Franck - Hertz Experiment

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***Abstract*—In this experiment, we have shown that the mercury atom can only absorb certain values of energy, which agrees with the quantum nature of atoms. To do that, we have accelerated electrons through a potential difference in a tube filled with mercury gas and collected the current as a function of the accelerating voltage. We have decided that the first excitation level of mercury is $\Delta E_{final} = 5.08eV \pm 0.07eV$, which is 2.57$\sigma$ away from the true value.**

I. Theory

The Franck - Hertz Experiment was first performed in 1914, by James Frank and Gustav Hertz. It was the first electrical measurement that clearly showed the quantum nature of atoms[1]. The results of the experiment also confirmed Bohr's quantum model. Bohr's model postulated that electrons orbit the nucleus at fixed energy levels and that they can be moved to a different energy level by addition of energy[2]. The amount of this energy is the difference between the two states of the atom. Then, in the experiment, when the accelerated electrons hit the mercury atoms, there are two possibilities. If the energy of the electron is lower than the energy difference between the ground state and the first excited state of the atom, the electron will simply bounce back and there will be an elastic collision. But if the energy of the electron is enough, there will be an inelastic collision and the mercury atom will move to an excited state. The energy levels of the atom are quantized, and the difference in energy between these levels can be represented by ΔE. Thus, the energy absorbed by the atom can be expressed as:

$$E_i = E_n + \Delta E \tag{1}$$

where E_n is the energy of the nth energy level of the atom and E_i is the energy of the electron before the collision. For the ground state, E_n is zero, then, the wavelength of the de-excitation is given by

$$\lambda = \frac{hc}{\Delta E} \tag{2}$$

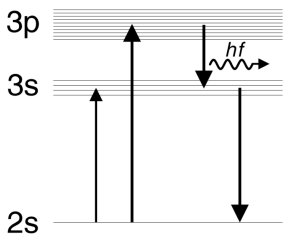

Fig. 1. Energy levels for Neon gas

In the experiment, the electrons are accelerated from the cathode in a tube filled with mercury gas, and a current is collected at the anode. The reason why mercury gas is used is because mercury forms a monoatomic gas, even though it is liquid metal[3]. In the gas form, it doesn't bind to each other or another atoms. So, it provides an environment with minimal reaction. Other suitable gases for the experiment would be neon or argon gases, since they are noble gases.

What we expect to observe in the experiment is that when the accelerating voltage reaches a certain value, 4.9V in the case of mercury, the current should reach a peak and decrease because the electrons excite an atom and their energy is absorbed, therefore they cannot reach the anode. The electrons are left with almost zero kinetic energy after the collision, but they can be accelerated again. When they reach 4.9 V, they will excite an atom again, that's why there will be current drops at the multiples of 4.9V. Since electrons lose all their kinetic energy when they reach 4.9 V, they cannot have an energy to excite an atom to the second excitation level.

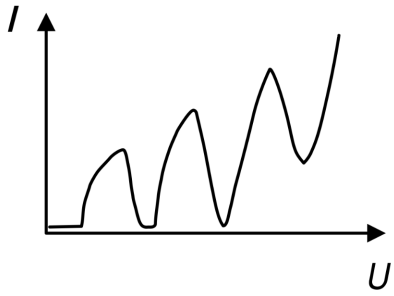

Fig. 2. Expected graph for Accelerating Voltage - Current

When a mercury atom is in the first excited state, it will de-excite after some time. The wavelength of it can be found with (2):

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.626 \times 10^{-34}\,\mathrm{J \cdot s}) \times (3.00 \times 10^{8}\,\mathrm{m/s})}{4.9\,\mathrm{eV}} = 254\,\mathrm{nm}$$

Since 254 nm is out of visible range, we cannot see a light when the de-excitation happens. If we were to use neon gas, the wavelength of the de-excitation would fall to the visible range, therefore we could observe it in a sufficiently darkened room with sensitive equipment[4].

II. The Experimental Setup

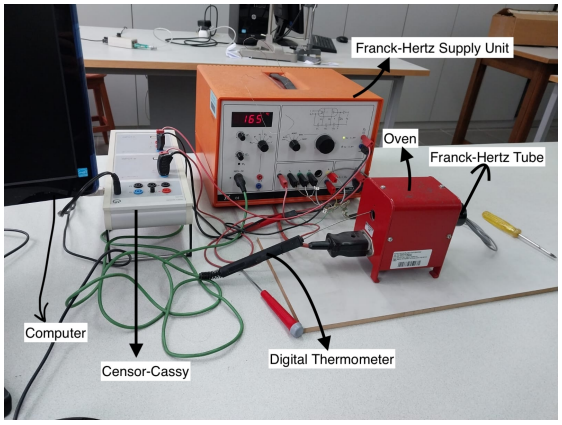

Fig. 3. The setup

- Franck-Hertz Supply Unit: It is the control and supply unit for the experiment. It supplies the potential difference for U1, U2 and U3. You can change and visualize the values of U1, U3 and the temperature, also set U2 as reset or automatic constant increase using the Franck-Hertz Supply Unit.
- Oven: It is used for heating the Franck-Hertz Tube.
- Digital Thermometer: It is used to measure the temperature of the Franck-Hertz Tube. It is also connected to the Franck-Hertz Supply Unit.
- Censor-Cassy: A digital multimeter connected to the computer
- Computer

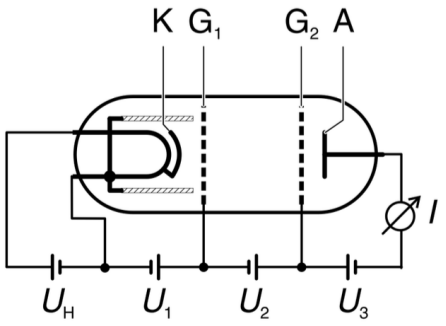

Fig. 4. Inside of the Franck-Hertz Tube

- Franck-Hertz Tube: The tube contains cathode (K), anode (A), two grids (G1 and G2) and a filament. Free electrons are generated from the heated filament and a low potential U1 is applied to the electrons as an initial push. Then, the main accelerating potential U2 is applied between the two grids. The electrons gain energy at this part of the tube to excite the mercury atoms. The final potential U3 is applied between G2 and the anode, which is slightly negative with respect to G2. It is applied to prevent the electrons with low kinetic energy reaching the anode.

III. Method

The procedure of the experiment is as follows:

1) The Franck-Hertz Tube is filled with mercury gas and connected to the Franck-Hertz Supply Unit.
2) The Franck-Hertz Tube is put inside the oven and heated to 164 Celsius.
3) U1 and U3 are set to a low value, U2 is initially set to reset.
4) We start taking data, which is the amount of the collector current as a function of the applied potential U2.
5) U2 is set to automatic constant increase.
6) When the data taking is over, we set U2 back to reset and save the data.
7) U1, U3 and the temperature are changed to different values for different datasets to observe their effect on the data.
8) We take 10 different datasets.

IV. The Data

Here are the plots for 10 datasets with different U1, U3 and temperature values. The uncertainty for the accelerator voltage is 0.15V and the uncertainty for the collector current is 0.005 nA.

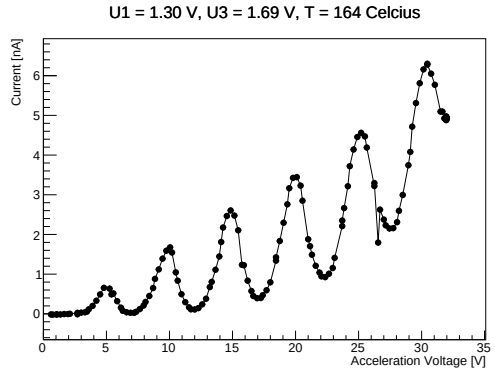

Fig. 5. Plot of current vs U2 at T = 164C, U1 = 1.30V, U3 = 1.69V

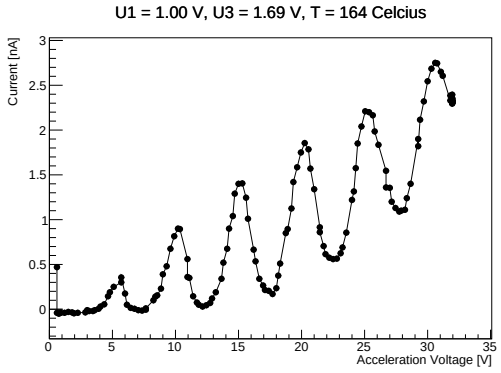

Fig. 6. Plot of current vs U2 at T = 164 °C, U1 = 1.00V, U3 = 1.69V

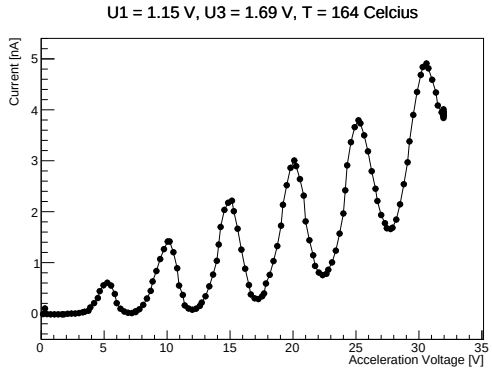

Fig. 7. Plot of current vs U2 at T = 164 °C, U1 = 1.15V, U3 = 1.69V

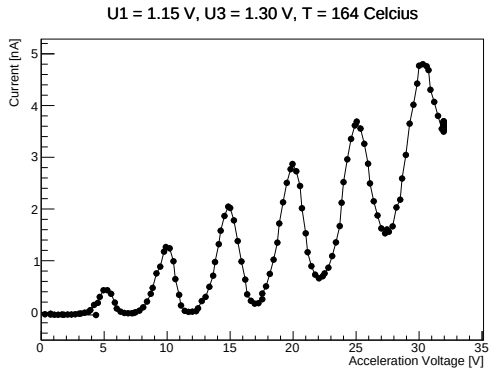

Fig. 8. Plot of current vs U2 at T = 164 °C, U1 = 1.15V, U3 = 1.30V

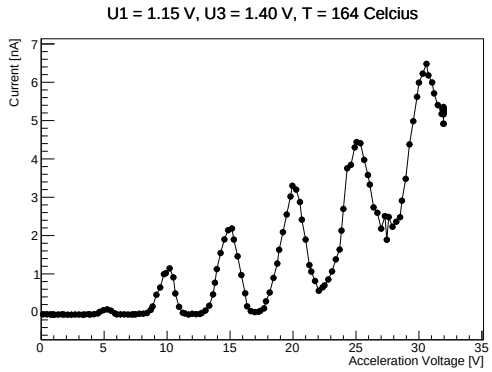

Fig. 9. Plot of current vs U2 at T = 164 °C, U1 = 1.15V, U3 = 1.40V

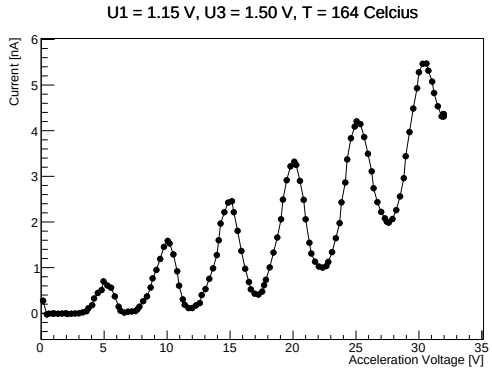

Fig. 10. Plot of current vs U2 at T = 164 °C, U1 = 1.15V, U3 = 1.50V

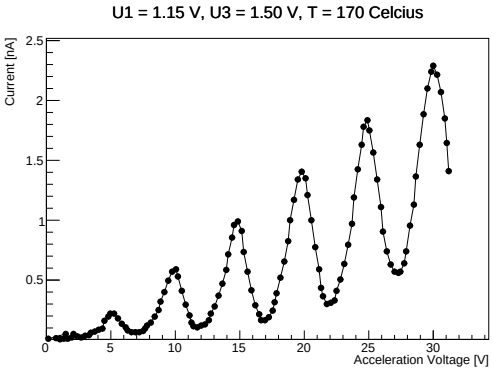

Fig. 11. Plot of current vs U2 at T = 170 °C, U1 = 1.15V, U3 = 1.50V

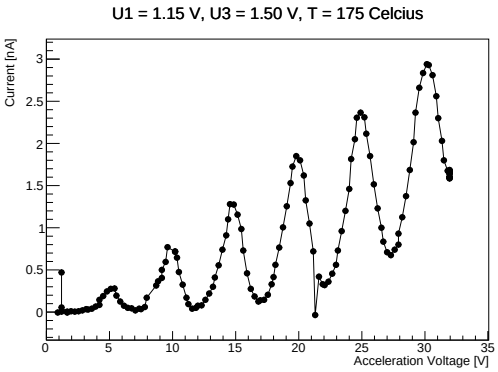

Fig. 12. Plot of current vs U2 at T = 175 °C, U1 = 1.15V, U3 = 1.50V

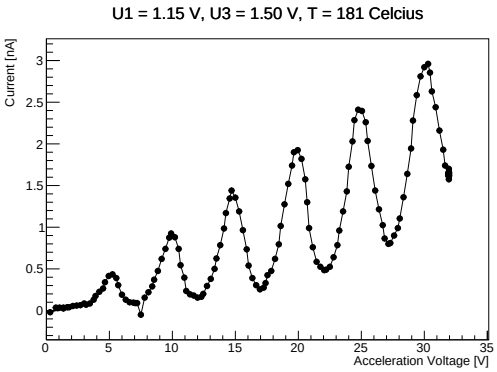

Fig. 13. Plot of current vs U2 at T = 181 °C, U1 = 1.15V, U3 = 1.50V

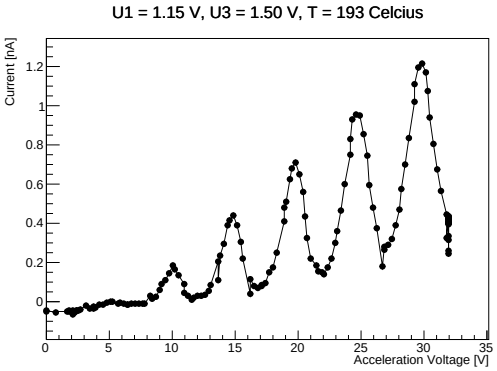

Fig. 14. Plot of current vs U2 at T = 193 °C, U1 = 1.15V, U3 = 1.50V

V. The Analysis

The analysis was made using CERN's framework ROOT, version v6.30.04. For a single dataset, we fitted Gaussian functions only to the peaks, because the whole function is not symmetric. We have recorded the mean and sigma values of the Gaussian fits with their uncertainties. The mean of the fit gives us the position of the peak. Then, we find the difference between the peaks using:

$$\Delta E_i = \mu_{i+1} - \mu_i \tag{3}$$

where i = 1, 2, 3, 4. The difference between the peaks will give us the energy of the excited levels. Then, we calculated the uncertainty of each peak.

For 5 peaks, we obtain 4 ΔE_i and 4 $\sigma_{\Delta E_i}$ values. To find $\Delta E_{i_{final}}$, we applied the weighted average formula.

To find the final uncertainty of a dataset $\sigma_{\Delta E_i}$, we also applied the weighted average formula to sigmas.

So, we have found the final energy difference value and its uncertainty for a single dataset. After doing this for each dataset, we ended up with 10 final energy difference values with their uncertainties. Finally, we plot a histogram with these values and fit a Gaussian to the histogram. The mean of the fit is our final first excitation level result and the sigma of the fit is our uncertainty for the result.

Here are the plots with peaks fitted:

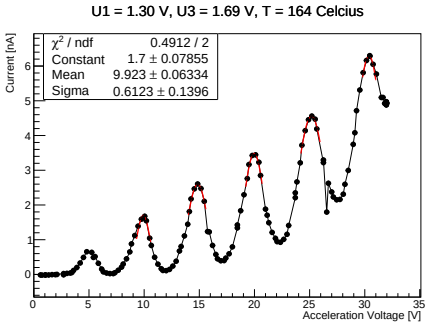

Fig. 15. Plot of current vs U2 at T = 164C, U1 = 1.30V, U3 = 1.69V

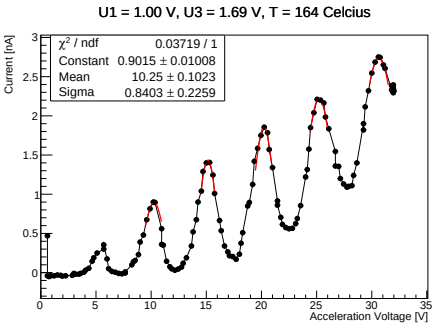

Fig. 16. Plot of current vs U2 at T = 164 °C, U1 = 1.00V, U3 = 1.69V

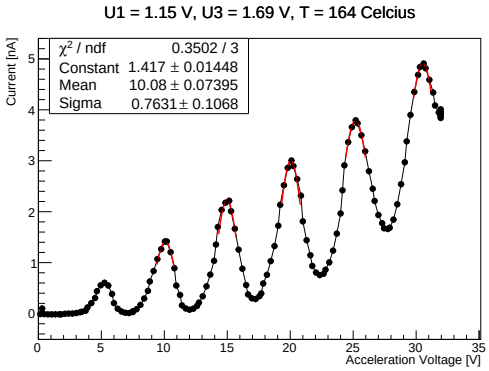

Fig. 17. Plot of current vs U2 at T = 164 °C, U1 = 1.15V, U3 = 1.69V

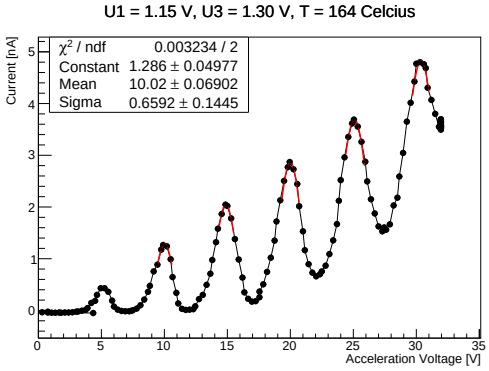

Fig. 18. Plot of current vs U2 at T = 164 °C, U1 = 1.15V, U3 = 1.30V

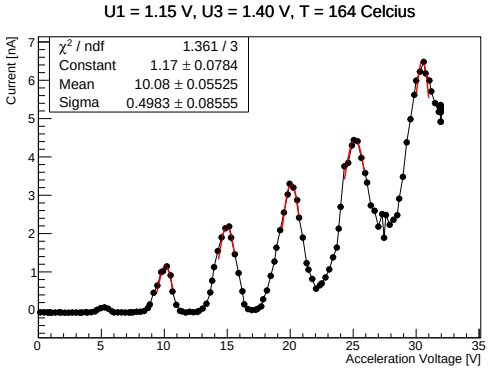

Fig. 19. Plot of current vs U2 at T = 164 °C, U1 = 1.15V, U3 = 1.40V

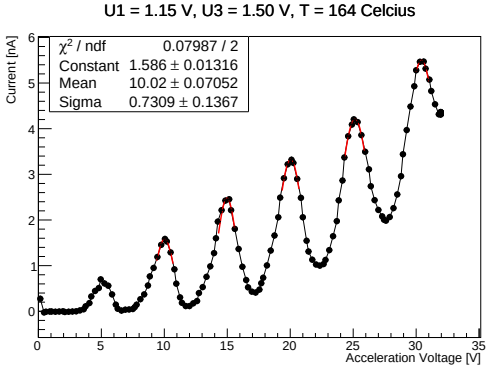

Fig. 20. Plot of current vs U2 at T = 164 °C, U1 = 1.15V, U3 = 1.50V

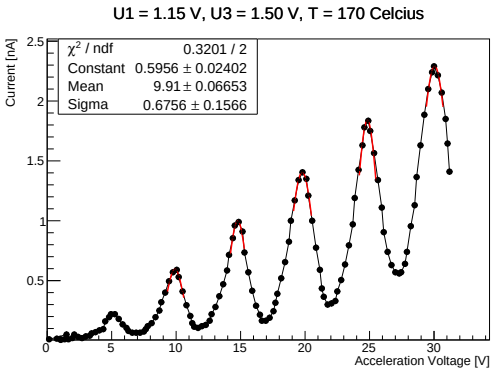

Fig. 21. Plot of current vs U2 at T = 170 °C, U1 = 1.15V, U3 = 1.50V

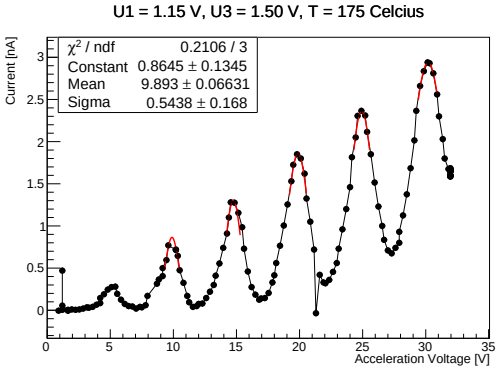

Fig. 22. Plot of current vs U2 at T = 175 °C, U1 = 1.15V, U3 = 1.50V

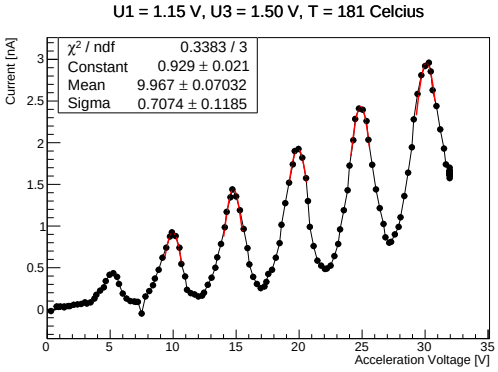

Fig. 23. Plot of current vs U2 at T = 181 °C, U1 = 1.15V, U3 = 1.50V

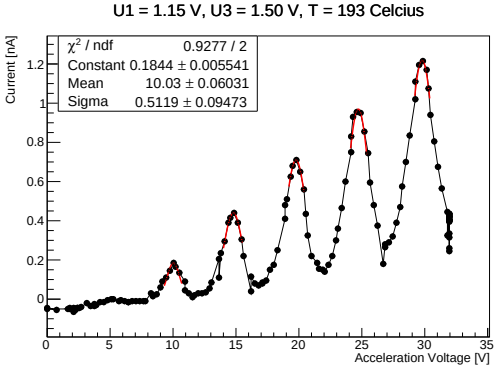

Fig. 24. Plot of current vs U2 at T = 193 °C, U1 = 1.15V, U3 = 1.50V

VI. The Result

After the data analysis, here is the final histogram:

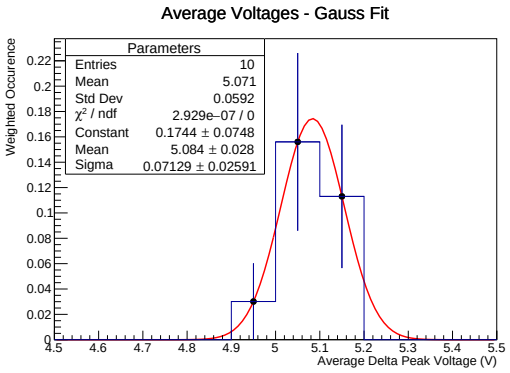

Fig. 25. Final Excitation Levels Histogram

The final result we have found is:

$$\Delta E_{final} = 5.08V \pm 0.07V \tag{7}$$

Finally, we have calculated our error by comparing it to the true value for mercury, which is 4.9V.

$$Error = \frac{|\Delta V_{\text{true}} - \Delta V_{\text{experimental}}|}{\sigma_{\Delta V}} = 2.57\sigma \tag{8}$$

VII. The Conclusion

Our result is only 2.57σ away from the true value, which is consistent with the theory, but it can still be improved. One of the reasons of error might be the approximation that the points near the peaks would have a Gaussian distribution. We have also seen that the result is closer to the true value at higher temperatures, such as 193 Celsius. The reason for this might be the higher concentration of mercury gas at higher temperatures. At very high temperatures, though, where the mercury concentration is too high, the electrons reaching the anode is minimal[4]. On the other hand, U1 and U3 dependency in our data was relatively minor, since we changed them in a narrow range, especially considering they are already smaller compared to U2. In addition, changes in experimental conditions like pressure and voltage fluctuations, as well as impurities in the mercury vapor or imperfections in our setup, might have affected our results.

To wrap it up, the experiment's results confirm that mercury atom must absorb a specific amount of energy to reach to the first excited state.

References

[1] *Franck-Hertz experiment - Wikipedia*. URL: https://en.wikipedia.org/wiki/Franck%E2%80%93Hertz_experiment.

[2] *Bohr Model*. URL: https://www.britannica.com/science/Bohr-model.

[3] *Mercury (element)*. URL: https://en.wikipedia.org/wiki/Mercury_(element).

[4] E. Gülmez. *Advanced Physics Experiments*. 1st. Boğaziçi University Publications, 1999.

[5] E. Gülmez. *Statistics Book*.

VIII. Appendix

The formula for calculating the uncertainty of each peak[5]:

$$\sigma_{\Delta E_i} = \sqrt{\sigma_{\mu_i}^2 + \sigma_{\mu_{i+1}}^2} \tag{4}$$

We use this formula because when we perform mathematical operations involving measurements with uncertainties, the uncertainties in the original measurements propagate through the calculations and contribute to the uncertainty in the final result. This formula provides a convenient way to estimate that combined uncertainty.

In a weighted average, each measurement contributes to the final average proportionally to its reliability, represented by the inverse of its variance.

The formula of the weighted average for energy differences[5]:

$$\Delta E_{\text{final}} = \frac{\sum_{i=1}^{N} \frac{\Delta E_i}{\sigma_i^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}} \tag{5}$$

The formula of the weighted average for uncertainties:

$$\sigma_{\Delta E_{i_{final}}} = \sqrt{\frac{1}{\sum_{i=1}^{N} \sigma_i^2}} \tag{6}$$

```
#include <iostream>
#include <vector>
#include <string>
#include <cmath>
#include <utility>

{
   std::vector<std::string> filenames = {
      "u1_1.30-u3_1.69_T_164.txt",
      "u1_1.00-u3_1.69_T_164.txt",
      "u1_1.15-u3_1.69_T_164.txt",
      "u1_1.15-u3_1.3_T_164.txt",
      "u1_1.15-u3_1.4_T_164.txt",
      "u1_1.15-u3_1.5_T_164.txt",
      "u1_1.15-u3_1.5_T_170.txt",
      "u1_1.15-u3_1.5_T_175.txt",
      "u1_1.15-u3_1.5_T_181.txt",
      "u1_1.15-u3_1.5_T_193.txt",
   };

   std::vector<std::vector<std::pair<double,
       double>>> allSetRanges = {
      {{9.3, 10.6}, {14, 15.6}, {19.2, 20.7},
          {24.2, 25.95}, {29.8, 31}},
      {{9.5, 10.94}, {14.5, 15.8}, {19.45,
          20.9}, {24.4, 25.98}, {29.8, 31.5}},
      {{9.4, 10.8}, {14.3, 15.7}, {19.3,
          20.8}, {24.4, 25.98}, {29.8, 31.2}},
      {{9.4, 10.6}, {14.35, 15.5}, {19.3,
          20.7}, {24.4, 25.9}, {29.7, 31}},
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          20.7}, {24.3, 25.9}, {30, 31}},
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          20.5}, {24.2, 25.5}, {29.4, 30.7}},
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          20.55}, {24.3, 25.55}, {29.4, 30.9}},
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          {24.1, 25.6}, {29.3, 30.8}},
      {{9.3, 10.7}, {14, 15.5}, {19.2, 20.4},
          {24.1, 25.4}, {29.2, 30.4}}
   };

   std::vector<std::vector<double>> allMeans;
   std::vector<std::vector<double>> allSigmas;
   std::vector<std::vector<double>>
       allDeltaMeans;
   std::vector<std::vector<double>>
       allUncertainties;
   std::vector<std::vector<double>> allStds;
   std::vector<double> finalmeans;
   std::vector<double> results;
   std::vector<double> rsigmas;
   TF1 *fnew = new TF1("fnew", "gaus");

   for (size_t fileIndex = 0; fileIndex <
       filenames.size(); ++fileIndex) {

      std::string filename =
          filenames[fileIndex];
      std::stringstream titleStream;

      size_t u1Pos = filename.find("u1_");
      size_t u3Pos = filename.find("-u3_");
      size_t tPos = filename.find("_T_");

   if (u1Pos != std::string::npos && u3Pos !=
       std::string::npos && tPos !=
       std::string::npos) {
      double u1, u3, t;
      std::istringstream(filename.substr(u1Pos
          + 3, u3Pos - (u1Pos + 3))) >> u1;
      std::istringstream(filename.substr(u3Pos
          + 4, tPos - (u3Pos + 4))) >> u3;
      std::istringstream(filename.substr(tPos
          + 3)) >> t;

      titleStream << "U1 = " << std::fixed <<
          std::setprecision(2) << u1 << " V,
          U3 = " << u3 << " V, T = " <<
          std::fixed << std::setprecision(0)
          << t << " Celcius";} else {
      titleStream << "Upper Title Here";
   }

   std::string upperTitle = titleStream.str();

   TGraphErrors
       gr(filenames[fileIndex].c_str());
   gStyle->SetOptFit(1);
   gStyle->SetStatX(0.48);
   gStyle->SetStatY(0.9);

   gr.GetXaxis()->SetTitle("Acceleration
       Voltage [V]");
   gr.GetYaxis()->SetTitle("Current [nA]");
   gr.SetTitle(upperTitle.c_str());
   gr.SetLineColor(kBlack);
   gr.SetMarkerStyle(20);



   for (int i = 0; i < gr.GetN(); ++i) {
      gr.SetPointError(i, 0.15, 0.005);
```

```
}

std::vector<double> means;
std::vector<double> sigmas;
std::vector<double> deltaMeans;
std::vector<double> uncertainties;
double stdmean[5] = {0, 0, 0, 0, 0};
double stdsigma[5] = {0, 0, 0, 0, 0};

for (int i = 0; i < 5; i++) {
   double rangeFirst =
       allSetRanges[fileIndex][i].first;
   double rangeSecond =
       allSetRanges[fileIndex][i].second;
   fnew->SetRange(rangeFirst, rangeSecond);
   gr.Fit("fnew", "QSR+");
   means.push_back(fnew->GetParameter(1));
   sigmas.push_back(fnew->GetParameter(2));
   stdmean[i] = fnew->GetParError(1);
   stdsigma[i] = fnew->GetParError(1);
}

TCanvas *c1 = new TCanvas();
gr.Draw();
std::string finalfilename =
    std::to_string(fileIndex) + ".pdf";
c1->Print(finalfilename.c_str());


for (size_t i = 1; i < means.size(); ++i) {
   double deltaMean = means[i] - means[i -
       1];
   double uncertainty = std::sqrt(sigmas[i]
       * sigmas[i] + sigmas[i - 1] *
       sigmas[i - 1]);
   deltaMeans.push_back(deltaMean);
   uncertainties.push_back(uncertainty);
}

double wmean, wsigma;
double totmean = 0;
double totsigma = 0;
double totweight = 0;
for (int i = 0; i < 4; i++) {
   wmean = 1. / (stdmean[i] * stdmean[i]);
   wsigma = 1. / (stdsigma[i] *
       stdsigma[i]);
   totweight += wmean;
   totmean += deltaMeans[i] * wmean;
   totsigma += uncertainties[i] * wsigma;
}

totmean /= totweight;
double finalsigma = 1. /
    std::sqrt(totsigma);

allMeans.push_back(means);
allSigmas.push_back(sigmas);
allDeltaMeans.push_back(deltaMeans);
allUncertainties.push_back(uncertainties);
results.push_back(totmean);
rsigmas.push_back(finalsigma);

}

TCanvas *c2 = new TCanvas();
TH1D *h1 = new TH1D("Parameters","Average
    Voltages - Gauss Fit",10,4.5,5.5);
for (int i=0; i<10; i++)
    h1->Fill(results[i] ,rsigmas[i]);
h1->SetMarkerStyle(kFullCircle);
h1->GetXaxis()->SetTitle("Average Delta
    Peak Voltage (V)");
h1->GetYaxis()->SetTitle("Weighted
    Occurence");
fnew->SetRange(4.9, 5.2);
h1->Fit("fnew", "Q");
h1->Draw("HIST same E");


double result = fnew->GetParameter(1);
double sd_result = fnew->GetParameter(2);
   for(int i = 0; i < 10; i++){
   cout << results[i] << " +- " <<
       rsigmas[i] << endl;
}

cout << "First Excitation Energy = " <<
    result << " +- "
<< sd_result << " eV" <<endl;
c2->Print("Franck-Hertz.pdf")


}
```