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Chapter 3 HW.

Review Exercise for Chapter 3.

1. Determine for which values of k the vectors form a basis for R^4

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ -2 & 1 & 0 & 3 \\ 0 & -1 & 1 & 4 \\ 2 & 3 & 4 & k \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 11 \\ 0 & 3 & 4 & k-2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & 11 \\ 0 & 0 & 0 & k-69 \end{bmatrix}$$

↳ determinant $\Rightarrow k-69$

R^4의 basis이기 위해서는 linearly independent 한 vector이어야 함. $\therefore k \neq 69$

3. Let

$$S = \left\{ \begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix} \middle| \ a, b, c \in R \right\}$$

a. Show that S is a subspace of $M_{2\times2}$.

S의 임의의 원소 합이 2×2, 스칼라 곱도 2×2 행렬이므로 S는 $M_{2\times2}$ 의 Subspace

b. Is $\begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$ in S?

$a-b=5 \quad a=3$
$b+c=-2 \quad a-c=3$ $\quad \therefore \begin{Bmatrix} a \\ b \\ c \end{Bmatrix} = \begin{Bmatrix} 3 \\ -2 \\ 0 \end{Bmatrix}$, Yes

c. Find a basis B for S.

$\because (a,b,c) \ni \{(1,0,0), (0,1,0), (0,0,1)\}$

$$A = \underbrace{a\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} + b\begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix}}_{\text{선형독립}}$$

Basis B for S = $\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} \right\}$

d. Give a 2×2 matrix that is not in S.

For example, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$a-b=1 \quad b=-1$
$a=0$
$b+c=0 \quad c=1$
$a-c=1 \quad c=-1$) contradiction

5. Suppose that $S = \{v_1, v_2, v_3\}$ is a basis for a vector space V.

a. Determinant whether the set $T = \{v_1, v_1+v_2, v_1+v_2+v_3\}$ is a basis for V.

$C_1 v_1 + C_2(v_1+v_2) + C_3(v_1+v_2+v_3) = 0$
↳ $(C_1+C_2+C_3)v_1 + (C_2+C_3)v_2 + C_3 v_3 = 0$

$\therefore C_1+C_2+C_3 = C_2+C_3 = C_3 = 0$ (Linearly Independent)

V는 T의 기저벡터이다.

b. Determine whatever the set $W = \{-v_2+v_3, 3v_1+2v_2+v_3, v_1-v_2+2v_3\}$ is a basis for V.

$C_1(-v_2+v_3) + C_2(3v_1+2v_2+v_3) + C_3(v_1-v_2+2v_3) = 0$
↳ $(3C_2+C_3)v_1 + (-C_1+2C_2-C_3)v_2 + (C_1+C_2+2C_3)v_3 = 0$

$3C_2+C_3 = -C_1+2C_2-C_3 = C_1+C_2+2C_3 = 0$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 0 & 3 & 1 \\ -1 & 2 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

$R_2+R_3 = [0 \ 3 \ 1] \rightarrow$ linearly dependent

V는 W의 기저벡터가 아니다.

7. Suppose span$\{v_1, \dots, v_n\} = V$ and $C_1v_1 + C_2v_2 + \dots + C_nv_n = 0$ which $C_1 \neq 0$. Show that span$\{v_2, \dots, v_n\} = V$.

$C_1 \neq 0$ 이므로 위 식은

$C_1v_1 = -(C_2v_2 + \dots + C_nv_n)$

$v_1 = \dfrac{-C_2}{C_1}v_2 + \dots + \dfrac{-C_n}{C_1}v_n$

↳ $v_2, v_3, \dots, v_n$ 의 스칼라곱의 선형결합

$\therefore V = \text{span}\{v_2, v_3, \dots, v_n\}$

9. Let

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

Such that $u \cdot v = 0$, $\sqrt{u_1^2+u_2^2} = 1 = \sqrt{v_1^2+v_2^2}$

a. Show that $B = \{u, v\}$ is a basis for R^2

u, v가 Linearly independent 라고 할 때, $v=0$ 이라고 가정 $(u \cdot v = 0)$

$au+bv=0$ 을 v에 대입

$u \cdot (au+bv)$
$= a u^2 + b(u v)$
$= a(u_1^2+u_2^2) = 0 \quad \therefore a = 0$

u, v는 선형독립, B는 R^2의 basis다

b. Find the coordinates of the vector $w = \begin{bmatrix} x \\ y \end{bmatrix}$ relative to the ordered basis B.

$w = \begin{bmatrix} a \\ b \end{bmatrix} \rightarrow au + bv = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} au_1+bv_1 \\ au_2+bv_2 \end{bmatrix}$

Cramer Rule을 적용하면

$$a = \frac{\begin{vmatrix} x & v_1 \\ y & v_2 \end{vmatrix}}{\begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix}}, \quad b = \frac{\begin{vmatrix} u_1 & x \\ u_2 & y \end{vmatrix}}{\begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix}}$$

$$a = \frac{xv_2 - yv_1}{u_1v_2 - u_2v_1}, \quad b = \frac{yu_1 - xu_2}{u_1v_2 - u_2v_1}$$

(단 $u_1v_2 \neq u_2v_1$)

Chapter 3: Chapter Test

1. C에 C_1+C_2를 대입

$(C_1+C_2) \odot x = x + C_1 + C_2$?

$= C_1x + C_2x \Rightarrow C_1 + x + C_2 + x$

False

5. det $\Rightarrow 1(1-0) - 0(2-0) - 2(4-3) = -1$

True

9. $P_3 \Rightarrow \{1, x, x^2, x^3\}$ standard.

True

13. $\underbrace{S, T}$ → Linearly dependent

True

17. 만약 V가 차원보다 많은 vector를 가진다면 성립 X

False

21. True

25. $B_1 S = B_2$

$B_1 E = B_2 S^{-1}$

True

29. $\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 1 & -1 \end{bmatrix} = B_2$

True

33. True