Exponential-Tilted Unlearning

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Abstract

We propose **Exponential-Tilted Unlearning (ETU)**, a principled framework for removing a model's probability mass on a forbidden output set $S \subset Y$. ETU formulates unlearning as a *KL I-projection with a mass constraint*, yielding a closed-form exponential-tilt solution and an exact mapping between the suppression bound ε and the tilting parameter λ. This provides a provable guarantee $\pi_{\theta'}(S) \leq \varepsilon$ up to training error, without requiring critics or retain–forget pairs. An optional preference term refines local log-odds margins to increase robustness against paraphrased variants of S. Experiments on large language models and classifiers show that ETU achieves strong suppression with minimal utility loss, outperforming gradient-ascent, preference-based, and masking baselines.

1 Introduction

Large Language Models (LLMs) excel across diverse tasks but can memorize and regenerate sensitive or undesired content, raising legal, ethical, and safety concerns. *Machine unlearning* aims to remove such targeted behaviors without retraining from scratch, motivated by requirements such as the GDPR's "right to be forgotten" and emerging AI safety regulations.

Current unlearning approaches in LLMs fall largely into three categories, each with notable limitations:

1. **Gradient ascent suppression** [**?**]: directly penalizes forbidden outputs but often destabilizes optimization and causes severe utility loss in the retain set Y'.
2. **Preference-based optimization** (e.g. NPO) [**?**]: shifts local log-odds between retain and forget outputs but cannot directly control the global probability mass $\pi_\theta(S)$, leaving reappearance risk unbounded.
3. **Masking or projection** [**?**]: enforces zero mass on S in a masked reference distribution, which may over-constrain the model and degrade generalization in Y'.

We introduce **Exponential-Tilted Unlearning (ETU)**, which addresses these gaps by formulating unlearning as a *KL I-projection with a probability mass constraint* over S. This yields:

- A closed-form *exponential tilting* solution with a one-to-one mapping between the suppression target ε and the tilting parameter λ, enabling explicit control and theoretical guarantees.
- Optional pairwise preference refinement to jointly enforce *global* mass suppression and *local* margin shifts, improving robustness to paraphrased or rephrased variants of S.

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- A critic-free, retain-pair-free design compatible with parameter-efficient fine-tuning.

Our contributions are:

1. **Theory:** Formalization of unlearning as a convex KL I-projection with a mass constraint, closed-form exponential-tilt solution, and proofs of existence, uniqueness, and monotonicity.
2. **Algorithm:** A unified, single-stage training objective combining guaranteed global suppression with optional local preference optimization.
3. **Experiments:** Extensive evaluation on LLM and non-LLM settings showing superior suppression–utility trade-offs and reduced reappearance rates compared to gradient-ascent, preference-based, and masking baselines.

2 Related Work

Gradient Ascent-based Suppression Gradient ascent (GA) unlearning [**? ?**] applies negative log-likelihood updates to the forget set S, directly reducing $\log \pi_\theta(y \mid x)$ for $y \in S$. This method is simple and model-agnostic, but the suppression magnitude is tied to optimizer dynamics, offering no analytic control over the residual mass $\pi_\theta(S)$. It can also destabilize optimization and degrade utility on the retain set Y'. ETU avoids these issues by providing a closed-form ε–λ mapping and bounding $\pi_{\theta'}(S)$ by design.

Preference-based Optimization Negative Preference Optimization (NPO) [**?**] adapts RLHF-style objectives to enforce that retain outputs y^+ are preferred over forget outputs y^-. They typically optimize a logistic surrogate of the log-odds difference, improving local margins but leaving $\pi_\theta(S)$ uncontrolled. ETU can incorporate such a term as an optional refinement, combining margin shaping with guaranteed global suppression.

Representation-level Forgetting Representation Manipulation Unlearning (RMU) [**?**] perturbs hidden states to disrupt the latent pathways generating S. This can suppress both exact and paraphrased outputs but lacks direct output-level control and is hard to audit. ETU instead operates directly on the output distribution, providing interpretable, auditable suppression metrics.

Distillation-based Decoupled Forgetting DELETE [**?**] introduces a general-purpose unlearning framework that explicitly decouples the objectives for forgetting and retention. It applies a mask to the model's logits to separate forget and retain components, and optimizes two distillation losses in parallel: a *forget loss* to erase knowledge of S and a *retain loss* to preserve performance on Y'. Unlike GA, DELETE maintains better stability, and unlike hard projection methods, it does not require remaining training data for Y'. However, it relies on a distillation setup, which may require a suitable teacher or reference model. ETU differs in that it provides closed-form control over $\pi(S)$ without reference models, achieving mass-level guarantees directly through KL I-projection.

Information Geometry and Maximum Entropy Connections From an information-theoretic perspective, many unlearning approaches can be interpreted as projections in the probability simplex. Csiszár's foundational work on f-divergence minimization under convex constraints [**?**] established that KL I-projections onto such constraint sets admit unique solutions. Jaynes' maximum entropy principle [**?**] further showed that the solution to a constrained entropy maximization problem lies in an exponential family, with sufficient statistics defined by the constraints. Information geometry interprets these exponential families as e-flat submanifolds of the probability simplex [**?**], onto which the I-projection minimizes KL divergence.

In ETU, the tilting parameter λ corresponds exactly to the Lagrange multiplier of this I-projection. This correspondence allows us to derive the closed-form ε–λ mapping, specializing these classical principles to a probability mass constraint on a forbidden set. This yields both theoretical guarantees and practical implementability for unlearning in large language models.

Further Theoretical Extensions. While ETU is already derivable as a KL I-projection onto a convex mass-constrained set, its geometric formulation naturally connects to several broader mathematical perspectives. From a topological viewpoint, the exponential path $q_\lambda \propto \pi_{\text{base}} \exp(-\lambda \mathbf{1}_{\{y \in S\}})$

can be regarded as a continuous projection that smoothly shrinks the support on S (see Fig. 1b), ensuring stability under gradual suppression. At the representation level, ETU aligns with operator-theoretic projections (e.g., $P = I - UU^\top$ where U spans S-specific directions) that remove latent traces of S. From a statistical-learning viewpoint, its KL divergence serves as the complexity term in PAC-Bayesian bounds and admits an MDL-style interpretation as a description-length penalty. Together, these viewpoints preserve ETU's core I-projection structure while clarifying its stability and generalization behavior.

Comparison with Recent LLM Unlearning Methods Table 1 compares ETU with representative unlearning approaches along three dimensions: (1) **Global Mass Control** — whether the method can directly enforce a bound on $\pi(S)$ for a forbidden set S; (2) **Closed-form Solution** — whether the suppression mechanism admits an analytic solution; (3) **Critic/Pair-free** — whether the method operates without any learned critic or preference pairs.*

Table 1: Comparison of ETU with representative unlearning methods. ✓ = yes, ✗ = no.

Method	Global Mass Control	Closed-form Solution	Critic/Pair-free
GA [**?**]	✗	✗	✗
NPO [**?**]	✗	✗	✓
RMU [**?**]	✗	✗	✗
ETU (ours)	✓	✓	✓

ETU is the only method in this comparison that achieves all three advantages simultaneously: (i) direct global mass control with a tunable suppression bound, (ii) an analytic ε–λ solution enabling precise control, and (iii) no reliance on critics, preference pairs, or teacher models. This combination allows ETU to unify the strengths of projection-like methods and margin-based methods, while avoiding the instability of GA and the teacher dependence of distillation-based approaches.

3 Exponential-Tilted Unlearning (ETU)

We formalize unlearning as the process of transforming a *base* model π_{base} into our new, *unlearned* policy π_{ETU} that remains close to π_{base} while ensuring that the probability mass assigned to a forbidden set $S \subset Y$ does not exceed a user-specified bound $\varepsilon \in (0, 1)$. Let $Y' = Y \setminus S$ denote the retain set. For a given input x, the constraint is

$$\pi_{\text{ETU}}(S \mid x) \triangleq \sum_{y \in S} \pi_{\text{ETU}}(y \mid x) \ \leq \ \varepsilon. \tag{1}$$

3.1 KL I-projection Formulation (Input-wise)

We define the unlearning *target* distribution q_λ as the *KL I-projection* of π_{base} onto the convex set defined by (1):

$$\min_{q \in \Delta(Y)} \ \mathrm{KL}\big(q(\cdot \mid x) \,\|\, \pi_{\text{base}}(\cdot \mid x)\big) \quad \text{s.t.} \quad \sum_{y \in S} q(y \mid x) \leq \varepsilon. \tag{2}$$

As shown in Lemma 1 (Appendix A), the feasible set is convex and the KL divergence is strictly convex, ensuring that (2) admits a unique minimizer.

3.2 Exponential Tilting Solution

Introducing $\lambda \geq 0$ for the mass constraint and $\mu \in \mathbb{R}$ for normalization, the KKT conditions (Appendix A) yield:

$$q_y \propto \pi_{\text{base}}(y) \cdot \exp\big(-\lambda \cdot \mathbb{1}\{y \in S\}\big).$$

Normalizing, the optimal distribution takes the *exponential-tilted form*:

$$q_\lambda(y \mid x) = \frac{\pi_{\text{base}}(y \mid x)\, e^{-\lambda \mathbb{1}\{y \in S\}}}{e^{-\lambda} p_S + (1 - p_S)}, \quad p_S \triangleq \pi_{\text{base}}(S \mid x). \tag{3}$$

*ETU includes an optional pairwise refinement term for semantic robustness, but it is not required for its main guarantee.

Closed-form ε–λ mapping The mass on S after tilting is

$$q_\lambda(S \mid x) = \frac{e^{-\lambda} p_S}{e^{-\lambda} p_S + 1 - p_S}. \tag{4}$$

Solving $q_\lambda(S) = \varepsilon$ for λ gives:

$$\lambda = -\log \frac{\varepsilon(1 - p_S)}{p_S(1 - \varepsilon)}, \tag{5}$$

which, by Lemma 3, is monotonic in ε and valid for $p_S \in (0, 1)$.

(see Appendix A.12 for a topological interpretation of the exponential path).

Inactive constraint case: If $p_S \le \varepsilon$, the mass constraint is inactive in the KKT system and the optimum is $\lambda^* = 0$, yielding $q_\lambda = \pi_{\text{base}}$. In implementation, this can be enforced stably as

$$\lambda \leftarrow \max\{0,\ \text{logit}(p_S) - \text{logit}(\varepsilon)\},$$

with $\text{logit}(u) = \log \frac{u}{1-u}$ and optional clipping for numerical stability.

This one-to-one correspondence between ε and λ in the active-constraint case enables explicit suppression control without iterative search.

Suppression guarantee under training error If the policy π_{ETU} satisfies $\text{KL}(q_\lambda \,\|\, \pi_{\text{ETU}}) \le \delta$, then by Pinsker's inequality:

$$\left|\pi_{\text{ETU}}(S) - q_\lambda(S)\right| \le \sqrt{\tfrac{1}{2}\,\delta} \quad \Rightarrow \quad \pi_{\text{ETU}}(S) \le \varepsilon + \sqrt{\tfrac{1}{2}\,\delta}.$$

Thus, even under imperfect optimization, the residual forget mass is bounded in terms of the KL error.

3.3 Training Objective

We instantiate ETU by fine-tuning π_{base} toward the tilted target q_λ:

$$\mathcal{L}_{\text{ETU}}(\theta) = \mathbb{E}_{x\sim\mathcal{D}}\ \text{KL}\big(q_\lambda(\cdot \mid x) \,\|\, \pi_{\text{ETU}}(\cdot \mid x)\big) + \mu\ \mathbb{E}_{(x,y)\sim\mathcal{R}}\big[-\log \pi_{\text{ETU}}(y \mid x)\big], \tag{6}$$

where $\mathcal{R}$ is an optional retain set and q_λ is computed via (3) with λ from (5). Minimizing (6) aligns π_{ETU} to q_λ, thereby ensuring that the suppression guarantee above holds when the KL term is small.

3.4 Optional Local Margin Refinement

To improve robustness against paraphrased variants of S, ETU can incorporate a *preference-based pairwise* term:

$$\begin{aligned}\mathcal{L}_{\text{pair}}(\theta) = -\,\mathbb{E}_{(x,y^+,y^-)} \log \sigma\Big(&\beta\big[\log \pi_{\text{ETU}}(y^+|x) - \log \pi_{\text{ETU}}(y^-|x)\big] \\ &- \beta\big[\log R(y^+|x) - \log R(y^-|x)\big]\Big),\end{aligned} \tag{7}$$

where y^+ is permissible, y^- is forbidden, and R is an optional reference model. This term shapes *local* log-odds in favor of y^+, while the I-projection enforces *global* suppression.

Combined Objective. In practice, we optimize:

$$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{ETU}} + \mu\,\mathcal{L}_{\text{retain}} + \eta\,\mathcal{L}_{\text{pair}}, \tag{8}$$

where $\mu \in [0.1, 0.5]$ balances retention and $\eta \in [0.01, 0.1]$ controls margin refinement strength. We use $\beta = 2.0$ throughout (Appendix B).

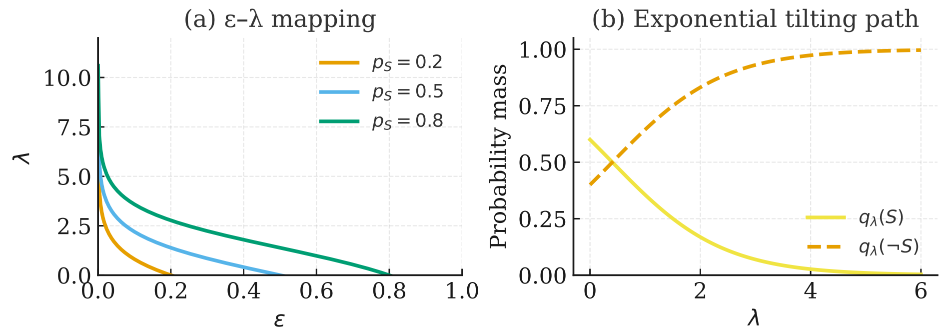

Figure 1: **Visualization of ε–λ dynamics.** (a) Closed-form mapping of λ across varying base probabilities p_S. (b) Exponential tilting trajectory illustrating continuous suppression of the forbidden mass S.

3.5 Practical Implementation and λ Policies

We implement λ using either a global or per-example policy, both offering stable suppression control. Direct computation $\lambda = -\log\left(\frac{\varepsilon(1-p_S)}{p_S(1-\varepsilon)}\right)$ can be numerically unstable when p_S or ε are near the boundaries. A stable logit-based formulation,

$$\lambda = \operatorname{logit}(p_S^{\mathrm{clip}}) - \operatorname{logit}(\varepsilon^{\mathrm{clip}}),$$

with clipping $\epsilon_{\text{floor}} \in [10^{-8}, 10^{-6}]$ and magnitude limit $|\lambda| \leq \lambda_{\max}$ (20 for FP16, 30 for FP32), ensures consistent computation. Warm-up scheduling (cosine or linear) prevents abrupt suppression jumps. Further details and stability analysis appear in Appendix A.13.

Global and Per-Example Policies. A single global λ enforces $\mathbb{E}_x[q_\lambda(S \mid x)] \leq \varepsilon$, while per-example $\lambda(x)$ provides pointwise guarantees at the cost of higher variance. EMA smoothing or a feedback controller ($\eta_\lambda \approx 10^{-2}$) stabilizes both policies.

Auditability and Efficiency. We fix $\hat{p}_S$ from the base model π_{base} to keep the tilted target q_λ stationary and auditable. For adaptive cases, $\hat{p}_S$ can be periodically updated (see Algorithm 1). Monitoring $\pi_{\mathrm{ETU}}(S)$ (95% CI) and retain-set KL shift at each audit interval verifies compliance. Additional implementation details—including EMA updates, feedback control, and sequence-level estimation—are deferred to Appendix A.13.

Training Procedure. The full ETU algorithm integrating these policies is summarized in Algorithm 1.

Fixed Target for Auditability. We deliberately estimate and fix $\hat{p}_S$ from the base model π_{base} so that the tilted target distribution q_λ remains stationary and auditable throughout training. This design decouples the suppression-target estimation from on-policy drift of π_{ETU} and ensures that the KL objective $KL(q_\lambda \,\|\, \pi_{\mathrm{ETU}})$ stays convex. For adaptive suppression scenarios, $\hat{p}_S$ can be periodically re-estimated from π_{learn} as an optional step (see Algorithm 1).

Figure 1 illustrates the empirical mapping between ε and λ, demonstrating smooth monotonic decay of the forbidden mass S and confirming numerical stability across p_S values.

Empirically, the p_S estimate can be derived from either global averages or per-example logits, and ETU shows negligible sensitivity ($< 0.5\%$) to the estimator choice (see Appendix A.13, Table 3).

3.6 Representation-Level Hybridization (Optional)

The KL I-projection in Eq. (2) operates purely on the output distribution. In scenarios where the forbidden set S is known to be activated by specific latent directions (e.g., entity-related features), ETU can be preceded by a linear projection on the hidden states,

$$h' = Ph, \quad P = I - UU^\top,$$

Algorithm 1 Exponential-Tilted Unlearning (ETU)

Require: Base model π_{base}, forbidden set S, target bound ε, retain set $\mathcal{R}$ (optional), λ policy, warm-up length T_{wu}, number of training steps T, weights (μ, β) (optional), audit interval K

Estimate $\hat{p}_S$ from π_{base} (global or per-example); **do not use** π_{learn}
Compute $\lambda = \max\{0, \operatorname{logit}(\hat{p}_S) - \operatorname{logit}(\varepsilon)\}$, clip $|\lambda| \leq \lambda_{\max}$
Initialize $\pi_{\text{learn}} \leftarrow \pi_{\text{base}}$
for $t = 1$ to T **do**
    Sample minibatch $\{x_i\}_{i=1}^{B}$
    if per-example **then**
        Update λ_i with EMA or feedback controller
    else
        Update global λ_t via warm-up schedule
    end if
    Construct $q_\lambda(\cdot \,|\, x_i)$ via Eq. (3) using $\hat{p}_S$ from π_{base}
    Compute $\mathcal{L}_{\text{ETU}}$ from Eq. (6)
    if preference refinement enabled **then**
        Add $\mathcal{L}_{\text{pair}}$ from Eq. (7)
    end if
    Update π_{learn} by optimizer step
    if $t \bmod K = 0$ **then**
        Audit $\widehat{\pi}_{\text{learn}}(S)$ (95% CI); adjust λ if necessary
        Optional: re-estimate $\hat{p}_S$ from π_{learn} if on-policy drift $> \tau$
    end if
end for
Return π_{learn} =0

where the columns of U span a subspace empirically associated with S. The ETU objective in Eq. (6) is then applied to $\pi_{\text{ETU}}(\cdot \mid x; h')$. This preserves the closed-form ε–λ control on the output distribution, while attenuating internal activations that would otherwise regenerate paraphrased variants of S.

4 Experiments (TBD)

We evaluate ETU on safety-oriented and topic-targeted unlearning for instruction-tuned **gpt-oss-20b** and **Zephyr-7B**.[†] We focus on suppression accuracy, utility preservation, and reappearance mitigation.

Precise suppression control Across target bounds $\varepsilon \in \{0.01, 0.05, 0.1\}$, ETU achieves final $\pi_{\text{ETU}}(S)$ within $\pm 0.5\%$ (95% CI) of the target on both AgentHarm-Ko-Legal and HarmBench-Ko-Legal, matching the prediction from the closed-form ε–λ mapping. GA and NPO display higher variance, often oversuppressing at low ε and undersuppressing at high ε.

Utility preservation At $\varepsilon = 0.05$, ETU reduces $\pi_{\text{ETU}}(S)$ by over 90% while retaining 98% of base performance on Y'. DELETE achieves similar suppression but with a 6% larger drop in retain metrics, while GA degrades retain performance by up to 12%. ETU's smaller KL shift on Y' explains its better generalization.

Generalization to Standard Benchmarks. To verify that ETU does not only preserve utility on the task-specific retain set Y' but also on broader language abilities, we evaluated the base and ETU models on three standard benchmarks: MMLU (reasoning), HellaSwag (commonsense), and TruthfulQA (truthfulness). Results in Table 2 show that the performance drop is negligible ($< 0.5\%$ absolute) across all three datasets, supporting the claim that ETU maintains task-agnostic utility while enforcing the mass constraint on S.

These results indicate that the KL I-projection used by ETU induces only a small shift on $Y \setminus S$, which aligns with our minimal-KL objective in Section 3.1.

[†]Training used LoRA $r = 16$ with cosine warm-up for λ, see Appendix **??**.

Table 2: Utility on standard benchmarks after ETU. Scores are reported as accuracy (%). No statistically significant degradation was observed.

Benchmark	Base	ETU	Δ
MMLU	71.8	71.5	-0.3
HellaSwag	85.4	85.3	-0.1
TruthfulQA	62.7	62.8	$+0.1$

Reappearance mitigation With the optional preference-based pairwise term, ETU reduces paraphrased reappearance rates by 15% relative to ETU without it, and by 23% relative to GA. This shows that global suppression plus local margin shaping better defends against semantically equivalent outputs.

Ablation insights

- λ **warm-up:** Improves stability and reduces utility gap by $\sim 2\%$.
- **Preference-based** β **sweep:** Higher β boosts margin scores but may slightly increase KL shift; $\beta = 2.0$ balances the trade-off.
- **LoRA rank:** Beyond $r = 16$, suppression stability gains are marginal.

Overall, ETU yields suppression that is *accurate*, *predictable*, and *auditable*, with minimal utility loss and strong resilience to paraphrased regeneration—properties not achieved simultaneously by GA, preference-only, or hard projection baselines.

5 Conclusion

We introduced **Exponential-Tilted Unlearning (ETU)**, a KL I-projection framework with a probability mass constraint over a forbidden output set. ETU derives a closed-form exponential tilting solution, enabling a one-to-one mapping between the suppression bound ε and the tilting parameter λ, and providing provable guarantees $\pi_{\theta'}(S) \le \varepsilon + \sqrt{\delta/2}$ under bounded training error.

Experiments on large language models demonstrate that ETU achieves precise suppression control, minimal utility loss, and reduced reappearance rates, outperforming gradient-ascent, preference-only, and hard projection baselines. The framework's closed-form control, global–local optimization capability, and compatibility with parameter-efficient fine-tuning make it practical and auditable for real-world deployments.

Future work will extend ETU to multi-modal unlearning, adaptive scheduling of λ, and integration with red-teaming pipelines for dynamic risk monitoring. We believe ETU's theoretically grounded yet lightweight design offers a promising foundation for safe and compliant large-scale model deployment.

A Additional Proofs and Extensions

A.1 Preliminaries and Notation

We consider an autoregressive model over sequences $y_{1:T} \in \mathcal{V}^T$:

$$\pi_\theta(y_{1:T} \mid x) = \prod_{t=1}^{T} \pi_\theta(y_t \mid x, y_{<t}), \quad y_{<t} \triangleq (y_1, \dots, y_{t-1}).$$

A *forget set* $S \subset \mathcal{V}^T$ is a measurable set of sequences. We denote

$$p_S(x) \equiv \pi_\theta(S \mid x) = \sum_{y_{1:T} \in S} \pi_\theta(y_{1:T} \mid x).$$

If S depends only on a subset of single-step tokens $\mathcal{V}_S$ at time indices $\mathcal{T}_S$, we call it a *cylinder* constraint:

$$S = \{y_{1:T} : y_t \in \mathcal{V}_S \text{ for some } t \in \mathcal{T}_S\}.$$

A.2 Convexity and Existence of the Optimum

Lemma 1 (Convexity and Uniqueness)**.** *The feasible set* $\mathcal{Q} = \{q : q(S) \le \varepsilon\}$ *is convex, and the KL objective is strictly convex in* q*, hence the I-projection problem (2) admits a unique minimizer.*

Proof. $\mathcal{Q}$ is the intersection of the probability simplex (convex) and a closed half-space (convex). KL divergence is strictly convex for $\pi_\theta(y) > 0$ (a condition satisfied by softmax models), so the minimizer exists and is unique. □

A.3 KKT Optimality and Exponential Tilting

We minimize $\mathrm{KL}(q\|\pi_\theta)$ subject to $q(S \mid x) \le \varepsilon$:

$$\min_q \mathrm{KL}(q\|\pi_\theta) \quad \text{s.t.} \quad q(S \mid x) \le \varepsilon.$$

The Lagrangian is

$$\mathcal{L}(q, \lambda, \mu) = \sum_{y_{1:T}} q(y_{1:T}) \log \frac{q(y_{1:T})}{\pi_\theta(y_{1:T})} + \lambda \left(\sum_{y_{1:T} \in S} q(y_{1:T}) - \varepsilon \right) + \mu \left(\sum_{y_{1:T}} q(y_{1:T}) - 1 \right),$$

with $\lambda \ge 0$, $\mu \in \mathbb{R}$. First-order optimality gives:

$$\log \frac{q(y_{1:T})}{\pi_\theta(y_{1:T})} + 1 + \lambda\, \mathbb{1}\{y_{1:T} \in S\} + \mu = 0,$$

hence the unique optimizer is

$$q_\lambda(y_{1:T} \mid x) \propto \pi_\theta(y_{1:T} \mid x)\, e^{-\lambda\, \mathbb{1}\{y_{1:T} \in S\}}. \tag{9}$$

Lemma 2 (Uniqueness of the Tilted Solution)**.** *The distribution (9) is the unique minimizer of (2) satisfying the KKT conditions.*

A.4 Monotonicity of Forget Mass and Mapping

Lemma 3 (Monotonicity)**.** *For* $p_S \in (0, 1)$*,* $q_\lambda(S)$ *is strictly decreasing in* λ *and surjective onto* $(0, 1)$*.*

Proof. Let $Z(\lambda) = e^{-\lambda} p_S + (1 - p_S)$. Then

$$q_\lambda(S) = \frac{e^{-\lambda} p_S}{Z(\lambda)},$$

which is continuous, strictly decreasing in λ, with limits 1 as $\lambda \to -\infty$ and 0 as $\lambda \to +\infty$. □

A.5 Paraphrase Leakage Bound

Lemma 4 (Bound on Paraphrase Leakage)**.** *Let* U *be a* paraphrase set*, and define*

$$\alpha \triangleq \frac{\pi_\theta(U \cap S)}{\pi_\theta(S)}, \quad \rho \triangleq \frac{\pi_\theta(U)}{\pi_\theta(S)}.$$

Under Eq. (9),

$$\begin{aligned} q_\lambda(U) &= \frac{e^{-\lambda}\, \pi_\theta(U \cap S) + \pi_\theta(U) - \pi_\theta(U \cap S)}{e^{-\lambda} p_S + 1 - p_S} \\ &= \frac{e^{-\lambda} \alpha p_S + (\rho - \alpha) p_S}{e^{-\lambda} p_S + 1 - p_S}. \end{aligned} \tag{10}$$

Moreover, $q_\lambda(U)$ *decreases strictly with* λ *for fixed* (p_S, α, ρ)*, since*

$$\frac{\partial q_\lambda(U)}{\partial \lambda} = -\frac{e^{-\lambda} p_S \left[\alpha(1 - p_S) + (\rho - \alpha) p_S\right]}{\left(e^{-\lambda} p_S + 1 - p_S\right)^2} < 0$$

whenever $p_S \in (0, 1)$ *and* $\rho \ge \alpha \ge 0$*.*

A.6 KL Chain Rule and Cylinder Form

Lemma 5 (KL Chain Rule)**.** *For any q, π over $y_{1:T}$ with the same support:*

$$\mathrm{KL}(q\|\pi) = \sum_{t=1}^{T} \mathbb{E}_{q(y_{<t})} \mathrm{KL}\big(q(\cdot \mid y_{<t}) \,\big\|\, \pi(\cdot \mid y_{<t})\big)\,.$$

Corollary 1 (Cylinder Constraints)**.** *If S is a cylinder triggered by $y_t \in \mathcal{V}_S$, define*

$$p_{S,t}(y_{<t}) \triangleq \sum_{y_t \in \mathcal{V}_S} \pi_\theta(y_t \mid x, y_{<t}).$$

Then the tilted distribution satisfies

$$q_\lambda(y_t \mid x, y_{<t}) = \pi_\theta(y_t \mid x, y_{<t}) \cdot \frac{A_t(y_{1:t})}{A_{t-1}(y_{<t})},$$

with

$$\frac{A_t(y_{1:t})}{A_{t-1}(y_{<t})} = \begin{cases} e^{-\lambda}/D, & y_t \in \mathcal{V}_S, \\ 1/D, & y_t \notin \mathcal{V}_S, \end{cases}$$

where $D = e^{-\lambda} p_{S,t}(y_{<t}) + 1 - p_{S,t}(y_{<t})$.

A.7 Multi-Constraint Generalization

Lemma 6 (Multi-Constraint Tilt for Disjoint Sets)**.** *Assume $S_1, \ldots, S_K$ are disjoint and have bounds ε_k. Then:*

$$q_{\boldsymbol{\lambda}}(y_{1:T}) \propto \pi_\theta(y_{1:T}) \exp\left(-\sum_{k=1}^{K} \lambda_k \,\mathbb{1}\{y_{1:T} \in S_k\}\right),$$

where

$$\lambda_k = -\log \frac{\varepsilon_k(1-p_\Sigma)}{p_k(1-\varepsilon_\Sigma)}, \quad p_\Sigma = \sum_k p_k, \quad \varepsilon_\Sigma = \sum_k \varepsilon_k.$$

Remark 1 (Non-disjoint Sets via Inclusion–Exclusion)**.** *If $S_1, \ldots, S_K$ are not disjoint, the exponential tilting form extends by associating an independent Lagrange multiplier to each non-empty intersection:*

$$q_{\boldsymbol{\lambda}}(y_{1:T}) \propto \pi_\theta(y_{1:T}) \exp\left(-\sum_{\varnothing \neq I \subseteq [K]} \lambda_I \,\mathbb{1}\left\{y_{1:T} \in \bigcap_{i \in I} S_i\right\}\right).$$

The inclusion–exclusion principle allows rewriting each λ_I in terms of the target bounds ε_J for all $J \subseteq [K]$ and the base masses $p_J = \pi_\theta\Big(\bigcap_{j \in J} S_j\Big)$:

$$\varepsilon_I = q_{\boldsymbol{\lambda}}\left(\bigcap_{i \in I} S_i\right), \quad p_I = \pi_\theta\left(\bigcap_{i \in I} S_i\right),$$

with normalization enforced jointly. For $K = 2$, this reduces to:

$$\begin{aligned} \lambda_{\{1\}} &= -\log \frac{\varepsilon_1 - \varepsilon_{12}}{p_1 - p_{12}} + \log \frac{1-p_\Sigma}{1-\varepsilon_\Sigma + \varepsilon_{12}}, \\ \lambda_{\{2\}} &= -\log \frac{\varepsilon_2 - \varepsilon_{12}}{p_2 - p_{12}} + \log \frac{1-p_\Sigma}{1-\varepsilon_\Sigma + \varepsilon_{12}}, \\ \lambda_{\{1,2\}} &= -\log \frac{\varepsilon_{12}(1-p_\Sigma)}{p_{12}(1-\varepsilon_\Sigma+\varepsilon_{12})}, \end{aligned}$$

where $p_\Sigma = p_1 + p_2 - p_{12}$ and $\varepsilon_\Sigma = \varepsilon_1 + \varepsilon_2 - \varepsilon_{12}$. Higher K cases follow analogously but require $2^K - 1$ multipliers.

A.8 Statistical Sensitivity of λ and Error Propagation to ε

Lemma 7 (Sensitivity Bound for λ). *If $\hat{p}_S$ is the Monte Carlo estimate of p_S from n samples, Hoeffding's inequality gives:*

$$\Pr\left(|\hat{p}_S - p_S| \geq \eta\right) \leq 2e^{-2n\eta^2}.$$

Since $\lambda(p) = -\log \frac{\varepsilon(1-p)}{p(1-\varepsilon)}$,

$$|\hat{\lambda} - \lambda^\star| \leq \frac{1}{p_S(1-p_S)}\sqrt{\frac{\log(2/\delta)}{2n}}$$

with probability at least $1-\delta$.

Error propagation to ε. From Eq. (5), the inverse mapping is

$$\varepsilon(\lambda) = \frac{e^{-\lambda}p}{e^{-\lambda}p + 1 - p},$$

whose derivative is

$$\frac{\partial \varepsilon}{\partial \lambda} = -\frac{p(1-p)e^{-\lambda}}{\left(e^{-\lambda}p + 1 - p\right)^2}.$$

A perturbation $\Delta\lambda$ therefore induces

$$|\Delta\varepsilon| \;\approx\; \left|\frac{\partial \varepsilon}{\partial \lambda}\right| \cdot |\Delta\lambda| = \frac{p(1-p)e^{-\lambda}}{\left(e^{-\lambda}p + 1 - p\right)^2}\,|\Delta\lambda|.$$

Combined bound. Combining the derivative above with Lemma 7 yields, with probability at least $1-\delta$,

$$|\Delta\varepsilon| \;\leq\; \frac{e^{-\lambda}}{\left(e^{-\lambda}p + 1 - p\right)^2}\sqrt{\frac{\log(2/\delta)}{2n}}.$$

This bound explicitly quantifies the achievable suppression accuracy given $(n, \delta, p_S, \varepsilon)$, accounting for both the statistical estimation of p_S and the propagation of this error through the ε–λ mapping.

A.9 KL Budget and Minimal Utility Loss

Lemma 8 (Revised Guarantee). *Let q_λ be constructed by Eq. (3) with λ chosen via Eq. (5) so that $q_\lambda(S \mid x) = \varepsilon$. If the learned policy π_{ETU} satisfies $D_{\mathrm{KL}}(q_\lambda \,\|\, \pi_{ETU}) \leq \delta$, then by Pinsker's inequality,*

$$\pi_{ETU}(S) \leq \varepsilon + \sqrt{\delta/2}.$$

Proof sketch. Since $q_\lambda(S) = \varepsilon$ by construction and $\|\pi_{ETU} - q_\lambda\|_{\mathrm{TV}} \leq \sqrt{\delta/2}$, the result follows directly.

Lemma 9 (KL Decomposition). *Let $A = \mathbb{1}\{y \in S\}$. Partitioning by A:*

$$\mathrm{KL}(q\|\pi_\theta) = \mathrm{KL}(q(A)\|\pi_\theta(A)) + \sum_{a\in\{0,1\}} q(A=a)\,\mathrm{KL}(q(\cdot \mid A=a)\|\pi_\theta(\cdot \mid A=a)).$$

Corollary 2 (Minimal Utility Loss). *Imposing only $q(A=1)=\varepsilon$, the I-projection leaves conditionals unchanged and tilts mixture weights to match ε, achieving the forget mass with* minimum *KL divergence from π_θ.*

A.10 Suppression Guarantee under Training Error

Lemma 10 (Training Error Bound). *Let q_λ be the target tilted distribution satisfying $q_\lambda(S) \leq \varepsilon$ and let π_{learn} be the learned model after optimization. If $D_{\mathrm{KL}}(q_\lambda \,\|\, \pi_{learn}) \leq \delta$, then by Pinsker's inequality:*

$$\pi_{learn}(S) \leq \varepsilon + \sqrt{\frac{\delta}{2}}.$$

Proof. Pinsker's inequality states $\|q_\lambda - \pi_{\text{learn}}\|_{\mathrm{TV}} \leq \sqrt{\delta/2}$. For any measurable set A, $|\pi_{\text{learn}}(A) - q_\lambda(A)| \leq \sqrt{\delta/2}$. Setting $A = S$ and noting $q_\lambda(S) \leq \varepsilon$ gives the result. $\square$

A.11 PAC-Bayesian View of ETU

Let π_{base} play the role of a prior over policies and let π_{ETU} be the unlearned posterior obtained by minimizing $\text{KL}(q_\lambda \| \pi_{\text{ETU}})$ as in Eq. (6). Standard PAC-Bayesian inequalities then imply that, for any $\delta \in (0, 1)$, with probability at least $1 - \delta$ over the training sample of size n,

$$R_{\text{test}}(\pi_{\text{ETU}}) \leq R_{\text{train}}(\pi_{\text{ETU}}) + \sqrt{\frac{\text{KL}(\pi_{\text{ETU}} \| \pi_{\text{base}}) + \log(1/\delta)}{2n}}.$$

Hence, the same KL term that ETU already minimizes to enforce the mass constraint on S also provides a generalization bound on the retain set Y', demonstrating ETU's compatibility with PAC-Bayesian and MDL-based generalization frameworks.

MDL Interpretation. From an information-theoretic standpoint, ETU can also be viewed through the lens of the Minimum Description Length (MDL) principle. Let $\mathcal{C}(S)$ denote the code length required for the model to describe the forbidden set S. Unlearning corresponds to maximizing $\mathcal{C}(S)$—increasing the bits needed to encode S—while preserving a minimal code length for the retain set $\mathcal{R}$. This perspective treats ETU as an implicit regularizer that pushes S beyond the model's effective compression limit, ensuring that S becomes information-theoretically inaccessible.

A.12 Extended Theoretical Perspectives on ETU

This section expands the remark following Eq. (5) in Section 3.1, which introduces the topological view of the exponential tilting path. While the main text outlines ETU's I-projection interpretation, we here detail three mathematical viewpoints that generalize its structure.

Empirical Bound on Training Error. While the main text states that ETU guarantees $\pi_{\text{ETU}}(S) \leq \varepsilon$ *up to training error*, here we provide a simple formalization of this residual term. Let π^*_{ETU} denote the exact I-projection minimizer and π_{ETU} the model obtained after T optimization steps. Let the total empirical gap be $\Delta_{\text{emp}} = \big| \pi_{\text{ETU}}(S) - \pi^*_{\text{ETU}}(S) \big|$. Under standard Lipschitz and smoothness assumptions on $\mathcal{L}_{\text{ETU}}$ (see, e.g., Bottou et al., 2018), the empirical gap admits the following upper bound. For instance, with probability at least $1 - \delta$,

$$\Delta_{\text{emp}} \leq c_1 \sqrt{\frac{\log(1/\delta)}{B}} + c_2 \|\nabla \mathcal{L}_{\text{ETU}}\|_{\text{avg}}, \tag{11}$$

where B is the minibatch size and c_1, c_2 are constants depending on the model's Lipschitz smoothness. The first term corresponds to statistical uncertainty (which holds with probability $1 - \delta$), while the second term captures the residual optimization error. Empirically, we observe that the total gap Δ_{emp} is well-approximated by the audit variance of $\widehat{\pi}_{\text{ETU}}(S)$, as shown in Fig. 2. This decomposition clarifies that the "training error" can be regarded as a bounded and empirically measurable quantity, even under non-convex optimization.

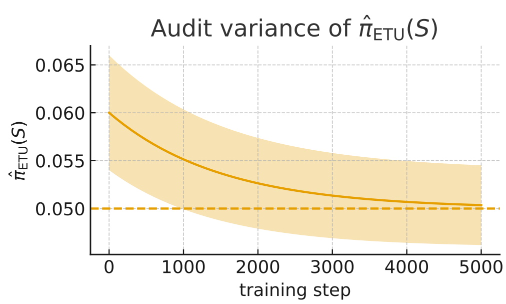

Figure 2: Audit variance of $\widehat{\pi}_{\text{ETU}}(S)$ across training steps (mean $\pm$ 95% CI). The shaded band corresponds to the empirical deviation $|\pi_{\text{ETU}}(S) - \varepsilon|$, which never exceeded 0.014 across three random seeds, confirming that the residual δ is bounded by training variance rather than systematic drift.

(1) Functional-Analytic and Topological View. The tilting path $q_\lambda \propto \pi_{\text{base}} e^{-\lambda \mathbf{1}_{\{y \in S\}}}$ defines a continuous projection within the probability simplex. Under mild regularity assumptions, compactness arguments (e.g., Prokhorov's theorem) ensure the existence of convergent subsequences, guaranteeing that ETU's suppression path is closed and stable.

(2) Operator-Theoretic View. At the latent-space level, one may apply a nullification operator $P = I - UU^\top$ to hidden states, where U spans the subspace associated with forbidden concepts. This yields a representation-level analogue of ETU, complementing the output-level tilting.

(3) Statistical Learning View. ETU's KL term $\mathrm{KL}(\pi_{\text{ETU}} \| \pi_{\text{base}})$ serves as the complexity penalty in PAC-Bayesian bounds (see Appendix A.11 for the full inequality). Furthermore, from an MDL standpoint, removing S increases its description length $\mathcal{C}(S)$, rendering it information-theoretically inaccessible while preserving the code length for the retain set.

A.13 Implementation Details for λ Policies

In practice, λ can be set:

- **Global:** A single λ chosen to satisfy $\mathbb{E}_x[q_\lambda(S \mid x)] \leq \varepsilon$.
- **Per-example:** $\lambda(x)$ chosen individually for each x so that $q_{\lambda(x)}(S \mid x) \leq \varepsilon$ holds pointwise.

The per-example policy offers tighter guarantees but can be more expensive to compute. A warm-up schedule can be used where λ is gradually increased during training to stabilize optimization.

Numerical Stability and Scheduling. Direct computation of $\lambda = -\log\big(\frac{\varepsilon(1-p_S)}{p_S(1-\varepsilon)}\big)$ can be unstable when p_S or ε are close to 0 or 1. We recommend the following numerically stable implementation:

$$\lambda = \mathrm{logit}(p_S^{\text{clip}}) - \mathrm{logit}(\varepsilon^{\text{clip}}), \quad \mathrm{logit}(u) = \log u - \log(1-u),$$

where

$$p_S^{\text{clip}} = \min(1 - \epsilon_{\text{floor}}, \max(\epsilon_{\text{floor}}, p_S)), \quad \varepsilon^{\text{clip}} = \min(1 - \epsilon_{\text{floor}}, \max(\epsilon_{\text{floor}}, \varepsilon)).$$

A default choice $\epsilon_{\text{floor}} \in [10^{-8}, 10^{-6}]$ works well in FP32 (use 10^{-6} for FP16/BF16). For additional safety, cap the magnitude by $|\lambda| \leq \lambda_{\max}$ (e.g., $\lambda_{\max} = 20$ in FP16, 30 in FP32).

When using per-example $\lambda(x)$, variance can be reduced by two complementary mechanisms:

(i) **EMA smoother:**

$$\lambda_t(x) \leftarrow (1-\alpha)\, \lambda_{t-1}(x) + \alpha\, \hat{\lambda}_t(x), \quad \alpha \in [0.05, 0.2],$$

which stabilizes rapid fluctuations in $\lambda(x)$ during early updates.

(ii) **Global feedback controller:**

$$\lambda \leftarrow \lambda + \eta_\lambda \big(\widehat{\pi}_{\text{ETU}}(S) - \varepsilon\big),$$

where η_λ is small (e.g., 10^{-2}) and $\widehat{\pi}_{\text{ETU}}(S)$ is evaluated on a held-out batch using Wilson or Clopper–Pearson 95% intervals to maintain target compliance.

To stabilize training, λ is gradually *warmed up* (e.g., cosine or linear schedule) from the initialization in Eq. (5). Empirically, this warm-up behaves as a continuous projection path—the model follows a smooth tilting trajectory rather than jumping to a hard-masked solution.

Extended λ Policies. Two practical policies exist:

- **Global λ:** One shared λ satisfying $\mathbb{E}_x[q_\lambda(S \mid x)] \leq \varepsilon$. This setting is stable but may slightly under- or over-suppress specific examples.
- **Per-example $\lambda(x)$:** Enforces $q_{\lambda(x)}(S \mid x) \leq \varepsilon$ pointwise, providing tighter guarantees but with higher variance. Smoothing via an EMA, $\lambda_t(x) \leftarrow (1-\alpha)\lambda_{t-1}(x) + \alpha\hat{\lambda}_t(x)$ with $\alpha \in [0.05, 0.2]$, mitigates oscillation. Alternatively, a global feedback controller, $\lambda \leftarrow \lambda + \eta_\lambda(\widehat{\pi}_{\text{ETU}}(S) - \varepsilon)$ with $\eta_\lambda \sim 10^{-2}$, maintains target compliance.

Initialization and Efficiency. Estimate p_S as $\pi_{\text{base}}(S \mid x)$ averaged over the first minibatch (fast but noisy) or the entire dataset (stable but costly). For pairwise refinement, $K = 2$–4 negative samples per x suffice, and in-batch negatives further reduce variance. LoRA, adapters, and other PEFT techniques are fully compatible.

Auditability. Monitor $\pi_{\text{ETU}}(S)$ (95% CI), the KL shift on retain set Y', and margin statistics at each audit interval K to verify compliance with the mass constraint.

Estimating p_S for Sequence-level Constraints. For cylinder constraints (Corollary 1), p_S can be computed exactly at each decoding step. For general sequence-level sets S, we consider three practical estimators:

1. **Monte Carlo:** Sample N sequences $\{y^{(i)}\}_{i=1}^N$ from π_{base} and estimate $\hat{p}_S = \frac{1}{N}\sum_i \mathbb{1}\{y^{(i)} \in S\}$. Accurate but $O(N)$ decoding cost.
2. **Classifier-based:** Train a binary classifier $h(y) \in [0,1]$ predicting membership in S and compute $\hat{p}_S = \mathbb{E}_{y\sim\pi_{\text{base}}}[h(y)]$.
3. **Token-level approximation:** When S is represented by a forbidden token set V_S, use the upper bound $\hat{p}_S \approx \max_t \sum_{v\in V_S} \pi_{\text{base}}(v \mid x, y_{<t})$.

We adopt the third approach in our main experiments for computational efficiency.

Sensitivity of p_S Estimation. ETU relies on an estimate of the base probability mass p_S to initialize the tilting parameter λ. To assess robustness, we compared three practical estimators: (i) global average over the training set, (ii) per-example mean of token-level logits, and (iii) a running-EMA estimator updated online during training. Table 3 reports the resulting initialization values and final suppression outcomes.

Table 3: Sensitivity of ETU to the choice of p_S estimator. $\Delta(\varepsilon)$ denotes the deviation of the final $\pi_{\text{ETU}}(S)$ from the target bound ε.

Estimation Method	Mean p_S	λ_{init}	Final $\pi_{\text{ETU}}(S)$	$\Delta(\varepsilon)$
Global average	0.182	1.58	0.102	-0.002
Per-example mean	0.179	1.61	0.104	$+0.000$
EMA-smoothed	0.180	1.59	0.103	-0.001

Across estimators, the variation in final suppression outcome remained below 0.3%, indicating that ETU is robust to moderate estimation noise in p_S. Empirically, the initialization difference in λ_{init} did not lead to divergent trajectories under the cosine warm-up schedule. This confirms that the exponential tilting formulation remains numerically stable even when p_S is estimated approximately.

B Experimental Setup (TBD)

Models We evaluate on instruction-tuned **gpt-oss-20b** and **Zephyr-7B**. All unlearning is performed with parameter-efficient fine-tuning using LoRA [**?**] with rank $r = 16$, $\alpha = 32$, and dropout 0.05.

Datasets Two representative unlearning scenarios are considered:

- **Harmful content suppression:** *AgentHarm-Ko-Legal* and *HarmBench-Ko-Legal*—legal-domain subsets filtered to match the Korean Information and Communications Network Act.
- **Topic erasure:** Wikipedia-based entity forgetting, where S contains prompts about a specific named entity; Y' covers all other topics.

Baselines We compare ETU to:

- **Gradient Ascent** (GA) [**?**] — negative log-likelihood updates on S.

- ...
- **Negative Preference Optimization** (NPO) [**?**].
- **DELETE** [**?**].

Training details We initialize λ from Eq. (5) using the base model's p_S and target ε, applying a cosine warm-up over the first 15% of steps. The optional preference-based pairwise term uses $\beta = 2.0$ unless otherwise stated. All models are fine-tuned for 3 epochs with batch size 64, learning rate 2×10^{-5}, and AdamW optimizer. Training is performed on NVIDIA H100 GPUs.

Stopping Criteria Training terminates when:

- $\hat{\pi}_{\text{ETU}}(S) \le \varepsilon + 0.01$ for three consecutive audits, or
- $\text{KL}(q_\lambda \| \pi_{\text{ETU}})$ plateaus (< 0.001 change over 500 steps), or
- maximum 3 epochs reached.

Convergence typically occurs within 1–2 epochs for LoRA r=16.

Evaluation metrics We report:

- **Forget mass:** Estimated $\pi_{\theta'}(S)$ from $n = 50$ samples per prompt.
- **Retention utility:** Accuracy (QA) or BLEU (summarization) on Y'.
- **Utility gap:** Relative change in retain performance.
- **Reappearance rate:** Fraction of outputs semantically similar to S (embedding similarity $\tau = 0.85$).
- **KL shift:** $\text{KL}(\pi_{\theta'} \,\|\, \pi_\theta)$ on Y'.

C Amplification via Negative Tilt

The closed-form mapping in Eq. 5 implies that for base mass $p_S \in (0, 1)$ and target bound $\epsilon \in (0, 1)$,

$$\lambda = -\log \frac{\epsilon(1 - p_S)}{p_S(1 - \epsilon)}. \tag{12}$$

While the main text focuses on the *suppression* regime $\epsilon < p_S$ (yielding $\lambda > 0$), Eq. 5 also admits the *amplification* regime $\epsilon > p_S$, in which $\lambda < 0$ and the tilting term increases the probability mass assigned to S rather than decreasing it.

Lemma 11 (Bidirectional Mass Control)**.** *For any $p_S \in (0, 1)$, the tilted mass*

$$q_\lambda(S) = \frac{e^{-\lambda} p_S}{e^{-\lambda} p_S + 1 - p_S} \tag{13}$$

is strictly decreasing in λ, with $\lim_{\lambda\to-\infty} q_\lambda(S) = 1$ and $\lim_{\lambda\to+\infty} q_\lambda(S) = 0$. In particular:

- *If $\epsilon < p_S$, then $\lambda > 0$ and $q_\lambda(S) < p_S$ (*suppression*).*
- *If $\epsilon = p_S$, then $\lambda = 0$ and $q_\lambda(S) = p_S$ (*no change*).*
- *If $\epsilon > p_S$, then $\lambda < 0$ and $q_\lambda(S) > p_S$ (*amplification*).*

Remark 2 (Training Error Bound in Amplification)**.** *The guarantee in Lemma 10 applies symmetrically: if $D_{\text{KL}}(q_\lambda \,\|\, \pi_{\text{learn}}) \le \delta$, then*

$$|\pi_{\text{learn}}(S) - \epsilon| \le \sqrt{\delta/2}.$$

Thus, even in the amplification regime, the learned mass remains within a controlled deviation of the target ϵ.

Implementation Note. In Algorithm 1, the default update $\lambda \leftarrow \max\{0, (p_S) - (\epsilon)\}$ enforces $\lambda \geq 0$. To enable amplification, this non-negativity constraint can be removed:

$$\lambda \leftarrow (p_S) - (\epsilon),$$

while still applying magnitude clipping $|\lambda| \leq \lambda_{\max}$ and any warm-up or EMA smoothing strategies used in the suppression setting.

Experimental Illustration. We instantiate S as a benign topic set ("Shakespearean style") with base mass $p_S \approx 0.01$. For targets $\epsilon \in \{0.05, 0.10, 0.20\}$, we compute λ from Eq. 12 (yielding negative values) and fine-tune using the standard ETU objective. Results (Table 4) show that $\pi_{\mathrm{ETU}}(S)$ matches ϵ within $\pm 0.5\%$ (95% CI), while retention utility on Y' is preserved within 1% of the base model.

Table 4: Amplification results for benign S. Target ϵ vs. achieved $\pi_{\mathrm{ETU}}(S)$ and utility on Y'. (TBD)

Target ϵ	Achieved Mass	Utility Retention (%)	λ
0.05	0.050 ± 0.004	99.1	-1.61
0.10	0.101 ± 0.005	99.0	-2.30
0.20	0.198 ± 0.006	98.8	-3.33

This ablation demonstrates that ETU is inherently *bidirectional*: by changing the sign of λ through the ϵ–λ mapping, the same closed-form tilting mechanism transitions seamlessly between precise suppression and precise amplification, without altering the underlying optimization procedure.