ENGN2340 Final Project

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December 2023

Part 1

The Matlab files used in this section are `mesh_hole.m` and `ENGN2340_2023_project_template`; the plots comparing the results of different mesh sizes were created using `plots.m`. All of these files are included in the attached `.zip` file. Using quarter symmetry, the sections below are the FEM solutions for this problem with meshes of increasing resolution. The stress concentration factor was found by dividing the output stress (from the first integration point) at the top corner of the hole by the applied traction, which was set to 1N. The energy norm was calculated in lines 198-240 of the main template code by the formula:

$$L_2 = \sum_{\text{all nodes}} \left[\frac{1}{2} * (\sigma_{\text{FEM}} - \sigma_{\text{exact}}) (\epsilon_{\text{FEM}} - \epsilon_{\text{exact}}) \right]^{1/2}$$

where the value for σ_{11} was used for σ_{FEM}, and the values for strain were calculated by $\epsilon = D^{-1}\sigma$, with D as the matrix of material properties.

Results

6x6

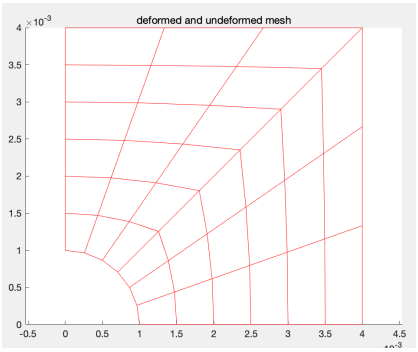
Starting with `meshn = 6`, the mesh and stress contour plots are below in Figure 1. The calculated stress concentration factor on the top corner of the hole was **2.4243**. The calculated energy norm was **1.7380e-8**.

10x10

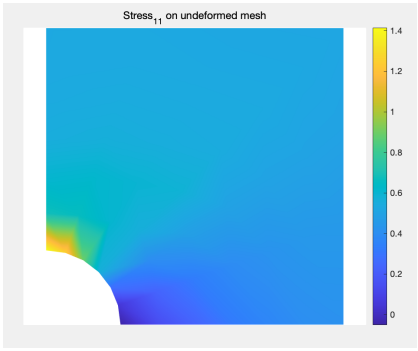
With `meshn = 10`, the mesh and stress contour plots are shown in Figure 2. The calculated stress concentration factor on the top corner of the hole was **2.7062**. The calculated energy norm was **5.6594e-09**.

20x20

With `meshn = 20`, the mesh and stress contour plots are below in Figure 3. The calculated stress concentration factor on the top corner of the hole was **2.8845**. The calculated energy norm was **1.3115e-09**.

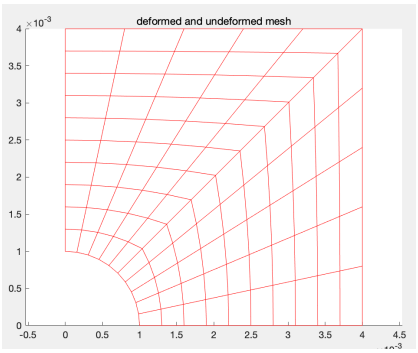


(a) The deformed 6x6 mesh.

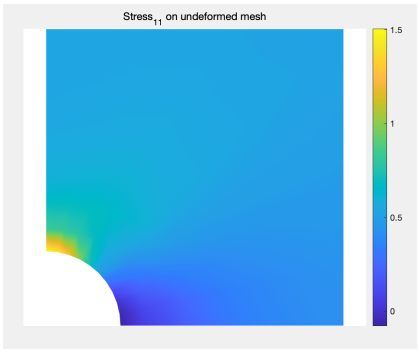


(b) The stress contour for the 6x6 mesh.

Figure 1: FEM solution for a 6x6 mesh.



(a) The deformed 10x10 mesh.



(b) The stress contour for the 10x10 mesh.

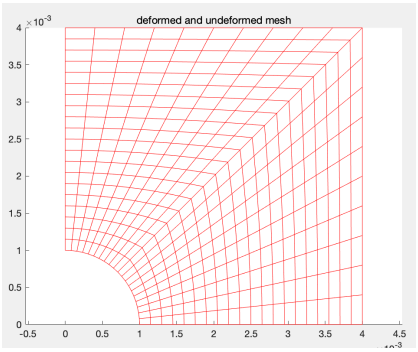
Figure 2: FEM solution for a 10x10 mesh.

40x40

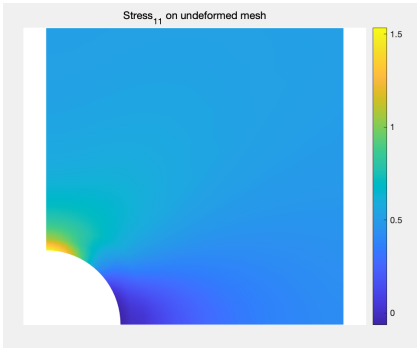
With `meshn = 40`, the mesh and stress contour plots are shown in Figure 4. The calculated stress concentration factor on the top corner of the hole was **2.9520**. The calculated energy norm was **6.0108e-10**.

80x80

This mesh (`meshn = 80`) was completed without plotting to save computational costs (attempts to plot it with OSCAR were unsuccessful as well). The calculated stress concentration factor on the top corner of the hole was **2.9985**. The calculated energy norm was **5.2006e-10**.

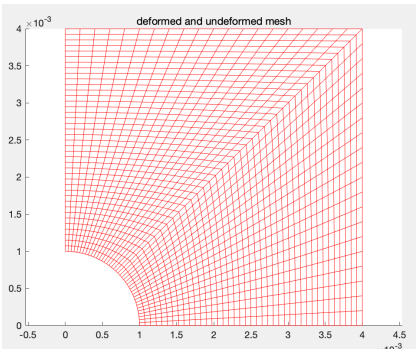


(a) The deformed 20x20 mesh.

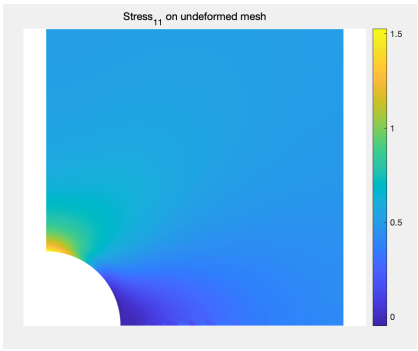


(b) The stress contour for the 20x20 mesh.

Figure 3: FEM solution for a 20x20 mesh.



(a) The deformed 40x40 mesh.



(b) The stress contour for the 40x40 mesh.

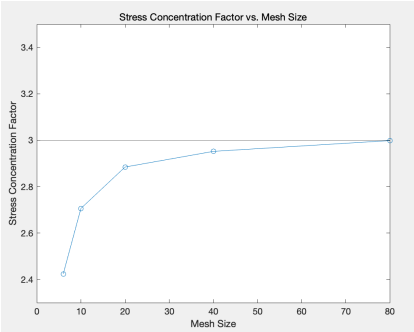
Figure 4: FEM solution for a 40x40 mesh.

Discussion

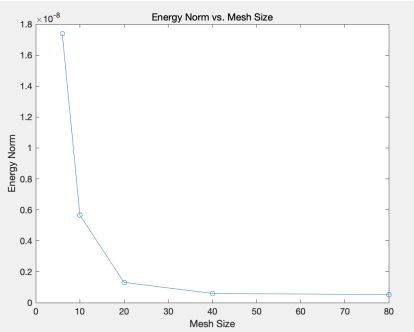
The stress concentration factors at the corner element of the hole for each mesh resolution are compared in Table 1. These values are converging towards the stress concentration factor of 3, which agrees with the commonly accepted value given by the exact solution. Additionally, the L2 energy norms are decreasing as the mesh resolution increases, indicating that the error is getting smaller and smaller, as expected.

Mesh Size	Stress Concentration Factor	Energy Norm
6x6	2.4243	1.7380e-08
10x10	2.7062	5.6594e-09
20x20	2.8845	1.3115e-09
40x40	2.9520	6.0108e-10
80x80	2.9985	5.2006e-10

Table 1: Stress concentration factor and energy norms for each chosen mesh resolution.



(a) Stress Concentration Factor vs. Mesh Size.



(b) Energy Norm vs. Mesh Size.

Figure 5: Plots of SCF at corner of hole and Energy Norm vs. Mesh Size, given a Poisson ratio of $\nu = 0.3$.

Part 2

The Matlab files used in this section are `mesh_hole.m`, `improved_ENGN2340_2023_project_template`, and `plots.m`, all from the attached `.zip` file. For the final part examining the Cook's Membrane problem, the file `mesh_Cook.m` is also needed. The explicit material properties are recalled in lines 1074-1082 in the improved project template, and can be commented out for each run depending on the problem to be solved.

FEM Solutions With vs Without B-Bar Improvement

Just as in the first problem, the FEM solutions were calculated with the Poisson's ratio $\nu = 0.4999$. Since ν is so close to 0.5, the traditional calculation for the bulk modulus goes to infinity, and resulting FEM calculations are unreasonable and inaccurate. The B-Bar correction is required to advoiding dividing by 0 to get reasonable simulation results for near-incompressible bodies.

Results

The same iteration process as in the previous problem was run with the B-Bar method, and the energy norm for each mesh resolution was recorded and compared.

Discussion

Similarly to the previous part, Table 2 is a table comparing the energy norm for each chosen mesh resolution, given the new Poisson ratio and computation method.

B-Bar Mesh Size	Energy Norm
6x6	7.3354e-04
10x10	2.8361e-04
20x20	7.3420e-05
40x40	1.9210e-05
80x80	4.7639e-06

Table 2: Energy norm for each chosen mesh resolution when $\nu = 0.4999$, using the B-Bar method.

These values are plotted in Figure 6. This error convergence confirms that the B-Bar method is a reliable way to work with nearly-incompressible materials.

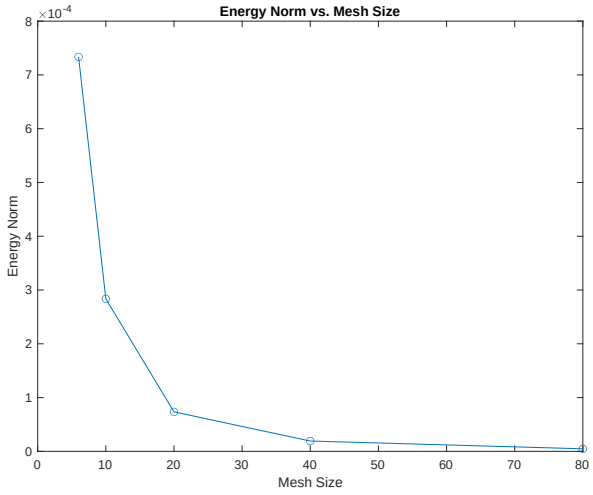


Figure 6: Energy Norm vs Mesh Size for the updated value of ν, utilizing the B-Bar method.

Cook's Membrane Test

Below in Figures 7, 8, 9, 10, and 11 are the deflections of the membrane for each chosen mesh size. Every figure in this study was produced using the B-Bar method on a material with $\nu = 0.4999$.

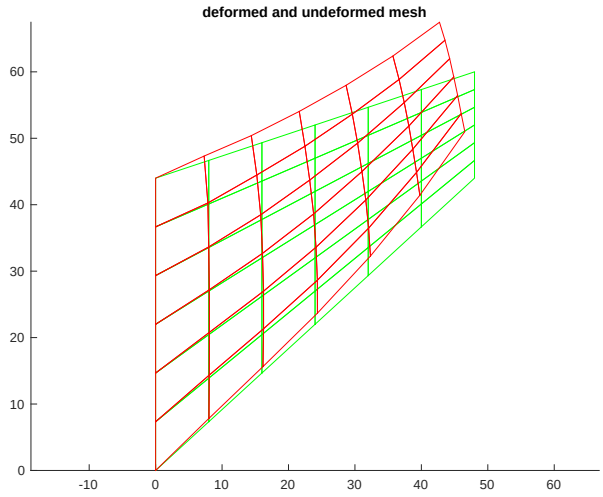


Figure 7: Deformed and undeformed 6x6 mesh of the Cook's membrane, utilizing the B-Bar method.

The displacement of the top right corner of the membrane versus the resolution of the mesh is plotted in Figure 12. This figure shows that the displacement of this point converges, agreeing with the given reference paper studying this problem.

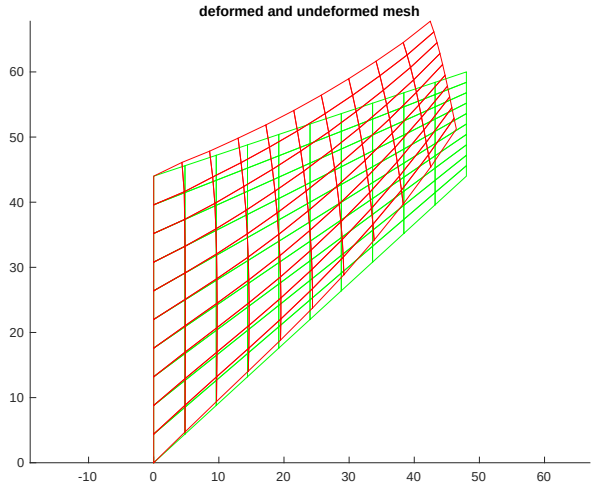


Figure 8: Deformed and undeformed 10x10 mesh of the Cook's membrane, utilizing the B-Bar method.

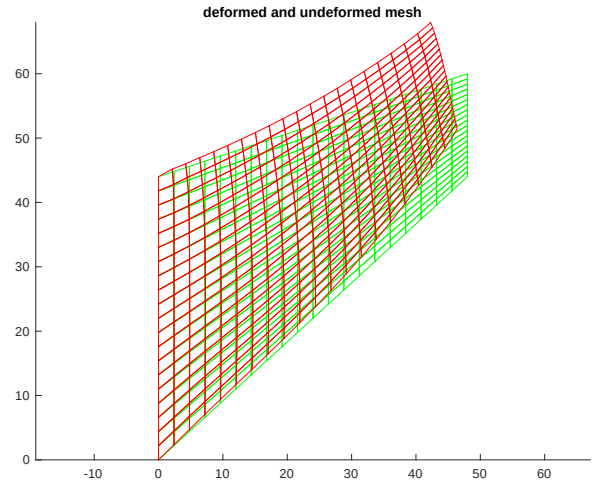


Figure 9: Deformed and undeformed 20x20 mesh of the Cook's membrane, utilizing the B-Bar method.

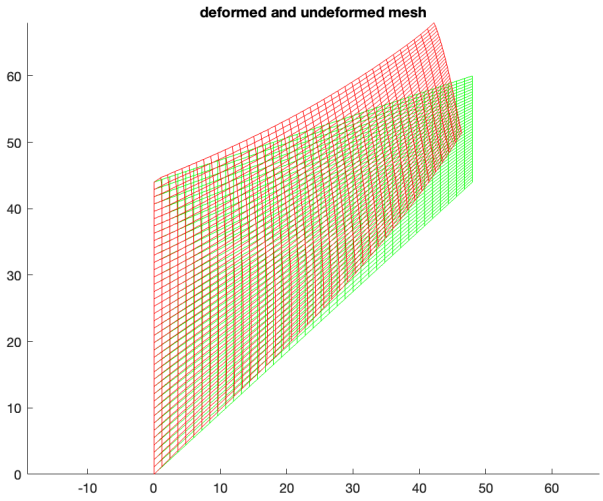


Figure 10: Deformed and undeformed 40x40 mesh of the Cook's membrane, utilizing the B-Bar method.

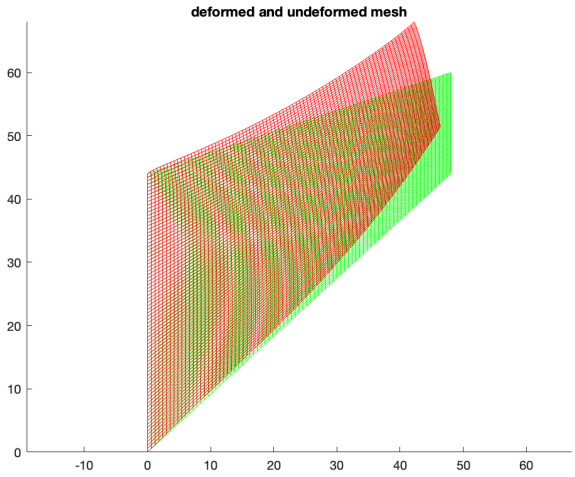


Figure 11: Deformed and undeformed 80x80 mesh of the Cook's membrane, utilizing the B-Bar method.

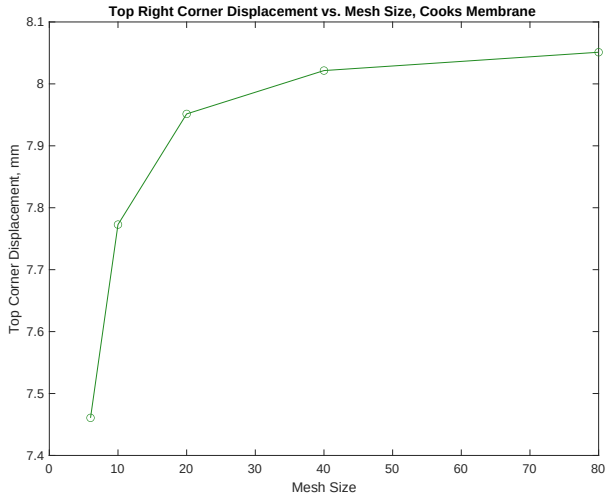


Figure 12: Displacement of top-right corner of membrane versus the size of the mesh, using the B-Bar method.