased principal (receiver). The principal first decides the bias of the intermediary. Next, the expert talks to the intermediary, who then talks to the principal. The paper shows that the principal should set the intermediary's bias in the opposite direction of the expert's, so that, roughly speaking, the biases offset each other. In comparison, whereas Ivanov (2010) lets the receiver determine the intermediary's bias, we let the seller make that decision. The result is that the seller-optimal contract makes the intermediary as biased as possible in favor of the seller while maintaining its talk credible.

Finally, our work is related to a burgeoning strand of literature on mediated communication and persuasion. On the one hand, models of mediated communication, such as those by Goltsman et al. (2009), Ganguly and Ray (2012), and Ambrus, Azevedo, and Kamada (2013), involve three or more agents with exogenous conflicts of interest engaging in cheap talk sequentially. In these cases, intermediaries typically do not improve on the direct communication between the initial sender and the final receiver unless the conflict of interest between the two are already sufficiently small. On the other hand, models of mediated persuasion, such as those by Arieli, Babichenko, and Sandomirskiy (2022) and Zapechelnyuk (2022), grant the initial sender the commitment power to truthfully report the results of their experiments, leaving little room for an improvement by an intermediary. In contrast with these two extreme modes of mediation, the intermediary in our model emerges successful despite the large conflict of interest between the seller and the buyer. This success is due to the fact that our seller, who cannot credibly communicate directly, can instead make the mediator credible by strategically designing the mediator's incentives to align sufficiently with the buyer's.

1.3 Outline

We organize the rest of our paper as follows. In Section 2, we define the model, the equilibrium, and the seller-optimal contract. In Section 3, we derive a necessary condition for any contract with a positive success rate. In Section 4, we characterize the seller-optimal contract. In Section 5, we discuss the seller's optimal outcome compared to those without an intermediary, concluding the paper.

[3]Notably, Antic and Persico (2020) allows endogenous conflict of interest but not endogenous information acquisition.

2 Model

There are Seller (S), Intermediary (I), and Buyer (B). The seller (he) is an entrepreneur whose firm has an uncertain state $\omega \in \Omega = \{0, 1\}$ of either good ($\omega = 1$) or bad ($\omega = 0$). There is a publicly known, objective prior probability $\pi(\omega) \in (0, 1)$ for each state $\omega \in \Omega$, namely, $\pi(0) = 1 - p$ and $\pi(1) = p$. The intermediary (she) is an investment bank that can acquire private information about the firm's state and send a cheap message to the buyer. The buyer (he) is a consortium of investors whose final action is $a \in A = \{0, 1\}$ for *investing* (or *buying*, $a = 1$) and *not investing* (or *not buying*, $a = 0$) in the seller's firm with a fixed amount $\kappa \in (p, 1)$.

The seller designs a mechanism between the intermediary and the buyer through the following arrangement. He first makes a take-it-or-leave-it offer of a *contract* $t = (t_0, t_1) \in \mathbb{R}^2$ to the intermediary, where t_ω is the amount of transfer from the buyer to the intermediary when the buyer invests ($a = 1$) and the realized state is ω. If the intermediary rejects the offer, all agents receive zero payoffs. If he accepts the offer, the intermediary and the buyer enter a two-player game called the *initial public offering (IPO) mechanism.*

The IPO mechanism consists of two stages that involve only the intermediary and the buyer. In Stage 1 (the "due diligence stage"), the intermediary publicly chooses an *information structure* or a *due diligence structure*, a map $\sigma : \Omega \longrightarrow \Delta(S)$ where $S = \{s_0, s_1, \ldots, s_{J-1}\}$ is a set of signals. We write $\sigma(s|\omega)$ to represent the conditional probability of signal s given the state ω, and write the set of all due diligence structures as $\Sigma = (\Delta(S))^\Omega$. After choosing σ, the intermediary privately observes a realized signal $s \in S$ from the probability distribution $\sigma(\cdot|\omega)$. Consequently, the probability of receiving a signal s under the due diligence structure σ given a prior belief p is

$$\varphi(s|\sigma) = \sum_{\omega \in \Omega} \sigma(s|\omega)\pi(\omega) = (1 - p)\sigma(s|0) + p\sigma(s|1). \tag{1}$$

By Bayes' rule and consistency with the prior beliefs, the intermediary's private posterior belief on a state ω given a due diligence structure σ and a realized signal $s \in S$ is $\pi(\cdot|s, \sigma)$ such that, for all $\omega \in \Omega$,

$$\pi(\omega|s, \sigma) = \begin{cases} \frac{\sigma(s|\omega)\pi(\omega)}{\varphi(s|\sigma)} & \text{if } \varphi(s|\sigma) > 0, \text{ and} \\ \pi(\omega) & \text{if } \varphi(s|\sigma) = 0. \end{cases} \tag{2}$$

For all j, we say that the due diligence structure σ *induces* a posterior q_j with probability $\varphi(s_j|\sigma)$ if $q_j = \pi(1|s_j, \sigma)$.

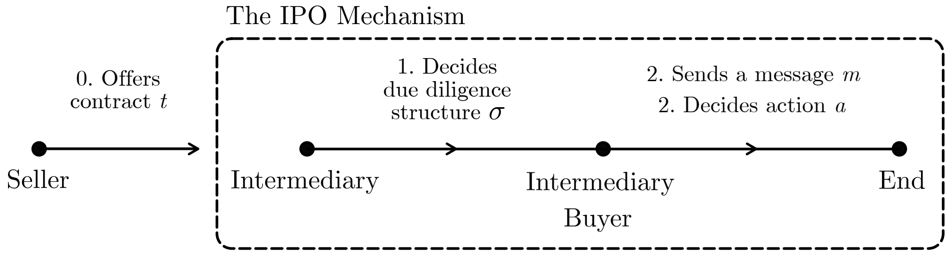

Figure 1: Summary of timing

In Stage 2 (the "roadshow stage"), the intermediary sends a message $m \in M = S$ to the buyer.[4] The message is cheap: it is (a) costless, (b) non-verifiable, and (c) non-binding. After receiving the message, the buyer chooses an action $a \in \{0, 1\}$. Finally, the state ω is publicly revealed. Figure 1 shows the summary of the game's timing.

After the IPO mechanism ends, the seller's payoff is

$$u^S(\omega, a) = a. \tag{3}$$

The intermediary's payoff is

$$u^I_{t,\sigma}(\omega, a) = at_\omega - c(\sigma), \tag{4}$$

where at_ω is the intermediary's revenue and $c : \Sigma \longrightarrow \mathbb{R}$ is the *information cost function.* The information cost function is the expected reduction in Shannon entropy, scaled by a constant $\lambda > 0$:

$$c(\sigma) = \lambda \left[H(\pi) - \sum_{s \in S} H(\pi|s, \sigma)\varphi(s|\sigma) \right], \tag{5}$$

where $H(\cdot)$ and $H(\cdot|s, \sigma)$ are functionals such that

$$H(\omega) = \sum_{\omega \in \Omega} \pi(\omega) \log(\pi(\omega)), \text{ and} \tag{6}$$

$$H(\omega|s, \sigma) = \sum_{\omega \in \Omega} \pi(\omega|s, \sigma) \log(\pi(\omega|s, \sigma)). \tag{7}$$

The buyer's payoff is

$$u^B_t(\omega, a) = (-\kappa + \omega - t_\omega)a. \tag{8}$$

Since $\kappa \in (p, 1)$, such payoff means that the buyer is unwilling to invest *ex ante* under a null contract $t = (0, 0)$. This assumption lets us focus on the interesting case in which communication can potentially improve the outcome.

[4]Letting $S = M$ is without loss of generality because, if $S \neq M$, we can redefine the signal space S' and message space M' as $S' = M' = S \cup M$.

In the IPO mechanism, the intermediary's strategy is $(\sigma, \boldsymbol{\mu})$, where $\boldsymbol{\mu} = \{\mu_\sigma\}_{\sigma \in \Sigma}$ is a collection of *message rules* $\mu_\sigma : S \longrightarrow \Delta(M)$ that assigns each signal to a probability distribution over messages. We write $\mu_\sigma(m|s)$ to represent the conditional probability of the message m given the signal s, under the due diligence structure σ. The buyer's strategy is $\boldsymbol{\alpha} = \{\alpha_\sigma\}_{\sigma \in \Sigma}$, a collection of *action rules* $\alpha_\sigma : M \longrightarrow A$. A *strategy profile* of the IPO mechanism is the triple $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$.

Define functionals U^S, U^I_t, and U^B_t, such that

$$U^S(\sigma, \mu, \alpha) = \sum_{\omega \in \Omega} \sum_{s \in S} \sum_{m \in M} u^S(\omega, \alpha(m)) \mu(m|s) \sigma(s|\omega) \pi(\omega), \tag{9}$$

$$U^I_t(\sigma, \mu, \alpha) = \sum_{\omega \in \Omega} \sum_{s \in S} \sum_{m \in M} u^I_{t,\sigma}\big(\omega, \alpha(m)\big) \mu(m|s) \sigma(s|\omega) \pi(\omega), \text{ and} \tag{10}$$

$$U^B_t(\sigma, \mu, \alpha) = \sum_{\omega \in \Omega} \sum_{s \in S} \sum_{m \in M} u^B_t\big(\omega, \alpha(m)\big) \mu(m|s) \sigma(s|\omega) \pi(\omega), \tag{11}$$

for every due diligence structure σ, message rule μ, and action rule α. The interpretation is that $U^S(\sigma, \mu, \alpha)$, $U^I_t(\sigma, \mu, \alpha)$, and $U^B_t(\sigma, \mu, \alpha)$ are the expected payoffs of the seller, the intermediary, and the buyer, where the expectation is taken over all realizations of the states $\omega \in \Omega$, the signals $s \in S$, and the messages $m \in M$.

Definition 1 (Equilibrium)**.** A strategy profile $(\sigma^*, \boldsymbol{\mu}^*, \boldsymbol{\alpha}^*)$ is an *equilibrium* of the IPO mechanism under a contract $t = (t_0, t_1)$ if it satisfies the following conditions (a)–(c).
(a) ($\boldsymbol{\mu}^*$ and $\boldsymbol{\alpha}^*$ are mutual best responses) For every $\sigma \in \Sigma$, $\mu : S \longrightarrow \Delta(M)$, and $\alpha : M \longrightarrow A$,

$$U^I_t(\sigma, \mu^*_\sigma, \alpha^*_\sigma) \geq U^I_t(\sigma, \mu, \alpha^*_\sigma) \quad \text{and} \quad U^B_t(\sigma, \mu^*_\sigma, \alpha^*_\sigma) \geq U^B_t(\sigma, \mu^*_\sigma, \alpha). \tag{12}$$

(b) ($(\boldsymbol{\mu}^*, \boldsymbol{\alpha}^*)$ is not Pareto-dominated) There exists no pair $(\boldsymbol{\mu}, \boldsymbol{\alpha})$ such that the pair satisfies the condition (a) and, for some $\sigma \in \Sigma$,

$$U^I_t(\sigma, \mu^*_\sigma, \alpha^*_\sigma) \leq U^I_t(\sigma, \mu_\sigma, \alpha_\sigma) \quad \text{and} \quad U^B_t(\sigma, \mu^*_\sigma, \alpha^*_\sigma) \leq U^B_t(\sigma, \mu_\sigma, \alpha_\sigma), \tag{13}$$

with at least one strict inequality.
(c) (σ^* is a best response of $(\boldsymbol{\mu}^*, \boldsymbol{\alpha}^*)$) For every $\sigma \in \Sigma$,

$$U^I_t(\sigma^*, \mu^*_{\sigma^*}, \alpha^*_{\sigma^*}) \geq U^I_t(\sigma, \mu^*_\sigma, \alpha^*_\sigma), \text{ and} \tag{14}$$

$$U^I_t(\sigma^*, \mu^*_{\sigma^*}, \alpha^*_{\sigma^*}) \geq 0. \tag{15}$$

We note two points about this definition. First, the condition (b) demands that the

Nash equilibria of the talking stage (Stage 2) subgame be Pareto-efficient. That is, we only consider cases in which the intermediary and buyer communicate efficiently in the talking stage and rule out, for example, babbling equilibria whenever there exists a mutually beneficial, informative one. This requirement shrinks the set of implementable outcomes to reasonable ones, thus preventing the designer from being too powerful.[5] Second, the condition (c) includes both the incentive compatibility and individual rationality constraints (14)–(15) for the intermediary in Stage 1.

We are ready to define a *seller-optimal investment banking contract* or simply *seller-optimal contract*, whose characterization is a main aim of our study.

Definition 2 (Seller-optimal contract)**.** Suppose $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$ is an equilibrium under a contract $t = (t_0, t_1)$. Suppose $\rho = U^S(\sigma, \mu_\sigma, \alpha_\sigma)$. Then the contract t *implements* a success rate ρ with $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$. A contract $t^* = (t_0^*, t_1^*)$ is *seller-optimal* (or *optimal*) if it implements a success rate ρ^* with some equilibrium $(\sigma^*, \boldsymbol{\mu}^*, \boldsymbol{\alpha}^*)$ and there exists no contract that implements some $\rho' > \rho^*$ with any equilibrium.

3 Successful investment banking contracts

This section derives the properties of investment banking contracts required for successful outcomes to the seller. We say that a contract is *successful* if it implements a positive success rate in an equilibrium. In any successful contract, the intermediary necessarily shares the risk of loss with the buyer. More precisely, we state and prove the following result.

Theorem 1. *Suppose a contract $t = (t_0, t_1)$ implements a success rate $\rho > 0$ with an equilibrium. Then $t_0 \in [-\kappa, 0)$ and $t_1 \in (0, 1-\kappa]$.*

This result restricts the intermediary's and the buyer's payoffs to have the same signs for any successful contract, as the buyer's payoff is $-\kappa - t_0 \leq 0$ in the bad state and $1-\kappa-t_1 \geq 0$ in the good state. This restriction implies that a "best-efforts" contract—compensating the intermediary with a fixed transfer ($t_0 = t_1$)—cannot be successful. Rather, a successful contract must make the intermediary incur losses (gains) whenever the buyer incurs losses (gains). Figure 2 illustrates a typical contract satisfying this requirement. The two ends of the dashed line segment indicate the sum of the intermediary's and the buyer's payoffs in the bad and good states, respectively. The left ends of the solid line segments, indicating the payoffs in the bad state, are below the x-axis. The right ends of the solid line segments, indicating the payoffs in the good state, are above the x-axis.

[5] For example, without this requirement, the seller can implement any due diligence structure σ as long as it satisfies the participation constraint (15), by choosing the uninformative outcome for every off-path equilibrium.

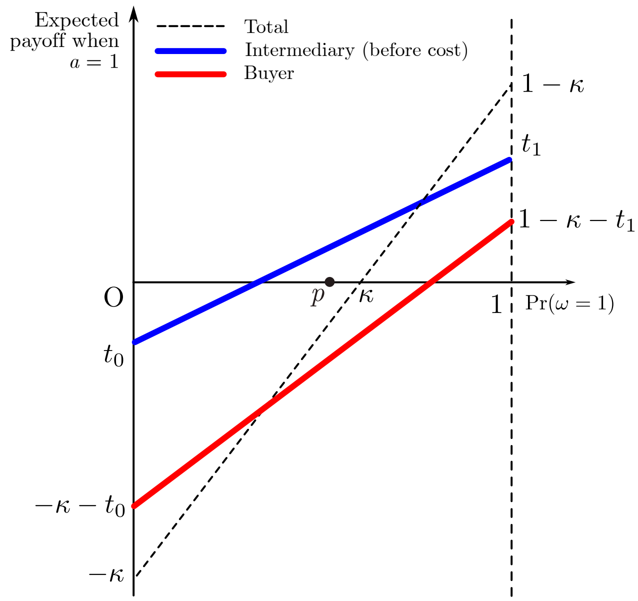

Figure 2: A successful contract requires a joint risk of loss

Before proving this theorem, we establish several definitions and lemmas. Let a contract $t = (t_0, t_1)$ be given. Let V_t^I and V_t^B denote functions such that, for all $q \in [0, 1]$,

$$V_t^I(q) = (1-q)t_0 + qt_1, \text{ and} \tag{16}$$

$$V_t^B(q) = (1-q)(-\kappa - t_0) + q(1-\kappa-t_1). \tag{17}$$

The functions are interpreted as the Stage-2 expected payoffs given a posterior q for the intermediary and the buyer, respectively, when the buyer chooses to invest. As an example, the solid line segments in Figure 2 are the graphs of functions V_t^I and V_t^B under a successful contract.

Definition 3. A due diligence structure $\sigma \in \Sigma$ is *incentive-aligned* under a contract t if, for all posteriors q induced by σ with positive probability, $V_t^I(q)V_t^B(q) \geq 0$.

In other words, a due diligence structure σ is incentive-aligned if both players weakly prefer the same actions in all probable realizations of the signals s from σ.

Definition 4. For every due diligence structure $\sigma \in \Sigma$ and every message rule $\mu : S \longrightarrow M$, define the product $\mu\sigma : \Sigma \longrightarrow M$ such that

$$(\mu\sigma)(\tilde{s}|\omega) = \sum_{s \in S} \mu(m|s)\sigma(s|\omega), \quad \text{for all } m \in M \text{ and } \omega \in \Omega. \tag{18}$$

Thus defined, $\mu\sigma$ represents the distribution of messages $m \in M$ conditional on each states $\omega \in \Omega$. Since $M = S$, $\mu\sigma$ itself is a due diligence structure.

Lemma 1. *There exists no contract that implements a success rate of 1 with any equilibrium.*

Proof. On the contrary, suppose a contract t implements a success rate of 1 with equilibrium $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$. Then all posteriors q induced by $\mu_\sigma\sigma$ with positive probability must satisfy $V_t^B(q) \geq 0$. Then $V_t^B(p) \geq 0$ because p is an weighted average of all posteriors induced by $\mu_\sigma\sigma$. Observe that

$$V_t^I(p) + V_t^B(p) = (1-p)(-\kappa) + p(1-\kappa) < 0, \tag{19}$$

so $V_t^I(p) < 0$. Then

$$U_t^I(\sigma, \mu_\sigma, \alpha_\sigma) = \sum_{j=0}^{J-1} V_t^I(q_j)\varphi(s_j|\mu_\sigma\sigma) = V_t^I(p) < 0, \tag{20}$$

where $q_0, q_1, \dots, q_{J-1}$ are the posteriors induced by $\mu_\sigma\sigma$, the first equality is from the definitions of U_t^I and V_t^I, and the second equality is due to the fact that V_t^I is affine. Then $U_t^I(\sigma, \mu_\sigma, \alpha_\sigma) < 0$, thus $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$ is not an equilibrium, a contradiction. □

A key step in the above proof is applying Bayes-plausibility (Kamenica and Gentzkow, 2011), that the mean of the posteriors q equals the prior p. A success rate of 100% implies that the buyer is willing to invest in all realized posteriors q. Then by Bayes-plausibility, the buyer is willing to invest *ex ante*, contradicting our assumption on p.

The next lemma establishes a crucial fact that a successful contract entails the intermediary choosing a binary due diligence structure and a truth-telling message rule.

Lemma 2. *Suppose a contract t implements a success rate $\rho \in (0,1)$ with an equilibrium $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$. Then the equilibrium satisfies*

1. *binary due diligence: σ induces exactly two posteriors with positive probability.*

2. *fully revealing messages: Let $\tilde{\sigma} = \mu_\sigma\sigma$. Then $\tilde{\sigma}$ induces the same posteriors with positive probability as σ.*

Proof. Suppose t implements $\rho \in (0,1)$ with an equilibrium $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$ in an IPO mechanism. Let $\tilde{q}_0, \tilde{q}_1, \dots, \tilde{q}_{J-1}$ denote the posteriors induced by $\tilde{\sigma}$. Define subsets of messages $M_d = \{m \in M : \alpha_\sigma(m) = d\}$ for $d = 0, 1$. Since $\rho \in (0,1)$, M_0 and M_1 are nonempty.

Step 1: Show that $\tilde{\sigma}$ is incentive-aligned. On the contrary, suppose that $\tilde{\sigma}$ is not incentive-aligned: there exists j such that $\varphi(s_j|\tilde{\sigma}) > 0$ and $V_t^I(\tilde{q}_j)V_t^B(\tilde{q}_j) < 0$. First, suppose

$V_t^I(\tilde{q}_j) < 0$ and $V_t^B(\tilde{q}_j) > 0$. Since α_σ is a best response to μ_σ, $\alpha_\sigma(s_j) = 1$. Then μ_σ is not a best response to α_σ, as decreasing $\mu_\sigma(s_j|s)$ and increasing $\mu_\sigma(s'|s)$ for some $s' \in M_0$ by the same amount makes the intermediary strictly better off, a contradiction. Second, suppose $V_t^I(\tilde{q}_j) > 0$ and $V_t^B(\tilde{q}_j) < 0$. Since α_σ is a best response to μ_σ, $\alpha_\sigma(s_j) = 0$. Then μ_σ is not a best response to α_σ, as decreasing $\mu_\sigma(s_j|s)$ and increasing $\mu_\sigma(s'|s)$ for some $s' \in M_1$ by the same amount makes the intermediary strictly better off, a contradiction.

Step 2: Construct a binary due diligence structure σ'. Let $\sigma' \in \Sigma$ such that, for all $\omega \in \Omega$,

$$\sigma'(s_j|\omega) = \sum_{s \in M_j} \tilde{\sigma}(s|\omega), \quad \text{for } j = 0, 1, \tag{21}$$

and $\sigma'(s_j|\omega) = 0$ for $j = 2, 3, \ldots, J-1$. Then σ' induces two posteriors with positive probabilities: for $j = 0, 1$,

$$q'_j = \frac{p\sigma'(s_j|\omega)}{\varphi(s_j|\sigma')} = \frac{\sum_{s \in M_j} p\tilde{\sigma}(s|\omega)}{\varphi(s_j|\sigma')}, \tag{22}$$

with probabilities

$$\varphi(s_j|\sigma') = \sum_{\omega \in \Omega} \sigma'(s_j|\omega)\pi(\omega) = \sum_{\omega \in \Omega} \sum_{s \in M_j} \tilde{\sigma}(s|\omega)\pi(\omega). \tag{23}$$

Observe that, for all $j = 0, 1$, q'_j is a weighted average of posteriors $\tilde{q}_k$ whose signals belong to M_j. Namely, for all $j = 0, 1$,

$$q'_j = \sum_{s_k \in M_j} \tilde{q}_k \frac{\varphi(s_k|\sigma')}{\varphi(s_j|\sigma')}. \tag{24}$$

We show that the constructed posteriors q'_0 and q'_1 are distinct in Step 4.

Step 3: Show that σ' is incentive-aligned. Since $\tilde{\sigma}$ is incentive-aligned, $V_t^I(\tilde{q}_k)V_t^B(\tilde{q}_k) \geq 0$ for all posteriors $\tilde{q}_k$ with positive probabilities. Observe that, by construction, $\alpha_\sigma(s_k) = 0$ for all $s_k \in M_0$, so $V_t^B(\tilde{q}_k) \leq 0$ for all k such that $s_k \in M_0$. Then $V_t^I(\tilde{q}_k) \leq 0$ for all k such that $s_k \in M_0$. Similarly, by construction, $\alpha_\sigma(s_k) = 1$ for all $s_k \in M_1$, so $V_t^B(\tilde{q}_k) \geq 0$ for all k such that $s_k \in M_1$. Then $V_t^I(\tilde{q}_k) \geq 0$ for all k such that $s_k \in M_1$.

Observe that V_t^I and V_t^B are affine. Since q'_0 is a weighted average of posteriors $\tilde{q}_k$ for all k such that $s_k \in M_0$, $V_t^I(q'_0) \leq 0$ and $V_t^B(q'_0) \leq 0$. Similarly, since q'_1 is a weighted average of posteriors $\tilde{q}_k$ for all k such that $s_k \in M_1$, $V_t^I(q'_1) \geq 0$ and $V_t^B(q'_1) \geq 0$. Therefore, $V_t^I(q'_j)V_t^B(q'_j) \geq 0$ for all $j = 0, 1$.

Step 4: Show that $q'_0 \neq q'_1$. On the contrary, suppose $q'_0 = q'_1$. Recall from Step 3 that, for each $d = 0, 1$, q'_d is an weighted average of all $\tilde{q}_k$ such that $s_k \in M_d$. Also recall that $V_t^B(\tilde{q}_k) \le 0$ for all $\tilde{q}_k$ such that $s_k \in M_0$. Similarly, $V_t^B(\tilde{q}_k) \ge 0$ for all $\tilde{q}_k$ such that $s_k \in M_1$. By the linearity of V_t^B, these inequalities imply that $\tilde{q}_k = p$ for all $k = 0, 1, \dots, J-1$. Then $\rho = 0$ or $\rho = 1$, a contradiction.

Step 5: Characterize $\mu_{\sigma'}$ and $\alpha_{\sigma'}$. Consider $\mu_{\sigma'}$ and $\alpha_{\sigma'}$. Because $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$ is an equilibrium, we know that $\mu_{\sigma'}$ and $\alpha_{\sigma'}$ are mutual best responses given σ' and that the pair is not Pareto-dominated by other such pairs of mutual best responses. Let $M'_d = \{m \in M : \alpha_{\sigma'}(m) = d\}$ for $d = 0, 1$. We claim that M'_0 and M'_1 are nonempty, and $\mu_{\sigma'}$ satisfies

$$\sum_{m \in M_d} \mu_{\sigma'}(m|s_d) = 1, \quad \text{for } d = 0, 1, \tag{25}$$

which implies that $\mu_{\sigma'}$ is fully revealing. To show that M'_0 and M'_1 are nonempty, suppose that one of them is empty.

First, suppose M'_0 is empty. Then $\alpha_{\sigma'}(m) = 1$ for all $m \in M$, implying that $V_t^B(q'_0) \ge 0$ and $V_t^B(q'_1) \ge 0$. Let q'' denote the minimum of the two posteriors q'_0 and q'_1. Then $q'' < p$ because $q'_0 \ne q'_1$. From the definitions of V_t^I and V_t^B, $V_t^B(q'') \ge 0$ implies that

$$V_t^I(q'') \le (1 - q'')(-\kappa) + q''(1 - \kappa). \tag{26}$$

The right-hand side of this inequality is strictly negative because $q'' < p < \kappa$. Then $V_t^I(q'') < 0$ whereas $V_t^B(q'') \ge 0$. Recall that σ' is aligned, so $V_t^I(q'')V_t^B(q'') \ge 0$ in particular. Then $V_t^B(q'') = 0$. However, $(\mu_{\sigma'}, \alpha_{\sigma'})$ is Pareto-dominated by a pair (μ', α') where

$$\mu'(s|s) = 1, \quad \text{for all } s \in S, \tag{27}$$

$$\alpha'(m) = 1, \quad \text{if and only if } m = s_1. \tag{28}$$

Then $\alpha_{\sigma'}$ is not a best response, a contradiction. Second, suppose M'_1 is empty. Then both the intermediary and the buyer earns zero payoffs at this stage, whereas (μ', α') earns the buyer a nonnegative payoff and the intermediary a positive payoff. Then $(\mu_{\sigma'}, \alpha_{\sigma'})$ is Pareto-dominated by (μ', α'), a contradiction.

To show equation (25), suppose that this equation is not true: there exists $m \notin M_d$ such that $\mu_{\sigma'}(m|s_d) > 0$ for some $d = \{0, 1\}$. Then $\mu_{\sigma'}$ is not a best response because the intermediary strictly gains by reducing $\mu_{\sigma'}(m|s_d) > 0$ and increasing $\mu_{\sigma'}(m'|s_d)$ for some $m' \in M_d$ by the same amount.

Conversely, let us show that if M'_0 and M'_1 are nonempty and equation (25) is true, then $\mu_{\sigma'}$ and $\alpha_{\sigma'}$ are mutual best responses and the pair is not Pareto-dominated. Since $\mu_{\sigma'}$ is fully revealing and σ' is incentive-aligned, $\mu_{\sigma'}$ and $\alpha_{\sigma'}$ are mutual best responses. Moreover, because $\mu_{\sigma'}$ is fully revealing, no other mutually best-responding pair gives a higher expected payoff to the intermediary or the buyer.

Step 6: Show that $U_t^I(\sigma', \mu_{\sigma'}, \alpha_{\sigma'}) = U_t^I(\sigma, \mu_\sigma, \alpha_\sigma)$. By the incentive alignment of σ and the affineness of V_t^I,

$$\sum_{j=0}^{1} V_t^I(q_j)\varphi(s_j|\sigma') = \sum_{j=0}^{J-1} V_t^I(q_j)\varphi(s_j|\tilde{\sigma}) \tag{29}$$

Moreover, by construction, σ' is a garbling of $\tilde{\sigma}$ and $\tilde{\sigma}$ is a garbling of σ. Thus,

$$c(\sigma') \leq c(\tilde{\sigma}) \leq c(\sigma). \tag{30}$$

Observe that $\Pi(\sigma', \mu_{\sigma'}, \alpha_{\sigma'})$ equals the left-hand side of (29) minus the cost $c(\sigma')$. Also, $\Pi(\sigma, \mu_\sigma \alpha_{\sigma'})$ equals the right-hand side of (29) minus the cost $c(\sigma)$. Then $U_t^I(\sigma', \mu_{\sigma'}, \alpha_{\sigma'}) \geq U_t^I(\sigma, \mu_\sigma, \alpha_\sigma)$. Since σ is a best response to $(\boldsymbol{\mu}, \boldsymbol{\sigma})$, $U_t^I(\sigma', \mu_{\sigma'}, \alpha_{\sigma'}) \leq U_t^I(\sigma, \mu_\sigma, \alpha_\sigma)$. Then $U_t^I(\sigma', \mu_{\sigma'}, \alpha_{\sigma'}) = U_t^I(\sigma, \mu_\sigma, \alpha_\sigma)$.

Step 7: Show that $\sigma = \sigma'$. Observe that, if σ is binary, then $\sigma = \sigma'$ by construction. Suppose σ is not binary, inducing more than two posteriors with positive probabilities. Then σ' is strictly less Blackwell-informative than σ, so $c(\sigma') < c(\sigma)$. This implies that $U_t^I(\sigma, \mu_\sigma, \alpha_\sigma) < U_t^I(\sigma, \mu_\sigma, \alpha_\sigma)$, which is impossible. So $\sigma = \sigma'$. Observe also from Step 5 that $\mu_{\sigma'}$ is fully revealing, thus μ_σ is fully revealing. Therefore, the equilibrium $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$ satisfies both binary due diligence and fully revealing messages. □

The first important step in the proof of Lemma 2 is the observation that the buyer's information structure is a garbling of the intermediary's in an equilibrium with a positive success rate. We illustrate this idea in Figure 3. On the one hand, the solid arrows indicate the distribution of posteriors for the intermediary after observing a signal from her due diligence. On the other hand, the dashed arrows indicate the distribution of posteriors for the buyer after observing a message from the intermediary. Since the intemediary's message rule specifies a distribution of messages for each due diligecne signal, the dashed arrows form a mean-preserving contraction of the solid arrows. Then the intermediary has no reason to collect more information than necessary, as the buyer will make their investment decision based on the dashed arrows anyway, and collecting more information is costly. Thus, the intermediary's message is fuly revealing. This step is similar to the result by Pei (2015), who finds that the sender in a cheap talk game with endogenous information acquisition always fully reveals his private information.

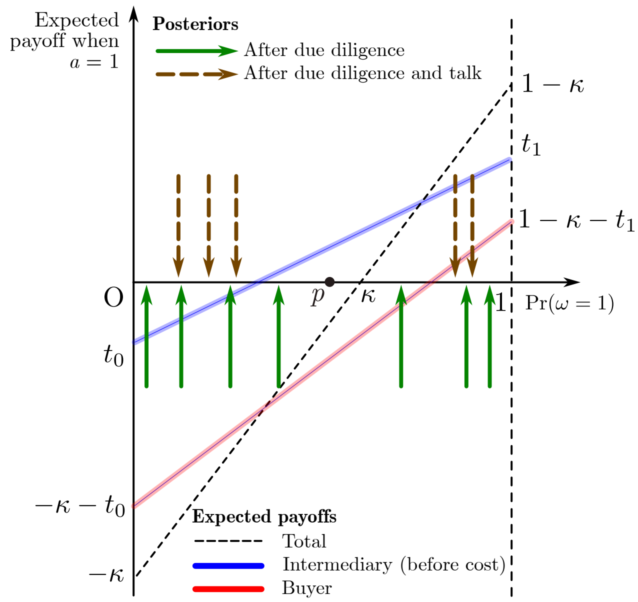

Figure 3: Distribution of the intermediary's and the buyer's posteriors

The second important step in the proof is that the buyer's information structure must be incentive-aligned. That is, all of the dashed arrows should lie on the x-axis where the intermediary and the buyer prefer the same action. Otherwise, the intermediary would want to deviate to a different message and obtain her preferred outcome. For example, if a dashed arrow were to be sufficiently close to the prior p in Figure 3, the intermediary wants the buyer to invest whereas the buyer is unwilling to do so. Once this is the case, the intermediary has no reason to chooses anything more informative than a binary due diligence, because the eventual action by the buyer is either to invest or not. This result is similar to Corollary 1 in Matějka and McKay (2015) which shows that, in a rational inattention problem with finitely many actions, we can restrict our attention to information structures that map each state to a probability distribution over the actions.

The next lemma establishes that the intermediary' incentives are balanced, in the sense that she prefers that the buyer does not invest when her posterior is pessimistic and prefers that the buyer does invest when her posterior is optimistic.

Lemma 3. *Suppose a contract t implements a success rate $\rho \in (0,1)$ with some equilibrium. Let (ℓ, r) denote the pair of posteriors induced by the equilibrium due diligence. Then*

1. *the interemediary's and the buyer's incentives are aligned: $V_t^I(q)V_t^B(q) \geq 0$ for both $q \in \{\ell, r\}$, and*

2. *the intermediary's incentives are balanced: $V_t^I(\ell) \leq 0$ and $V_t^I(r) > 0$.*

Proof. Let $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$ denote the said equilibrium. By Lemma 2, the message rule μ_σ is fully revealing. That is, $\mu_\sigma \sigma$ induces the same two posteriors with positive probability as σ. Let $(\ell, r) \in [0, p) \times (p, 1]$ denote the pair of those posteriors. Then that σ is incentive-aligned follows from Steps 3 and 7 in the proof of Lemma 2.

Now suppose that the intermediary's incentives are not strictly balanced. Then $V_t^I(\ell)V_t^I(r) \geq 0$. Consider the following two cases. First, suppose $V_t^I(\ell) \leq 0$ and $V_t^I(r) \leq 0$. Then the intermediary's profit is negative, so $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$ cannot be an equilibrium, a contradiction. Second, suppose $V_t^I(\ell) > 0$. Then $V_t^B < 0$, because $V_t^I(\ell) + V_t^B(\ell) < V_t^I(p) + V_t^B(p) < 0$. Then σ is not aligned, a contradiction. Therefore, $V_t^I(\ell) \leq 0$ and $V_t^I(r) > 0$. $\square$

We are now ready to prove Theorem 1.

Proof of Theorem 1. Let (ℓ, r) denote the pair of posteriors with positive probability induced by the equilibrium due diligence structure. By the definitions of V_t^I and V_t^B, for all $q \in [0, 1]$,

$$(t_1 - t_0)q \geq -t_0 \quad \text{if and only if} \quad V_t^I(q) \geq 0, \text{ and} \tag{31}$$

$$(1 - t_1 + t_0)q \geq \kappa + t_0 \quad \text{if and only if} \quad V_t^B(q) \geq 0, \tag{32}$$

which also hold when the inequalities are strict. By Lemma 3, we know that (ℓ, r) must satisfy $V_t^I(q)V_t^B(q) \geq 0$ for both $q \in \{\ell, r\}$ (*incentive alignment*) as well as $V_t^I(\ell) \leq 0$ and $V_t^I(r) > 0$ (*incentive balancedness*). We show that all of the following four cases violate these conditions.

Case 1: Suppose $t_0 < -\kappa$. First, if $1 - t_1 + t_0 \geq 0$, then the strict inequality (32) implies that $V_t^B(q) > 0$ for all $q \in [0, 1]$. In particular, $V_t^B(\ell) > 0$ and $V_t^B(r) > 0$. Then incentive alignment implies that $V_t^I(\ell) \geq 0$ and $V_t^I(r) \geq 0$. Then (ℓ, r) does not satisfy incentive balancedness. Second, if $1 - t_1 + t_0 < 0$, then $t_1 - t_0 > 1$. From (31)–(32), we have

$$q \geq \frac{-t_0}{t_1 - t_0} \quad \text{if and only if} \quad V_t^I(q) \geq 0, \text{ and} \tag{33}$$

$$q \leq \frac{\kappa + t_0}{1 - t_1 + t_0} \quad \text{if and only if} \quad V_t^B(q) \geq 0. \tag{34}$$

Then incentive alignment requires that both posteriors ℓ and r lie in the interval $\left[\frac{\kappa+t_0}{1-t_1+t_0}, \frac{-t_0}{t_1-t_0}\right]$. Then $V_t^I(\ell) \geq 0$ and $V_t^I(r) \geq 0$, violating incentive balancedness.

Case 2: Suppose $t_0 \geq 0$. First, if $t_1 \geq t_0$, then the inequality (31) implies that $V_t^I(q) \geq 0$ for all $q \in [0, 1]$. In particular, $V_t^I(\ell) \geq 0$ and $V_t^I(r) \geq 0$, violating the incentive balancedness.

Second, if $t_1 < t_0$, then $1 - t_1 + t_0 > 1$. From (31)–(32), we have

$$q \leq \frac{-t_0}{t_1 - t_0} \qquad \text{if and only if} \quad V_t^I(q) \geq 0, \text{ and} \tag{35}$$

$$q \geq \frac{\kappa + t_0}{1 - t_1 + t_0} \quad \text{if and only if} \quad V_t^B(q) \geq 0. \tag{36}$$

Then, as in Case 1, incentive alignment requires that both posteriors ℓ and r lie on the interval $\left[\frac{\kappa+t_0}{1-t_1+t_0}, \frac{-t_0}{t_1-t_0}\right]$. Then $V_t^I(\ell) \leq 0$ and $V_t^I(r) \leq 0$, violating incentive balancedness.

Case 3: Suppose $t_1 \leq 0$. We know from Cases 1–2 that $t_0 \in [-\kappa, 0)$. Then $V_t^I(q) = (1-q)t_0 + qt_1 \leq 0$ for all $q \in [0,1]$. Then $V_t^I(\ell) \leq 0$ and $V_t^I(r) \leq 0$, which violates incentive balancedness.

Case 4: Suppose $t_1 > 1 - \kappa$. As before, we know from Cases 1–2 that $t_0 \in [-\kappa, 0)$. Then $V_t^B(q) = (1-q)(-\kappa - t_0) + q(1 - \kappa - t_1) < 0$ for all $q \in (0,1]$. In particular, $V_t^B(r) < 0$. Then incentive alignment requires that $V_t^I(r) \leq 0$, which violates incentive balancedness.

We know from Cases 1–2 that $t_0 \in [-\kappa, 0)$. In addition, we know from Cases 3–4 that $t_1 \in (0, 1-\kappa]$. These results imply that a contract that implements a success rate $\rho > 0$ satisfies $(t_0, t_1) \in [-\kappa, 0) \times (0, 1-\kappa]$. □

Corollary 1. *Suppose a contract $t \in T$ implements a success rate $\rho > 0$. Then $V_t^I(q)$ is strictly increasing in q and $V_t^B(q)$ is increasing in q.*

Proof. From Theorem 1, $(t_0, t_1) \in [-\kappa, 0) \times (0, 1-\kappa)$. Then $V_t^I(0) = t_0 < t_1 = V_t^I(1)$ and V_t^I is affine in q, hence V_t^I is strictly increasing. Also, $V_t^B(0) = -\kappa - t_0 \leq 1 - \kappa - t_1 = V_t^B(1)$ and V_t^B is affine in q, thus V_t^B is strictly increasing. □

4 Optimal investment banking contracts

We have shown in the previous section that, for a contract to be successful, the intermediary must share the risk of loss with the buyer. In this section, we go further and characterize the optimal contract from the seller's perspective. Specifically, a contract is seller-optimal if and only if it maximizes the intermediary's bias in favor of the seller while maintaining a minimal alignment with the buyer's interests.

To make this statement precise, we introduce additional notation. Let us write $T = [-\kappa, 0) \times (0, 1-\kappa]$, the set of contracts that satisfy the necessary condition of a successful contract from Theorem 1. Without loss of generality, let $\widehat{\Sigma}$ denote the set of all $\sigma \in \Sigma$ such that, for all $\omega \in \Omega$ and for all $j = 2, 3, \ldots, J-1$, we have $\sigma(s_j|\omega) = 0$ and $\sigma(s_0|1) < \sigma(s_1|1)$. That is, $\widehat{\Sigma}$ is the set of all binary due diligence structures such that s_0 is a pessimistic signal and s_1 is an optimistic signal. In this context, we say that $\sigma \in \widehat{\Sigma}$ *induces* the pair of posteriors $(\ell, r) \in [0, p) \times (p, 1]$ if $\ell = \pi(1|s_0, \sigma)$ and $r = \pi(1|s_1, \sigma)$.

Definition 5 (Incentive alignment)**.** Let a contract $t \in T$ be given. Suppose $\sigma \in \widehat{\Sigma}$ induces (ℓ, r). We say that σ is *incentive-aligned* if $V_t^I(\ell)V_t^B(\ell) \geq 0$ and $V_t^I(r)V_t^B(r) \geq 0$. We say that σ is *minimally incentive-aligned* if it is incentive-aligned and $V_t^B(r) = 0$. We write $\Lambda(t)$ to denote the set of all $\sigma \in \widehat{\Sigma}$ that are incentive-aligned given t, and $\underline{\Lambda}(t)$ to denote the set of all $\sigma \in \widehat{\Sigma}$ that are minimally incentive-aligned given t.

This definition of minimal incentive alignment is sensible because it means that, even when the buyer receives an optimistic message, they are indifferent between investing and not investing. Note that the definition of incentive alignment (not necessarily minimal) is simply the application of the same term defined in the previous section to binary due diligence structures.

Definition 6 (Incentive compatibility)**.** Let a contract $t \in T$ be given. Let μ be a fully revealing message rule and let $\alpha(s_a) = a$ for all $a \in A$. A binary due diligence structure $\sigma \in \Lambda(t)$ is *incentive-compatible* if $U_t^I(\sigma, \mu, \alpha) \geq U_t^I(\sigma', \mu, \alpha)$ for all $\sigma' \in \Lambda(t)$. We write $\Gamma(t)$ to denote the set of all $\sigma \in \Lambda(t)$ that are incentive-compatible given t.

Theorem 2. *Suppose a contract $t^* \in T$ implements a success rate $\rho^* > 0$ and does not implement any success rate higher than ρ^*. The contract t^* is seller-optimal if and only if $\rho^* = \max\{\varphi(s_1|\sigma) : \sigma \in \underline{\Lambda}(t) \cap \Gamma(t), t \in T\}$.*

To better understand the statement of this theorem, let us illustrate what it means for a binary due diligence structure to be both incentive-aligned and incentive-compatible. Panel (a) of Figure 4 shows an example structure $\sigma \in \widehat{\Sigma}$ that satisfies both the incentive alignment and incentive compatibility conditions, for a given contract $t \in T$. The points ℓ and r on the x-axis are the posteriors induced by σ. Since V_t^I (dashed line) and V_t^B (solid red line) are such that both the intermediary and the buyer prefer $a = 0$ when the realized posterior is ℓ and prefer $a = 1$ when it is r, the due diligence structure incentive-aligned.

Moreover, the example due diligence structure represented in Figure 4(a) is incentive-compatible. To see this, let us represent the information cost as the weighted average

$$c(\sigma) = \varphi(s_0|\sigma) \cdot \lambda[h(p) - h(\ell)] + \varphi(s_1|\sigma) \cdot \lambda[h(p) - h(r)], \tag{37}$$

where $h(q) = (1-q)\log(1-q) + q\log q$. We interpret the terms $\lambda[h(p)-h(\ell)]$ and $\lambda[h(p)-h(r)]$ as the costs *conditional* on the realizations of the posteriors ℓ and r, respectively. Then the

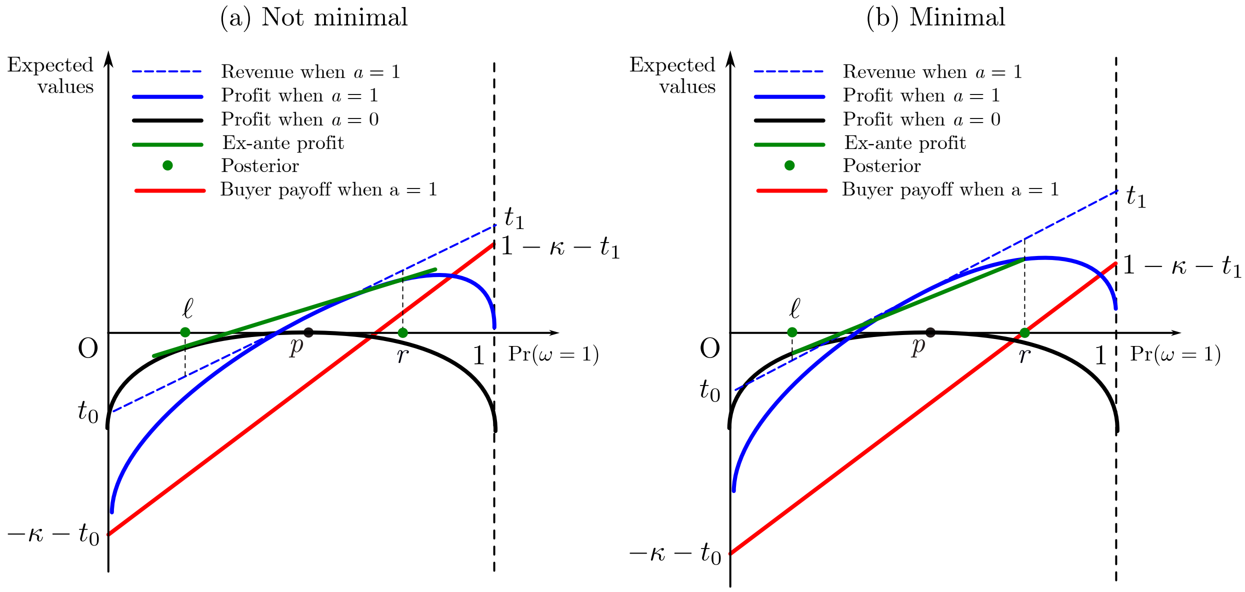

Figure 4: Incentive alignment may or may not be minimal in an equilibrium

two curves represent the intermediary's expected profits Π_0 and Π_1, functions defined as

$$\Pi_a(q) = a \cdot V_t^I(q) - \lambda[h(p) - h(q)], \tag{38}$$

for the buyer choosing to not invest ($a = 0$) or invest ($a = 1$), respectively. Then the intermediary's *ex ante* expected profit, $\varphi(s_0|\sigma)\Pi_0(\ell) + \varphi(s_1|\sigma)\Pi_1(r)$, is the height of the line that passes through the points $(\ell, \Pi_0(\ell))$ and $(r, \Pi_1(r))$. The due diligence structure represented in this figure is incentive-compatible, because the points ℓ and r are such that this line is tangent to both curves, maximizing the intermediary's expected profit. In this case, the incentive alignment condition does not bind, and the incentive-compatible due diligence structure yields a solution similar to that by Matějka and McKay (2015).

In contrast, the incentive alignment condition binds in another example, in Panel (b) of Figure 5. That is, $\sigma \in \partial\Lambda(t)$ rather than $\sigma \in \Lambda^\circ(t)$. Here, the pair of unconstrained-optimal posteriors (ℓ', r') by the intermediary—whose line segment would be tangent to the two profit curves—would violate incentive alignment, as r' would be to the left of r where the intermediary still prefers $a = 1$ but the buyer prefers $a = 0$. Such due diligence structure would, however, lead to fruitless communication in the roadshow stage (the talking stage). It is hence compatible with the intermediary's interest to settle with (ℓ, r) represented in the figure. This due diligence structure is *minimally* incentive-aligned, in the sense that the buyer is just indifferent between investing and not investing. Therefore, Theorem 2 means that any optimal contract must result in outcomes of this kind, with binding incentive alignment.

To prove Theorem 2, we introduce further notation and establish several lemmas. For

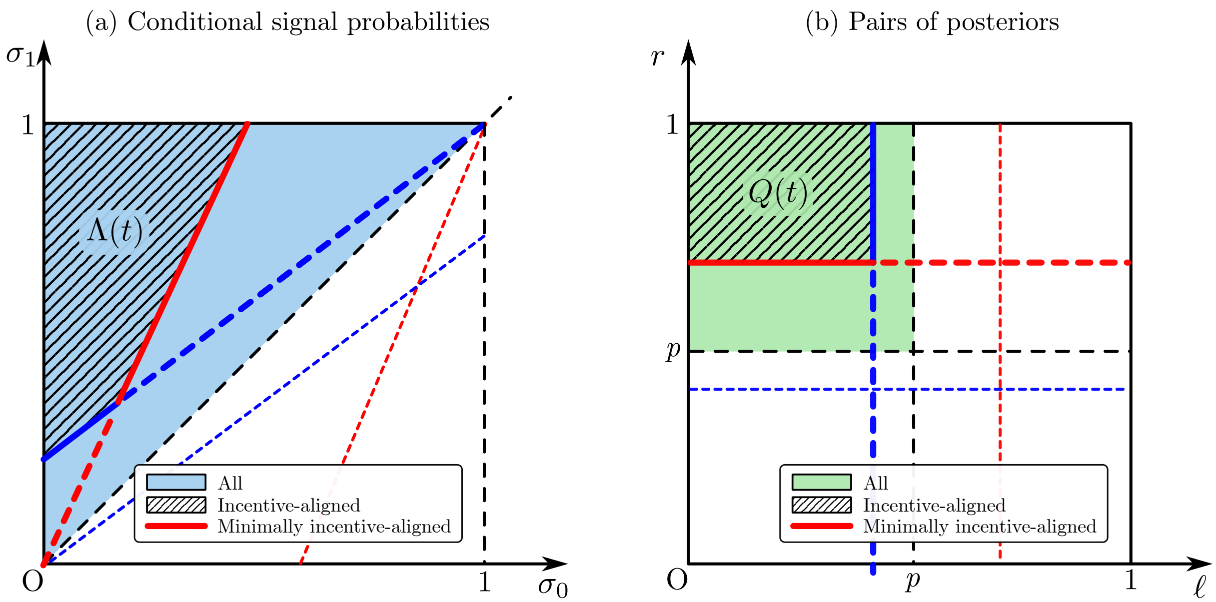

Figure 5: The set of incentive-aligned due diligence structures

any $\sigma \in \widehat{\Sigma}$, let σ_ω denote $\sigma(s_1|\omega)$, the conditional probability of the optimistic signal. Then $\sigma_0 = \sigma(s_1|0)$ and $\sigma_1 = \sigma(s_1|1)$. Consider a metric $d : \widehat{\Sigma} \times \widehat{\Sigma} \longrightarrow \mathbb{R}$ such that

$$d(\sigma, \sigma') = \sqrt{(\sigma_0 - \sigma_0')^2 + (\sigma_1 - \sigma_1')^2}, \tag{39}$$

which is the Euclidean distance between (σ_0, σ_1) and (σ_0', σ_1') in $\mathbb{R}^2$. For the rest of our paper, we use this metric for the distance between any two binary due diligence structures.

For any contract $t \in T$, we write $\Lambda^\circ(t)$ to represent the set of interior points of $\Lambda(t)$ in the metric space $(\widehat{\Sigma}, d)$. Furthermore, for any contract $t \in T$, let us define $Q(t)$ as the set of all pairs of posteriors $(\ell, r) \in [0, p) \times (p, 1]$ induced by some $\sigma \in \Lambda(t)$. Let us define $\underline{Q}(t)$ as the set of all pairs of posteriors $(\ell, r) \in [0, p) \times (p, 1]$ induced by some $\sigma \in \underline{\Lambda}(t)$. We define $Q^\circ(t)$ as the interior of $Q(t)$. Figure 5 illustrates the sets $\Lambda(t)$, $\underline{\Lambda}(t)$, $Q(t)$ and $\underline{Q}(t)$.

The mapping from the set $\Lambda(t)$ of incentive-aligned due diligence structures and the set $Q(t)$ of their induced posteriors is invertible, resulting in the following lemma:

Lemma 4. *Let $t \in T$ and $\sigma \in \Lambda(t)$ be given. Let $(\ell, r) \in [0, p) \times (p, 1]$ denote the pair of posteriors induced by σ. Then $\sigma \in \Lambda^\circ(t)$ if and only if $(\ell, r) \in Q^\circ(t)$.*

Proof. Let us write $\sigma_\omega = \sigma(s_1|\omega)$ for both $\omega \in \Omega$. Because (ℓ, r) is the pair of posteriors induced by σ, it satisfies

$$\ell = \frac{(1 - \sigma_1)p}{(1 - \sigma_0)(1 - p) + (1 - \sigma_1)p} \quad \text{and} \quad r = \frac{\sigma_1 p}{\sigma_0(1 - p) + \sigma_1 p}. \tag{40}$$

Equivalently, we have

$$\sigma_0 = \frac{1-r}{1-p} \cdot \frac{p-\ell}{r-\ell} \quad \text{and} \quad \sigma_1 = \frac{r}{p} \cdot \frac{p-\ell}{r-\ell}. \tag{41}$$

So there exists a continuous and invertible map $f : \Lambda(t) \longrightarrow Q(t)$ such that $f(\sigma) = (\ell, r)$. Then $x \in Q^\circ(t)$ implies $f(x) \in \Lambda^\circ(t)$. Similarly, $\sigma \in \Lambda^\circ(t)$ implies $f^{-1}(\sigma) \in Q^\circ(t)$. □

Recall from Lemma 3 that if $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$ is an equilibrium with a positive success rate, then σ is incentive-aligned. The following lemma generalizes this result to any binary due diligence structure σ not necessarily on the equilibrium path.

Lemma 5. *Suppose a message rule μ and an action rule α are mutual best responses given $(t, \sigma) \in T \times \widehat{\Sigma}$. Then $U_t^I(\sigma, \mu, \alpha) \geq 0$ only if $\sigma \in \Lambda(t)$.*

Proof. Let (ℓ, r) denote the pair of posteriors induced by σ. Let $\tilde{\sigma} = \mu\sigma$ so that, for all $\omega \in \Omega$ and $s \in S$,

$$\tilde{\sigma}(\tilde{s}|\omega) = \sum_{s \in S} \mu(\tilde{s}|s)\sigma(s|\omega). \tag{42}$$

Let $q_0, q_1, \ldots, q_{J-1}$ denote the posteriors induced by $\tilde{\sigma}$. Let $M_a = \{m \in M : \alpha(m) = a\}$ for all $a \in A$.

Step 1. Show that the success rate $U_t^S(\sigma, \mu, \alpha)$ lies within the open interval $(0, 1)$. Observe that, because $\sigma \in \widehat{\Sigma}$ and thus $c(\sigma) > 0$, the intermediary's expected revenue is strictly positive. That is, $U_t^I(\sigma, \mu, \alpha) + c(\sigma) > 0$. First, suppose $U_t^S(\sigma, \mu, \alpha) = 0$. Then $m \in M_0$ for all messages $m \in M$ such that $\pi(m|\tilde{\sigma}) > 0$. Then the intermediary's expected revenue is zero, a contradiction. Second, suppose $U_t^S(\sigma, \mu, \alpha) = 1$. Since α is a best response, this implies that $V_t^B(q) \geq 0$ for all posteriors induced by $\tilde{\sigma}$ with positive probability. As in the proof of Lemma 1, this implies that $U_t^I(\sigma, \mu, \alpha) < 0$, a contradiction.

Step 2. Show that $\tilde{\sigma}$ is incentive-aligned. We proceed in the same way as in Step 1 of the proof of Lemma 2. Observe that neither M_0 nor M_1 are empty because $U_t^S(\sigma, \mu, \alpha) \in (0, 1)$. Suppose $\tilde{\sigma}$ is not incentive-aligned. Then there exists a posterior q_j induced by $\tilde{\sigma}$ with positive probability such that $V_t^I(q_j)V_t^B(q_j) < 0$. First, suppose $V_t^I(q_j) > 0$ and $V_t^B(q_j) < 0$. Because α is a best response, $\alpha(s_j) = 0$. Then μ is not a best response, a contradiction. Second, suppose $V_t^I(q_j) < 0$ and $V_t^B(q_j) > 0$. Because α is a best response, $\alpha(s_j) = 1$. Then μ is not a best response, a contradiction.

Step 3. Show that σ is incentive-aligned. Since $\tilde{\sigma}$ is a garbling of σ, $\ell \leq \min\{q_0, q_1, \ldots, q_{J-1}\}$ and $r \geq \max\{q_0, q_1, \ldots, q_{J-1}\}$. Since $\tilde{\sigma}$ is incentive-aligned and the intermediary's revenue is strictly positive, $V_t^I(q_j) < 0$ for some j and $V_t^I(q_k) > 0$ for some k. Since, from Corollary

1, V_t^I and V_t^B are both increasing functions, we have $V_t^I(\ell) < 0$, $V_t^I(r) > 0$, $V_t^B(\ell) \leq 0$, and $V_t^B(r) \geq 0$. Therefore, $V_t^I(\ell)V_t^B(\ell) \geq 0$ and $V_t^I(r)V_t^B(r) \geq 0$, meaning that $\sigma \in \Lambda(t)$. $\square$

The next lemma shows that the intermediary' expected payoffs are smooth and strictly concave. A critical requirement for this result is that the collection of message rules and action rules are not Pareto-dominated. This condition ensures that the equilibrium of the talking-stage subgame is the informative one whenever it is available, leading to truthful revelation and mutually preferred actions.

Lemma 6. *Let a contract $t \in T$ be given. Let $(\boldsymbol{\mu}, \boldsymbol{\alpha}) = \{(\mu_\sigma, \alpha_\sigma)\}_{\sigma \in \Sigma}$ be given such that it is a mutual best response and is not Pareto-dominated. The map $\sigma \mapsto U_t^I(\sigma, \mu_\sigma, \alpha_\sigma)$ with domain $\Lambda(t)$ is continuously differentiable and strictly concave.*

Proof. Observe that

$$U_t^I(\sigma, \mu_\sigma, \alpha_\sigma) = \sum_{\omega \in \Omega} \sum_{s \in S} \sum_{m \in M} t_\omega \alpha_\sigma(m) \mu_\sigma(m|s) \sigma(s|\omega) \pi(\omega) - c(\sigma) \tag{43}$$

Let $M_a = \{m \in M : \alpha_\sigma(m) = a\}$ for all $a = 0, 1$. As in Step 5 of Proof of Lemma 2, the fact that $\sigma \in \Lambda(t)$, $(\mu_\sigma, \alpha_\sigma)$ is a mutual best response and is not Pareto-dominated implies that $\mu(m|s_a) = 1$ for all $m \in M_a$ for all $a = 0, 1$. Then

$$U_t^I(\sigma, \mu_\sigma, \alpha_\sigma) = \sum_{\omega \in \Omega} \sum_{a \in A} a t_\omega \sigma(s_a|\omega) \pi(\omega) - c(\sigma). \tag{44}$$

The first term on the right-hand side of the above equation is affine in σ. The second term, $c(\sigma)$, is continuously differentiable and strictly convex in σ. Therefore, $U_t^I(\sigma, \mu_\sigma, \alpha_\sigma)$ is strictly concave in σ on the domain $\Lambda(t)$. $\square$

The next lemma shows that, whenever a due diligence structure satisfies non-binding incentive alignment, the solution to the intermediary's optimization problem is the standard solution of a Rational Inattention problem by Matějka and McKay (2015).

Lemma 7. *Let a contract $t \in T$ and a due diligence structure $\sigma \in \Lambda^\circ(t)$ be given. Let (ℓ, r) denote the pair of posteriors induced by σ. Then $\sigma \in \Gamma(t)$ if and only if $1 - r = (1 - \ell)e^{t_0/\lambda}$ and $r = \ell e^{t_1/\lambda}$.*

Proof. We first show the "only if" part of the statement. Suppose $\sigma \in \Gamma(t)$. By definition, σ is a best response to some pair $(\boldsymbol{\mu}, \boldsymbol{\alpha})$ that is a mutual best response and not Pareto-dominated. Since $\sigma \in \Lambda^\circ(t)$ is a best response, it satisfies

$$\sigma \in \operatorname*{argmax}_{\sigma' \in \Lambda^\circ(t)} U_t^I(\sigma', \mu_{\sigma'}, \alpha_{\sigma'}). \tag{45}$$

Observe from Proof of Lemma 6 that, because $\sigma \in \Lambda(t)$ and $(\boldsymbol{\mu}, \boldsymbol{\alpha})$ is a mutual best response,

$$U_t^I(\sigma, \mu_\sigma, \alpha_\sigma) = \sum_{\omega \in \Omega} \sum_{a \in A} a t_\omega \sigma(s_a|\omega)\pi(\omega) - c(\sigma). \tag{46}$$

Moreover, observe from the definition of c that

$$c(\sigma) = \lambda \cdot \sum_{\omega \in \Omega} \sum_{a \in A} \sigma(s_a|\omega)\pi(\omega) \log\left(\frac{\sigma(s_a|\omega)}{\sum_{\omega \in \Omega} \sigma(s_a|\omega)\pi(\omega)}\right). \tag{47}$$

Since $\Lambda^\circ(t)$ is open, σ^* is an interior point of $\Lambda^\circ(t)$. Then this problem is a special case of the model of rational inattention by Matějka and McKay (2015), whose first-order necesssary conditions for σ^* are, for all $\omega \in \Omega$,

$$\frac{\pi(\omega|s_a, \sigma^*)}{e^{a t_\omega/\lambda}} = \frac{\pi(\omega|s_b, \sigma^*)}{e^{b t_\omega/\lambda}}, \tag{48}$$

for actions $a = 0$ and $b = 1$. Recall that $\ell = \pi(1|s_0, \sigma^*)$ and $r = \pi(1|s_0, \sigma^*)$. Then the first-order conditions imply that $1 - r = (1 - \ell)e^{t_0/\lambda}$ and $r = \ell e^{t_1/\lambda}$.

We now show the "if" part of the statement. Suppose that $1 - r = (1 - \ell)e^{t_0/\lambda}$ and $r = \ell e^{t_1/\lambda}$. Take any pair $(\boldsymbol{\mu}, \boldsymbol{\alpha})$ of mutual best responses. By Lemma 6, the map $\sigma' \mapsto U_t^I(\sigma, \mu_\sigma, \alpha_\sigma)$ is continuously differentiable and strictly concave in σ. Then σ satisfies the necessary and sufficient condition for an optimum for the problem $\max_{\sigma \in \Lambda^\circ(t)} U_t^I(\sigma, \mu_\sigma, \alpha_\sigma)$, thus $\sigma \in \Gamma(t)$. □

Proof of Theorem 2. Let $t^* = (t_0^*, t_1^*) \in T$ denote a contract implementing ρ^* with equilibrium $(\sigma^*, \boldsymbol{\mu}, \boldsymbol{\alpha})$. By construction, $t^* = (t_0^*, t_1^*) \in T$ is optimal if and only if $\rho^* = \max\{\varphi(s_1|\sigma) : \sigma \in \Gamma(t), t \in T\}$. We show that an optimal t^* requires that $\sigma^* \in \underline{\Lambda}(t^*) \cap \Gamma(t^*)$, meaning that t^* is optimal if and only if $\rho^* = \max\{\varphi(s_1|\sigma) : \sigma \in \underline{\Lambda}(t) \cap \Gamma(t), t \in T\}$.

Since $U_t^I(\sigma^*, \mu_{\sigma^*}, \alpha_{\sigma^*}) \geq 0$, we have $\sigma^* \in \Lambda(t)$ by Lemma 5. Let (ℓ^*, r^*) denote the pair of posteriors induced by σ^*. Since $(\mu_{\sigma^*}, \alpha_{\sigma^*})$ is a mutual best response,

$$\rho^* = U_t^S(\sigma^*, \mu_{\sigma^*}, \alpha_{\sigma^*}) = \varphi(s_1|\sigma^*) = \frac{p - \ell^*}{r^* - \ell^*}. \tag{49}$$

Since σ^* is a best response, $\sigma^* \in \Gamma(t^*)$.

Step 1: Show that σ^* cannot lie on the boundary of $\Lambda(t^*)$ such that $\sigma_0^* = 0$, $\sigma^* = 1$,$V_t^I(r^*) = 0$, or $V_t^B(\ell^*) = 0$. This is achieved by showing that $\sigma_0 > 0$, $\sigma_1 < 1$, $V_t^I(r^) > 0$, and $V_t^B(\ell^*) < 0$. Since σ^* is incentive-compatible with the intermediary and its marginal cost of information accuracy approaches infinity as $\sigma_0^* \longrightarrow 0$ or $\sigma_1^* \longrightarrow 1$, we know that $\sigma_0^* > 0$ and $\sigma_1^* < 1$. Also, by Lemma 3, we know that $V_t^I(r^*) > 0$. Furthermore, we see that Lemma 3 implies that $V^B(r^*) \geq 0$, which then implies $V_t^B(\ell^*) < 0$, by using the fact that $V_t^I(\ell^*) \leq 0$ and that

$$\ell^* = \frac{(1-\sigma_1^*)p}{(1-\sigma_0^*)(1-p) + (1-\sigma_1^*)p} \quad \text{and} \quad r^* = \frac{\sigma_1^* p}{\sigma_0^*(1-p) + \sigma_1^* p}. \tag{50}$$

Step 2: Show that σ^* cannot lie on the interior of $\Lambda(t^*)$. Suppose $\sigma^* \in \Lambda^\circ(t^*)$, equivalently $(\ell^*, r^*) \in Q^\circ(t^*)$ by Lemma 4. By Lemma 7, (ℓ^*, r^*) satisfies $1-r^* = (1-\ell^*)e^{t_0^*/\lambda}$ and $r^* = \ell^* e^{t_1^*/\lambda}$. Equivalently,

$$\ell^* = \frac{1-e^{t_0^*/\lambda}}{e^{t_1^*/\lambda} - e^{t_0^*/\lambda}} \quad \text{and} \quad r^* = \frac{e^{-t_0^*/\lambda} - 1}{e^{-t_0^*/\lambda} - e^{-t_1^*/\lambda}}. \tag{51}$$

From Theorem 1, we know that $(t_0^*, t_1^*) \in [-\kappa, 0) \times (0, 1-\kappa]$. In particular, $t_0^* < 0$.

Consider a contract $t' = (t_0', t_1')$ such that $t_0' = t_0^* + \varepsilon$ and $t_1' = t_1^*$, for an arbitrarily small $\varepsilon > 0$. Define ℓ' and r' as the two expressions in (51) after replacing (t_0^*, t_1^*) with (t_0', t_1'). Because $(\ell^*, r^*) \in \Lambda^\circ(t)$ and the expressions in (51) are continuous in t_0^*, there exists $\varepsilon > 0$ such that $(\ell', r') \in \Lambda^\circ(t)$. Because both expressions in (51) are decreasing in t_0, we have $\ell' < \ell^*$ and $r' < r^*$. Let $\sigma' \in \widehat{\Sigma}$ such that

$$\sigma'(s_1|0) = \frac{1-r'}{1-p} \cdot \frac{p-\ell'}{r-\ell'} \quad \text{and} \quad \sigma'(s_1|1) = \frac{r'}{p} \cdot \frac{p-\ell'}{r-\ell'}. \tag{52}$$

Observe that, by construction, σ' induces the pair of posteriors (ℓ', r'). Also by construction, $1 - r' = (1-\ell')e^{t_0'/\lambda}$ and $r' = \ell' e^{t_1'/\lambda}$. Then by Lemma 7, $\sigma' \in \Gamma(t')$, so the contract t' implements a success rate $\rho' = U_t^S(\sigma', \mu_{\sigma'}, \alpha_{\sigma'})$ with equilibrium $(\sigma', \boldsymbol{\mu}, \boldsymbol{\alpha})$. Then we have

$$\rho^* = U_t^S(\sigma^*, \mu_{\sigma^*}, \alpha_{\sigma^*}) = \frac{p-\ell^*}{r^*-\ell^*} < \frac{p-\ell'}{r'-\ell'} = U_t^S(\sigma', \mu_{\sigma'}, \alpha_{\sigma'}) = \rho'. \tag{53}$$

Then t^* is not optimal, a contradiction.

Step 3: Show that σ^* cannot lie on the boundary of $\Lambda(t^*)$ such that $V_t^I(r^*) = 0$ unless $\sigma^* \in \underline{\Lambda}(t^*)$. From Lemma 3, we know that $V_t^I(r^*) \leq 0$. Since $\sigma^* \in \Gamma(t)$, it solves the problem

$$\max_{\sigma\in\Lambda(t)} U_t^I(\sigma,\mu_\sigma,\alpha_\sigma) \tag{54}$$

$$\text{subject to} \quad \sigma(s_0|\omega)\pi(\omega)\leq 0, \text{ and} \tag{55}$$

$$\sum_{a\in A}\sigma(s_a|\omega)=1, \text{ for all } \omega\in\Omega. \tag{56}$$

The first-order necessary conditions of this problem reveal that the Lagrange multiplier on the constraint (55) is zero. Thus, the solution to this problem is identical to that when $\sigma^*\in\Lambda^\circ(t^*)$. We have shown in Step 2 that the contract t^* cannot be optimal given this σ^*.

Steps 1 through 3 imply that $\sigma^*\in\underline{\Lambda}(t^*)$. □

The key idea behind the proof of Theorem 2 is the following. A contract that leads to a non-binding incentive alignment—such as the one illustrated in Panel (a) of Figure 4—between the intermediary and the buyer cannot be optimal for the seller, because we can always find a slightly different contract that maintains the alignment of incentives and raises the success rate. Specifically, by raising t_0, contract designer (the seller) reduces the intermediary's loss in the bad state, thereby bringing the intermediary's interest closer to the seller's own. If a contract already has binding incentive alignment, such deviation is impossible as illustrated in Panel (b) of of the same figure.

While the previous theorem characterizes the seller-optimal contract in a qualitative sense, the following corollary provides a procedure to compute one.

Corollary 2. *A contract $t^*=(t_0^*,t_1^*)\in T$ that implements a positive success rate is optimal if and only if*

$$(1-r^*)t_0^*+r^*t_1^*=(1-r^*)(-\kappa)+r^*(1-\kappa), \tag{57}$$

$$(1-\ell^*)t_0^*+\ell^*t_1^*\geq -\lambda\left[(1-\ell^*)\log\frac{1-\ell^*}{1-r^*}+\ell^*\log\frac{\ell^*}{r^*}\right], \text{ and} \tag{58}$$

$$(1-\ell^*)t_0^*+\ell^*t_1^*\leq 0, \tag{59}$$

where (ℓ^*,r^*) is a solution to the problem $\max_{(\ell,r)\in[0,p)\times(p,1]}\frac{p-\ell}{r-\ell}$ subject to

$$\lambda\left[(1-r)\log\frac{1-r}{1-\ell}+r\log\frac{r}{\ell}\right]=(1-r)(-\kappa)+r(1-\kappa). \tag{60}$$

Proof. Suppose a contract t^* implements a positive success rate in an equilibrium $(\sigma^*,\boldsymbol{\mu},\boldsymbol{\alpha})$. From Theorem 2, we know that t^* is optimal if and only if $(t^*,\sigma^*)\in\operatorname{argmax} U_t^I(\sigma,\mu_\sigma,\alpha_\sigma)$

subject to $\sigma \in \underline{\Lambda}(t) \cap \Gamma(t)$ and $t \in T$. Consider the set $\underline{\Lambda}(t^*) \cap \Gamma(t^*)$. Observe that

$$\underline{\Lambda}(t^*) \cap \Gamma(t^*) = \operatorname*{argmax}_{\sigma \in \underline{\Lambda}(t^*)} U^I_{t^*}(\sigma, \mu_\sigma, \alpha_\sigma). \tag{61}$$

Equivalently, we have

$$\begin{aligned} \underline{\Lambda}(t^*) \cap \Gamma(t^*) = &\operatorname*{argmax}_{\sigma \in \Lambda(t^*)} U^I_{t^*}(\sigma, \mu_\sigma, \alpha_\sigma) \\ &\text{subject to} \quad \sum_{\omega\in\Omega}\sum_{a\in A} a(\omega-\kappa-t^*_\omega)\sigma(s_a|\omega)\pi(\omega) = 0, \text{ and} \quad (62) \\ &\qquad\qquad\qquad \sum_{a\in A}\sigma(s_a|\omega) = 1, \text{ for all } \omega\in\Omega. \quad (63) \end{aligned}$$

From Lemma 6, $U^I_{t^*}(\sigma, \mu_\sigma, \alpha_\sigma)$ is continuously differentiable and strictly concave in σ. Thus, the first-order necessary conditions for the above problem and the condition $\sigma \in \Lambda(t^*)$ are sufficient for $\sigma^* \in \underline{\Lambda}(t^*) \cap \Gamma(t^*)$. Let μ denote the Lagrangian multiplier for the constraint (62) and let γ_ω denote the Lagrangian multiplier for the constraint (63). Then the first-order necessary conditions are

$$at^*_\omega - \lambda \log\left(\frac{\sigma(s_a|\omega)}{\varphi(s_a|\sigma)} + 1\right) + \mu a(\omega-\kappa-t^*_\omega) - \gamma_\omega = 0, \tag{64}$$

for all $(\omega, a) \in \Omega \times A$. After eliminating γ_ω, we have

$$t^*_\omega - \lambda \log\left[\frac{\sigma^*(s_1|\omega)}{\sigma^*(s_0|\omega)} \cdot \frac{\varphi(s_0|\sigma^*)}{\varphi(s_1|\sigma^*)}\right] = (\omega-\kappa-t^*_\omega)\mu, \tag{65}$$

for all $\omega \in \Omega$. Let (ℓ^*, r^*) denote the pair of posteriors induced by σ^*. Recall from Theorem 2 that t^* being optimal implies $V^B_{t^*}(r^*) = 0$. That is,

$$(1-r^*)(-\kappa-t_0) + r^*(1-\kappa-t_1) = 0. \tag{66}$$

By applying this fact and the mapping of σ^* to (ℓ^*, r^*) to (65), we get

$$\lambda\left[(1-r^*)\log\frac{1-r^*}{1-\ell^*} + r^*\log\frac{r^*}{\ell^*}\right] = (1-r^*)(-\kappa) + r^*(1-\kappa). \tag{67}$$

Furthermore, applying the fact that $V^B_{t^*}(\ell^*) \le 0$ and that $\mu \ge 0$, we get

$$-\lambda\left[(1-\ell^*)\log\frac{1-\ell^*}{1-r^*} + \ell^*\log\frac{\ell^*}{r^*}\right] \le (1-\ell^*)t^*_0 + \ell^* t^*_1 \le 0. \tag{68}$$

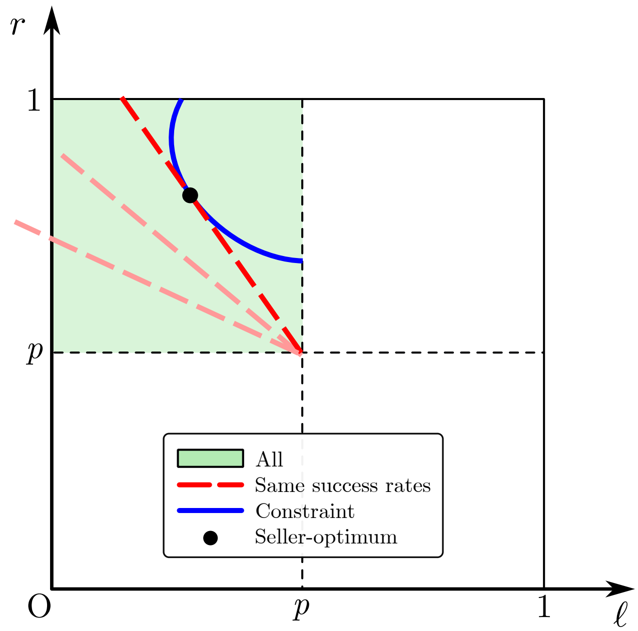

Figure 6: Finding the due diligence structure for a seller-optimal contract

Therefore, $\sigma^* \in \underline{\Lambda}(t^*) \cap \Gamma(t^*)$ if and only if (ℓ^*, r^*) and t^* satisfies (66)–(68). Then t^* is optimal if and only if $(\ell^*, r^*) \in \operatorname{argmax}_{(\ell,r)\in[0,p)\times(p,1]} \frac{p-\ell}{r-\ell}$ subject to (66)–(68). $\square$

Figures 6 and 7 illustrate how to find seller-optimal contracts as characterized in Corollary 2. The solid blue curve in Figure 6 represents the constraint (60) in the space of pairs of posteriors induced by binary due diligence structures. Because the left-hand side of the constraint is strictly concave in (ℓ, r) while the rigt-hand side is affine in (ℓ, r), the resulting curve is a convex curve. Each dashed red line represents the set of pairs (ℓ, r) that results in the same success rates; the flatter its slope, the greater the success rate. Therefore, finding the due diligence structure (ℓ^*, r^*) that maximizes the success rate entails finding the point of tangency between the solid curve and the dashed line. Once we know (ℓ^*, r^*), we can find the seller-optimal contracts that implements this due diligence structure from the equation (57) and inequalities (58)–(59), as illustrated by the thick red line segment in Figure 7.

All three constraints that determine the set of optimal contracts—one equality and two inequalities—have economic meaning. First, most importantly, the equality constraint (57) or the green solid line in Figure 7 requires that the buyer is just indifferent between investing and not investing, when he receives a truthful optimistic message. As a result, such contracts provide the intermediary higher rates of return on investment (ROI) than that to the buyer,

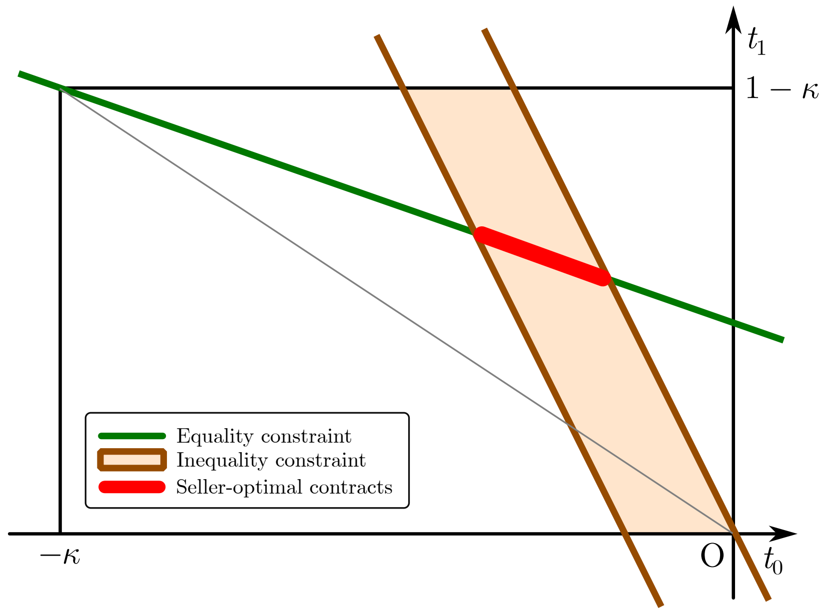

Figure 7: Seller-optimal contracts promise higher returns on investment to the intermediary than to the buyer

where we define the (gross) return on investment as

$$\text{Return on investment (ROI)} = \frac{\text{Dollars earned in the good state}}{\text{Dollars lost in the bad state}}. \tag{69}$$

Specifically, for an optimal contract $t^* = (t_0^*, t_1^*)$, the intermediary's ROI is $\frac{t_1^*}{-t_0^*}$ whereas the buyer's ROI is $\frac{1-\kappa-t_1^*}{\kappa+t_0^*}$; the overall ROI is $\frac{1-\kappa}{\kappa}$. With equation (57), we see that the intermediary's ROI is higher than both the buyer's ROI and the overall ROI. This result is illustrated by the fact that the solid green line is above the thin diagonal line that passes through the top-left and bottom-right corners of the rectangle in Figure 7. This result helps explain why entrepreneurs often choose to pay significant fees to investment banks: to give the intermediary a sufficiently high rate of return to incentivize a successful sale.

Second, the inequality (58) requires that the the chosen due-diligence structure is incentive-compatible with the intermediary. That is, the contracts to the left of the shaded area in Figure 7 cannot implement the desired success rate because, the contractual terms have such high stakes that the intermediary would want to collect more information. Namely, the intermediary would want to increase the optimistic posterior r to the right, deviating from the seller's optimum.

Third, the inequality (59) requires that the desired due-diligence structure remains incentive-aligned between the intermediary and the buyer. That is, the contracts to the right of the shaded area in Figure 7 biases the intermediary's incentives too much in favor of the seller—

the intermediary would prefer the buyer to invest regardless of the information collected under the chosen due diligence structure.

We conclude this section by presenting a sufficient condition for the existence of an optimal contract. In essence, an optimal contract exists when the cost of acquiring information is not prohibitively high:

Corollary 3. *There exists a threshold $\overline{\lambda} > 0$ such that an optimal contract exists if $\lambda \leq \overline{\lambda}$.*

Proof. We prove the statement by constructing such $\overline{\lambda}$. Let L denote the set of $\lambda > 0$ such that there exists some $(\ell, r) \in [0, p) \times (p, 1]$ that satisfies the equation (60). Let $t \in T$ be a contract. Fix any $\lambda' \in (0, \sup L)$, and let (ℓ^*, r^*) denote the solution to (60) with λ. Let $\sigma^* \in \widehat{\Sigma}$ denote the binary due diligence structure that induces (ℓ^*, r^*). Let t^* denote the unique contract that satisfies (57) and (59) with equality. Let (μ, α) be any best-responding, non-Pareto dominated pair of collections of message rules and action rules. Then there exists $\overline{\lambda} \in (0, \lambda']$ such that $U_t(\sigma, \mu_\sigma, \alpha_\sigma) \geq 0$ when the cost parameter is $\lambda \leq \overline{\lambda}$. Then t^* is optimal with any such λ, implementing a positive success rate $\frac{p-\ell^*}{r^*-\ell^*}$ with the equilibrium $(\sigma, \boldsymbol{\mu}, \boldsymbol{\alpha})$. □

5 Discussion

Our results make concrete predictions about investment banking contracts in a model in which an investment bank (intermediary) serves as a delegated communicator for an entrepreneur (seller) to an investor (buyer). Our first main result (Theorem 1) says that any successful contract must exhibit a shared risk of loss between the intermediary and the buyer. Our second main result (Theorem 2) says that a seller-optimal contract maximizes the intermediary's incentives in favor of the seller while maintaining a minimal alignment with the buyer's incentives. A corollary to the second result shows how to compute the set of optimal contracts and shows that optimal contracts provide a higher return to the intermediary than to the buyer.

It is straightforward to see from our analysis how this arrangement with an intermediary can strictly benefit the seller compared to scenarios without one. For instance, on the one hand, consider an alternative model in which the seller himself serves as the investment bank. The seller enters the IPO mechanism as the intermediary, with a critical restriction that $t_0 \geq 0$ and $t_1 \geq 0$—in other words, the entrepreneur cannot make any negative ex-post transfers to the buyer. This scenario is both legally and economically reasonable, as most firms impose limited liability on their shareholders, and the typical amount of capital raised in

IPOs far exceeds the net worth of the sellers. However, in this scenario, the seller's incentives are imbalanced, as he stands to gain a positive amount in both good and bad states after the buyer invests. Consequently, there is no due diligence structure that aligns the seller's and the buyer's incentives, meaning that there exists no equilibrium with positive success rate. This outcome holds true even if the entrepreneur, with insider knowledge about his own firm, has significantly lower cost of information than any investment bank. Therefore, intermediaries acting as delegated talkers are valuable when the seller himself cannot credibly commit to a shared risk of loss. Unlike the entrepreneur, investment banks with large capital can make the necessary commitment to bear significant losses.

On the other hand, consider another alternative model, in which the investor serves as the investment bank: the original buyer plays both the intermediary and the buyer in the IPO mechanism. Then the choice of the contract (t_0, t_1) is irrelevant because it represents the amount of transfers from the buyer to the buyer himself. Regardless of the contract, the buyer gets the payoff $-\kappa$ in the bad state and $1 - \kappa$ in the good state. As a result, the buyer collects information as the intermediary would in the original model under the contract $(t_0, t_1) = (-\kappa, 1 - \kappa)$ without the incentive-alignment constraint. As long as $\kappa < 2p$ and λ is sufficiently small, this arrangement cannot be seller-optimal, because the success rate approaches 0.5 as $\lambda \longrightarrow 0$ in this alternative model whereas the success rate in the original model approaches $p/\kappa > 0.5$ as $\lambda \longrightarrow 0$. In other words, letting the buyer carry out the due diligence results in collecting information more fairly than desired by the seller.

All in all, our findings demonstrate the critical role played by investment banks in initial public offerings (IPOs). Rather than assuming their expertise or reputational concerns, we show that their risk-bearing capacity makes their cheap talk credible and informative. Such delegated communicators are especially valuable when the seller cannot be held liable for ex-post losses to the buyer, which is precisely the case in IPOs as the buyers are purchasing ownership of the entrepreneur's company.

Although our paper has primarily focused on investment banks and IPOs as the most salient example, one can apply this model to similar settings where two parties have a substantial conflicts of interest. For instance, consider a high school student seeking admission to a college and the college's admission committee that aims to accept only the most promising students. Given the incentives of both, the student's application essay claiming his genius would not be credible. As a result, he needs an intermediary—his teacher or counselor—to evaluate him and send a recommendation letter. In this case, a school policy that optimal for the student would involve incentivizing the teacher to be maximally biased in the student's favor while maintaining minimal alignment with the admission committee's interests.

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