

# Causality and Causal Misperception in Dynamic Games

Sungmin Park

The Ohio State University

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# Motivation

Limited observation of reality  $\Rightarrow$  Varying perceptions of causality

- People have **different perceptions** about how **actions** affect **outcomes**



Smoking



Education



Police size



Social media

- Subjects in **lab experiments** look at the same data and tell different causal narratives (Kendall and Charles, 2022)
- Yet, most applications of game theory continue to assume **Rational Expectations (RE)**

# What I do

**Question**      What is a useful solution concept to incorporate people's **misperceptions** about **causality** in extensive-form games?

**Answer**          Let each player best respond to a **belief** about Nature and others' strategies **consistent with observed outcomes**

**Even better**    + let each player's belief be the **simplest explanation** consistent with observation

“**Maximum-entropy** Observation-consistent Equilibrium” (**MOE**)

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# Main Results

## Does it Exist?

Every **finite extensive-form game** with perfect recall and **observational constraint** has an MOE

## Is it Useful?

MOE captures **common causal misperceptions** such as

- Correlation neglect
- Omitted-variable bias (selection neglect)
- Simultaneity bias (reverse causality bias)

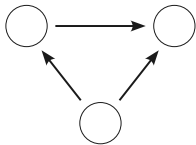
## Is it Compatible with RE?

If agents have **perfect observation** of outcomes,

- **OE**  $\Leftrightarrow$  Self-confirming equilibrium
- **MOE**  $\Leftrightarrow$  Perfect Bayesian Equilibrium (PBE)

# Literature

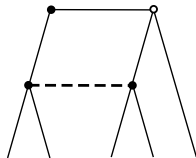
## Bridging behavioral theory and standard game theory



### Behavioral theory

(e.g. Spiegel, 2020, 2021)

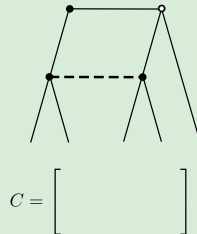
- Single-person decisions
- Directed Acyclic Graphs
- Subjective best responses



### Standard game theory

(e.g. Kreps and Wilson, 1982)

- Multiple players
- Rational expectations
- Objective best responses



### My paper (MOE)

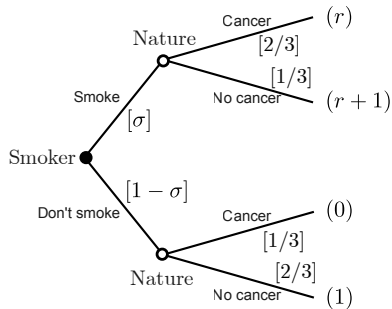
- Multiple players
- Observational structure ( $C$ )  
+ maximum entropy
- Subjective best responses



## Simplest Example

## Simplest example

- Player chooses to **smoke** ( $s = 1$ ) or **not** ( $s = 0$ ).
  - If he smokes, he gets cancer with prob  $\pi_1 = 2/3$ .
  - If not, he gets cancer with prob  $\pi_0 = 1/3$ .
  - He gets  $r < \frac{1}{3}$  if he smokes and loses 1 if he gets cancer.
- Player's **strategy** is the prob  $\sigma \in [0, 1]$  of smoking.
- Player's **belief** is  $\beta = (\beta_0, \beta_1)$  where  $\beta_s$  is the subjective probability of getting cancer given  $s$ .



**Smoker's Problem**

$\Rightarrow$  Under **RE**, one shouldn't smoke because the **causal effect** of smoking on cancer ( $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ ) is larger than the **reward**  $r$

# Observational consistency

**Assumption** Player observes only the marginal prob of cancer.

## Definition

Given strategy  $\sigma \in [0, 1]$ , a belief  $\beta \in [0, 1]^2$  is **observation-consistent** if

$$\underbrace{\sigma\beta_1 + (1 - \sigma)\beta_0}_{\text{perceived marginal prob of cancer}} = \underbrace{\sigma \cdot \frac{2}{3} + (1 - \sigma) \cdot \frac{1}{3}}_{\text{actual marginal prob of cancer}}$$

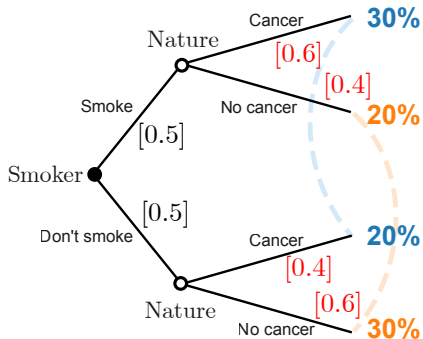
**Interpretation** Player sees a population of players choosing  $\sigma$  and sees the overall **rate of cancer** patients, but do not know the **conditional probabilities**.

**Problem** There are many observation-consistent beliefs.

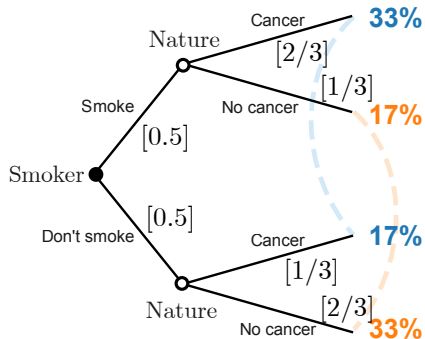
# Illustration of an observational consistency

Suppose I smoke half of the time ( $\sigma = 0.5$ ).

What I **think** Nature does



What Nature **really** does



# Principle of Maximum Entropy

## Notation

- $\mathbf{p}(\sigma, \beta)$ : vector of probabilities over the 4 terminal nodes.
- $G(\cdot)$ : Shannon entropy function, i.e.  $G(\mathbf{q}) = \sum -q \log q$

## Definition

Given strategy  $\sigma \in (0, 1)$ , an observation-consistent belief  $\beta^* \in [0, 1]^2$  **maximizes the entropy** if

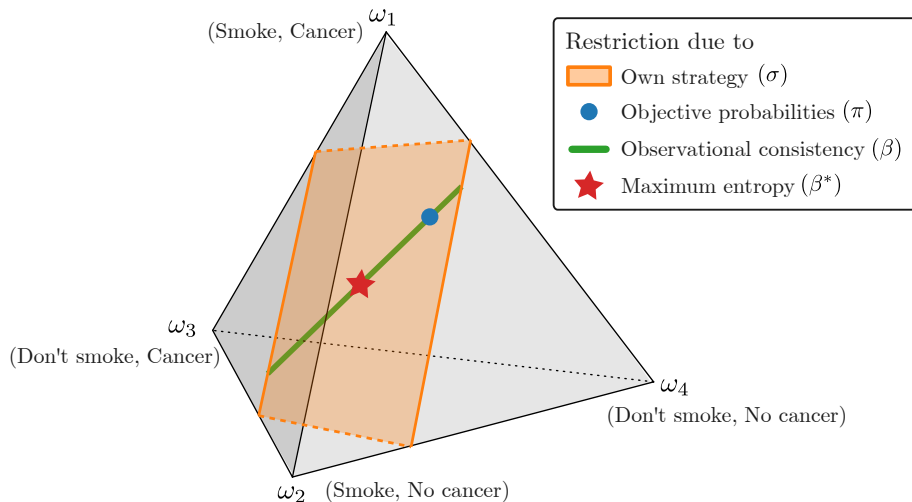
$$\beta^* \in \underset{\beta \text{ is observation-consistent}}{\operatorname{argmax}} G(\mathbf{p}(\sigma, \beta)).$$

## Interpretation

- Among many worldviews consistent with observation, the agent believes in the the one that **assumes the least information**

# Illustration of maximum entropy

A point prediction on belief



# Maximum entropy $\Rightarrow$ correlation neglect

## Claim

For every  $\sigma \in (0, 1)$ , the maximum-entropy belief  $\beta^*$  satisfies

$$\beta_0^* = \beta_1^* = (1 - \sigma) \cdot \frac{1}{3} + \sigma \cdot \frac{2}{3}.$$

**Meaning** Player doesn't think smoking **causes** cancer

**Intuition** Player observes **no evidence** of dependence between smoking and cancer, so he **believes in none**.

**General result** (Shore and Johnson, 1980; Csiszar, 1991)

**Correlation neglect**  $\Leftrightarrow$  **maximum entropy**, whenever agents observe only the marginal prob. distribution between two variables

# Equilibrium

Defined just for the Smoker's Problem

## Definition

A strategy-belief pair  $(\sigma, \beta)$  is an **observation-consistent equilibrium (OE)** if

- ① Given the belief  $\beta$ , the strategy  $\sigma$  is a **best response**, and
- ② Given the strategy  $\sigma$ , the belief  $\beta$  is **observation-consistent**.

## Interpretation

- OE is a **prediction** of how the smoker **behaves**, given his possibly wrong but observationally consistent belief



# OE is too permissive

Every strategy is rationalizable by some observation-consistent belief

## Claim

Every strategy  $\sigma$  has a belief  $\beta$  such that  $(\sigma, \beta)$  is an OE.

**Note:** Specifically, the OCE equilibria are

- ①  $\sigma = 0$ ,  $\beta_0 = \frac{1}{3}$ , and  $\beta_1 - \beta_0 \geq r$ ,
- ②  $\sigma = 1$ ,  $\beta_1 = \frac{2}{3}$ , and  $\beta_1 - \beta_0 \leq r$ , and
- ③  $\sigma \in (0, 1)$ ,  $\beta_0 = \sigma \cdot (\frac{2}{3} - r) + (1 - \sigma) \cdot \frac{1}{3}$ , and  $\beta_1 = \sigma \cdot \frac{2}{3} + (1 - \sigma)(\frac{1}{3} + r)$ .

**Idea** Because there are many observation-consistent beliefs, there are many OEs.

# Definition of MOE

## Definition

An OE  $(\sigma, \beta)$  is a **maximum-entropy observation-consistent equilibrium (MOE)** if  $\beta$  maximizes the entropy given  $\sigma \in (0, 1)$ .

- \* For  $\sigma \notin (0, 1)$ , an OE is an MOE if some  $\{(\sigma^k, \beta^k)\}_{k=1}^{\infty} \rightarrow (\sigma, \beta)$  and each  $\beta^k$  maximizes the entropy given  $\sigma^k$

## Interpretation

- **MOE** is an OE with the extra requirement that the smoker believes in the **simplest explanation** consistent with observation

# MOE provides a sharper prediction

## Claim

A strategy-belief pair  $(\sigma, \beta)$  is an MOE if and only if

$$\sigma = 1 \quad \text{and} \quad \beta_0 = \beta_1 = \frac{2}{3}.$$

## Meaning

- Player **keeps smoking** while thinking that smoking **doesn't cause cancer**

## Intuition

- Maximum-entropy belief features **correlation neglect**, so no other strategy is a best response.

## General Framework

# General framework

## Model

$(\Gamma, C)$  where

- $\Gamma$ : a finite extensive-form game with perfect recall, and
- $C$ : **observational structure**, a linear map from **outcomes**  $(\Delta(\Omega))$  to **observable outcomes**  $(\mathbb{R}^\ell)$

## Observational consistency

Given a strategy  $\sigma_i$ , a belief  $\beta_i$  is **observation-consistent** if

$$C\mathbf{p}(\sigma_i, \beta_i) = C\mathbf{p}(\sigma_i, (\sigma_{-i}, \pi)).$$

## Equilibrium (MOE)

A profile of **strategies**, **beliefs**, and **posterior functions** such that

- each strategy is **(subjectively) sequentially rational**,
- each belief **maximizes the entropy** s.t. obs consistency, and
- each posterior function satisfies **Bayes rule**

# Existence of MOE

## Theorem

Every finite extensive-form game with perfect recall and observational constraint has an MOE.

## Meaning

- There always exists a prediction where everyone **best responds** to what they **think** how others play, with a belief that assumes **the least information** beyond observation.

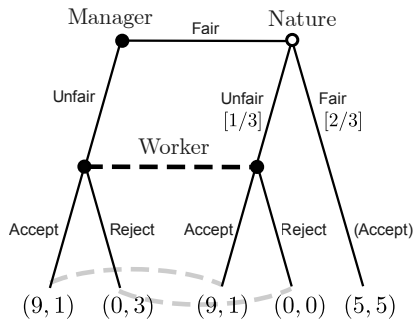
## Key proof step

- With  **$\epsilon$ -constrained strategies**, mappings from a strategy profile  $\sigma$  to a maximum-entropy beliefs  $\beta_i$  and posterior functions are well-behaved.

## Example: An ultimatum-game-like scenario

### Manager-Worker game

- Manager decides a **fair** or **unfair** bonus to Worker
- Even if Manager chooses a fair bonus, Nature might change it to **unfair** or keep it **fair**
- If Worker receives fair bonus, he accepts. If not, he either **accepts** or **rejects**.
  - He gets a thrill for rejecting an unfair Manager
- Worker doesn't know how likely Manager treats him unfairly in the **interim** or **ex post** (in a population)



$$C = \begin{bmatrix} 1 & \cdot & 1 & \cdot & \cdot \\ \cdot & 1 & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}$$

# Standard prediction

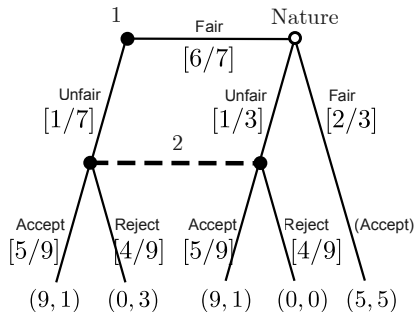
Manager often treats Worker unfairly

## Claim

In the unique **Perfect Bayesian Equilibrium (PBE)**,

- Manager offers an **unfair** bonus 1 out of 7 times
- Worker **accepts** an unfair bonus 5 out of 9 times
  - He infers (correctly) that any unfair offer is due to Manager 1 out of 3 times

## Perfect Bayesian Equilibrium



## Intuition

- There is **no causal misperception**, because there is no ex-ante uncertainty about others' strategies



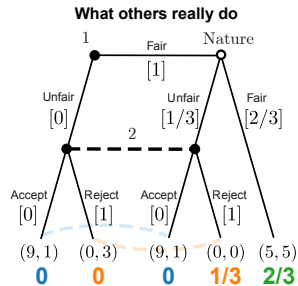
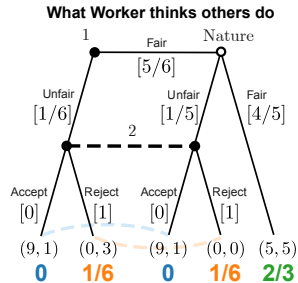
# MOE prediction: Manager always tries to be fair

## Claim

In the unique MOE,

- Manager always offers the **fair** bonus
  - She believes (correctly) that Worker will reject any unfair offer.
- Worker always **rejects** an unfair offer.
  - He believes (incorrectly) that Manager offers the unfair bonus 1 out of 6 times
  - He infers (incorrectly) that any unfair offer is caused by Manager 1 out of 2 times

**Intuition** Worker has **no clue** about the causes of his unfair treatment



## Discussion: How to test MOE in the lab

**Ideal experiment** Have lab subjects play a game with **different observational structures** (perfect and imperfect)

- 1 Randomly assign subjects into **Control** and **Treated** groups
- 2 Within each group, **randomly match** each subject with another and let them play 1 round of the game
- 3 Control players receive **perfect feedback** about all Control outcomes; Treated players receive **imperfect feedback** about all Treated outcomes
- 4 **Repeat** steps 2–3 for sufficiently many rounds



**Example** A simplified poker game (work in progress)

## MOE and Common Causal Misperceptions

- ① Correlation neglect
- ② Omitted-variable bias (selection neglect)
- ③ Simultaneity bias (reverse causality bias)

# 1. A two-stage game of correlated consequences

Players	$N = \{1, 2, \dots, n\}$
Stages	<ol style="list-style-type: none"><li>1. Players choose <b>actions</b> <math>x = (x_i)_{i \in N}</math>.</li><li>2. Nature chooses a <b>consequence</b> <math>y = (y_1, y_2)</math> with conditional probability <math>\pi(y x) &gt; 0</math> for all <math>(x, y)</math>.</li></ol>
Payoffs	$u_i(x, y)$
Obs. structure	Marginal probabilities of pairs $(x, y_1)$ and $(x, y_2)$

# Correlation neglect

## Proposition

An OE  $(\sigma, \beta, \mu)$  is a MOE if and only if for every player  $i$ ,

$$\beta_i(x_{-i}) = \sigma_{-i}(x_{-i}) \quad \text{for all } x_{-i}, \text{ and}$$

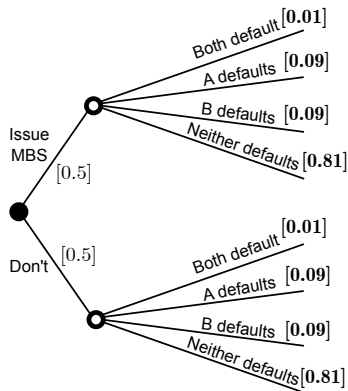
$$\beta_i(y_1, y_2 | x) = \pi(y_1 | x) \pi(y_2 | x) \quad \text{for all } x \text{ and } (y_1, y_2).$$

**Meaning** In an MOE, players believe  $y_1$  and  $y_2$  remain (conditionally) **independent** regardless of their actions  $x$ .

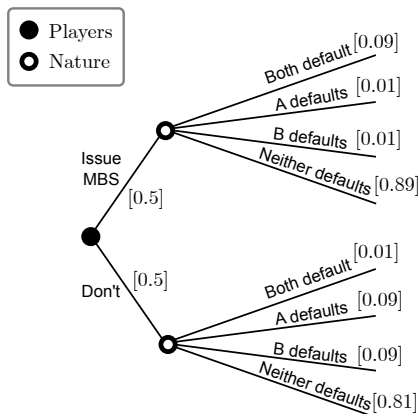
**Example (Acharya and Richardson, 2009)** Financial regulators **neglect the correlation** between **bank failures** under lenient regulation

# Stylized example of correlation neglect

What I think how mortgage-backed securities (MBS) work



How they really work



**Result** Regulators neglect that issuing MBS **causes** correlated defaults

## 2. An omitted-variable game

Players

$$N = \{1, 2, \dots, n\}$$

Stages

1. Nature assigns a **state**  $t$  with probability  $\pi(t)$ .
2. Players see the state  $t$  and choose **actions**  $x = (x_i)_{i \in N}$ .
3. Nature chooses a **consequence**  $y$  with probability  $\pi(y|t, x)$ .

Payoffs

$$u_i(t, x, y)$$

Obs. structure

Marginal probabilities of pairs  $(t, x)$  and  $(x, y)$

# Omitted-variable bias (selection neglect)

## Proposition

An OE  $(\sigma, \beta, \mu)$  is an MOE if and only if every player's belief  $\beta_i$  satisfies

$$\beta_i(t) = \pi(t),$$

$$\beta_i(x_{-i}|t) = \sigma_{-i}(x_{-i}|t), \text{ and}$$

$$\beta_i(y|t, x) = \sum_{t' \in \mathcal{T}} \pi(y|t', x) w(t', x) \quad \text{for all } (t, x, y).$$

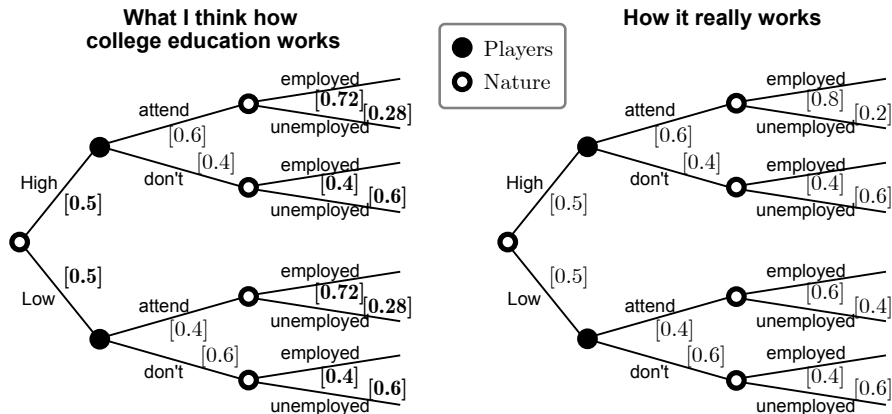
**Note:**  $w(\cdot)$  is a weight function  $w(t', x) = \lim_{k \rightarrow \infty} \frac{\sigma^k(x|t')\pi(t')}{\sum_{t'' \in \mathcal{T}} \sigma^k(x|t'')\pi(t'')},$

**Meaning** Players believe the **effect** of  $x$  on  $y$  is the **same** across states  $t$

**Example** High school graduates may **overestimate** or **underestimate** the value of college education



# Stylized example of omitted-variable bias



**Result** High-ability students underestimate the value of college education.

Low-ability students overestimate it.

### 3. Game with simultaneous causality

**Players**  $N = \{1, 2, \dots, n\}$

**Stages** (1) Nature assigns a **state**  $t \in \{\text{Forward}, \text{Reverse}\}$  with probability  $\pi(t)$ .

If  $t = F$ , (2) players learn  $t$  and choose **actions**  $x = (x_i)_{i \in N}$  and

(3) Nature chooses **consequence**  $y$  with prob  $\pi(y|F, x)$ .

If  $t = R$ , (2) Nature chooses **consequence**  $y$  with prob  $\pi(y|R)$  and

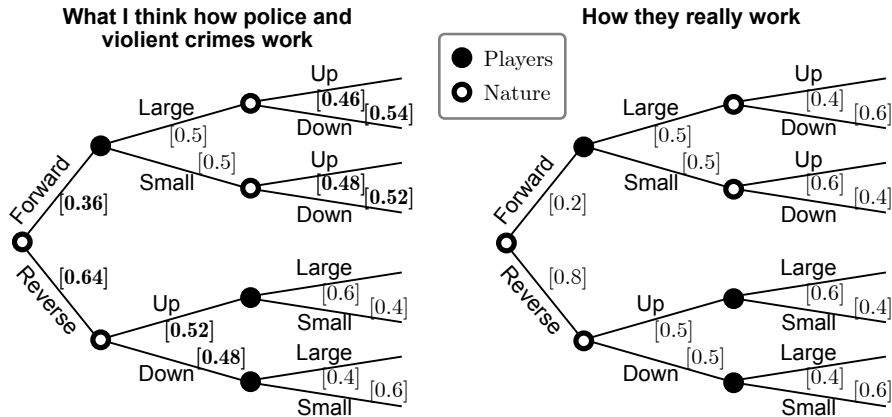
(3) players learn  $(t, y)$  and choose **actions**  $x = (x_i)_{i \in N}$ .

**Payoffs**  $u_i(t, x, y)$

**Obs. structure** Marginal probabilities of the pair  $(x, y)$

**Example** City mayors may **misperceive** the effects of police on reducing crimes

# Stylized example of simultaneity (reverse causality) bias



**Result** Mayor **underestimates** the effect of police on reducing crime

# Discussion: Implications for structural econometrics

## Rational expectations (RE) assumption

- “Ubiquitous” even though it’s a “very strong assumption”  
(Aguirregabiria and Mira, 2010)
- Relaxing it requires modeling and estimating beliefs  
(e.g., Aguirregabiria and Magesan, 2020)

## MOE assumption

- A viable alternative to RE by providing a point-prediction on beliefs
- Only requires an existing model + observational structure  $C$
- Example application: Models of education and occupational choice  
(e.g., Keane and Wolpin, 1997)

# Rest of the paper and takeaway

## Rest of the paper

- Comparison with related concepts ▶ Comparison
- Game-theoretic definition of causality ▶ Causality
- Cooperation in Centipede games ▶ Centipede game
- Games with infinite time horizons ▶ Markov games

**Takeaway:** MOE is useful if you want to

- allow **causal misperception** in a dynamic model,
- let misperception arise **endogenously** from the observational structure, and
- want **narrow predictions**.

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Thank you!

## Appendix

## Precise definitions in the general framework

**Strategy**  $\sigma_i \in \mathcal{S}_i$        $\sigma_i(a|I_i)$  is player  $i$ 's **objective prob** of action  $a$  by  $i$  at info set  $I_i$

**Belief**  $\beta_i \in \mathcal{S}_{-i}$        $\beta_i(a|I_j)$  is player  $i$ 's **subjective prob** of action  $a$  by Nature or an opponent at info set  $I_j$ .

**Posterior function**  $\mu_i$        $\mu_i(h|I_i)$  is player  $i$ 's **subjective prob** of **history**  $h \in I_i$  given  $I_i$ .

**“Assessment”**       $(\sigma, \beta, \mu) = \{(\sigma_i, \beta_i, \mu_i)\}_{i \in N}$



## Definition of OE

**Notation**  $\mathbf{p}(\sigma_i, \beta_i)$  is the subjective probability distribution over  $\Omega$

### Definition

An assessment  $(\sigma, \beta, \mu)$  is an **observation-consistent equilibrium (OE)** if for every player  $i$ ,

- 1 the strategy  $\sigma_i$  is (subjectively) sequentially rational given  $(\beta_i, \mu_i)$ ,
- 2 the belief  $\beta_i$  is observation-consistent given the strategy profile  $\sigma$ :

$$C\mathbf{p}(\sigma_i, \beta_i) = C\mathbf{p}(\sigma_i, (\sigma_{-i}, \pi)), \text{ and}$$

- 3 the posterior function  $\mu_i$  is Bayes-consistent given  $(\sigma_i, \beta_i)$ .

## Definition of MOE

Given a strategy profile  $\sigma$ , a player's observation-consistent belief  $\beta_i$  **maximizes the entropy** if

$$\beta_i \in \operatorname{argmax}_{\beta'_i \text{ is obs-cons}} G(\mathbf{p}(\sigma_i, \beta'_i)).$$

### Definition

An OE  $(\sigma, \beta, \mu)$  is a **maximum-entropy observation-consistent equilibrium (MOE)** if there exists a sequence

$$\{\sigma^k, \beta^k\}_{k=1}^{\infty} \longrightarrow (\sigma, \beta)$$

where each  $\sigma^k$  is a totally mixed strategy profile and each player's belief  $\beta_i^k$  maximizes the entropy given  $\sigma^k$ .

## OE and MOE nest standard concepts as special cases

### Proposition

Under **perfect observation** of outcomes ( $C = \text{identity}$ ),

$$\begin{array}{lll} \text{OE} & \iff & \text{Self-confirming equilibrium}^*, \text{ and} \\ \text{MOE} & \iff & \text{Perfect Bayesian equilibrium.} \end{array}$$

\* Version with sequential rationality.

### Implication

- Varying the extent of misperception is straightforward: Take an **existing model** and vary the **observational structure**  $C$ .

## Other related concepts

### Analogy-based expectation equilibrium (ABEE)

Jehiel (2005); Jehiel and Koessler (2008); Jehiel (2022)

- Players believe others **behave the same** in “analogous” situations

### Cursed (sequential) equilibrium

Eyster and Rabin (2005, **CE**); Fong, Lin and Palfrey (2023, **CSE**); Cohen and Li (2022, **SCE**)

- Players believe others **behave the same** regardless of their types/info

### Berk-Nash equilibrium

Esponda and Pouzo (2016)

- Players' beliefs about the **game** are **misspecified**

## Wait... what do I even mean by causality?

**Notation**  $p(\sigma_i, \beta_i)(E|h)$  is the subjective probability of **event**  $E \subset \Omega$  given **history**  $h$ , **strategy**  $\sigma_i$ , and **belief**  $\beta_i$ .

### Definition

Let  $(\sigma, \beta, \mu)$  be an OE. An action  $a$  instead of  $b$  is a **subjective cause** of an event  $E \subset \Omega$  given history  $h$  to player  $i$  if

$$p(\sigma_i, \beta_i)(E|h, a) > p(\sigma_i, \beta_i)(E|h, b).$$

An action  $a$  instead of  $b$  is an **objective cause** of an event  $E \subset \Omega$  given history  $h$  to player  $i$  if

$$p(\sigma_i, (\sigma_{-i}, \pi))(E|h, a) > p(\sigma_i, (\sigma_{-i}, \pi))(E|h, b).$$

## Example: A centipede game

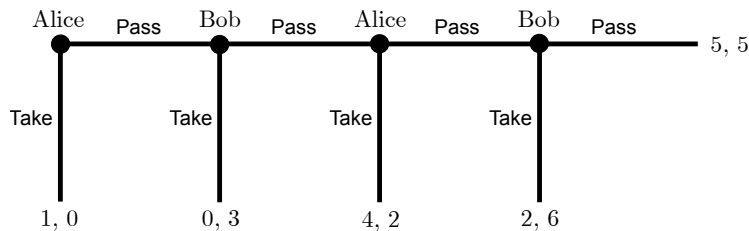


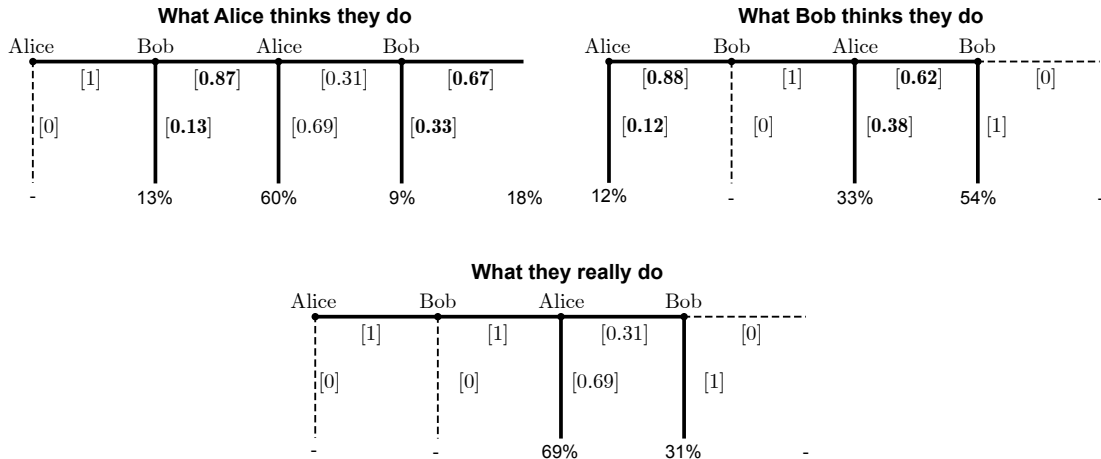
Figure: A four-node centipede game

### Claim

Suppose players observe only the average number of passes ( $C = [0 \ 1 \ 2 \ 3 \ 4]$ ). There exists no MOE in which Alice Takes immediately.

# Unique MOE of the centipede game

Each thinks the other mixes more than they really do



## Extension: Stochastic (Markov) Games

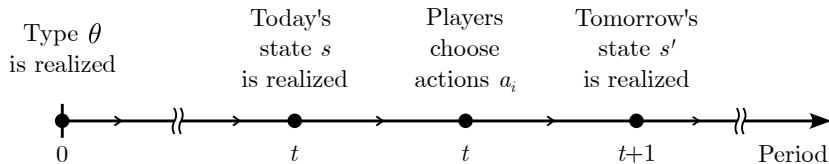


Figure: Stochastic game with permanent game types  $\theta$

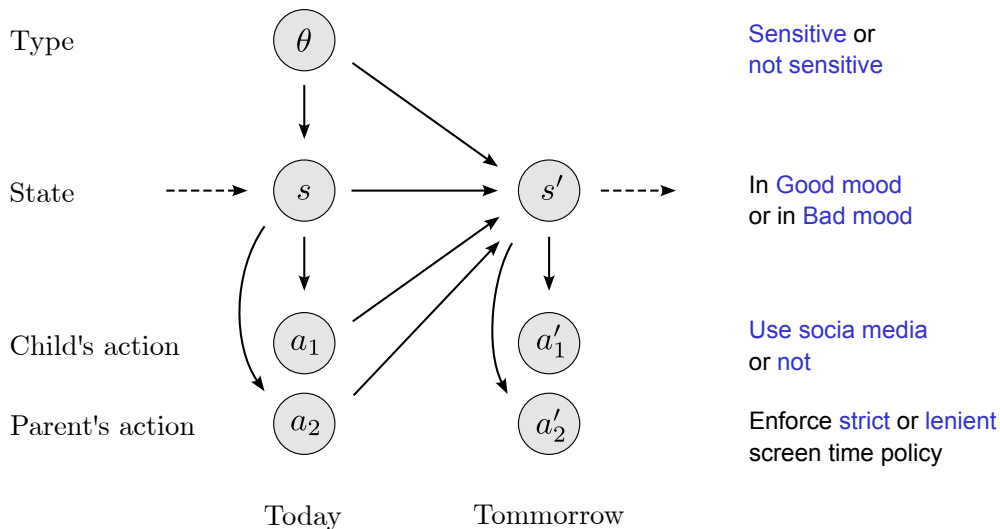
### Proposition

If players perfectly observe steady-state outcomes  $(\theta, s, a, s')$ ,

MOE  $\iff$  Markov perfect equilibrium (MPE).



## Illustration: Parent-Child game of social media use



## Equilibrium in the Parent-Child game

Equilibrium	Type ( $\theta$ )	Child's strategy ( $\sigma_1$ )		Parent's strategy ( $\sigma_2$ )	
		Bad mood	Good mood	Bad mood	Good mood
MPE	Not sensitive	Use	Use	Lenient	Lenient
	Sensitive	Don't	Use	Lenient	Lenient
MOE	Not sensitive	Use	Use	Strict	Lenient
	Sensitive	Use	Use	Strict	Lenient

**Note:** MPE refers to Markov perfect equilibrium. MOE refers to maximum-entropy observation-consistent equilibrium.

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