Introduction to Optimization of Digital Logic Design

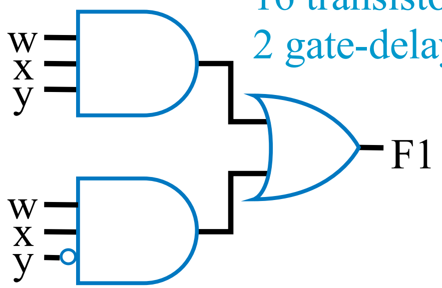
– Chapter 6 –

Introduction

- We now know how to build digital circuits
  - How can we build ***better*** circuits?
- Let's consider two important design criteria
  - *Delay* – the time from inputs changing to new correct stable output
  - *Size* – the number of transistors
  - For quick estimation, assume
    - Every gate has delay of "1 gate-delay"
    - Every gate *input* requires 2 transistors
    - Ignore inverters

Transforming F1 to F2 represents an ***optimization***: Better in all criteria of interest

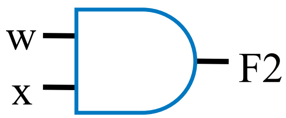
16 transistors
2 gate-delays



$F1 = wxy + wxy'$

(a)

4 transistors
1 gate-delay

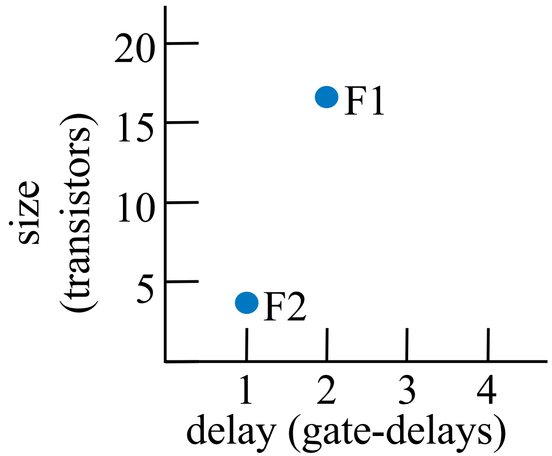


$F2 = wx$

(b)

$= wx(y+y') = wx$

a



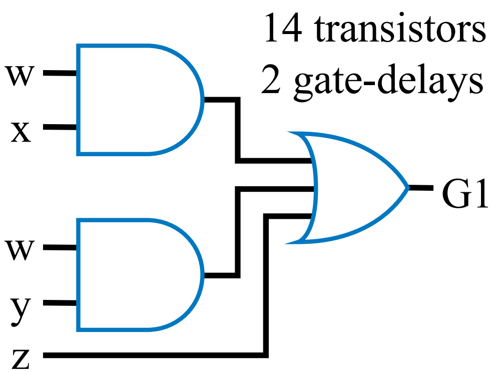
(c)

Note: Slides with animation are denoted with a small red "a" near the animated items

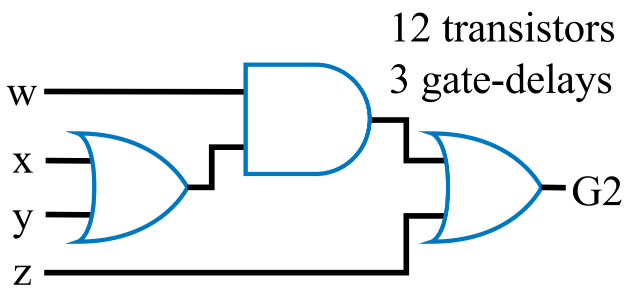
Tradeoff

- Tradeoff
  - Improves some, but worsens other, criteria of interest

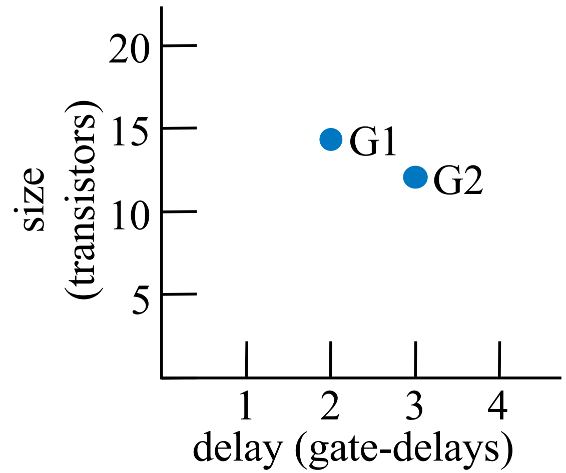
Transforming G1 to G2 represents a ***tradeoff***: Some criteria better, others worse.



G1 = wx + wy + z



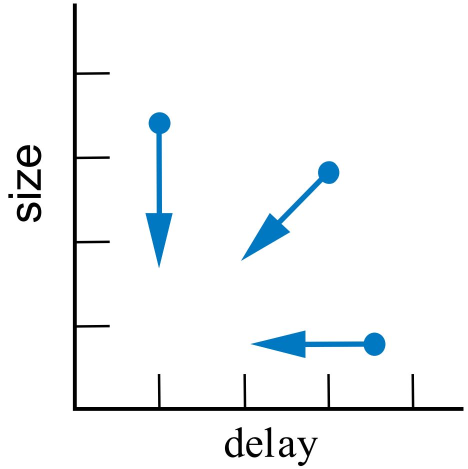
G2 = w(x+y) + z

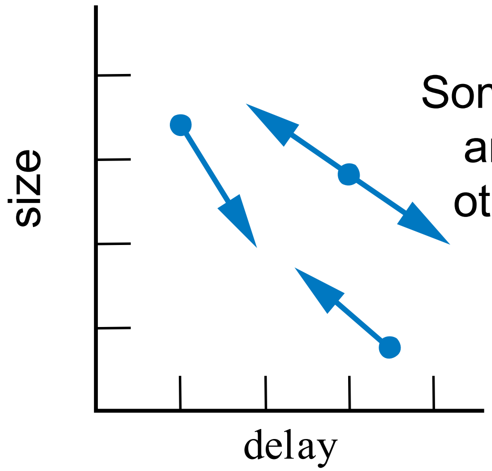


Optimization and Tradeoff

Optimizations

All criteria of interest are improved (or at least kept the same)





Tradeoffs

Some criteria of interest are improved, while others are worsened

- We obviously prefer optimizations, but often must accept tradeoffs
  - You can't build a car that is the most comfortable, and has the best fuel efficiency, and is the fastest – you have to give up something to gain other things.

Combinational Logic Optimization and Tradeoffs

– 교재 6장 2절

Combinational Logic Optimization and Tradeoffs

- Two-level size optimization using algebraic methods
  - Goal: Two-level circuit (ORed AND gates) with fewest transistors
    - Though transistors getting cheaper (Moore's Law), still cost something
- Define problem algebraically
  - Sum-of-products yields two levels
    - F = abc + abc' is sum-of-products; G = w(xy + z) is not.
  - Transform sum-of-products equation to have *fewest literals and terms*
    - Each literal and term translates to a gate input, each of which translates to about 2 transistors (see Ch. 2)
    - For simplicity, ignore inverters

Example

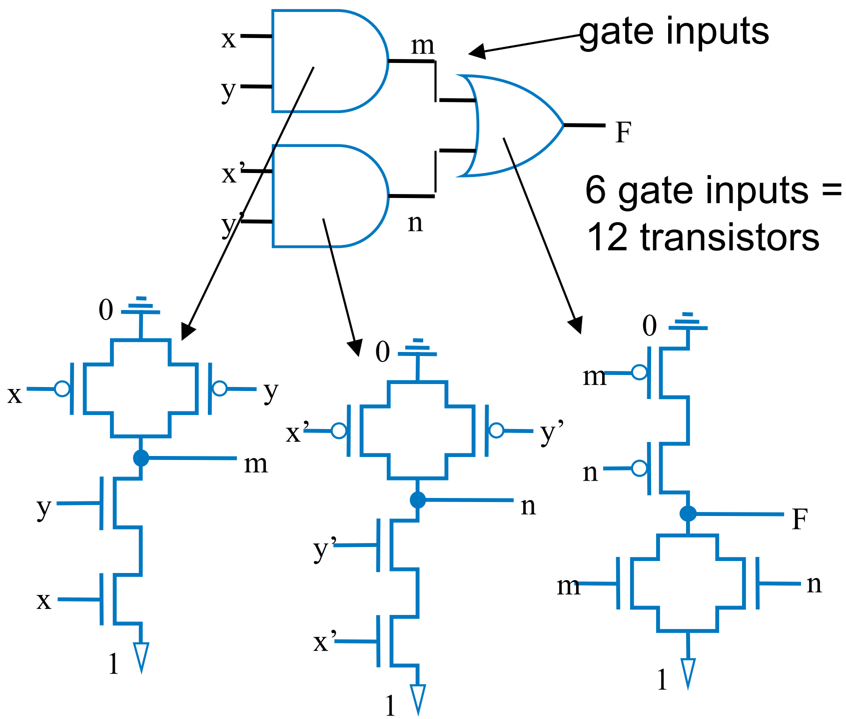
F = xyz + xyz' + x'y'z' + x'y'z

F = xy(z + z') + x'y'(z + z')

F = xy*1 + x'y'*1

F = xy + x'y'

4 literals + 2 terms = 6 gate inputs



Note: Assuming 4-transistor 2-input AND/OR circuits; in reality, only NAND/NOR use only 4 transistors.

Algebraic Two-Level Size Optimization

- Previous example showed common algebraic minimization method
  - (Multiply out to sum-of-products, then...)
  - Apply following as much as possible
    - $ab + ab' = a(b + b') = a*1 = a$
    - "*Combining terms to eliminate a variable*"
      - (Formally called the "*Uniting theorem*")
  - Duplicating a term sometimes helps
    - Doesn't change function
      - $c + d = c + d + d = c + d + d + d + d \ldots$
  - Sometimes after combining terms, can

$F = xyz + xyz' + x'y'z' + x'y'z$

$F = xy(z + z') + x'y'(z + z')$

$F = xy*1 + x'y'*1$

$F = xy + x'y'$

$F = x'y'z' + x'y'z + x'yz$

$F = x'y'z' + x'y'z + x'y'z + x'yz$

$F = x'y'(z+z') + x'z(y'+y)$

$F = x'y' + x'z$

$G = xy'z' + xy'z + xyz + xyz'$

$G = xy'(z'+z) + xy(z+z')$

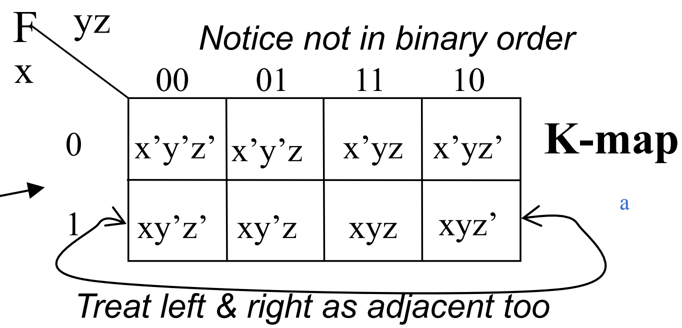
$G = xy' + xy$ *(now do again)*

$G = x(y'+y)$

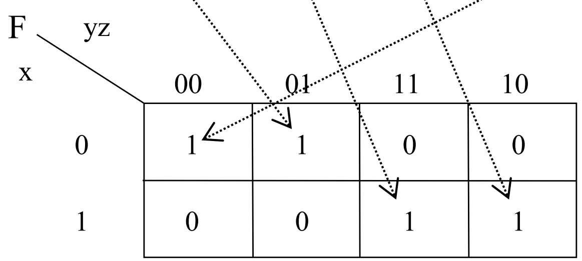
$G = x$

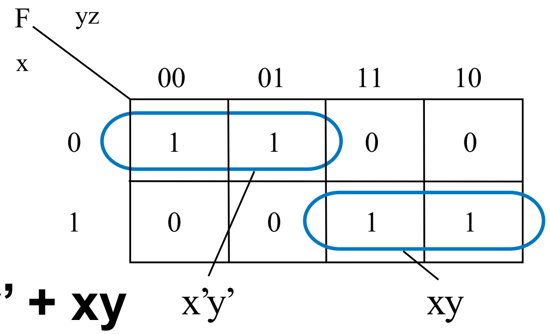
Karnaugh Maps for Two-Level Size Optimization

- Easy to miss possible opportunities to combine terms when doing algebraically
- *Karnaugh Maps (K-maps)*
  - **Graphical** method to help us find opportunities to combine terms
  - Minterms <u>differing in one variable</u> are *adjacent* in the map
  - Can clearly see opportunities to combine terms – look for adjacent 1s
    - For F, clearly two opportunities
    - Top left circle is *shorthand* for: $x'y'z' + x'y'z = x'y'(z'+z) = x'y'(1) = x'y'$
    - Draw circle, write term that has all the literals except the one that changes in the circle
      - Circle xy, x=1 & y=1 in both cells of the circle, but z changes (z=1 in one cell, 0 in the other)
    - Minimized function: OR the final terms



$F = x'y'z + xyz + xyz' + x'y'z'$





$\mathbf{F = x'y' + xy}$

Easier than algebraically:

$F = xyz + xyz' + x'y'z' + x'y'z$
$F = xy(z + z') + x'y'(z + z')$
$F = xy*1 + x'y'*1$
$F = xy + x'y'$

K-maps

- Four adjacent 1s means two variables can be eliminated
  - Makes intuitive sense – those two variables appear in all combinations, so one term *must* be true
  - Draw one big circle – *shorthand* for the algebraic transformations above

$G = xy'z' + xy'z + xyz + xyz'$

$G = x(y'z' + y'z + yz + yz')$ *(must be true)*

$G = x(y'(z'+z) + y(z+z'))$

$G = x(y'+y)$

$G = x$

G: x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	1

x

Draw the biggest circle possible, or you'll have more terms than really needed

G: x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	1

xy'

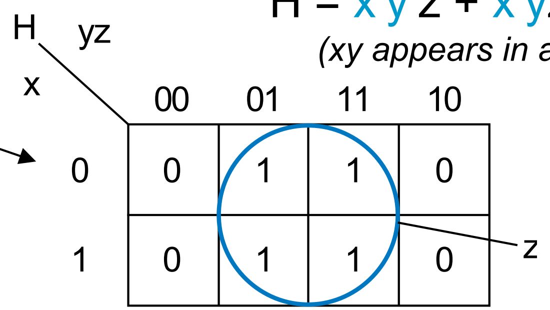
xy

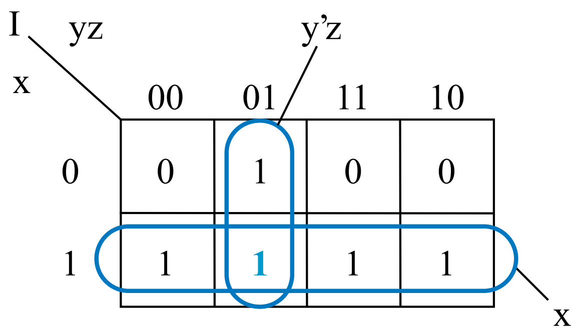
K-maps

- Four adjacent cells can be in shape of a square
- *OK* to cover a 1 twice
  - Just like duplicating a term
    - Remember, c + d = c + d + d
- No *need* to cover 1s more than once
  - Yields extra terms – not minimized

H = x'y'z + x'yz + xy'z + xyz

(xy appears in all combinations)





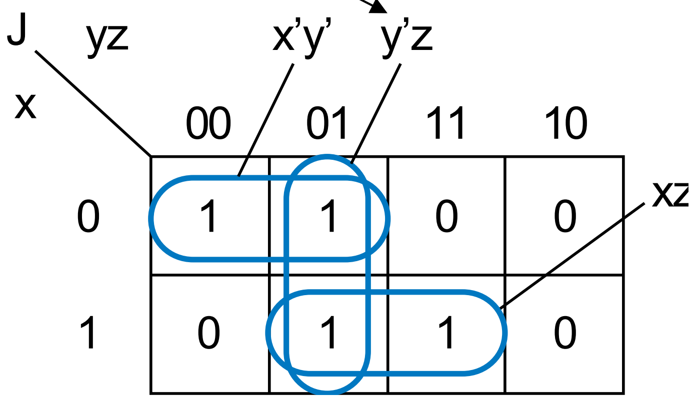
The two circles are shorthand for:

I = x'y'z + xy'z' + xy'z + xyz + xyz'

I = x'y'z + xy'z + xy'z' + xy'z + xyz + xyz'

I = (x'y'z + xy'z) + (xy'z' + xy'z + xyz + xyz')

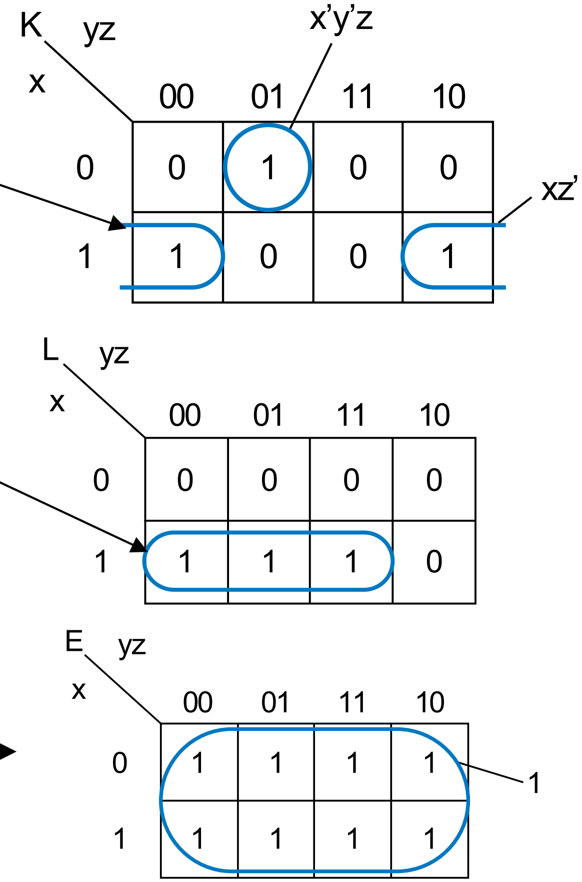
I = (y'z) + (x)



a

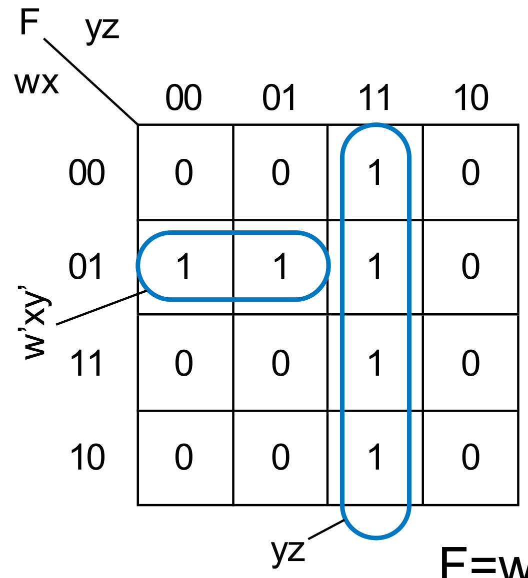
K-maps

- Circles can cross left/right sides
  - Remember, edges are adjacent
    - Minterms differ in one variable only
- Circles must have 1, 2, 4, or 8 cells – 3, 5, or 7 not allowed
  - 3/5/7 doesn't correspond to algebraic transformations that combine terms to eliminate a variable
- Circling all the cells is OK
  - Function just equals 1

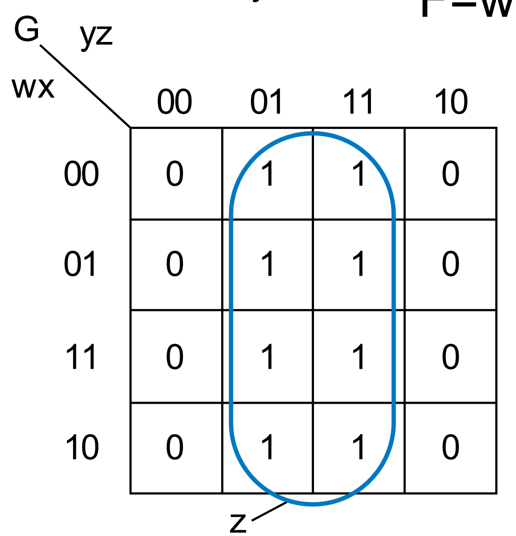


K-maps for Four Variables

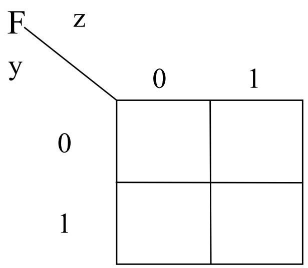
- Four-variable K-map follows same principle
  - Adjacent cells differ in one variable
  - Left/right adjacent
  - Top/bottom also adjacent
- 5 and 6 variable maps exist
  - But hard to use
- Two-variable maps exist
  - But not very useful – easy to do algebraically by hand



F=w'xy'+yz



G=z



Two-Level Size Optimization Using K-maps

General K-map method

1. *Convert* the function's equation into sum-of-minterms form
2. *Place* 1s in the appropriate K-map cells for each minterm
3. *Cover* all 1s by drawing the fewest largest circles, with every 1 included at least once; write the corresponding term for each circle
4. *OR* all the resulting terms to create the minimized function.

Two-Level Size Optimization Using K-maps

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Common to revise (1) and (2):

- Create *sum-of-products*
- Draw 1s for each product

Ex: $F = w'xz + yz + w'xy'z'$

w'xz

yz

w'xy'z'

F (wx \ yz)	00	01	11	10
00	0	0	1	0
01	1	1	1	0
11	0	0	1	0
10	0	0	1	0

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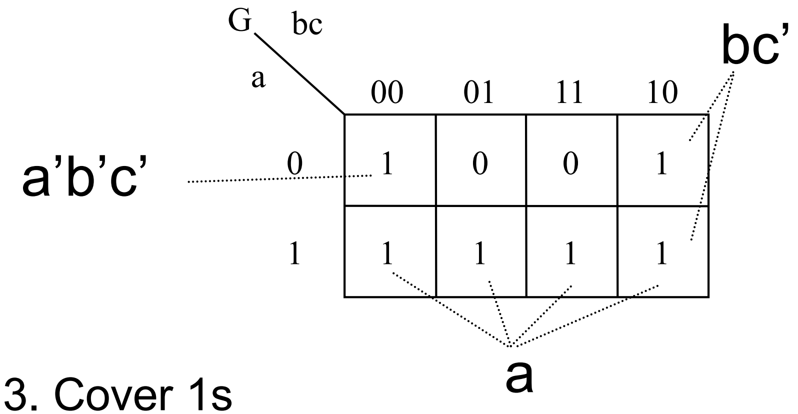
Example: Minimize:

G = a + a'b'c' + b*(c' + bc')

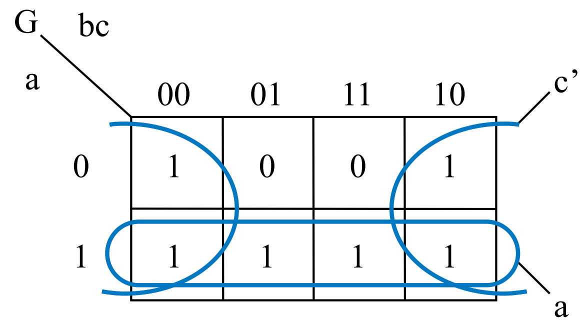
1. Convert to sum-of-products

G = a + a'b'c' + bc' + bc'

2. Place 1s in appropriate cells



3. Cover 1s



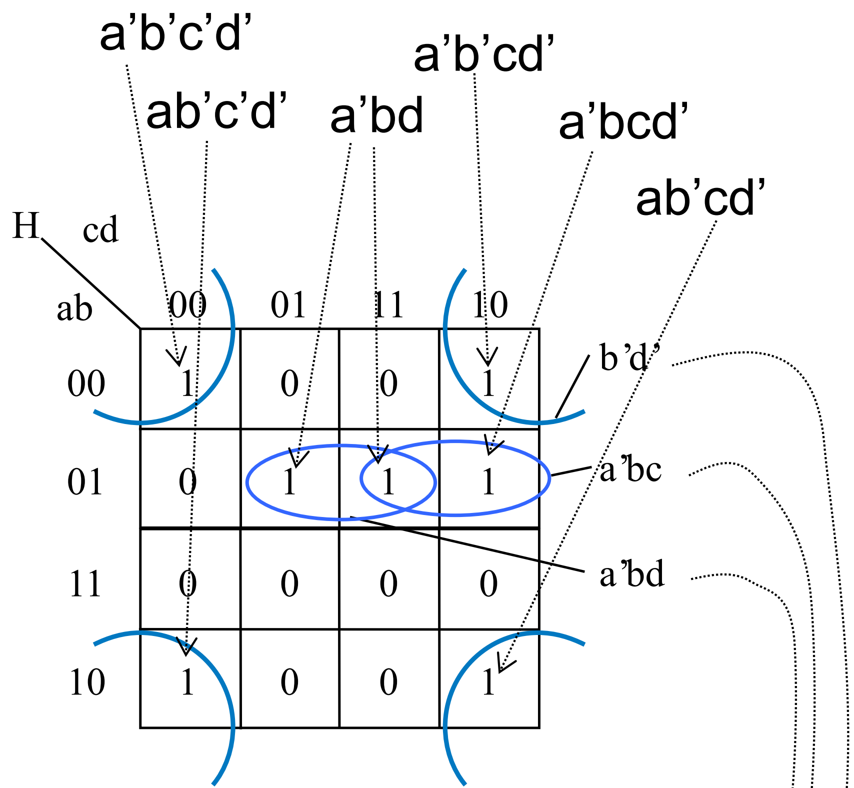
4. OR terms: **G = a + c'**

Two-Level Size Optimization Using K-maps

– Four Variable Example

- Minimize:
  - H = a’b’(cd’ + c’d’) + ab’c’d’ + ab’cd’ + a’bd + a’bcd’

1. Convert to sum-of-products:
   - H = a’b’cd’ + a’b’c’d’ + ab’c’d’ + ab’cd’ + a’bd + a’bcd’
2. Place 1s in K-map cells
3. Cover 1s
4. OR resulting terms

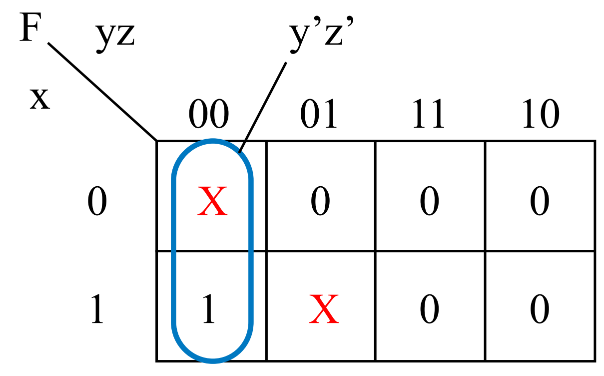


Funny-looking circle, but remember that left/right adjacent, and top/bottom adjacent

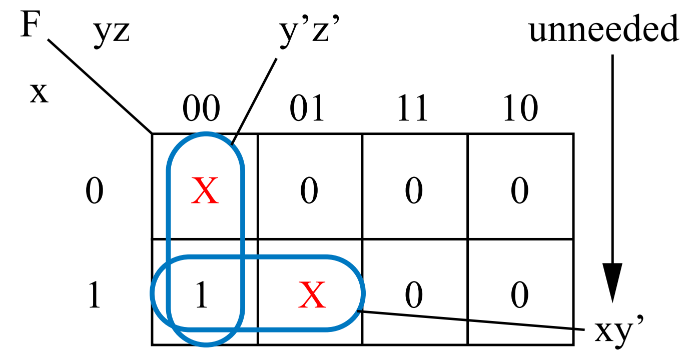
H = b’d’ + a’bc + a’bd

Don't Care Input Combinations

- **What if we know that particular input combinations can never occur?**
  - e.g., Minimize F = xy'z', given that x'y'z' (xyz=000) can *never* be true, and that xy'z (xyz=101) can *never* be true
  - So it doesn't matter what F outputs when x'y'z' or xy'z is true, because those cases *will never occur*
  - Thus, make F be 1 or 0 for those cases *in a way that best minimizes the equation*
- **On K-map**
  - Draw **X**s for don't care combinations
    - Include X in circle ONLY if minimizes equation
    - Don't include other Xs



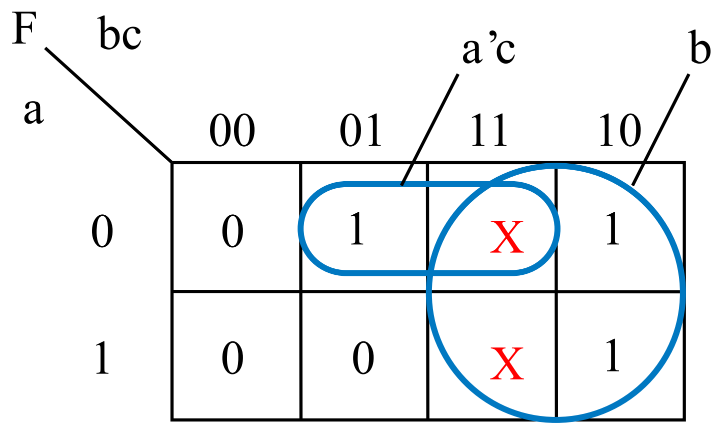
Good use of don't cares



Unnecessary use of don't cares; results in extra term

Optimization Example using Don't Cares

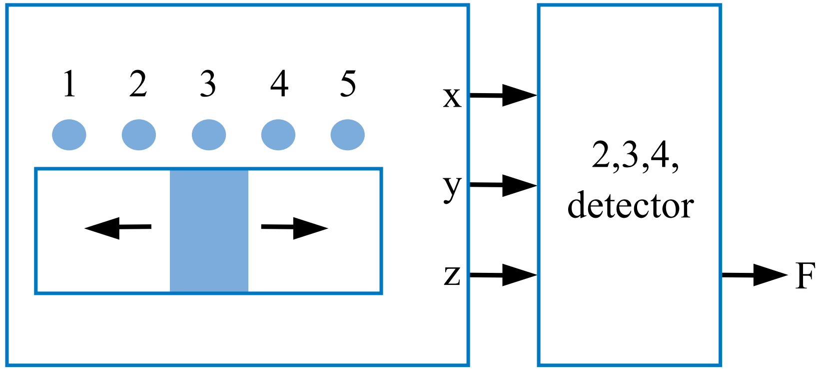
- Minimize:
  - F = a'bc' + abc' + a'b'c
  - Given don't cares: a'bc, abc
- Note: Introduce don't cares with caution
  - Must be *sure* that we really don't care what the function outputs for that input combination
  - If we do care, even the slightest, then it's probably safer to set the output to 0



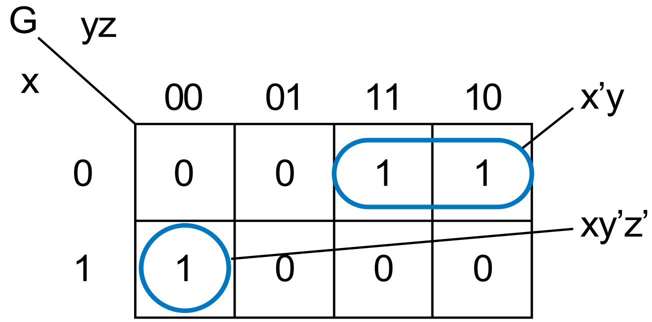
F = a'c + b

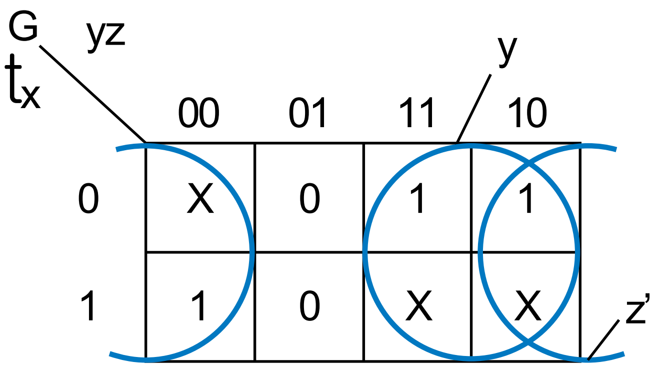
Optimization with Don't Cares Example: Sliding Switch

- Switch with 5 positions
  - 3-bit value gives position in binary
- Want circuit that
  - Outputs 1 when switch is in position 2, 3, or 4
  - Outputs 0 when switch is in position 1 or 5
  - Note that the 3-bit input can never output binary 0, 6, or 7
    - Treat as don't care input combinations



Without don't cares:
F = x'y + xy'z'





With don't cares:
F = y + z'