Week 7 HW

6.6 55μF capacitor

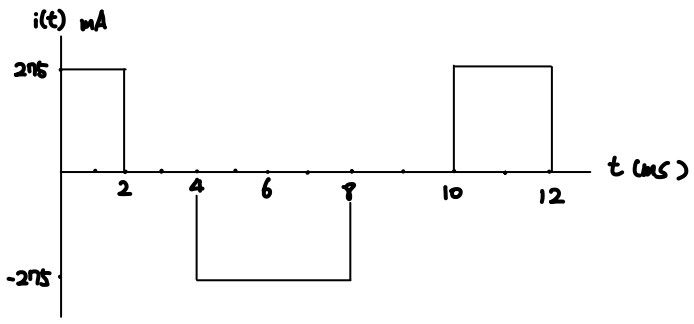
$i = C\frac{dv}{dt}$, $C = 55\times10^{-6}$

$$C\frac{dv}{dt} = \begin{cases} 5\text{ C} & (0\le t\le 2) \\ 0 & (2\le t\le 4) \\ -5\text{ C} & (4\le t\le 6) \\ -5\text{ C} & (6\le t\le 8) \\ 0 & (8\le t\le 10) \\ 5\text{ C} & (10\le t\le 12) \end{cases}$$

$5C = 275\times10^{-6} = \frac{275}{1000}\text{ mA} = 0.275\text{ mA}$

$-5C = -0.275\text{ mA}$

Answer:



6.8 4mF capacitor, initial current 2A

$$v = \begin{cases} 50V & (t\le 0) \\ Ae^{-100t} + Be^{-600t}\ V & (t\ge 0) \end{cases}$$

a) constant A, B

$i = C\frac{dv}{dt} = C\left(-100Ae^{-100t} - 600Be^{-600t}\right)$

$i(0) = -100C(A+6B) = 2$

$A+6B = \frac{2}{-100}\times\frac{1000}{4} = -5$

$v(0) = 50$

$v(0) = A+B$

$$\begin{bmatrix} 1 & 6 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} -5 \\ 50 \end{bmatrix}$$

$5B = -55$

$\therefore A = 61,\ B = -11$

b) energy stored at $t=0$.

$W = \frac{1}{2}Cv^2$, $t=0$

$\frac{1}{2}\cdot 4\times10^{-3}\ v(0)^2$

$= \frac{4\times2500}{2\times1000} = 5$ $\quad W = 5J$

c) capacitor current for $t>0$

$i = C(-100Ae^{-100t} - 600Be^{-600t})$

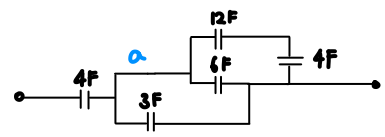
$\{A, B, C\} = \{61, -11, 10^{-3}\times4\}$

$i = \frac{4}{1000}(-6100e^{-100t} + 6600Be^{-600t})$

$\therefore i = -24.4e^{-100t} - 26.4e^{-600t}$

6.17 $C_{eq} = ?$

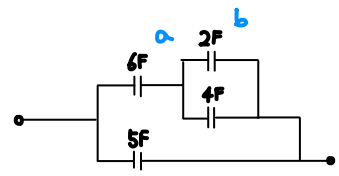
a)



$\frac{4a}{4+a}$, $a = 3+b$, $b = \frac{4\cdot12}{4+12} + 6$ $\quad b=9,\ a=12,\ C=3$

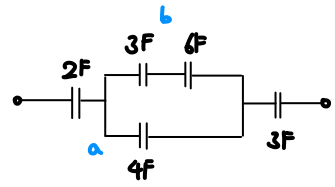
$C_{eq} = 3F$

b)



$a+5$, $a = \frac{6b}{6+b}$, $b=6$ $\quad a=3,\ C=8$ $\quad C_{eq} = 8F$

c)



$\frac{2a}{a+2}$, $a = 4+b$, $b = \frac{3\cdot6}{3+6} = 2$ $\quad a=6,\ \frac{2a}{a+2} = \frac{12}{8}$

$\frac{\frac{36}{8}}{\frac{12}{8}+3} = 1$ $\quad \therefore C_{eq} = 1F$

6.38 40-mH inductor, Find $v(t)$

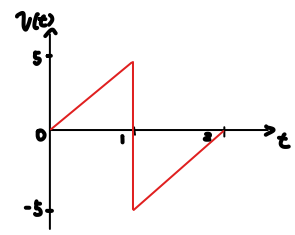
$$i(t) = \begin{cases} 0 & (t<0) \\ te^{-2t}\ A & (t>0) \end{cases}$$

$i = \frac{1}{L}\int_{t_0}^{t} i(\tau)d\tau + v(t_0)$, $v = L\frac{di}{dt}$

$v(t) = L(e^{-2t} - 2te^{-2t})dt$

$= (40-80t)e^{-2t}\ mA$, $t>0$ $\quad v(t) = (40-80t)e^{-2t}\ mV$, $(t>0)$

6.45 10mH inductor. Find $i(t)$, $i(0)=0$



$i = \frac{1}{L}\int_{t_0}^{t} v(\tau)d\tau + 0$

$0\le t<1$.

$\frac{1000}{10}\int_0^t 5t\,dt = 100\left[\frac{5}{2}t^2\right]_0^t$

$= 250t^2$

$1\le t<2$

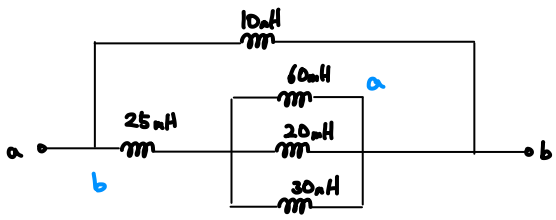
$100\int_1^t (5t-10)dt + i(1) = \int_1^t (500t - 1000)dt + 250$

$= \left(\frac{500}{2}t^2 - 1000t\right)_1^t + 250$

$= 250t^2 - 1000t + 1000$

$$\therefore i(t) = \begin{cases} 0.25t^2\ kA & (0\le t\le 1) \\ 0.25t^2 - t + 1\ kA & (1\le t\le 2) \end{cases}$$

6.51 L_{eq}=?

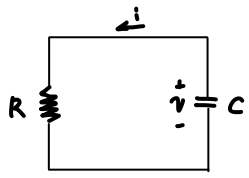


$L_{eq}=\frac{10b}{10+b}$, $b=25+a$, $a=\frac{1}{\frac{1}{25}+\frac{1}{20}+\frac{1}{60}}=10$

$L_{eq}=\frac{350}{45}$ $b=35$

$\therefore L_{eq}=\frac{350}{45}$ mH

7.1



$v(t)=56e^{-200t}$ V $(t>0)$

$i(t)=8e^{-200t}$ mA $(t>0)$

(a) Find R, C

$RC=\tau$

$v=56e^{-200t}$, $IR=8e^{-200t}\cdot R$, $R=\frac{V}{I}=\frac{56e^{-200t}}{8e^{-200t}\cdot 10^{-3}}=7000$

$i(t)=C\frac{dv}{dt}$, $-200\cdot 56e^{-200t}\cdot C=8e^{-200t}\cdot 10^{-3}$

$C=\frac{8e^{-200t}}{200\cdot 56e^{-200t}\cdot 1000}=\frac{1}{1400000}$

$\therefore R=7k\Omega$, $C=\frac{5}{7}\mu F$ $7\times\frac{1}{1000000}$

(b) τ=?

$RC=\frac{7000}{1400000}=\frac{1}{200}=\frac{1}{2}\times 10^{-2}=0.5\times 10^{-2}=5ms$

$\therefore \tau=5ms$

(c) hold, $t=0$

$v(t)=56e^{-200t}$, $v(0)=56$

$(50\%)\times 56=56e^{-200t}$

$\therefore e^{-200t}=\frac{1}{2}$, $e^{200t}=2$, $\ln 2=200t$

$\therefore t=\frac{\ln 2}{200}$