Causality and Causal Misperception in Dynamic Games

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Warning: This is a theory paper

What I do

Question What is a useful solution concept to incorporate people's misperceptions about causality in extensive-form games?

Answer Let each player best respond to a belief about Nature and others' strategies consistent with observed outcomes

Even better + let each player's belief be the simplest explanation consistent with observation

"Maximum-entropy Observation-consistent Equilibrium" (MOE)

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Motivation

Limited observation of reality $\Rightarrow$ Varying perceptions of causality

- People have different perceptions about how actions affect outcomes

- Subjects in lab experiments look at the same data and tell different causal narratives (Kendall and Charles, 2022)

- Yet, most applications of game theory continue to assume Rational Expectations (RE)

$\Rightarrow$ **Q**: How should we relax RE while maintaining sharp predictions?

Main Results

Does it Exist? Yes. Every finite extensive-form game with perfect recall and observational constraint has an MOE

Is it Useful? Yes. MOE captures common causal misperceptions such as

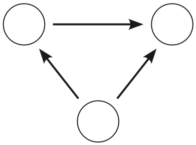
- Correlation neglect
- Omitted-variable bias (selection neglect)
- Simultaneity bias (reverse causality bias)

Is it Compatible with RE? Yes. If agents have perfect observation of outcomes,

- OE $\Leftrightarrow$ Self-confirming equilibrium
- MOE $\Leftrightarrow$ Perfect Bayesian Equilibrium (PBE)
- (with infinite horizons) MOE $\Leftrightarrow$ Markov Perfect Equilibrium (MPE)

Literature

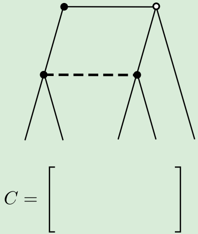
Bridging behavioral theory and standard game theory



Behavioral theory

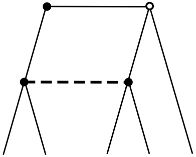
(e.g. Spiegler, 2020, 2021)

- Single-person decisions
- Directed Acyclic Graphs
- Maximum-entropy beliefs
- Subjective best responses



My paper (MOE)

- Multiple players
- Observational structure (C)
- Maximum-entropy beliefs
- Subjective best responses



Standard game theory

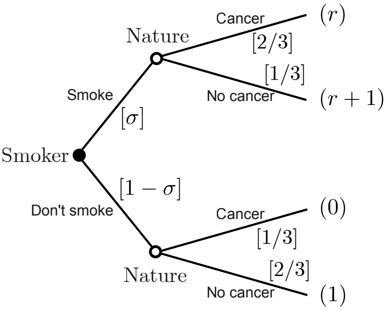
(e.g. Kreps and Wilson, 1982)

- Multiple players
- Perfect observation
- Correct beliefs
- Objective best responses

Simplest Example

Simplest example

- Smoker chooses to smoke ($s = 1$) or not ($s = 0$).
  - If he smokes, Nature gives him cancer with prob $\pi_1 = 2/3$.
  - If not, Nature gives him cancer with prob $\pi_0 = 1/3$.
- He gets $r < \frac{1}{3}$ if he smokes and loses 1 if he gets cancer.
- Smoker's **strategy** is the prob $\sigma \in [0, 1]$ of smoking.
- Smoker's **belief** is $\beta = (\beta_0, \beta_1)$ where β_s is the subjective probability of getting cancer given s.



Smoker's Problem

$\Rightarrow$ Under rational expectations, one shouldn't smoke because the causal effect of smoking on cancer ($\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$) is larger than the reward r

Observational consistency

Assumption Smoker observes only the marginal prob of cancer.

Definition

Given strategy $\sigma \in [0,1]$, a belief $\beta \in [0,1]^2$ is **observation-consistent** if

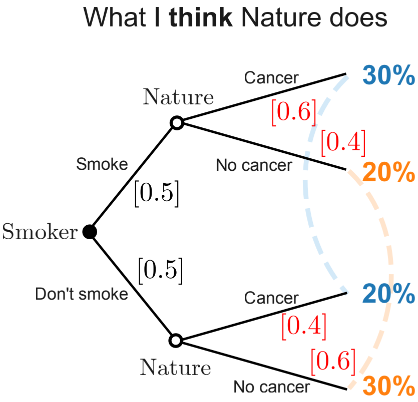
$$\underbrace{\sigma\beta_1 + (1-\sigma)\beta_0}_{\text{perceived marginal prob of cancer}} = \underbrace{\sigma \cdot \frac{2}{3} + (1-\sigma) \cdot \frac{1}{3}}_{\text{actual marginal prob of cancer}}$$

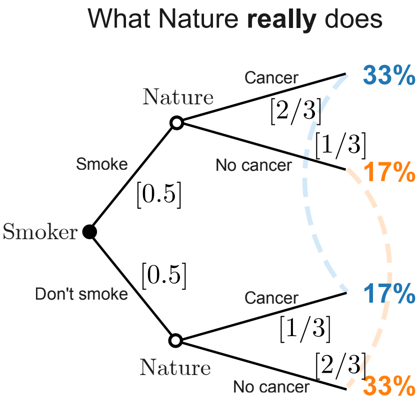
Interpretation Smoker sees a population of smokers choosing σ and sees the overall rate of cancer patients, but do not know the conditional probabilities.

Problem There are many observation-consistent beliefs.

Illustration of an observational consistency

Suppose I smoke half of the time ($\sigma = 0.5$).





Principle of Maximum Entropy

Notation

- $\mathbf{p}(\sigma, \beta)$: vector of probabilities over the 4 terminal nodes.
- $G(\cdot)$: Shannon entropy function.

Definition

Given strategy $\sigma \in (0, 1)$, an observation-consistent belief $\beta^* \in [0, 1]^2$ **maximizes the entropy** if

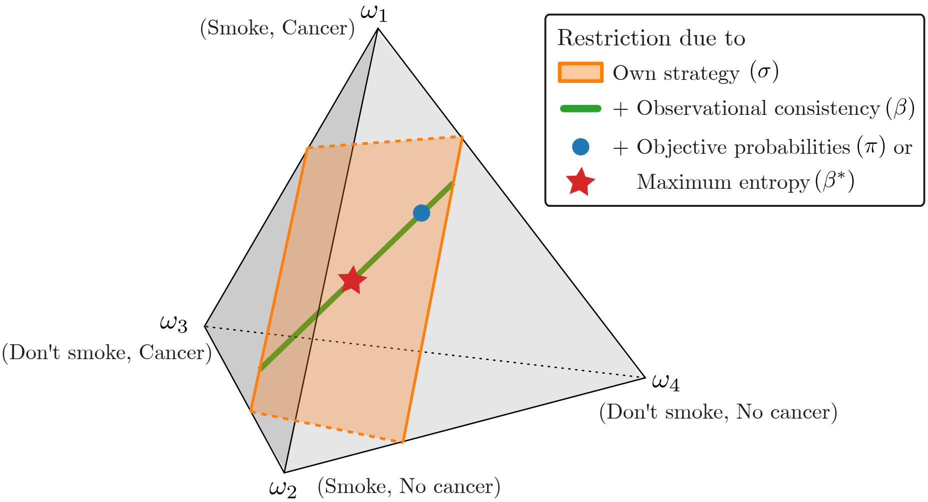
$$\beta^* \in \underset{\beta \text{ is observation-consistent}}{\operatorname{argmax}} G(\mathbf{p}(\sigma, \beta)).$$

Interpretation

- Among many worldviews consistent with observation, players believe in the the one that assumes the least information

Illustration of maximum entropy

A point prediction on belief



Maximum entropy $\Rightarrow$ correlation neglect

Claim

For every $\sigma \in (0,1)$, the maximum-entropy belief β^* satisfies

$$\beta_0^* = \beta_1^* = (1-\sigma)\cdot\frac{1}{3} + \sigma\cdot\frac{2}{3}.$$

Meaning The smoker doesn't think smoking causes cancer

Intuition The smoker observes no evidence of dependence between smoking and cancer, so he believes in none.

General result (Shore and Johnson, 1980; Csiszar, 1991)
Maximum entropy $\Leftrightarrow$ correlation neglect, whenever agents observe only the marginal prob. distribution between two variables

Equilibrium

Definition

A strategy-belief pair (σ, β) is an **observation-consistent equilibrium (OE)** if

1. Given the belief β, the strategy σ is a best response, and
2. Given the strategy σ, the belief β is observation-consistent.

Interpretation

- OE is a prediction of how the smoker behaves, given his possibly wrong but observationally consistent belief

Result on OE

Every strategy is rationalizable by some observation-consistent belief

Claim

Every strategy σ has a belief β such that (σ, β) is an OE.

Note: Specifically, the OCE equilibria are

1. $\sigma = 0$, $\beta_0 = \frac{1}{3}$, and $\beta_1 - \beta_0 \geq r$,
2. $\sigma = 1$, $\beta_1 = \frac{2}{3}$, and $\beta_1 - \beta_0 \leq r$, and
3. $\sigma \in (0, 1)$, $\beta_0 = \sigma \cdot (\frac{2}{3} - r) + (1 - \sigma) \cdot \frac{1}{3}$, and $\beta_1 = \sigma \cdot \frac{2}{3} + (1 - \sigma)(\frac{1}{3} + r)$.

Idea Because there are many observation-consistent beliefs, there are many OEs.

Definition of MOE

Definition

An OE (σ, β) is a **maximum-entropy observation-consistent equilibrium (MOE)** if there exists a sequence of strategy-belief pairs

$$\{(\sigma^k, \beta^k)\}_{k=1}^{\infty} \longrightarrow (\sigma, \beta)$$

such that each σ^k is a totally mixed strategy and each β^k maximizes the entropy given σ^k.

Interpretation

- MOE is an OE with the extra requirement that the smoker believes in the simplest explanation consistent with observation

Result on MOE

A sharper prediction

Claim

A strategy-belief pair (σ, β) is an MOE if and only if

$$\sigma = 1 \quad \text{and} \quad \beta_0 = \beta_1 = \frac{2}{3}.$$

Meaning

- Smoker keeps smoking while thinking that smoking doesn't cause cancer

Intuition

- Maximum-entropy belief features correlation neglect, so no other strategy is a best response.

Finite Extensive-form Games

1. Existence of MOE
2. Unique MOE in two example games
3. Interpretation and FAQ

General framework

A **finite extensive-form game with perfect recall** (Osborne and Rubinstein, 1994) and **observational constraint**

- N: set of players,
- H: set of histories (nodes)
  - Ω is the set of terminal histories
- ι: mapping of non-terminal histories to players,
- π: probability distribution of Nature's moves,
- $\mathcal{I}$: collection of information sets,
- u: payoff function, and
- C: **observational structure**, a linear map $\Delta(\Omega) \to \mathbb{R}^\ell$

Illustration of observational structure C

In Smoker's example, C is a matrix with $|\Omega| = 4$ columns.

Given a strategy σ, a belief β is **observation-consistent** if

$$C\mathbf{p}(\sigma, \beta) = C\mathbf{p}(\sigma, \pi).$$

Examples of C:

$$\begin{bmatrix} 1 & \cdot & 1 & \cdot \\ \cdot & 1 & \cdot & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}$$

Terminology in the general framework

Strategy $\sigma_i \in \mathcal{S}_i$ — $\sigma_i(a|I_i)$ is player i's objective prob of action a by i at info set I_i

Belief $\beta_i \in \mathcal{S}_{-i}$ — $\beta_i(a|I_j)$ is player i's subjective prob of action a by Nature or an opponent at info set I_j.

Posterior function μ_i — $\mu_i(h|I_i)$ is player i's subjective prob of history $h \in I_i$ given I_i.

"Assessment" — $(\sigma, \beta, \mu) = \{(\sigma_i, \beta_i, \mu_i)\}_{i \in N}$

Definition of OE

Notation $\mathbf{p}(\sigma_i, \beta_i)$ is the subjective probability distribution over Ω

Definition

An assessment (σ, β, μ) is an **observation-consistent equilibrium (OE)** if for every player i,

1. the strategy σ_i is (subjectively) sequentially rational given (β_i, μ_i),
2. the belief β_i is observation-consistent given the strategy profile σ:
$$C\mathbf{p}(\sigma_i, \beta_i) = C\mathbf{p}(\sigma_i, (\sigma_{-i}, \pi)), \text{ and}$$
3. the posterior function μ_i is Bayes-consistent given (σ_i, β_i).

Definition of MOE

Given a strategy profile σ, a player's observation-consistent belief β_i **maximizes the entropy** if

$$\beta_i \in \underset{\beta_i' \text{ is obs-cons}}{\operatorname{argmax}} \; G(\mathbf{p}(\sigma_i, \beta_i')).$$

Definition

An OE (σ, β, μ) is a **maximum-entropy observation-consistent equilibrium (MOE)** if there exists a sequence

$$\{\sigma^k, \beta^k\}_{k=1}^{\infty} \longrightarrow (\sigma, \beta)$$

where each σ^k is a totally mixed strategy profile and each player's belief β_i^k maximizes the entropy given σ^k.

Existence of MOE

Theorem

Every finite extensive-form game with perfect recall and observational constraint has an MOE.

Meaning

- There always exists a prediction where everyone best responds to what they think how others play, with a belief that assumes the least information beyond observation.

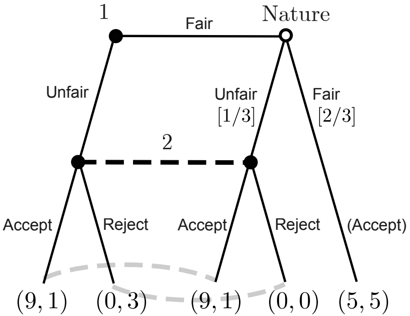
Key proof step

- With ϵ-constrained strategies, mappings from a strategy profile σ to a maximum-entropy belief profile β_i and posterior function β_i are well-behaved.

Example: Ultimatum-like game with causal misperception

Manager-Worker game

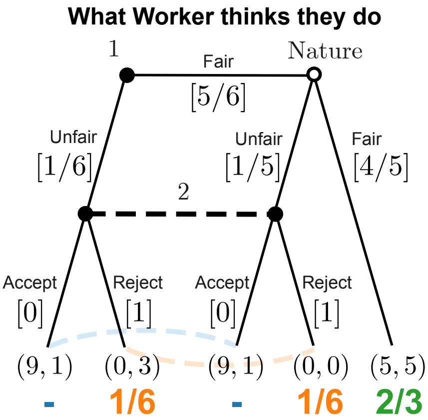
- Manager (Player 1) decides a fair or unfair bonus to Worker (Player 2)
- Even if Manager chooses a fair bonus, Nature might change it to unfair or keep it fair
- If Worker receives fair bonus, he accepts. If not, he either accepts or rejects.
    - He gets a thrill for rejecting an unfair Manager
- Worker doesn't know how likely Manager treats him unfairly in the interim or ex post (in a population)

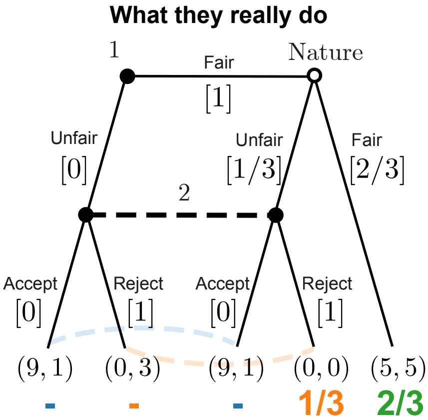


$$C = \begin{bmatrix} 1 & \cdot & 1 & \cdot & \cdot \\ \cdot & 1 & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix}$$

Unique MOE

Manager always tries to be fair





Example: A centipede game

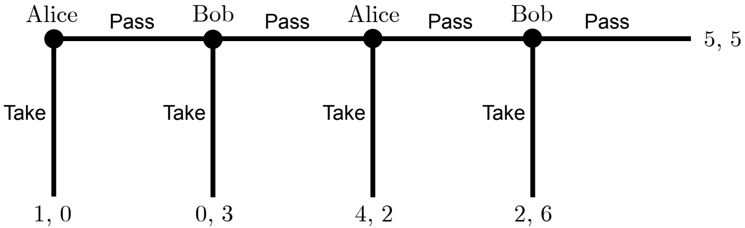


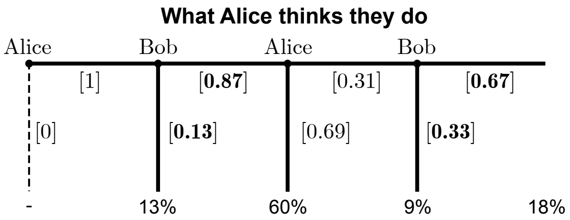
Figure: A four-node centipede game

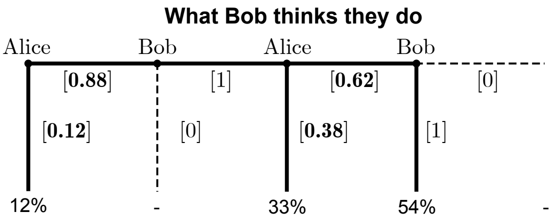
Claim

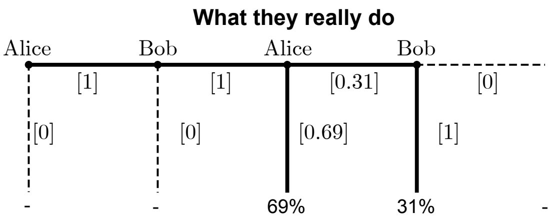
Let the observational structure be $C = [0\ 1\ 2\ 3\ 4]$. There exists no MOE in which Alice Takes immediately.

Unique MOE of the centipede game

Each thinks the other mixes more than they really do







How to interpret the observational structure C

Literal interpretation

- C represents the actual observable outcomes in a population of players

Metaphorical interpretation

- C represents how players psychologically process observable outcomes

Special case when players observe outcomes perfectly

Proposition

Suppose the observational structure C is the identity. Then

$$\begin{aligned} \text{OE} &\iff \text{Self-confirming equilibrium}^*, \quad \text{and} \\ \text{MOE} &\iff \text{Perfect Bayesian equilibrium}. \end{aligned}$$

***** Version with sequential rationality.

$\Rightarrow$ OE and MOE nest standard concepts as special cases

Frequently Asked Questions

How is MOE different from ______?

- Self-confirming equilibrium
  Battigalli and Guaitoli (1988); Battigalli (1997); Fudenberg and Levine (1993)
- Analogy-based expectation equilibrium
  Jehiel (2005); Jehiel and Koessler (2008); Jehiel (2022)
- (Sequential) Cursed equilibrium
  Eyster and Rabin (2005); Cohen and Li (2022); Fong et al. (2023)
- Berk-Nash equilibrium
  Esponda and Pouzo (2016)

MOE and Common Causal Misperceptions

1. Correlation neglect
2. Omitted-variable bias (selection neglect)
3. Simultaneity bias (reverse causality bias)

1. A two-stage game of correlated consequences

Players $N = \{1, 2, \ldots, n\}$

Stages

1. Players choose actions $x = (x_i)_{i \in N}$.
2. Nature chooses a consequence $y = (y_1, y_2)$ with conditional probability $\pi(y|x) > 0$ for all (x, y).

Payoffs $u_i(x, y)$

Obs. structure Marginal probabilities of pairs (x, y_1) and (x, y_2)

Correlation neglect

Proposition

An OE (σ, β, μ) is a MOE if and only if for every player i,

$$\beta_i(x_{-i}) = \sigma_{-i}(x_{-i}) \qquad \text{for all } x_{-i}, \text{ and}$$

$$\beta_i(y_1, y_2|x) = \pi(y_1|x)\pi(y_2|x) \qquad \text{for all } x \text{ and } (y_1, y_2).$$

Meaning In an MOE, players believe y_1 and y_2 remain (conditionally) independent regardless of their actions x.

Example Let x be whether an investment bank issues mortgage-backed securities or not. Let y be the default outcomes of two households.

Stylized example of correlation neglect

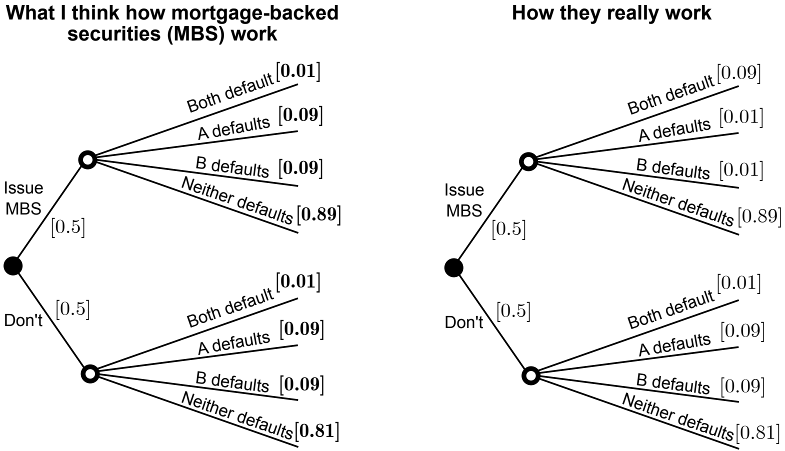


Figure: Effects of MBS on household default probabilities

2. An omitted-variable game

Players $N = \{1, 2, \ldots, n\}$

Stages

1. Nature assigns a state t with probability $\pi(t)$.

2. Players see the state t and choose actions $x = (x_i)_{i \in N}$.

3. Nature chooses a consequence y with probability $\pi(y|t, x)$.

Payoffs $u_i(t, x, y)$

Obs. structure Marginal probabilities of pairs (t, x) and (x, y)

Omitted-variable bias (selection neglect)

Proposition

An OE (σ, β, μ) is an MOE if and only if every player's belief β_i satisfies,

$$\beta_i(t) = \pi(t),$$

$$\beta_i(x_{-i}|t) = \sigma_{-i}(x_{-i}|t), \text{ and}$$

$$\beta_i(y|t,x) = \sum_{t' \in \mathcal{T}} \pi(y|t',x) w(t',x) \qquad \text{for all } (t,x,y).$$

Note: $w(\cdot)$ is a weight function such that $w(t',x) = \lim_{k\to\infty} \frac{\sigma^k(x|t')\pi(t')}{\sum_{t''\in\mathcal{T}} \sigma^k(x|t'')\pi(t'')}$, for some sequence $\{\sigma^k\}_{k=1}^{\infty}$ of totally mixed strategy profiles converging to σ.

Meaning Players believe the effect of x on y is the same across states t

Stylized example of omitted-variable bias

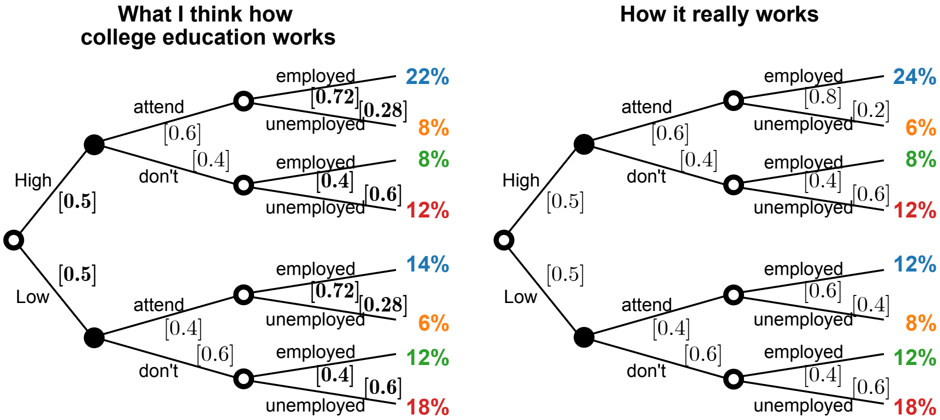


Figure: Effects of college education on employment

3. Simultaneity game

Players $N = \{1, 2, \ldots, n\}$

Stages **(1)** Nature assigns a state $t \in \{\text{Forward}, \text{Reverse}\}$ with probability $\pi(t)$.

If $t = F$, **(2)** players learn t and choose actions $x = (x_i)_{i \in N}$ and
(3) Nature chooses consequence y with prob $\pi(y|F, x)$.

If $t = R$, **(2)** Nature chooses consequence y with prob $\pi(y|R)$ and
(3) players learn (t, y) and choose actions $x = (x_i)_{i \in N}$.

Payoffs $u_i(t, x, y)$

Obs. structure Marginal probabilities of the pair (x, y)

Stylized example of simultaneity (reverse causality) bias

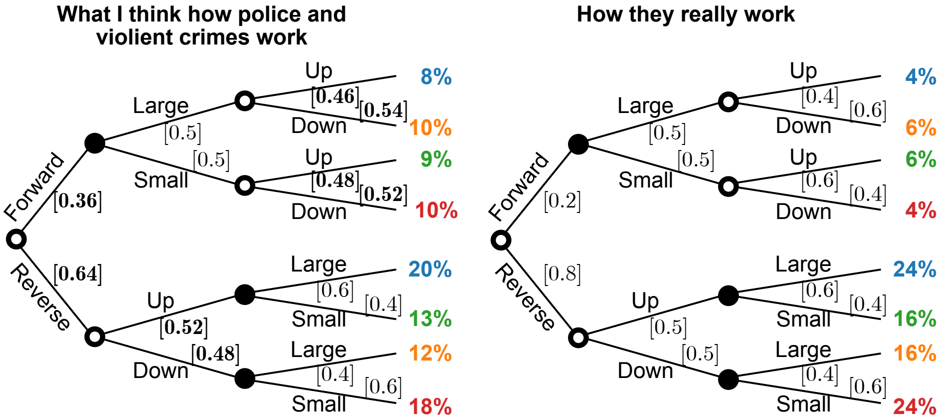


Figure: Effects of police size on violent crime rates

Wait... what do I even mean by causality?

Notation $p(\sigma_i, \beta_i)(E|h)$ is the subjective probability of event $E \subset \Omega$ given history h, strategy σ_i , and belief β_i.

Definition

Let (σ, β, μ) be an OE. An action a instead of b is a **subjective cause** of an event $E \subset \Omega$ given history h to player i if

$$p(\sigma_i, \beta_i)(E|h, a) > p(\sigma_i, \beta_i)(E|h, b).$$

An action a instead of b is an **objective cause** of an event $E \subset \Omega$ given history h to player i if

$$p(\sigma_i, (\sigma_{-i}, \pi))(E|h, a) > p(\sigma_i, (\sigma_{-i}, \pi))(E|h, b).$$

Extension to Infinite-horizon Games

Extension: Stochastic (Markov) Games

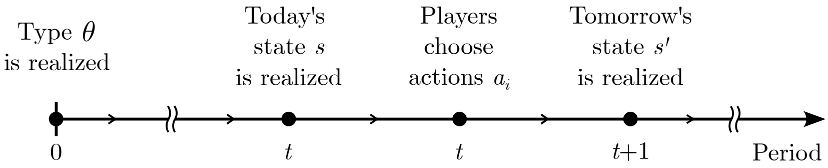


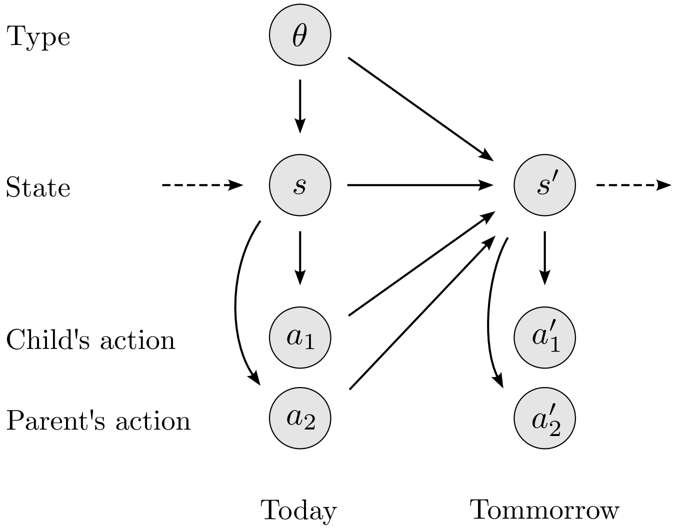
Figure: Stochastic game with permanent game types θ

Proposition

If players perfectly observe steady-state outcomes (θ, s, a, s'),

MOE $\iff$ Markov perfect equilibrium (MPE).

Illustration: Parent-Child game of social media use



Sensitive or
not sensitive

In Good mood
or in Bad mood

Use socia media
or not

Enforce strict or lenient
screen time policy

Equilibrium in the Parent-Child game

		Child's strategy (σ_1)		Parent's strategy (σ_2)	
Equilibrium	Type (θ)	Bad mood	Good mood	Bad mood	Good mood
MPE	Not sensitive	Use	Use	Lenient	Lenient
	Sensitive	Don't	Use	Lenient	Lenient
MOE	Not sensitive	Use	Use	Strict	Lenient
	Sensitive	Use	Use	Strict	Lenient

Note: MPE refers to Markov perfect equilibrium. MOE refers to maximum-entropy observation-consistent equilibrium.

Relation to dynamic stuctural dconometrics

Rational expectations (RE) assumption

- "Ubiquitous" even though it's a "very strong assumption" (Aguirregabiria and Mira, 2010)
- Relaxing it requires modeling and estimating beliefs (e.g., Aguirregabiria and Magesan, 2020)

Maximum-entropy belief assumption

- Offers a viable alternative to RE with a point-prediction on beliefs
- Only requires an existing model + observational structure C

Takeaway

If you want to

- allow causal misperception in a dynamic model,
- let misperception arise endogenously from the observational structure, and
- want narrow predictions, then ...

Takeaway

If you want to

- allow causal misperception in a dynamic model,
- let misperception arise endogenously from the observational structure, and
- want narrow predictions, then ...

... use MOE. Thank you!

Appendix

References I

Aguirregabiria, Victor and Arvind Magesan (2020) "Identification and estimation of dynamic games when players' beliefs are not in equilibrium," *The Review of Economic Studies*, 87 (2), 582–625.

Aguirregabiria, Victor and Pedro Mira (2010) "Dynamic discrete choice structural models: A survey," *Journal of Econometrics*, 156 (1), 38–67.

Battigalli, Pierpaolo (1997) "On rationalizability in extensive games," *Journal of Economic Theory*, 74 (1), 40–61.

Battigalli, Pierpaolo and Danilo Guaitoli (1988) *Conjectural equilibria and rationalizability in a macroeconomic game with incomplete information*: Università Commerciale L. Bocconi.

Cohen, Shani and Shengwu Li (2022) "Sequential Cursed Equilibrium," *arXiv preprint arXiv:2212.06025*.

Csiszar, Imre (1991) "Why least squares and maximum entropy? An axiomatic approach to inference for linear inverse problems," *The Annals of Statistics*, 2032–2066.

Esponda, Ignacio and Demian Pouzo (2016) "Berk–Nash equilibrium: A framework for modeling agents with misspecified models," *Econometrica*, 84 (3), 1093–1130.

References II

Eyster, Erik and Matthew Rabin (2005) "Cursed equilibrium," *Econometrica*, 73 (5), 1623–1672.

Fong, Meng-Jhang, Po-Hsuan Lin, and Thomas R. Palfrey (2023) "Cursed Sequential Equilibrium," 10.48550/ARXIV.2301.11971.

Fudenberg, Drew and David K Levine (1993) "Self-confirming equilibrium," *Econometrica: Journal of the Econometric Society*, 523–545.

Jehiel, Philippe (2005) "Analogy-based expectation equilibrium," *Journal of Economic Theory*, 123 (2), 81–104.

——— (2022) "Analogy-based expectation equilibrium and related concepts: Theory, applications, and beyond."

Jehiel, Philippe and Frédéric Koessler (2008) "Revisiting games of incomplete information with analogy-based expectations," *Games and Economic Behavior*, 62 (2), 533–557.

Kendall, Chad W and Constantin Charles (2022) "Causal narratives,"Technical report.

Kreps, David M and Robert Wilson (1982) "Sequential equilibria," *Econometrica: Journal of the Econometric Society*, 863–894.

References III

Osborne, Martin J and Ariel Rubinstein (1994) *A course in game theory*: MIT Press.

Shore, John and Rodney Johnson (1980) "Axiomatic derivation of the principle of maximum entropy and the principle of minimum cross-entropy," *IEEE Transactions on Information Theory*, 26 (1), 26–37.

Spiegler, Ran (2020) "Behavioral implications of causal misperceptions," *Annual Review of Economics*, 12, 81–106.

——— (2021) "Modeling players with random "data access"," *Journal of Economic Theory*, 198, 105374.