Bioen 485/585 Lab 1: Model Building

Reminder: Lab assignments do not require a polished document, but must answer all the questions.

1. **Electrical Model 1 (8 points)**. In lecture 2 (example 1), we created a model for a simple electrical circuit, deriving the equation

$$\frac{dI_2}{dt} = \frac{V_a(t)}{CR_2R_1} - \frac{R_1 + R_2}{CR_2R_1} I_2(t)$$

Solve this ODE model numerically, using the following values:

- R1 = 0.2 Ohm
- R2 = 0.2 Ohm
- C = 1 Farad
- I2(0) = 0
- Va(t) = 5 Volts, for t > 0.

   a. (3 points) Plot the response over a time frame appropriate for understanding the response. (*Hint: see the tutorial if needed and note the checklist item 3!*)

   b. (2 points) For this constant Va(t) situation, verify the result analytically. *Hint: find the expected steady state solution by setting the derivative to zero, and solving for I2.*

   c. (3 points) Plot the response when the voltage generator gives a sinusoidal voltage, Va(t) = Asin(bt), where A = 5 Volts and b = 10 sec-1. (*Hints: if you are using ODE solvers in MATLAB: you have passed the time variable to the solver, so make use of it. If you have a numeric artifact, fix it*)

2. **Fluid Model (10 points)**. In lecture 2, we created a diagram for a fluidic model (example 2).

   a. (4 points) Derive the mathematical form of the ODE model from the diagram and description in class. (*Hint: follow DIESE starting at I: from the description, make sure you know which are dependent variables, parameters, forcing functions, or intermediate variables that need to be removed.*)

   b. (4 points) Solve the model numerically, and plot both Q and Qf, assuming that the pump is turned off, with no flow in the channel, before t = 0, at which time the pump is turned on to a flow rate for 3 seconds, then turned off again. Use the following parameter values:

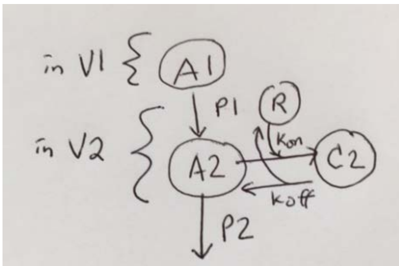
   - Rf = 5E9 Pa*s/m^3
   - Ct = 1E-10 m^3/Pa
   - Q = 2E-9 m^3/s

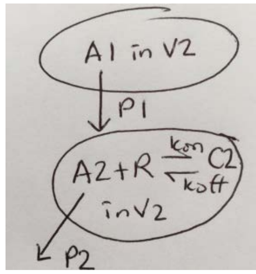
   Hint: odesolvers do not do well with sharp changes in input, as required for this step function. First see what you get, then read the last section of the MATLAB tutorial and decide what to do next.

   c. (2 points) Interpret your plot for 2b to describe what the model showed about the real system. (*Hint: Remember checklist item 6 on the lab report expectations document: for interpretation of a result, you must return to the original model description to remember what the variables and parameters mean, since you can't just describe the results using parameter and variable names. You also need to remember the original question, so you can interpret the plot to address that question.*)

3. **Chemical Model (10 points).** A drug, L, enters the stomach and is absorbed unidirectionally into the blood with permeability P1 (in L/s.) The stomach has a volume V1 and blood V2 (in L). In

the blood, the drug can bind to a receptor at rate kon in 1/M*1/s and unbind at rate koff in 1/s. The receptor is too big to be filtered by the kidneys, so the receptor-drug complex is not transported to the kidneys, but the free drug is, with permeability P2 (L/s). The total concentration of receptor in the blood is RT. Let A1 = concentration of free drug in the intestines, A2 = concentration of free drug in the blood, C2 = concentration of receptor-drug complex in the blood, and R = the concentration of free receptor in the blood. Initially, an amount of drug D in moles is dissolved in the stomach, and there is no drug anywhere else. This system is illustrated in the following diagrams, which show the same thing in two different ways.





a. <u>(4 points) Write a set of differential equations and initial conditions describing this system</u>. *Hint: you can remove R using an algebraic expression, leaving 3 equations*.

b. <u>(3 points) Solve the model and plot the three variables over time for the following parameters</u>: D = 20 µmoles, V1 = 1 L, V2 = 5 L, RT = 5 µM, koff = 0.001 1/s, kon = 10,000 1/s*1/M, P1 = 0.01 L/s, P2 = 0.0005 L/s.

c. <u>(3 points) Verify the steady state solution.</u> *(Hint: did you run it long enough?)*Can you use your intuition to get this same steady state prediction, assuming all parameters are nonzero.

4. **Mechanics Model (12 points)**. If you were an undergrad in the HAMM lab, you might do dunking experiments to determine the pCa curve for different states of muscle. Before lowering a small piece of muscle tissue in a bath of calcium ions and ATP, it must first be strung between two needle tips attached to a force transducer on one side, and a motor on the other. At time zero, the muscle is not stretched, so is at the equilibrium length. The muscle is pulled taut between them, with the force transducer stationary, and the motor applying a tensile force, F, to the system. Passive muscle tissue itself acts as a dashpot (viscous damping in tissue) in parallel with a spring (sarcolemma – muscle cell membrane). Let Fspring be the force on the spring and Fdamper that on the damper. Let x be the length (or change in length, but clarify) of the muscle.

   a. <u>(2 points) Diagram the model for this system.</u>

   b. <u>(3 points) Obtain a mathematical form of the model that relates the (change in) length of the muscle, x, to the force applied by the motor, F.</u>

   c. <u>(2 points) Perform the verifications described in lectures to see if your model makes sense:</u>

      i. is the model complete and appropriate for this problem description?

      ii. does your model pass a dimensional analysis?

   d. <u>(2 points) Plot the muscle length from t = 0 to t = 10 seconds for a piece of muscle tissue that is initially 2.5mm in length and is pulled with a motor force of 3mN.</u> Assume k = 1 N/m and b = 0.8 Ns/m and that the system is massless.

   e. <u>(3 points) Interpret your plot in words to describe what it shows about the muscle tissue.</u>