The Photoelectric Effect

Sungur Özkan

Boğaziçi University • Physics Department

Abstract—**We have investigated the particle-like nature of light that could not be explained by classical theory of light and led to the development of the modern description of electromagnetic radiation. We have shown the linear relationship between the maximum kinetic energy of the released electrons and the frequency of the incident light. Using our results we determined the work function of the anode material to be 1.923V $\pm$ 0.013V and the Planck constant to be** $3.60 \times 10^{-34} \pm 3.56 \times 10^{-36} J/Hz$ **which is 84.8**σ **away from the accepted value.**

I. THEORY

When light is shone on a metal, electrons can be ejected from the surface of that metal. This phenomenon is known as the photoelectric effect[1]. The photoelectric effect was first experimentally verified by Heinrich Rudolf Hertz in 1887. From his observations he concluded that some negatively charged particles were emitted by the zinc plate when exposed to ultraviolet light[2]. In 1897, J. J. Thomson found that these particles were electrons. But these physicists attempted to explain the photoelectric effect using classical physics.

In 1902, Philipp Lenard observed that the energy of individual emitted electrons was independent of the applied light intensity. This appeared to be at odds with Maxwell's wave theory of light, which predicted that the electron energy would be proportional to the intensity of the radiation[2]. In 1900, the German physicist Max Planck suggested that the energy carried by electromagnetic waves could only be released in packets of energy. In 1905, Albert Einstein published a paper advancing the hypothesis that light energy is carried in discrete quantized packets to explain experimental data from the photoelectric effect. Einstein theorized that the energy in each quantum of light was equal to the frequency of light multiplied by a constant, later called the Planck constant[2]. Einstein demonstrated that electromagnetic radiation, including light, has the characteristics of both a wave and, consistent with Planck's theory, a particle, which we now call photons. Finally, in 1914, Robert A. Milikan measured Planck constant accurately with the photoelectric effect and verified Einstein's explanation of the photoelectric effect 11 years later. Einstein was awarded the 1921 Nobel Prize in Physics for "his discovery of the law of the photoelectric effect", and Millikan was awarded the Nobel Prize in 1923 for "his work on the elementary charge of electricity and on the photoelectric effect" [3].

The energy of a photon can be calculated using the following equation:

$$E_{photon} = h\nu \tag{1}$$

where h is the proportionality constant, known as Planck's constant.

When light with frequency ν is incident on the metal surface, its energy is absorbed by the metal. We have the following equation for energy:

$$h\nu = K_{max} + \phi \tag{2}$$

where K_{max} is the maximum kinetic energy of the freed electron and ϕ is the work function. The work function is the minimum energy required to remove an electron from the surface of the material, the electrons are bound inside the metal surface with an energy ϕ. That's why work function differs from metal to metal. Work function is also equal to:

$$\phi = h\nu_0 \tag{3}$$

where ν_0 is the threshold frequency. It is the minimum frequency which can eject electrons from the surface. The kinetic energy of the electrons is directly proportional to the frequency of the light that liberates them.
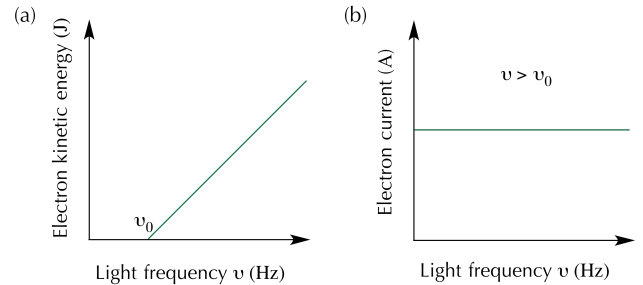

Fig. 1. Frequency Dependencies

If light of a given frequency can liberate electrons from a surface, increasing the intensity of the light will increase the number of electrons released from the surface. Light of smaller frequency cannot free electrons no matter how long it falls upon the surface or how great the intensity.
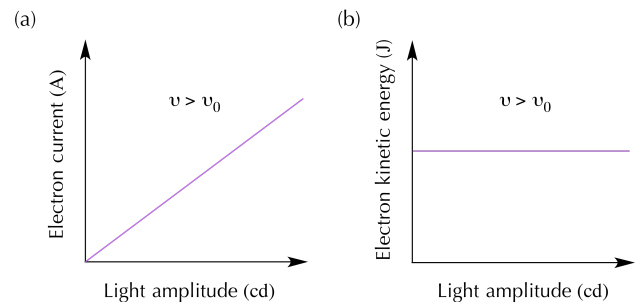

Fig. 2. Intensity Dependencies

To calculate the work function of a metal, a negative potential is applied, which is called the "stopping potential". When such a potential V is applied, only the electrons with kinetic energy $E_{kinetic} > eV$ can reach the anode. And for a potential V_s, no electrons arrive at the anode. This potential is called the stopping potential. This means that kinetic energy of the fastest electron electrons must be equal to eVs. We can summarize this as:

$$h\nu - \phi = eV_s = E_{kinetic} \qquad (4)$$

Then,

$$V_s = \frac{h}{e}\nu - \frac{1}{e}\phi \qquad (5)$$
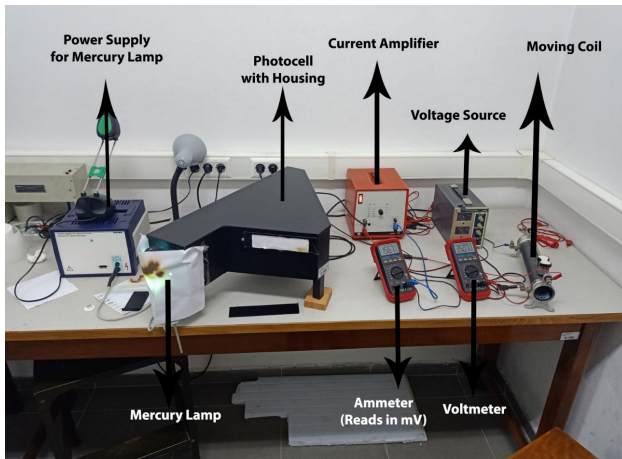
II. THE EXPERIMENTAL SETUP

Fig. 3. The Experimental Setup

- Photocell with housing
- Spectograph with a transmission grating
- High pressure mercury lamp with power supply
- Current Amplifier
- Moving Coil DC Voltmeter for the current amplifier
- DC Voltmeter (0-3 V)
- Power Supply (0-3 V)

III. METHOD

1) We darkened the room and turned on the power supplies.
2) We adjusted the orientation of the grating so that the yellow light fell on the photocell.
3) We began with zero retarding voltage and recorded the currents while slowly increasing the voltage.
4) We repeated these previous steps with four other colors.

IV. THE DATA

I have included the raw voltage and current data for the blue color here. The rest of the data is included in the appendix:

TABLE I
VOLT AND CURRENT FOR BLUE

Volt(mV)	Current(fA)
38.3	149.6
57.7	138.1
97.4	119.2
110.4	112.2
128.1	104.2
141.0	98.1
194.1	76.8
233.2	62.6
301.1	42.3
343.3	31.3
403.0	18.3
455.3	8.4
480.5	5.2
581.4	-6.4
676.9	-13.4
739.7	-16.2
819.7	-19.2
875.8	-20.6
1056.8	-23.4
1441.0	-27.4
1977.6	-29.7
2482.8	-31.3

V. THE ANALYSIS

The analysis was made using CERN's framework ROOT, version v6.30.04. The raw data is of the order of Milivolt and Femtoampere, so we converted these values to Volt and Ampere. Then, we have plotted the datasets for different colors. Then, we have used ROOT's built in function to fit a line to the datasets. Here are the fitted plots:
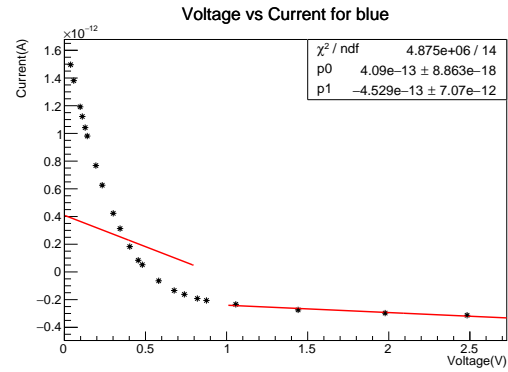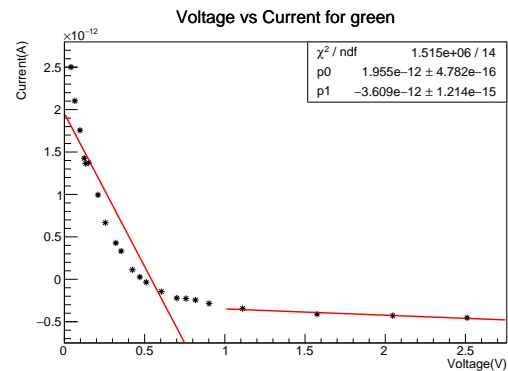

Fig. 4. Voltage - Current Plot For Blue

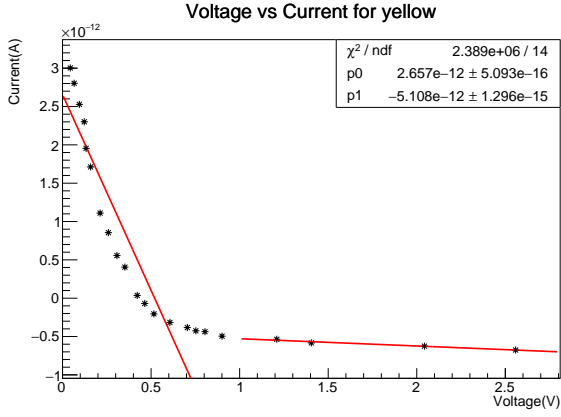Fig. 5. Voltage - Current Plot For Green

Voltage vs Current for yellow

Fig. 6. Voltage - Current Plot For Yellow

Voltage vs Current for violet
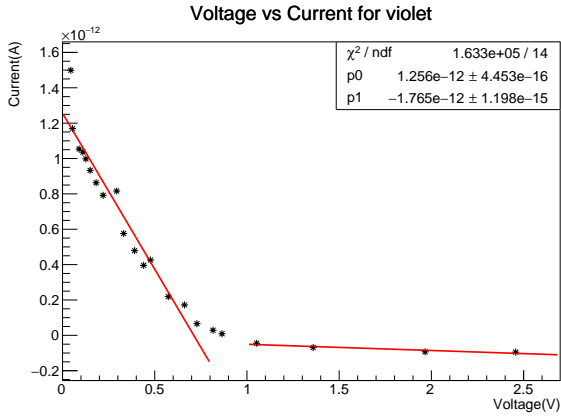

Fig. 7. Voltage - Current Plot For Violet
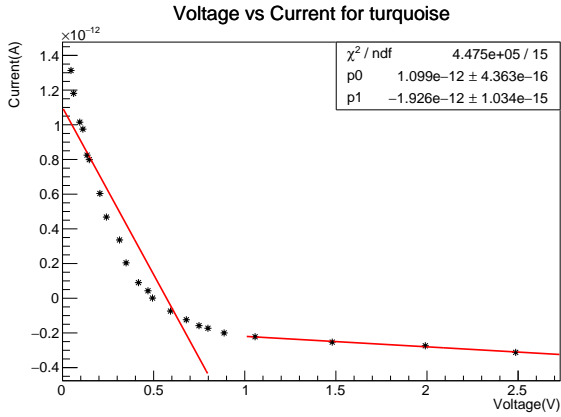
Voltage vs Current for turquoise

Fig. 8. Voltage - Current Plot For Turquoise

To determine the stopping potentials here, we cannot simply take the x intercept of the first line as the correct value, because there are both forward and reverse currents. The reverse current makes it harder to determine Vs. To find Vs accurately, we need to find the potential where the total current matches the reverse current we see at high voltages. This means the forward and reverse currents are balanced. But, even when they're balanced, there's still some forward current. To solve this, We find the intersection of the two lines.

$$y_1 = m_1 x + n_1 \tag{13}$$

$$y_2 = m_2 x + n_2 \tag{14}$$

By setting the two equations equal, we get the x value of the intercept, which is the stopping potential V_s :

$$V_s = x = \frac{n_2 - n_1}{m_1 - m_2} \tag{15}$$

The stopping potential V_s depends on 4 variables we have found earlier. Therefore, the errors on this variable should be propagated to the stopping potential. The formula for error propagation is given in the appendix. The results for stopping potentials are:

TABLE II
STOPPING POTENTIALS FOR DIFFERENT COLORS

	Stopping Potential (V)
Blue	1.491 ± 26.342
Green	0.631 ± 0.005
Yellow	0.616 ± 0.004
Violet	0.734 ± 0.001
Turquoise	0.674 ± 0.001

Then, we plotted the potentials as a function of frequencies and fitted a line:

Stopping Voltage vs Frequency
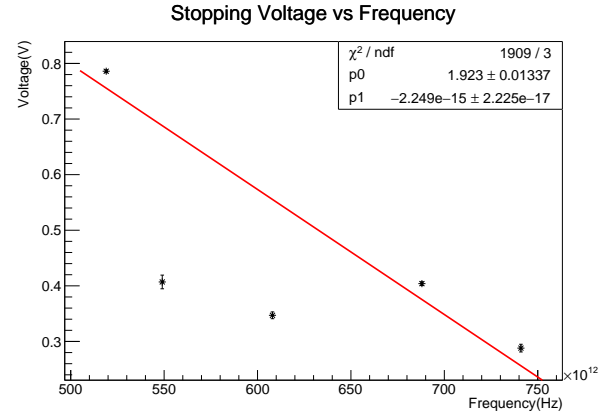
Fig. 9. Voltage - Frequency Plot

This line represents the Eq.5 described in the theory. The slope of the line gives $\frac{h}{e}$ and the y-intercept of the line gives $-\frac{1}{e}\phi$. From here, we can obtain the Planck's constant and the work function of the anode ϕ. The y-intercept is the work function of the anode, not the cathode. If we knew the difference of work functions of the anode and cathode, which we call contact potential difference cpd, we could also extract the work function of the cathode.

VI. THE RESULT

We have found the work function of the anode material to be:

$$\phi = 1.923V \pm 0.013V \tag{6}$$

The planck constant:

$$h = 3.60 \times 10^{-34} \pm 3.56 \times 10^{-36} J/Hz \tag{7}$$

VII. THE CONCLUSION

In this experiment we have seen that the photoelectric effect cannot be explained by classical theory of light, which holds light to be a wave. We have also observed that the maximum kinetic energy of the emitted electron is linearly related to the frequency of the incident light. Plotting the stopping potential versus the frequency of the incident light, we determined the Planck constant and the work function of the collector anode. Our result for the Planck constant could be estimated with a correct order of magnitude, which is a very promising result considering the sources of errors we dealt with. A possible source of error in the experiment can be the reverse current which prevents us from directly taking the horizontal axis intercept of the current as the stopping potential. One way to increase the certainty in determining the stopping potential is to disable the reverse current. Another source of error is the very small magnitude of the current. Even though we did the experiment in a dark room, there were still electromagnetic interactions and noises. We can also try to obtain a bigger dataset for the experiment to get a better result.

REFERENCES

[1] *Photoelectric effect, Khan academy*. URL: https://www.khanacademy.org/science/physics/quantum-physics/photons/a/photoelectric-effect.

[2] *The photoelectric effect - Wikipedia*. URL: https://en.wikipedia.org/wiki/Photoelectric_effect.

[3] *The Nobel Prize*. URL: https://www.nobelprize.org/prizes/physics/.

[4] E. Gülmez. *Advanced Physics Experiments*. 1st. Boğaziçi University Publications, 1999.

[5] E. Gülmez. *Statistics Book*.

VIII. APPENDIX

The formula that I have used for error propagation is[5]:

$$\sigma_{V_s}^2 = \left(\frac{n_2 - n_1}{(m_1 - m_2)^2}\right)^2 \sigma_{m_1}^2 +$$
$$\left(\frac{n_2 - n_1}{(m_1 - m_2)^2}\right)^2 \sigma_{m_2}^2 +$$
$$\left(\frac{1}{m_1 - m_2}\right)^2 \sigma_{n_1}^2 + \qquad (16)$$
$$\left(\frac{1}{m_1 - m_2}\right)^2 \sigma_{n_2}^2$$

The raw data for green, yellow, violet and turquoise:

TABLE III
VOLT AND CURRENT FOR GREEN

Volt(mV)	Current(fA)
41.3	250.1
64.8	210.3
97.1	175.6
124.0	142.7
132.5	136.5
150.9	137.4
209.4	99.5
255.4	66.7
320.3	42.8
353.9	33.2
424.2	11.2
470.1	2.7
510.1	-3.5
604.3	-14.5
700.2	-22.2
757.6	-22.7
816.9	-24.5
901.5	-28.5
1110.6	-34.5
1575.2	-41.1
2046.6	-42.8
2510.3	-45.4

TABLE IV
VOLT AND CURRENT FOR YELLOW

Volt(mV)	Current(fA)
43.8	300.3
65.7	280.3
95.0	252.8
122.8	230.1
133.0	195.4
158.6	171.2
212.3	111.0
258.9	85.5
307.3	55.6
352.1	40.5
421.7	3.6
464.3	-6.9
515.9	-20.5
606.4	-31.6
704.6	-38.3
752.7	-42.5
804.3	-43.7
900.6	-49.5
1209.7	-53.6
1404.9	-58.3
2043.4	-62.5
2558.2	-67.6

TABLE V
VOLT AND CURRENT FOR VIOLET

Volt(mV)	Current(fA)
44.7	149.9
54.2	116.9
89.9	105.4
110.3	103.6
126.5	99.8
149.8	93.3
182.1	86.3
220.1	79.2
293.5	81.6
331.5	57.6
390.8	47.9
439.8	39.6
477	42.6
573.6	21.9
661.4	17.2
728.9	6.6
815.9	2.9
864.7	0.9
1052.7	-4.5
1359	-6.9
1965.9	-9.3
2456.9	-9.5

TABLE VI
VOLT AND CURRENT FOR TURQUOISE

Volt(mV)	Current(fA)
46.5	131.3
60.8	118.1
94.6	101.5
111.7	97.5
134.7	82.5
146.9	79.9
205.0	60.4
240.9	46.8
312.9	33.6
348.8	20.4
416.7	9.0
468.9	4.3
493.8	0.1
592.6	-7.4
679.3	-12.4
748.9	-15.8
798.0	-17.3
886.8	-20.0
1056.7	-22.2
1480.0	-25.3
1990.8	-27.4
2486.7	-31.2

```
{

#include <iostream>
#include <vector>
#include <string>
#include <cmath>
#include <utility>

{

std::vector<std::string> filenames = {
    "blue.csv",
    "green.csv",
    "yellow.csv",
    "violet.csv",
    "turquoise.csv",
    };

std::vector<float> xintercepts;
std::vector<float> errors;


gStyle->SetOptFit(1);
gStyle->SetStatX(0.9);
gStyle->SetStatY(0.9);
TF1 *f1 = new TF1("line", "[0] + [1]*x");


for (int fileIndex = 0; fileIndex <
    filenames.size(); ++fileIndex) {
TTree *t = new TTree("t", "t");
t->ReadFile(filenames[fileIndex].c_str());
int n = t->GetEntries();
float * x, * y, * sx, * sy ;
x = new float[n];
y = new float[n];
sx = new float[n];
sy = new float[n];
float a,v;
t->SetBranchAddress("A", &a);
t->SetBranchAddress("V", &v);
string color = filenames[fileIndex].substr(0,
    filenames[fileIndex].find_last_of('.'));
string finalfilename = color + "fitted.pdf";

for (int i = 0; i < n; ++i) {
    t->GetEntry(i);
    x[i] = v * pow(10, -3);
    y[i] = a * pow(10, -14);
    sx[i] = 0.1 * pow(10, -3);
    sy[i] = 0.1 * pow(10, -14);
}

TGraphErrors gr(n, x, y, sx, sy);

gr.SetTitle(("Voltage vs Current for " +
    color).c_str());
gr.GetXaxis()->SetTitle("Voltage(V)");
gr.GetYaxis()->SetTitle("Current(A)");

f1->SetRange(0,0.8);
gr.Fit(f1,"R");

float yintercept1 = f1->GetParameter(0);
float slope1 = f1->GetParameter(1);
float sigmayintercept1 = f1->GetParError(0);
float sigmaslope1 = f1->GetParError(1);
```

```cpp
f1->SetRange(1,3.2);
gr.Fit(f1,"R+");

float yintercept2 = f1->GetParameter(0);
float slope2 = f1->GetParameter(1);
float sigmayintercept2 = f1->GetParError(0);
float sigmaslope2 = f1->GetParError(1);

float xintercept = (yintercept2 -
    yintercept1)/(slope1 - slope2);
xintercepts.push_back(xintercept);

float term1 = pow((yintercept2 - yintercept1)
    / pow((slope1 - slope2), 2), 2) *
    pow(sigmaslope1, 2);
float term2 = pow((yintercept2 - yintercept1)
    / pow((slope1 - slope2), 2), 2) *
    pow(sigmaslope2, 2);
float term3 = pow(1 / (slope1 - slope2), 2) *
    pow(sigmayintercept1, 2);
float term4 = pow(1 / (slope1 - slope2), 2) *
    pow(sigmayintercept2, 2);

float sigma_Vs = sqrt(term1 + term2 + term3 +
    term4);
errors.push_back(sigma_Vs);

cout << "Stopping potential for " << color <<
    " is equal to " << xintercept << "V +- "
    << sigma_Vs << "V"<< endl;

float freqs[5] = {5.19, 5.49, 6.08, 6.88,
    7.41};

TCanvas *c1 = new TCanvas();
gr.Draw("A*");
c1->Print(finalfilename.c_str());




}

const int ndata = 5;
double y2[ndata] = { 0.786, 0.407, 0.347,
    0.404, 0.288};
double x2[ndata] = {5.19, 5.49, 6.08, 6.88,
    7.41};
for (int i=0; i<5; i++){
   x2[i] = x2[i]*pow(10, 14);
}
double sy2[ndata] = {0.0026358, 0.0122895,
    0.00611123, 0.0034491, 0.00693817};
double sx2[ndata] = {0,0,0,0,0};


TGraphErrors gr2(ndata, x2, y2, sx2, sy2);
gr2.SetTitle("Stopping Voltage vs Frequency");
gr2.GetYaxis()->SetTitle("Voltage(V)");
gr2.GetXaxis()->SetTitle("Frequency(Hz)");
f1->SetRange(1,1*pow(10,15));
gr2.Fit(f1,"R");

float wf = f1->GetParameter(0);
float sigmawf = f1->GetParError(0);
float hq = f1->GetParameter(1);
float sigmahq = f1->GetParError(1);

float q = (-1)*1.602*pow(10,-19);
float h = hq*q;


cout << "Planck Constant: " << h << " J/Hz +-
    " << sigmahq*q << " J/Hz" << endl;
cout << "Work Function: " << wf << "V +- " <<
    sigmawf << "V" << endl;

TCanvas *c2 = new TCanvas();
gr2.Draw("A*");
c2->Print("freq.pdf");

}

}
```