Causality and Causal Misperception in Dynamic Games

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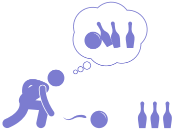
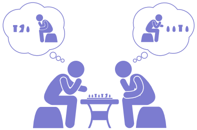
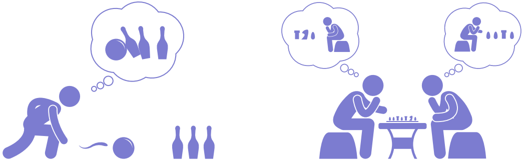




What I do

Question How should we capture players' misperceptions about causality in extensive-form games?

Answer Let each player best respond to a belief about Nature's and other players' strategies while requiring that belief to be observationally equivalent to how they actually play



"Observation-consistent expectations" (OCE) equilibrium

Motivation

Limited observation of reality $\Rightarrow$ Varying perceptions of causality

People have different perceptions about

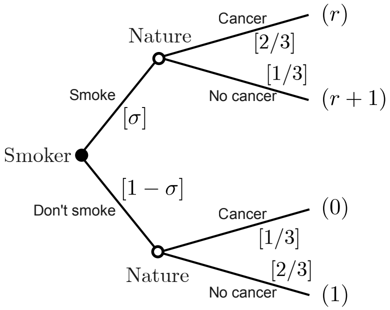
- Effects of smoking tobacco on cancer
- Effects of college degrees on labor market outcomes
- Effects of police presence on violent crimes
- Effects of social media on mental health

Subjects in lab experiments look at the same data and tell different causal narratives (Kendall and Charles, 2022)

$\Rightarrow$ **Q:** What is a useful solution concept that maintains sharp predictions while relaxing rational expectations?

Simplest example

- Smoker chooses to smoke ($s = 1$) or not ($s = 0$).
  - If he smokes, Nature gives him cancer with prob $\pi_1 = 2/3$.
  - If not, Nature gives him cancer with prob $\pi_0 = 1/3$.
- He gets $r < \frac{1}{3}$ if he smokes and loses 1 if he gets cancer.
- Smoker's **strategy** is the prob $\sigma \in [0, 1]$ of smoking.
- Smoker's **belief** is $\beta = (\beta_0, \beta_1)$ where β_s is the subjective probability of getting cancer given s.



Smoker's Problem

$\Rightarrow$ Under rational expectations, one shouldn't smoke because the causal effect of smoking on cancer ($\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$) is larger than the reward r

Observation-consistent expectations

Definition

Given strategy $\sigma \in [0,1]$, an **observation-consistent expectation (OCE)** is a belief $\beta \in [0,1]^2$ such that

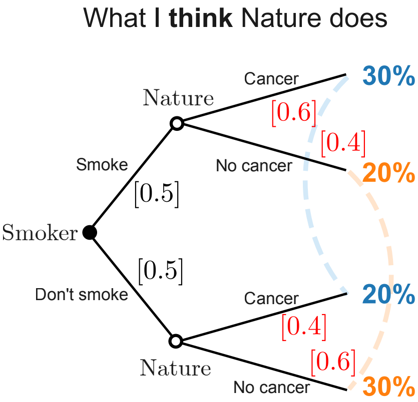
$$\underbrace{\sigma\beta_1 + (1-\sigma)\beta_0}_{\text{perceived marginal prob of cancer}} \quad = \quad \underbrace{\sigma \cdot \frac{2}{3} + (1-\sigma) \cdot \frac{1}{3}}_{\text{actual marginal prob of cancer}}$$

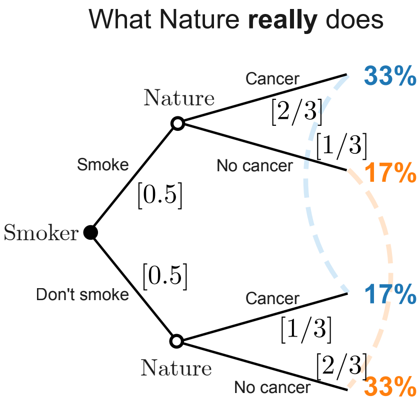
Interpretation

- Smoker sees a population of smokers choosing σ overall and sees the overall rate of cancer patients, but do not know the conditional probabilities
- What the smoker thinks Nature does (β_0, β_1) and what Nature really does $(\frac{1}{3}, \frac{2}{3})$ are observationally equivalent

Illustration of an OCE

Suppose I smoke half of the time ($\sigma = 0.5$).





Remark: There are many OCEs.

MaxEnt OCE

Let $\mathbf{p}(\sigma, \beta)$ be the vector of probabilities over the 4 terminal nodes.

Definition (MaxEnt OCE)

Given strategy $\sigma \in (0, 1)$, an OCE $\beta^* \in [0, 1]^2$ is a **maximum entropy (MaxEnt) OCE** if it satisfies

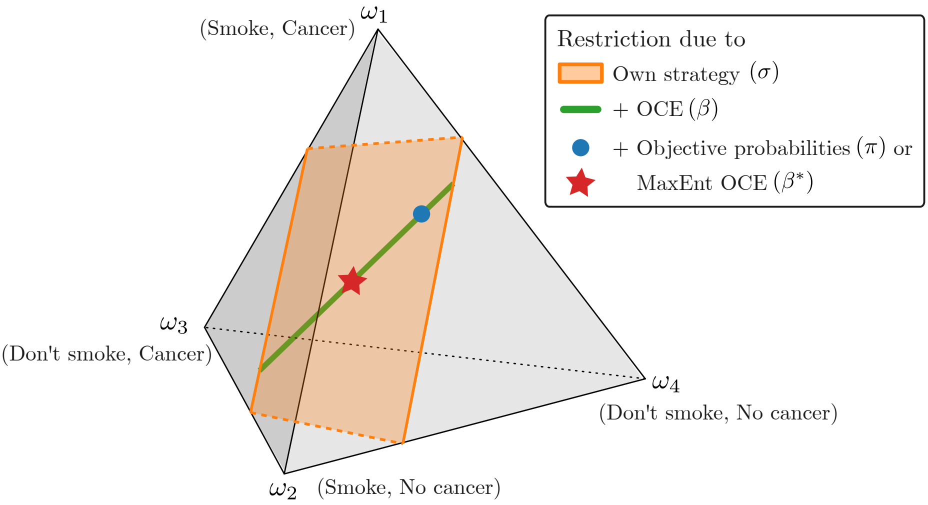
$$\beta^* \in \underset{\beta \in OCE(\sigma)}{\operatorname{argmax}} \; G(\mathbf{p}(\sigma, \beta))$$

where $G(\cdot)$ is the Shannon entropy function.

Interpretation

- MaxEnt OCE is the OCE with the least specific information.

Illustration of MaxEnt OCE



MaxEnt OCE $\Rightarrow$ correlation neglect

Claim

For every $\sigma \in (0,1)$, the MaxEnt OCE β^* satisfies

$$\beta_0^* = \beta_1^* = (1-\sigma)\cdot\frac{1}{3} + \sigma\cdot\frac{2}{3}.$$

Meaning The smoker doesn't think smoking causes cancer

Intuition The smoker observes no evidence of dependence between smoking and cancer, so he believes in none.

General result (Shore and Johnson, 1980; Csiszar, 1991)
MaxEnt OCE $\Leftrightarrow$ correlation neglect, whenever agents observe only the marginal prob. distribution between two variables

Definition of equilibrium

Motivation

- OCE and MaxEnt OCE take the strategy σ as given
- Is there an equilibrium where this σ is subjectively optimal?

Definition

A strategy-belief pair (σ, β) is an **OCE equilibrium** if

1. Given the belief β, the strategy σ is a best response, and
2. Given the strategy σ, the belief β is an OCE.

An OCE equilibrium (σ, β) is a **MaxEnt OCE equilibrium** if some $\{(\sigma^k, \beta^k)\}_{k=1}^{\infty} \to (\sigma, \beta)$ where each β^k is the MaxEnt OCE given σ^k.

Result: OCE equilibria

Every strategy is rationalizable with some OCE

Claim

Every strategy σ has a belief β such that (σ, β) is an OCE equilibrium.

Note: Specifically, the OCE equilibria are

1. $\sigma = 0$, $\beta_0 = \frac{1}{3}$, and $\beta_1 - \beta_0 \geq r$,
2. $\sigma = 1$, $\beta_1 = \frac{2}{3}$, and $\beta_1 - \beta_0 \leq r$, and
3. $\sigma \in (0, 1)$, $\beta_0 = \sigma \cdot (\frac{2}{3} - r) + (1 - \sigma) \cdot \frac{1}{3}$, and $\beta_1 = \sigma \cdot \frac{2}{3} + (1 - \sigma)(\frac{1}{3} + r)$.

Idea Because there are many OCEs, there are many OCE equilibria.

Result: MaxEnt OCE equilibrium

A sharper prediction

Claim

A strategy-belief pair (σ, β) is a MaxEnt OCE equilibrium if and only if

$$\sigma = 1 \quad \textit{and} \quad \beta_0 = \beta_1 = \frac{2}{3}.$$

Meaning

- Smoker keeps smoking while thinking that smoking doesn't cause cancer

Intuition

- MaxEnt OCE implies correlation neglect, so no other strategy is a best response.

Generalizing the observational constraint

Motivation

- Correlation neglect sounds too naïve. Can we make agents more sophisticated? Yes! Give them better observation

Definition

Given an **observation constraint matrix** C and strategy σ, an **OCE** is a belief β such that

$$C\mathbf{p}(\sigma, \beta) = C\mathbf{p}(\sigma, \pi).$$

Examples of C:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

Generalizing the approach to extensive-form games

Finite extensive-form game with perfect recall + observational constraint

- N: set of players,
- H: set of histories (nodes),
- ι: mapping of non-terminal histories to players,
- π: probability distribution of Nature's moves,
- $\mathcal{I}$: collection of information sets,
- u: payoff function, and
- C: **observational constraint matrix**

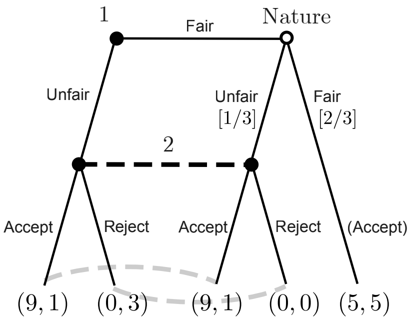
Theorem

Every finite extensive-form game with perfect recall and observational constraint has a MaxEnt OCE equilibrium.

Example: Ultimatum-like game with causal misperception

Manager-Worker game

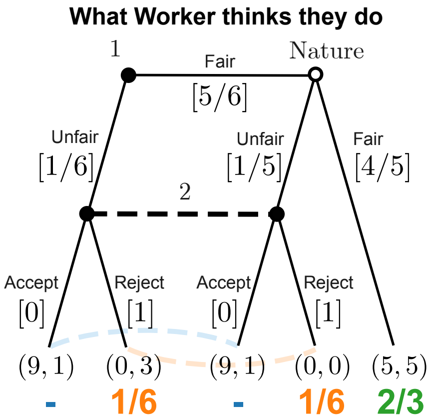
- Manager (Player 1) decides a fair or unfair bonus to Worker (Player 2)
- Even if Manager chooses a fair bonus, Nature might change it to unfair or keep it fair
- If Worker receives fair bonus, he accepts. If not, he either accepts or rejects.
    - He gets a thrill for rejecting an unfair Manager
- Worker doesn't know how likely Manager treats him unfairly in the interim or ex post

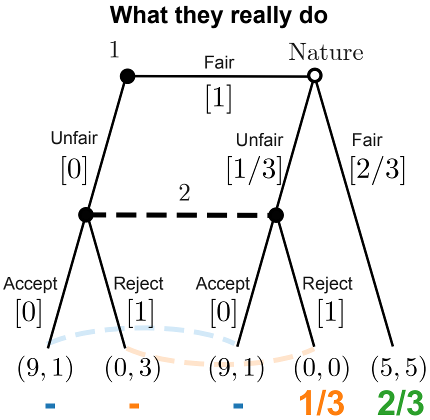


$$C = \begin{bmatrix} 1 & \cdot & 1 & \cdot & \cdot \\ \cdot & 1 & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}$$

Unique MaxEnt OCE equilibrium

Manager always tries to be fair





Example: A centipede game

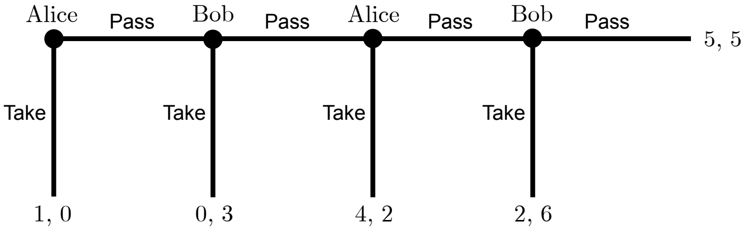


Figure: A four-node centipede game

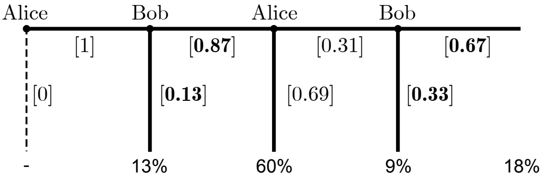
Claim

Let the observational structure be $C = [0\ 1\ 2\ 3\ 4]$. There exists no MaxEnt OCE equilibrium in which Alice Takes immediately.

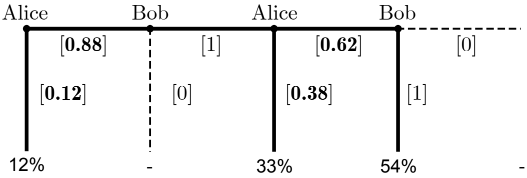
Unique equilibrium of the centipede game

Aligned with experimental results such as Healy (2017)

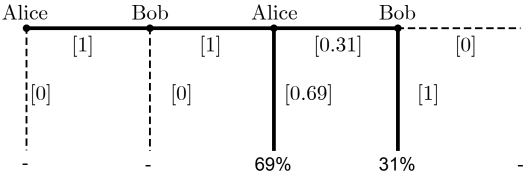
What Alice thinks they do

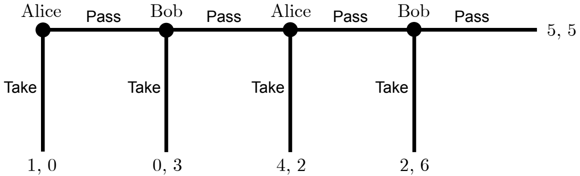


What Bob thinks they do



What they really do



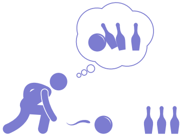


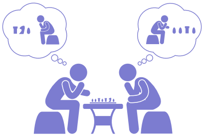
Takeaways

Use my solution concept if you want to . . .

- allow causal misperception in a dynamic model
- let misperception arise endogenously from observational constraints, and
- want narrow predictions

Thank you!





Appendix

References I

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