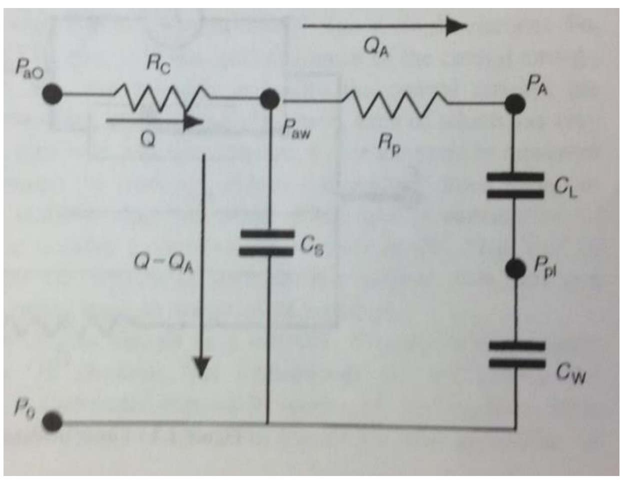
Bioen 485/585 Lab 2: Linear Systems

Lung Mechanics. This week we will analyze the response of a patient's lungs to mechanical ventilation. There are several modes of ventilation. Today we will model pressure-controlled ventilation, in which the ventilator controls the pressure, p(t), at the airway opening (pa0) relative to atmospheric pressure (p0), and the patient's lungs respond to determine q(t), the volumetric flow into the lungs. First we will use this system to further learn and illustrate the analytic and numeric linear systems analysis tools to this lab. Then, we will use our model and analysis tools to answer questions about ventilating diseased patients.

The model for the patient's lung mechanics is as follows: When air enters the lungs at rate q(t), it passes through the central airway, which has resistance R_c, but some air remains here due to a shunt compliance, C_s. The remaining air, $q_A(t)$, passes through the peripheral airways, with resistance Rp before arriving at the alveoli, where air exchange occurs. For the alveoli to fill with air, the lungs and chest wall must expand, and the amount of expansion is controlled by the compliance of the lungs, C_L and chest wall, C_W The air is modeled as a fluid, but has so little mass that we ignore fluid inertia. The diagram below, from Khoo chapter 2, illustrates this model.



As described briefly in Khoo, this model results in the following differential equation, which relates the p(t) to q(t) (the pressure of the ventilator at the airway opening to the volume delivered by the ventilator):

$$\frac{d^2p}{dt^2} + \frac{1}{R_pC_t}\frac{dp}{dt} = R_c\frac{d^2q}{dt^2} + \left(\frac{1}{C_s} + \frac{R_c}{R_pC_t}\right)\frac{dq}{dt} + \frac{1}{R_pC_s}\left(\frac{1}{C_L} + \frac{1}{C_W}\right)q$$

Where $\frac{1}{C_t} = \frac{1}{C_L} + \frac{1}{C_W} + \frac{1}{C_s}$. If you are interested in deriving this for yourself, see problem 3.

However, we are really most interested in the volume of air in the lungs, $v = \int q dt$, or $\frac{dv}{dt} = q$.

Use the parameter values for a healthy patient: Rc = 0.001; Rp = 0.0005; C_L=200; C_W =200; Cs = 5. (R's are in atm*s/L, and C's in L/atm.)

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1. Transfer function model, analytic work. (**10 points; 2 points each**)
   a. Find the transfer function, H(s)=V(s)/P(s), that can be used to calculate the transform of the volume of air in the lungs, V(s) to the transform of the ventilator-controlled pressure, P(s). *Give your answer in terms of the parameter names, not numbers.*
   b. For arbitrary parameter values, what is the steady state gain? *Hint: use the transfer function. Give your answer in terms of the parameter names, not numbers.*
   c. For the given parameter values, how much pressure should you use to produce an equilibrium volume of 0.5L? (The maximum lung capacity of a patient is typically 6 L, but a normal tidal volume is only 0.5 L.)
   d. For the given parameter values, is the system stable? *Hint: use the roots of the characteristic equation to answer this question, and give your answer in numeric form.*
   e. For the given parameter values, what is the slowest time constant? *Hint: use the roots of the characteristic equation , and give your answer in numeric form.*

2. Numeric model using transfer functions (SIMULINK): (**10 points; 2 points each**)
   a. Plot the time dependent response of this system to a step function, *Hint: represent your polynomials as vectors, use the commands we covered in lecture.*
   b. Plot the time dependent response of this system to a step function, this time using a simulink model of the system.
   c. Verify your work by explaining the relationship between the plots in 2a and 2b and the things you calculated in problem 1.
   d. Plot the response to a Periodic input. In the clinic, of course, you will not use a constant pressure, but instead will apply pressure and release it every few seconds. A normal adult breathes 18 to 22 times per minute, so use a period of 3 seconds. Use the pulse generator to apply the pressure you determined above, with a 1.5 second wide pulse starting every 3 seconds.
   e. Calculate the tidal volume for your periodic input. The main outcome of interest when we ventilate the patients is the tidal volume, which is the difference between the minimum and maximum volume. Note that this will may or may not be the same as the steady state gain. *Hint: You can determine this numerically from your outcomes using the min and max functions, but be sure you are doing this over an appropriate time frame to allow complete oscillations and avoid the early time points where the system is still affected by initial conditions*.

3. State space model. (**10 points**)

a. **(5 points)** Derive a state space model for the lung mechanics, in terms of two variables $v = \int q dt$ and $v_A = \int q_A dt$. That is, $z = [v; v_A]$. *Hint: use the diagram above, and the DIESE method from class. Make sure you replace all the q's with v's, and get rid of all the intermediate pressure variables. It should fall together fairly quickly. Show your work, but express your final answer by defining the matrices A, B, C, and D in terms of the parameters and defining x(t), y(t), and z(t) in terms of* $p, v,$ *and* v_A.

b. **(3 points)** For the given parameters, what are the stability, and time constants?

c. **(2 points)** For the given parameters, what is the steady state gain for v?

4. **(10 points)** Diseased patients.

   a. **(3 points)** Acute respiratory distress syndrome (ARDS), is a condition where the alveoli become inflamed. This decreases the compliance of the alveoli. Unfortunately, mechanical ventilation further decreases the compliance. To illustrate how this change affects the ventilation, plot the volume over time in response to the same ventilation protocol you used for a healthy patient, when the appropriate parameter is the same as a healthy patient or is decreased by 2-fold, 5-fold, and 10-fold

   b. (**5 points**) What is the fractional sensitivity of three outcomes (the steady state gain, the slowest time constant, and the tidal volume) to the compliance of the alveoli? *Hint: calculate and plot each of the three outcomes (steady state gain, time constant, and tidal volume) as a function of this parameter value. You can use any of the tools above, numeric or analytic, to do this. How do you calculate the fractional sensitivity from this?*

   c. (**2 points**)Explain if and how you would need to change the ventilation protocol P(t) for a patient with ARDS.

Simulink Hints:

- Parameter names that are defined in the MATLAB workspace (but not in the current function's workspace) are available when you call simulink from the command line or a script. You can call a simulink model from a MATLAB m-file with the command `tout = sim('modelname')` . The output variable tout is the time vector.
- To get any other values from your SIMULINK simulation, use the "simout" box in the "sink" menu of simulink, and rename it 'variablename'. After running the model, this variable will now be defined in the workspace, so you don't call it as an output. Important: the default output is a structure that has both the time and variable arrays, as well as other things. If you don't want to deal with structures, select the choice in the simout box that switches the type to an array. (If you aren't familiar with structures, you should check it out first - call the structure by typing 'variablename' on the

command line, and you will see a list of all theelements in the structure. To see more, add a dot and the element name, (like 'variablename.time') and you'll see what's in the element. Remember, an element can itself be a structure, in which case you will have two or more dots in the name you finally call.)

Other hints:

- For help with building this particular model in SIMULINK, see chapter 2.9 of Khoo.
- Most the things you need are in 'commonly used blocks' in the library browser. If you right click on a block you have added, you can change parameters, the visual format, etc.
- If you want to plot multiple solutions together on a plot, you can define: colors = ['b', 'c', 'g'] for example, and then use this to plot multiple results in a loop: plot(time,x, colors(i)). This will make the different plots different colors without you having to rewrite multiple lines of code. Alternatively, you can use subplot(# of rows,# of columns,plot #) for each plot to put them all on subsets of the same figure. For instance, subplot(2,3,1) would put the plot in the upper left of a block of six plots with two rows and three columns.