Radioactive Decay

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***Abstract*—In this experiment, we studied the decay of Radon gas, which is one of the elements produced in the Thorium decay series. We used a Wulf's electroscope and measured the time duration it takes to discharge. We found the half life of Radon gas to be $t_{1/2} = 67.24s \pm 0.10s$, which is 112$\sigma$ away from the accepted value of 55.6s.**

I. Theory

Radioactive decay is the process by which an unstable atomic nucleus loses energy by radiation. This is usually in the form of α particles (Helium nuclei), β particles (electrons or positrons) and γ rays (high energy photons) [1]. Radioactivity was first discovered in 1896, by the French scientist Henri Becquerel and Marie Curie while working on phosphorescent materials. Rutherford was the first to realize that all radioactive decay processes occur with the same mathematical exponential formula [2]. Radioactive decay is a random process, it is impossible to predict when a particular atom will decay according to quantum theory. However, for a significant number of identical atoms, the overall decay rate can be expressed as a decay constant or as half-life.

In the experiment, we study the decay of Radon gas (^{220}Rn), which is an intermediate step in the Thorium (^{232}Th) decay chain. The intermediates in the ^{232}Th decay chain are all relatively short-lived; the longest-lived intermediate decay products are ^{228}Ra and ^{228}Th, with half lives of 5.75 years and 1.91 years, respectively. The chain ends with ^{208}Pb, which is a stable atom [3].

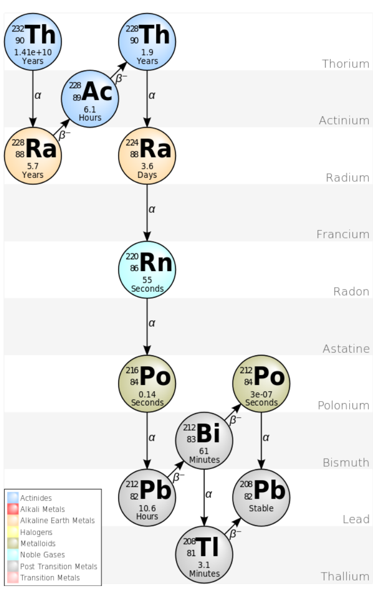


Fig. 1. Thorium decay chain [3]

The decay law obeyed by unstable nuclei is given by [4]:

$$N(t) = N_0 e^{-\lambda t} \tag{1}$$

where N_0 is the initial number of unstable isotope nuclei and λ is the decay constant. If we take the derivative of both sides:

$$\frac{dN(t)}{dt} = -N(t)\lambda \tag{2}$$

which means:

$$\frac{dN(t)}{dt} \propto N(t) \tag{3}$$

Radioactivity will be studied in this experiment through a simple way which makes use of ionizing capability of the charged particles produced in the process of radioactive decay. A Wulf's electroscope is used, which discharges automatically after reaching a certain threshold charge. The time between successive discharges are measured, and since the threshold charge necessary for the electroscope to discharge is fixed, the current is proportional to the inverse of the time elapsed to accumulate this amount of charge:

$$I = \frac{Q}{s} \quad \Rightarrow I \propto \frac{1}{s} \tag{4}$$

which means:

$$I \propto \frac{1}{s} \propto \frac{dN(t)}{dt} \propto N(t) \tag{5}$$

Then, we can write this equation:

$$\frac{1}{s_i} = \frac{1}{s_0} e^{-\lambda t} \tag{6}$$

where s_i is the time difference between two successive discharges. If we take the logarithm of both sides in (6), we get:

$$\ln(s_i) = \ln(s_0) + \lambda t \tag{7}$$

s_0 is a constant here. To find the half life, we can write t as:

$$t_{1/2} = \frac{t_{i+1} + t_i}{2} \tag{8}$$

We can extract the radioactive decay constant λ of Radon gas by using linear regression with this logarithmic version of the decay law. We can then, calculate the half life of Radon by using following equation:

$$t_{1/2} = \frac{\ln(2)}{\lambda} \tag{9}$$

II. The Experimental Setup

- Wulf's Electroscope
- Thorium Salt
- Ionisation Chamber
- HV Power Supply
- Stopwatch

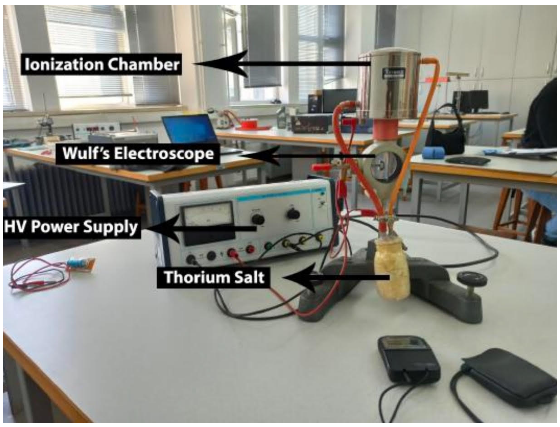


Fig. 2. The setup

III. Method

1) High voltage is turned off and the ionization chamber is opened to clean it up completely.
2) High Voltage is set to 3.0kV and the ionization chamber is closed.
3) The clamp is opened, the Thorium salt bottle is squeezed 2 times and the clamp is closed again. This way, the Radon gas is sent to the chamber.
4) The timer is started and the instants at which the electroscope discharges are recorded.
5) When the electroscope takes longer than one minute to discharge, we stop taking data.
6) High Voltage is set to 0 and the chamber is opened to evacuate the Radon gas.
7) This becomes one dataset, the steps until here are repeated with 3.0 kV, squeezing 3, 4 and 5 times.
8) After this step, 4 dataset is obtained. The steps until here are repeated for 3.5kV, 4.0kV and 4.5kV.
9) We obtain 16 datasets.

IV. The Data

When taking data, the uncertainty is our reflection time when we see the electroscope discharge. As the group doing the experiment, we have tested our reflection times and it came out as 0.30 second in average. So, the uncertainty for the each data is considered to be $\pm 0.30s$. For the plots, The y axis will be:

$$s_i = t_{i+1} - t_i \tag{10}$$

which corresponds to "interval" in our data. For the x axis, we take the middle point of two consecutive points:

$$T_i = (t_i + t_{i+1})/2 \tag{11}$$

Our data for 3.0 kV and 2 squeezes is here:

TABLE I
3.0 kV And 2 Squeezes Discharge Times

Discharge Times (s)	Intervals (s)
6.39	6.39
8.75	2.36
13.64	4.89
16.78	3.14
20.75	3.97
24.41	3.66
28.74	4.33
33.33	4.59
37.77	4.44
42.70	4.93
48.96	6.26
55.49	6.53
61.56	6.07
67.60	6.04
76.21	8.61
84.76	8.55
93.16	8.40
104.38	11.22
116.13	11.75
126.98	10.85
150.55	23.57
167.37	16.82
196.78	29.41
235.46	38.68
302.64	67.18

V. The Analysis

The analysis was made using CERN's framework ROOT, version v6.30.0. For the analysis part, we have plotted T versus 1/s. For the uncertainties, we have used the error propagation formula in the appendix.

$$\sigma_T = \sigma_t = 0.30s \tag{12}$$

$$\sigma_{1/s} = \sqrt{(-\frac{1}{s^2})^2 \sigma_t^2} = \frac{\sigma_t}{s^2} \tag{13}$$

Then, we have fitted an exponential function to the plot using ROOT's built-in function. The plot for 3.0 kV and 2 squeezes is:

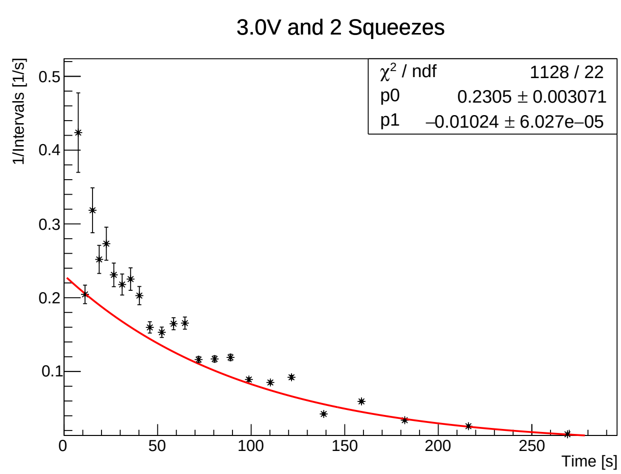


Fig. 3. 3.0V 2 Squeezes

The parameter "p1" corresponds to the decay constant for the dataset. After obtaining the decay constant for the 16 dataset, we have applied the weighted average formula which is given in the appendix and obtained a final decay constant with its uncertainty. Then, using Eq.9, we have obtained the life-time (half-life) of the Radon gas. Using the error propogation formula again, the sigma of the life time is:

$$\sigma_{t_{1/2}} = \frac{\sigma_\lambda}{\lambda^2} \tag{14}$$

Finally, we have plotted the decay constants as a function of number of squeeze and as a function of high voltage values:

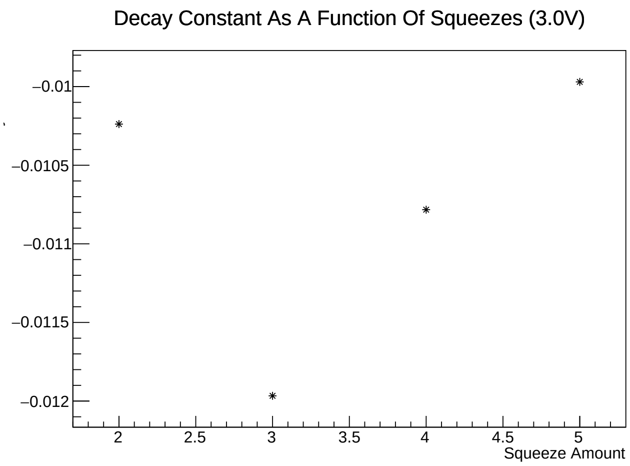


Fig. 4. Decay constants as a function of number of squeeze

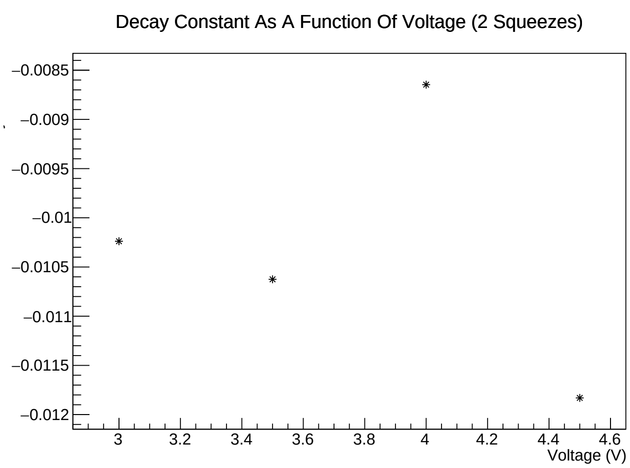


Fig. 5. Decay constants as a function of high voltage

VI. The Result

We have found the half-life of the Radon gas to be:

$$t_{1/2} = 67.24s \pm 0.10s \tag{15}$$

The error formula:

$$\text{Error } = \frac{\left|\left(t_{1/2}\right)_{\text{true}} - \left(t_{1/2}\right)_{\text{exp}}\right|}{\sigma_{\left(t_{1/2}\right)_{\text{exp}}}} \tag{16}$$

and the true value is 55.6 seconds. So, we have found our result to be 112σ away from the true value.

VII. The Conclusion

We have studied the radioactive decay of the Radon gas using a Wulf's electroscope and measuring the time intervals between two successive discharges. Our result came out to be 112σ away from the true value. We are considerably far away from the true value. One of the errors while doing the experiment could be failing to close the clamps properly and causing some Radon gas to leak into the chamber during the experiment. Another source of error could be that we have stopped taking data when each interval became longer than 1 minute, we also stopped when we reached 5 minutes in total. If we have waited for intervals longer than 1 minute, we could have a better result. Another simplification we made during our analysis was to obtain the data points by taking the middle of the interval. Therefore, we approximated the value at that time with the average value of the decay function over that interval. If the decay function were linear, this would not affect the results. Since the decay function is actually exponential we would expect a stronger decrease, and a greater slope. This would increase the λ in our calculations. As a result, our calculated half life would decrease and our final results would be closer to the true value.

References

[1] *Radioactive Decay - Isaac Physics*. URL: https://isaacphysics.org/concepts/cp_radioactive_decay?stage=all.

[2] *Radioactive decay - Wikipedia*. URL: https://en.wikipedia.org/wiki/Radioactive_decay.

[3] *Thorium-232 - Wikipedia*. URL: https://en.wikipedia.org/wiki/Thorium-232.

[4] E. Gülmez. *Advanced Physics Experiments*. 1st. Boğaziçi University Publications, 1999.

[5] E. Gülmez. *Statistics Book*.

VIII. Appendix

A. Formulas

The error propagation formula used in analysis [5]:

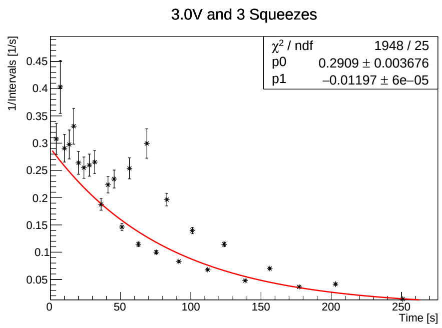
$$\sigma_f = \sqrt{\sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)^2 \sigma_{x_i}^2} \tag{17}$$

The weighted average formula is [5]:

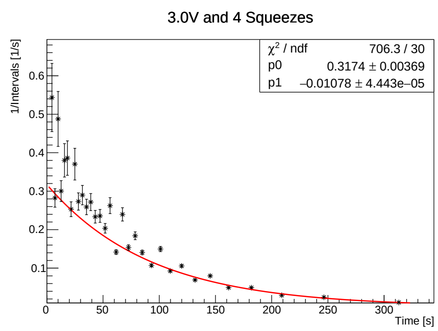
$$(t)_{\text{weighted}} = \frac{\sum_i \frac{(t)_i}{\sigma^2_{(t)_i}}}{\sum_i \frac{1}{\sigma^2_{(t)_i}}} \tag{18}$$

$$\sigma^2_{(t)_{\text{weighted}}} = \frac{1}{\sum_i \frac{1}{\sigma^2_{(t)_i}}} \tag{19}$$

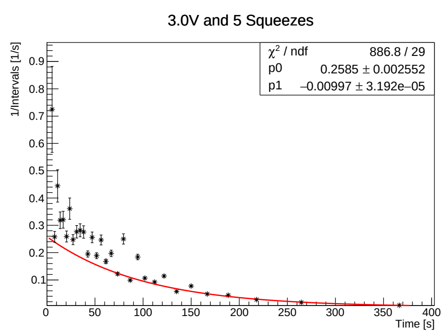
B. Plots



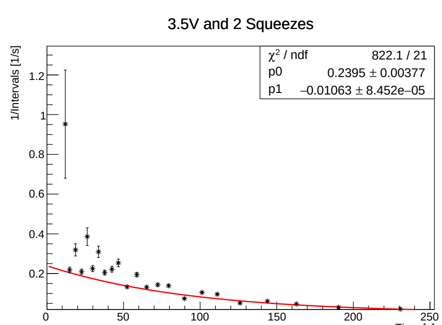
3.0V and 3 Squeezes
χ^2 / ndf 1948 / 25
p0 0.2909 ± 0.003676
p1 −0.01197 ± 6e−05
1/Intervals [1/s]
Time [s]



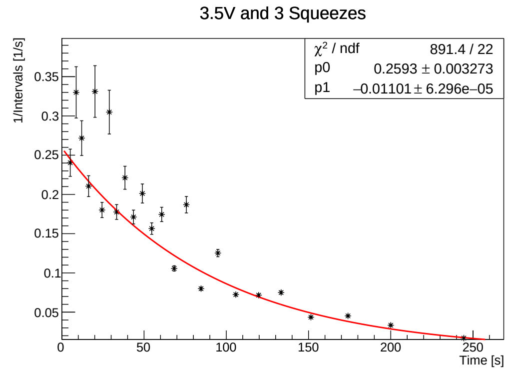
3.0V and 4 Squeezes
χ^2 / ndf 706.3 / 30
p0 0.3174 ± 0.00369
p1 −0.01078 ± 4.443e−05
1/Intervals [1/s]
Time [s]



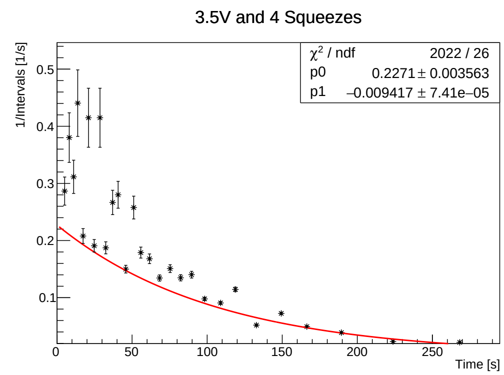
3.0V and 5 Squeezes
χ^2 / ndf 886.8 / 29
p0 0.2585 ± 0.002552
p1 −0.00997 ± 3.192e−05
1/Intervals [1/s]
Time [s]



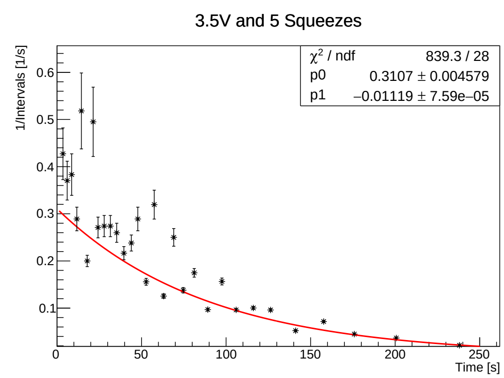
3.5V and 2 Squeezes
χ^2 / ndf 822.1 / 21
p0 0.2395 ± 0.00377
p1 −0.01063 ± 8.452e−05
1/Intervals [1/s]
Time [s]



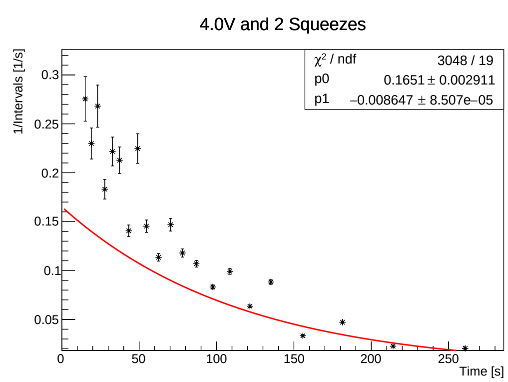
3.5V and 3 Squeezes
χ^2 / ndf 891.4 / 22
p0 0.2593 ± 0.003273
p1 −0.01101 ± 6.296e−05
1/Intervals [1/s]
Time [s]



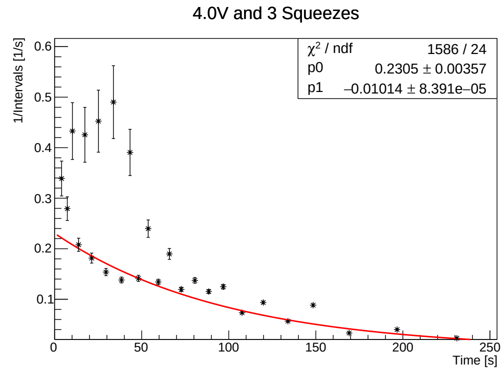
3.5V and 4 Squeezes
χ^2 / ndf 2022 / 26
p0 0.2271 ± 0.003563
p1 −0.009417 ± 7.41e−05
1/Intervals [1/s]
Time [s]

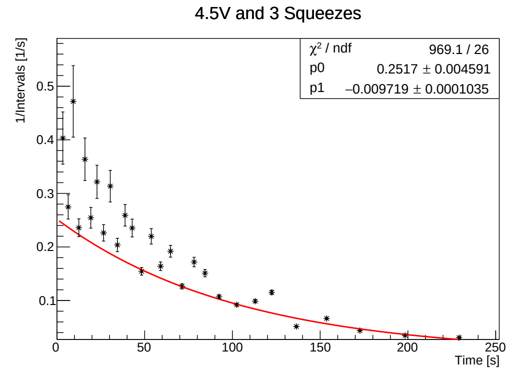


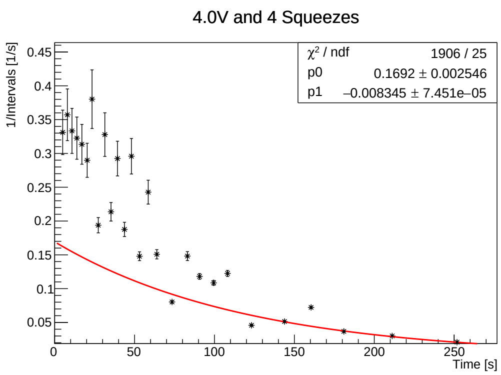
3.5V and 5 Squeezes
χ^2 / ndf 839.3 / 28
p0 0.3107 ± 0.004579
p1 −0.01119 ± 7.59e−05
1/Intervals [1/s]
Time [s]

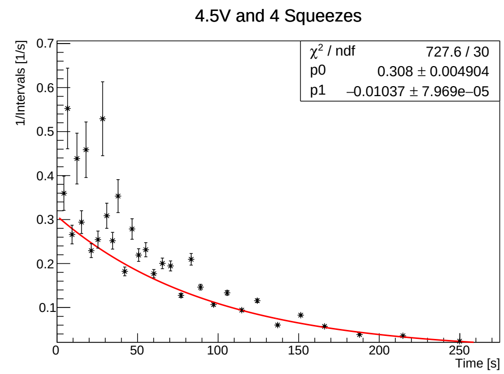


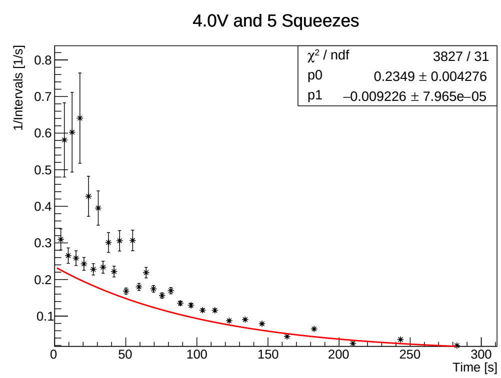
4.0V and 2 Squeezes
χ^2 / ndf 3048 / 19
p0 0.1651 ± 0.002911
p1 −0.008647 ± 8.507e−05
1/Intervals [1/s]
Time [s]

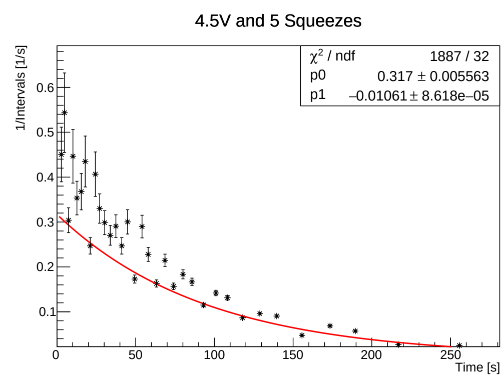


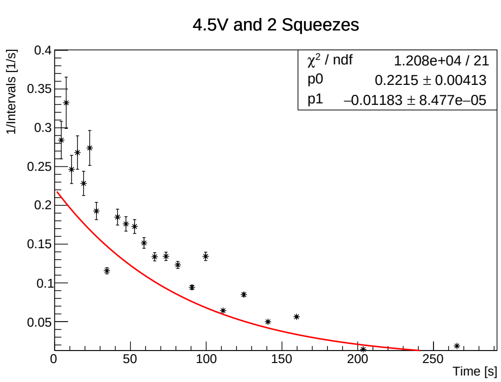












C. Codes

```
{
#include <iostream>
#include <vector>
#include <string>
#include <cmath>
#include <utility>
#include <sstream>


{
const float sigmat = 0.3;
const float ln2 = 0.6931;
const float truevalue = 55.6;
std::vector<float> results;
```

```
std::vector<float> resultsigmas;

TF1 *f1 = new TF1("f1","[0]*exp([1]*x)");
TF1 *f2 = new TF1("f2", "[0] + [1]*x");

gStyle->SetOptFit(1);
gStyle->SetStatX(0.9);
gStyle->SetStatY(0.9);
   std::vector<std::string> filenames = {
      "converted_data1.csv",
      "converted_data2.csv",
      "converted_data3.csv",
      "converted_data4.csv",
      "converted_data5.csv",
      "converted_data6.csv",
      "converted_data7.csv",
      "converted_data8.csv",
      "converted_data9.csv",
      "converted_data10.csv",
      "converted_data11.csv",
      "converted_data12.csv",
      "converted_data13.csv",
      "converted_data14.csv",
      "converted_data15.csv",
      "converted_data16.csv"
   };

for (size_t fileIndex = 0; fileIndex <
    filenames.size(); ++fileIndex) {
   TTree *t = new TTree("t", "t");
   t->ReadFile(filenames[fileIndex].c_str());
   int n = t->GetEntries();
   float * x, * y, * sx, * sy ;
   x = new float[n-1];
   y = new float[n-1];
   sx = new float[n-1];
   sy = new float[n-1];
   float time, interval;
   t->SetBranchAddress("T", &time);
   t->SetBranchAddress("I", &interval);

   for (int i = 0; i < n - 1; ++i) {
      t->GetEntry(i);
      float time_i = time;
      t->GetEntry(i+1);
      float time_ip1 = time;
      x[i] = (time_i + time_ip1) / 2;
      y[i] = 1 / interval;
      sx[i] = sigmat;
      sy[i] = sigmat / (interval * interval);
   }
   std::string voltage =
       std::to_string((fileIndex / 4) * 0.5 +
       3); // Voltage starts from 3V and
       increments by 0.5V
   std::ostringstream oss;
   oss << std::fixed << std::setprecision(1)
       << std::stof(voltage); // Convert to
       float and set precision to 1 decimal
       place
   std::string voltageString = oss.str();
   std::string squeezes =
       std::to_string((fileIndex % 4) + 2); //
       Squeezes start from 2 and increments by
       1
   std::string finalfilename = voltageString +
       "V_" + squeezes + "Squeezes_fitted.pdf";
   std::string finalgraphname = voltageString
       + "V and " + squeezes + " Squeezes";
   TGraphErrors gr(n-1, x, y, sx, sy);
   gr.GetXaxis()->SetTitle("Time [s]");
   gr.GetYaxis()->SetTitle("1/Intervals
       [1/s]");
   gr.SetTitle(finalgraphname.c_str());
   f1->SetRange(0, 500);
   f1->SetParameter(0, 0.25);
   gr.Fit(f1, "S");
   TCanvas *c1 = new TCanvas();
   gr.Draw("A*");
   // Constructing the filename

   c1->Print(finalfilename.c_str());
   float lambda = f1->GetParameter(1);
   float lambdasigma = f1->GetParError(1);
   results.push_back(lambda);
   resultsigmas.push_back(lambdasigma);
}

float uppersum;
float lowersum;

for (int i=0; i<16; i++){

uppersum += results[i]/pow(resultsigmas[i], 2);
lowersum += 1/pow(resultsigmas[i], 2);

}

float finallambda = uppersum/lowersum;
float finallambdasigma = sqrt(1/lowersum);

float halflife = -ln2/finallambda;
float halflifesigma =
    (ln2*finallambdasigma)/(finallambda*finallambda);

std::cout << "Half-life: " << halflife << "s
    +- " << halflifesigma << "s" << std::endl;
std::cout << "Our result is " <<
    (halflife-truevalue)/halflifesigma <<
    "sigma away from true value." << std::endl;
float * u, * v, *z, *q;
u = new float[4];
v = new float[4];
z = new float[4];
q = new float[4];

std::vector<float> squeezeconst;
std::vector<float> voltageconst;

for (int i=0; i<4; i++){
squeezeconst.push_back(results[i]);
voltageconst.push_back(results[i*4]);
}
for (int i=0; i<4; i++){
v[i] = squeezeconst[i];
z[i] = voltageconst[i];
u[i] = i+2;
q[i] = 0.5*i+3.0;
}


TCanvas *c11 = new TCanvas();
TGraphErrors gr1(4, u, v);
gr1.GetXaxis()->SetTitle("Squeeze Amount");
gr1.GetYaxis()->SetTitle("Decay Constant");
gr1.SetTitle("Decay Constant As A Function Of
    Squeezes (3.0V)");
```

```
gr1.Draw("A*");
c11->Print("squeeze.pdf");


TCanvas *c12 = new TCanvas();
TGraphErrors gr2(4, q, z);
gr2.GetXaxis()->SetTitle("Voltage (V)");
gr2.GetYaxis()->SetTitle("Decay Constant");
gr2.SetTitle("Decay Constant As A Function Of
    Voltage (2 Squeezes)");


gr2.SetMarkerStyle(20);
gr2.Draw("A*");
c12->Print("voltage.pdf");

}

}
```