Causality and Causal Misperception in Dynamic Games

Sungmin Park

The Ohio State University

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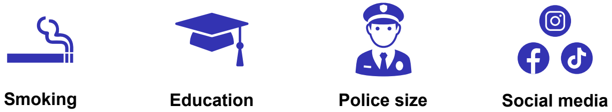
Warning: This is a theory paper

Motivation

Limited observation of reality $\Rightarrow$ Varying perceptions of causality

- People have different perceptions about how actions affect outcomes



- Subjects in lab experiments look at the same data and tell different causal narratives (Kendall and Charles, 2022)

- Yet, most applications of game theory continue to assume Rational Expectations (RE)

What I do

Question What is a useful solution concept to incorporate people's misperceptions about causality in extensive-form games?

Answer Let each player best respond to a belief about Nature and others' strategies consistent with observed outcomes

Even better + let each player's belief be the simplest explanation consistent with observation

"Maximum-entropy Observation-consistent Equilibrium" (MOE)

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Main Results

Does it Exist?

Every finite extensive-form game with perfect recall and observational constraint has an MOE

Is it Useful?

MOE captures common causal misperceptions such as

- Correlation neglect
- Omitted-variable bias (selection neglect)
- Simultaneity bias (reverse causality bias)

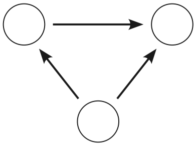
Is it Compatible with RE?

If agents have perfect observation of outcomes,

- OE $\Leftrightarrow$ Self-confirming equilibrium
- MOE $\Leftrightarrow$ Perfect Bayesian Equilibrium (PBE)

Literature

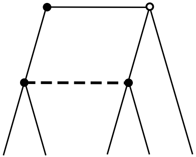
Bridging behavioral theory and standard game theory



Behavioral theory

(e.g. Spiegler, 2020, 2021)

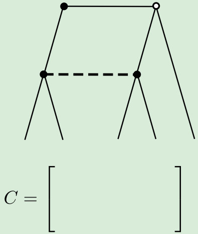
- Single-person decisions
- Directed Acyclic Graphs
- Subjective best responses



Standard game theory

(e.g. Kreps and Wilson, 1982)

- Multiple players
- Rational expectations
- Objective best responses



My paper (MOE)

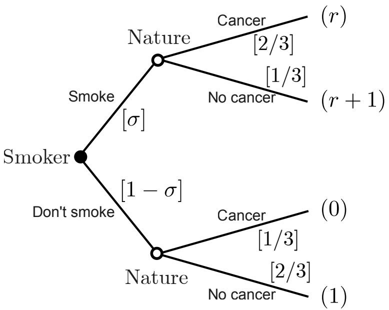
- Multiple players
- Observational structure (C)
- Subjective best responses

Simplest Example

Simplest example

- Smoker chooses to smoke ($s = 1$) or not ($s = 0$).
  - If he smokes, he gets cancer with prob $\pi_1 = 2/3$.
  - If not, he gets cancer with prob $\pi_0 = 1/3$.
  - He gets $r < \frac{1}{3}$ if he smokes and loses 1 if he gets cancer.
- Smoker's **strategy** is the prob $\sigma \in [0, 1]$ of smoking.
- Smoker's **belief** is $\beta = (\beta_0, \beta_1)$ where β_s is the subjective probability of getting cancer given s.

$\Rightarrow$ Under RE, one shouldn't smoke because the causal effect of smoking on cancer ($\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$) is larger than the reward r



Smoker's Problem

Observational consistency

Assumption Smoker observes only the marginal prob of cancer.

Definition

Given strategy $\sigma \in [0,1]$, a belief $\beta \in [0,1]^2$ is **observation-consistent** if

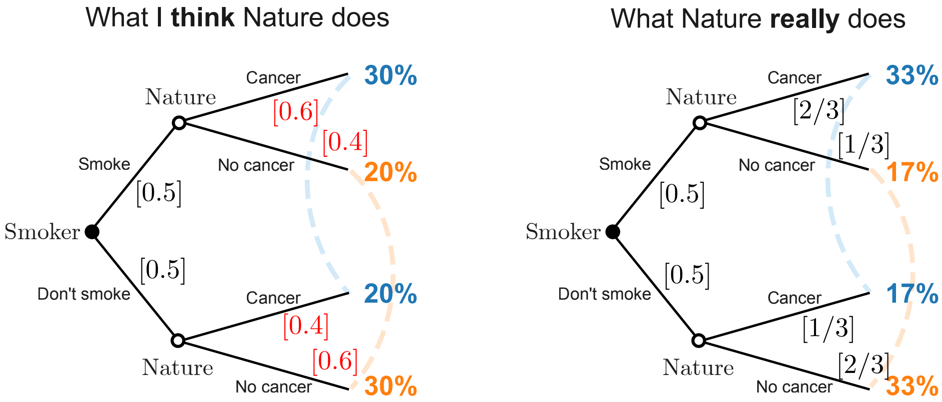
$$\underbrace{\sigma\beta_1 + (1-\sigma)\beta_0}_{\text{perceived marginal prob of cancer}} = \underbrace{\sigma \cdot \frac{2}{3} + (1-\sigma) \cdot \frac{1}{3}}_{\text{actual marginal prob of cancer}}$$

Interpretation Smoker sees a population of smokers choosing σ and sees the overall rate of cancer patients, but do not know the conditional probabilities.

Problem There are many observation-consistent beliefs.

Illustration of an observational consistency

Suppose I smoke half of the time ($\sigma = 0.5$).



Principle of Maximum Entropy

Notation

- $\mathbf{p}(\sigma, \beta)$: vector of probabilities over the 4 terminal nodes.
- $G(\cdot)$: Shannon entropy function, i.e. $G(\mathbf{q}) = \sum -q \log(q)$

Definition

Given strategy $\sigma \in (0,1)$, an observation-consistent belief $\beta^* \in [0,1]^2$ **maximizes the entropy** if

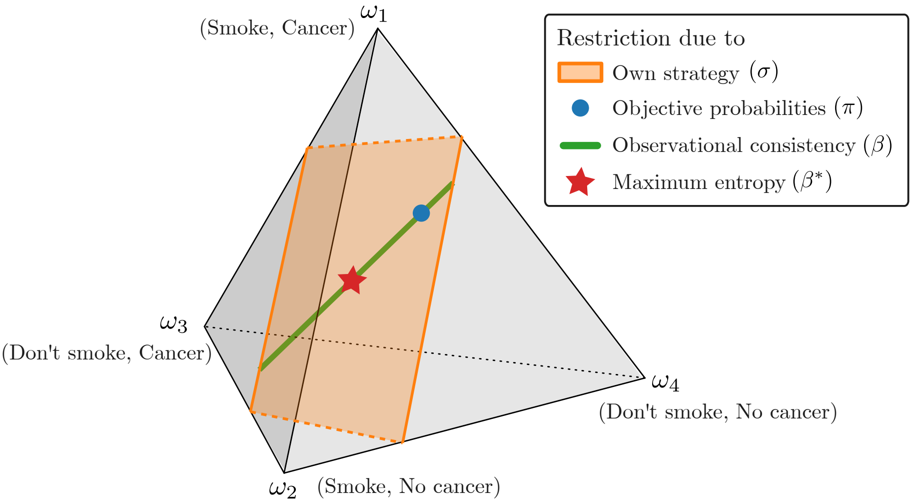
$$\beta^* \in \underset{\beta \text{ is observation-consistent}}{\operatorname{argmax}} G(\mathbf{p}(\sigma, \beta)).$$

Interpretation

- Among many worldviews consistent with observation, players believe in the the one that assumes the least information

Illustration of maximum entropy

A point prediction on belief



Maximum entropy $\Rightarrow$ correlation neglect

Claim

For every $\sigma \in (0,1)$, the maximum-entropy belief β^* satisfies

$$\beta_0^* = \beta_1^* = (1-\sigma)\cdot\frac{1}{3} + \sigma\cdot\frac{2}{3}.$$

Meaning The smoker doesn't think smoking causes cancer

Intuition The smoker observes no evidence of dependence between smoking and cancer, so he believes in none.

General result (Shore and Johnson, 1980; Csiszar, 1991)

Correlation neglect $\Leftrightarrow$ maximum entropy, whenever agents observe only the marginal prob. distribution between two variables

Equilibrium

Definition

A strategy-belief pair (σ, β) is an **observation-consistent equilibrium (OE)** if

1. Given the belief β, the strategy σ is a best response, and
2. Given the strategy σ, the belief β is observation-consistent.

Interpretation

- OE is a prediction of how the smoker behaves, given his possibly wrong but observationally consistent belief

OE is too permissive

Every strategy is rationalizable by some observation-consistent belief

Claim

Every strategy σ has a belief β such that (σ, β) is an OE.

Note: Specifically, the OCE equilibria are

1. $\sigma = 0$, $\beta_0 = \frac{1}{3}$, and $\beta_1 - \beta_0 \geq r$,
2. $\sigma = 1$, $\beta_1 = \frac{2}{3}$, and $\beta_1 - \beta_0 \leq r$, and
3. $\sigma \in (0, 1)$, $\beta_0 = \sigma \cdot (\frac{2}{3} - r) + (1 - \sigma) \cdot \frac{1}{3}$, and $\beta_1 = \sigma \cdot \frac{2}{3} + (1 - \sigma)(\frac{1}{3} + r)$.

Idea Because there are many observation-consistent beliefs, there are many OEs.

Definition of MOE

Definition

An OE (σ, β) is a **maximum-entropy observation-consistent equilibrium (MOE)** if β maximizes the entropy given $\sigma \in (0, 1)$.

* For $\sigma \notin (0, 1)$, an OE is an MOE if some $\{(\sigma^k, \beta^k)\}_{k=1}^{\infty} \to (\sigma, \beta)$ and each β^k maximizes the entropy given σ^k

Interpretation

- MOE is an OE with the extra requirement that the smoker believes in the simplest explanation consistent with observation

MOE provides a sharper prediction

Claim

A strategy-belief pair (σ, β) is an MOE if and only if

$$\sigma = 1 \quad \text{and} \quad \beta_0 = \beta_1 = \frac{2}{3}.$$

Meaning

- Smoker keeps smoking while thinking that smoking doesn't cause cancer

Intuition

- Maximum-entropy belief features correlation neglect, so no other strategy is a best response.

I. General Framework

General framework

Model (Γ, C) where

- Γ: a finite extensive-form game with perfect recall, and
- C: **observational structure**, a linear map from outcomes $(\Delta(\Omega))$ to observable outcomes $(\mathbb{R}^\ell)$

Observational consistency Given a strategy σ_i, a belief β_i is observation-consistent if

$$C\mathbf{p}(\sigma_i, \beta_i) = C\mathbf{p}(\sigma_i, (\sigma_{-i}, \pi)).$$

Equilibrium (MOE) A profile of strategies, beliefs, and posterior functions such that

- each strategy is (subjectively) sequentially rational,
- each belief maximizes the entropy s.t. obs consistency, and
- each posterior function satisfies Bayes rule

Existence of MOE

Theorem

Every finite extensive-form game with perfect recall and observational constraint has an MOE.

Meaning

- There always exists a prediction where everyone best responds to what they think how others play, with a belief that assumes the least information beyond observation.

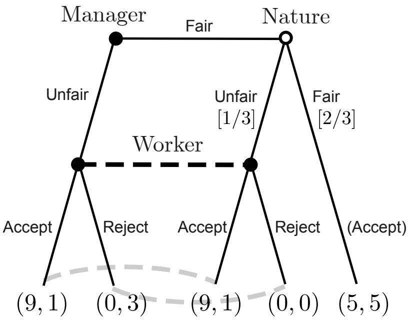
Key proof step

- With ϵ-constrained strategies, mappings from a strategy profile σ to a maximum-entropy beliefs β_i and posterior functions are well-behaved.

Example: An ultimatum-game-like scenario

Manager-Worker game

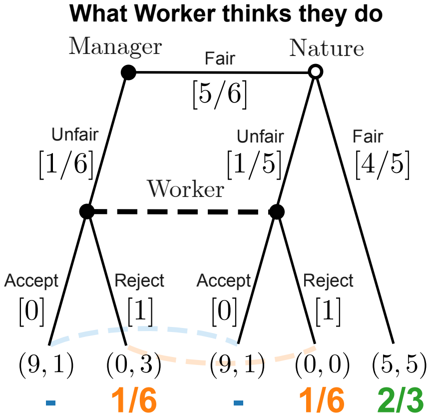
- Manager decides a fair or unfair bonus to Worker
- Even if Manager chooses a fair bonus, Nature might change it to unfair or keep it fair
- If Worker receives fair bonus, he accepts. If not, he either accepts or rejects.
  - He gets a thrill for rejecting an unfair Manager
- Worker doesn't know how likely Manager treats him unfairly in the interim or ex post (in a population)

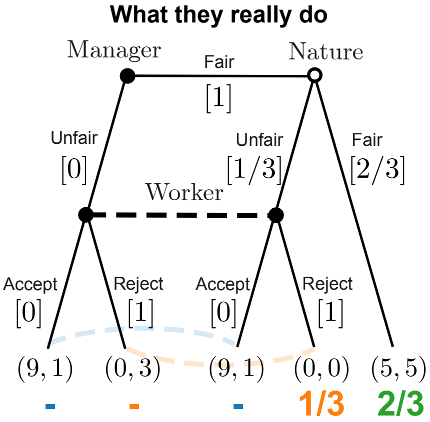


$$C = \begin{bmatrix} 1 & \cdot & 1 & \cdot & \cdot \\ \cdot & 1 & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}$$

Unique MOE

Manager always tries to be fair





Lesson With limited observation, players believe others mix more than they really do

II. MOE and Common Causal Misperceptions

1. Correlation neglect
2. Omitted-variable bias (selection neglect)
3. Simultaneity bias (reverse causality bias)

1. A two-stage game of correlated consequences

Players	$N = \{1, 2, \ldots, n\}$
Stages	**1.** Players choose actions $x = (x_i)_{i \in N}$. **2.** Nature chooses a consequence $y = (y_1, y_2)$ with conditional probability $\pi(y\|x) > 0$ for all (x, y).
Payoffs	$u_i(x, y)$
Obs. structure	Marginal probabilities of pairs (x, y_1) and (x, y_2)

Correlation neglect

Proposition

An OE (σ, β, μ) is a MOE if and only if for every player i,

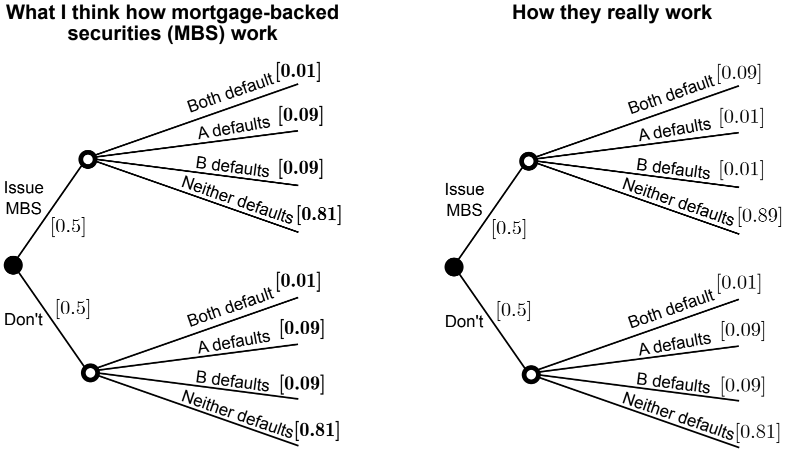
$$\beta_i(x_{-i}) = \sigma_{-i}(x_{-i}) \qquad \text{for all } x_{-i}, \text{ and}$$

$$\beta_i(y_1, y_2|x) = \pi(y_1|x)\pi(y_2|x) \qquad \text{for all } x \text{ and } (y_1, y_2).$$

Meaning In an MOE, players believe y_1 and y_2 remain (conditionally) independent regardless of their actions x.

Example Let x be whether an investment bank issues mortgage-backed securities (MBS) or not. Let y be the default outcomes of two households.

Stylized example of correlation neglect



Result Investment banks issue MBS, neglecting that those cause financial crises

2. An omitted-variable game

Players $N = \{1, 2, \dots, n\}$

Stages

1. Nature assigns a state t with probability $\pi(t)$.
2. Players see the state t and choose actions $x = (x_i)_{i \in N}$.
3. Nature chooses a consequence y with probability $\pi(y|t, x)$.

Payoffs $u_i(t, x, y)$

Obs. structure Marginal probabilities of pairs (t, x) and (x, y)

Omitted-variable bias (selection neglect)

Proposition

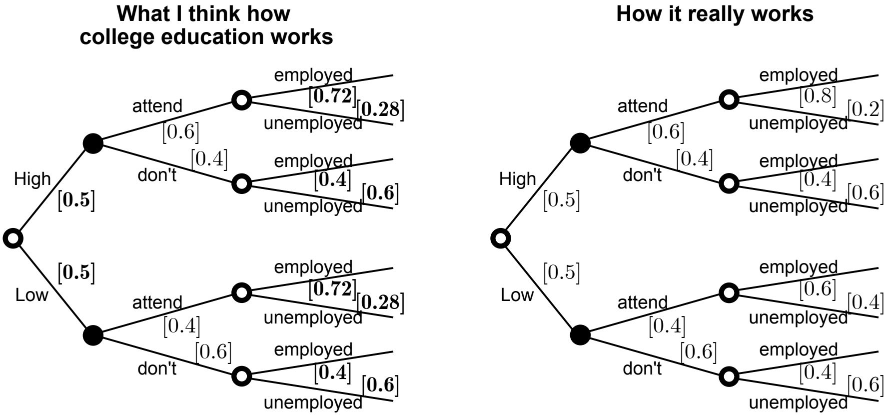
An OE (σ, β, μ) is an MOE if and only if every player's belief β_i satisfies

$$\beta_i(t) = \pi(t),$$
$$\beta_i(x_{-i}|t) = \sigma_{-i}(x_{-i}|t), \text{ and}$$
$$\beta_i(y|t,x) = \sum_{t' \in \mathcal{T}} \pi(y|t',x)w(t',x) \qquad \text{for all } (t,x,y).$$

Note: $w(\cdot)$ is a weight function such that $w(t',x) = \lim_{k\to\infty} \frac{\sigma^k(x|t')\pi(t')}{\sum_{t''\in\mathcal{T}} \sigma^k(x|t'')\pi(t'')}$, for some sequence $\{\sigma^k\}_{k=1}^{\infty}$ of totally mixed strategy profiles converging to σ.

Meaning Players believe the effect of x on y is the same across states t

Stylized example of omitted-variable bias



Result High-ability students underestimate the value of college education.

Low-ability students overestimate it.

3. Simultaneous causality game

Players $N = \{1, 2, \ldots, n\}$

Stages **(1)** Nature assigns a state $t \in \{\text{Forward}, \text{Reverse}\}$ with probability $\pi(t)$.

If $t = F$, **(2)** players learn t and choose actions $x = (x_i)_{i \in N}$ and
(3) Nature chooses consequence y with prob $\pi(y|F, x)$.

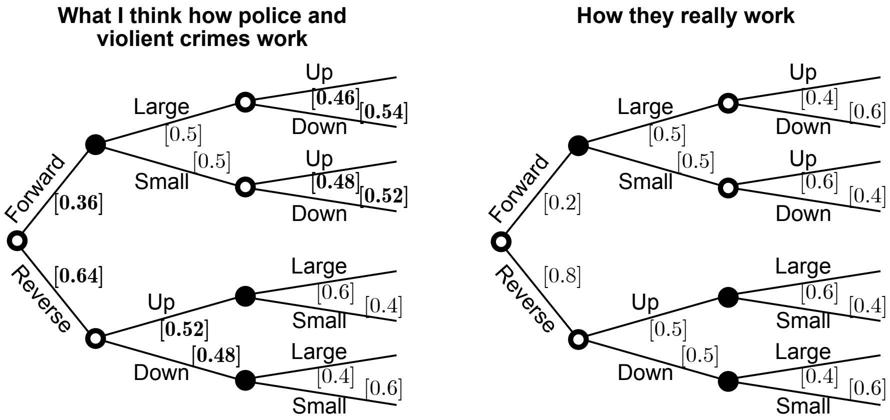
If $t = R$, **(2)** Nature chooses consequence y with prob $\pi(y|R)$ and
(3) players learn (t, y) and choose actions $x = (x_i)_{i \in N}$.

Payoffs $u_i(t, x, y)$

Obs. structure Marginal probabilities of the pair (x, y)

Stylized example of simultaneity (reverse causality) bias

Police size and violent crime rates



Result The police chief underestimates the effect of police on reducing crime

III. Relationship with existing solution concepts

OE and MOE nest standard concepts as special cases

Proposition

Under perfect observation of outcomes ($C =$ identity),

$$\begin{aligned} \text{OE} &\iff \text{Self-confirming equilibrium}^*, \quad \text{and} \\ \text{MOE} &\iff \text{Perfect Bayesian equilibrium}. \end{aligned}$$

***** Version with sequential rationality.

Implication

- Varying the extent of misperception is straightforward: Take an existing model and vary the observational structure C.

Other related concepts

Analogy-based expectation equilibrium (ABEE)

Jehiel (2005); Jehiel and Koessler (2008); Jehiel (2022)

- Players believe others behave the same in "analogous" situations

Cursed (sequential) equilibrium

Eyster and Rabin (2005, **CE**); Fong, Lin and Palfrey (2023, **CSE**); Cohen and Li (2022, **SCE**)

- Players believe others behave the same regardless of their types/info

Berk-Nash equilibrium

Esponda and Pouzo (2016)

- Players' beliefs about the game are misspecified

Takeaway

MOE is useful if you want to

- allow causal misperception in a dynamic model,
- let misperception arise endogenously from the observational structure, and
- want narrow predictions.

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Thank you!

Appendix

References I

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