Samantha Sun

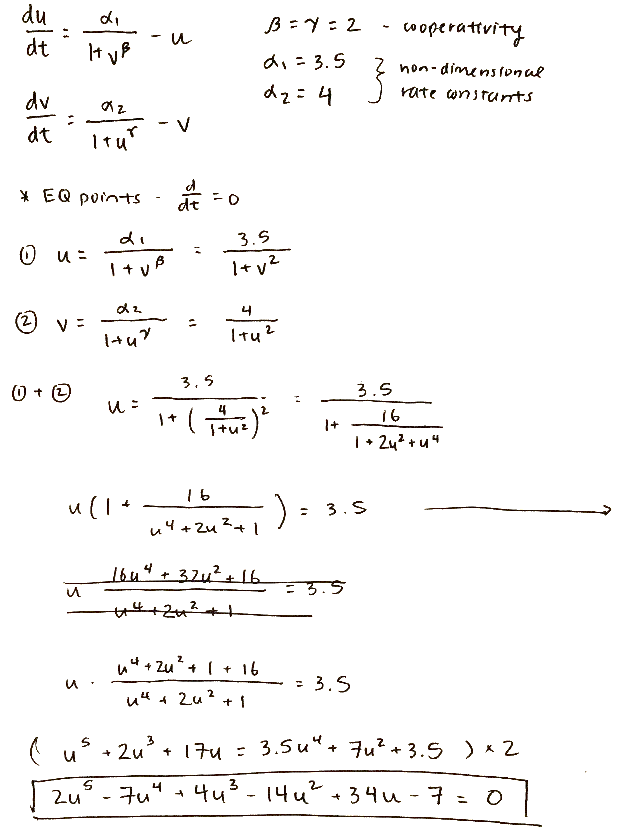
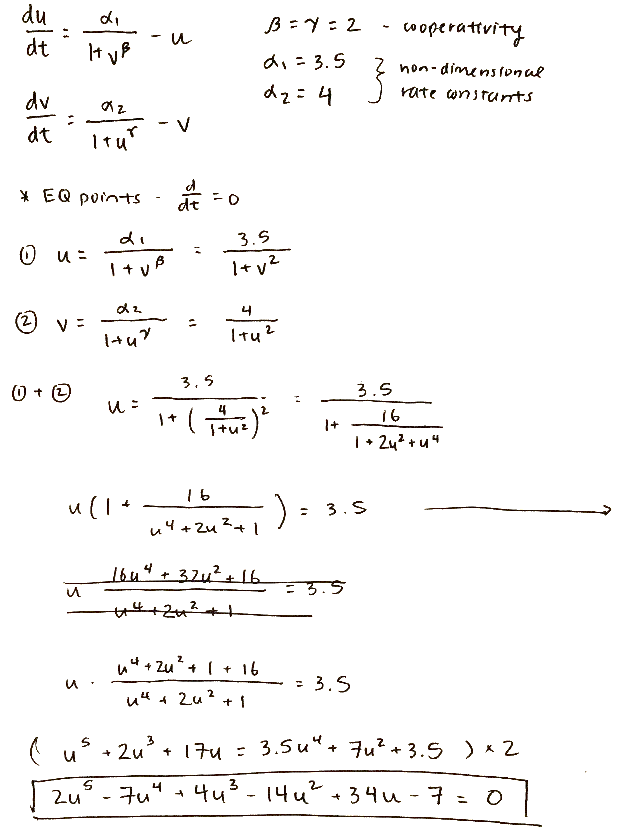
BIOEN585

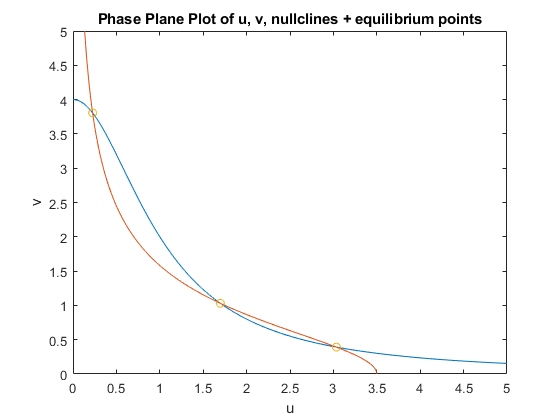
20190417

Lab 3: Nonlinear Analysis

# Question 1: Stability Analysis

1. Solve for nullclines and equilibrium points, plot on phase plane plot





1. Stability analysis: bi-stable switch?

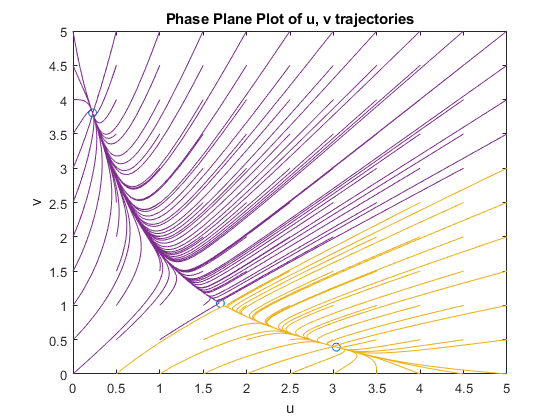
The figure on the left shows the two polynomial curves plotted based on solving for the nullclines for u and v. Where the two curves intersect are the equilibrium points.

There are three equilibrium points in this system, which can be seen by the three intersections in the figure. To determine the stability at each point, we create the Jacobian matrix and solve for the eigenvalues at each point.

At point 1 and point 3, the eigenvalues are both negative, which means they are stable equilibrium points. At point 2, there is one positive eigenvalue, which means it is an unstable equilibrium.

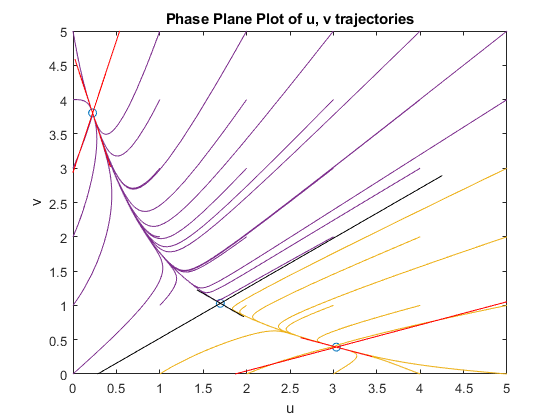
# Question 2: Phase Plane Analysis

1. Plot bi-stability trajectories



This figure shows the three equilibrium points identified previously, which are represented by three blue circles, and system trajectories with initial values at every 0.5 value of u and v from a range of 0 to 5. The trajectories in purple indicate that the system falls into the upper-left equilibrium point at steady-state, and the trajectories in yellow are those that have steady state at the lower right equilibrium point. None of the initial conditions led to a steady state at the unstable equilibrium point – the one in the middle. Overall, this figure confirms that the system is bi-stable, with two stable equilibrium points, where the initial condition has a big role in determining which equilibrium point the system will reach.

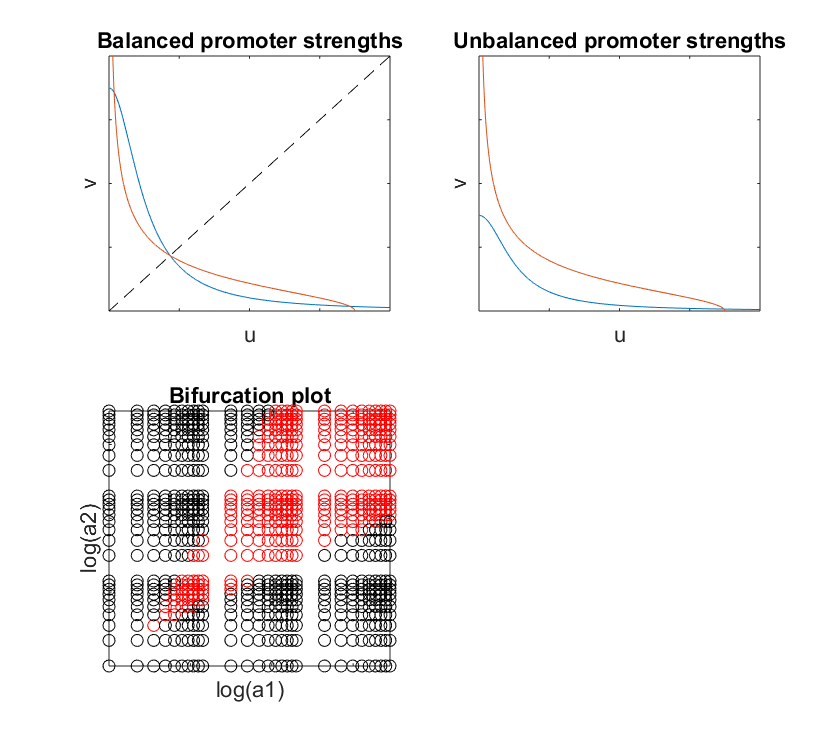
1. Plot eigenvectors



The eigenvectors have been plotted on a similar figure as above, where fewer initial conditions were used to plot the system trajectories. The lines for each vector are centered around each equilibrium point and scaled to the magnitude of the eigenvalue. Eigenvalues that were negative were plotted in red, otherwise they were plotted in black. We can see that for the two stable EQ point, the eigenvalues were all negative, and the unstable EQ point has one positive and one negative eigenvalue (not seen in figure but fixed bug in code…). We observe that one of these eigenvectors are parallel to the curve formed by the trajectories that pass through each EQ point, and the other vector is parallel to the trajectories along that line.

# Question 3: Bifurcation Analysis

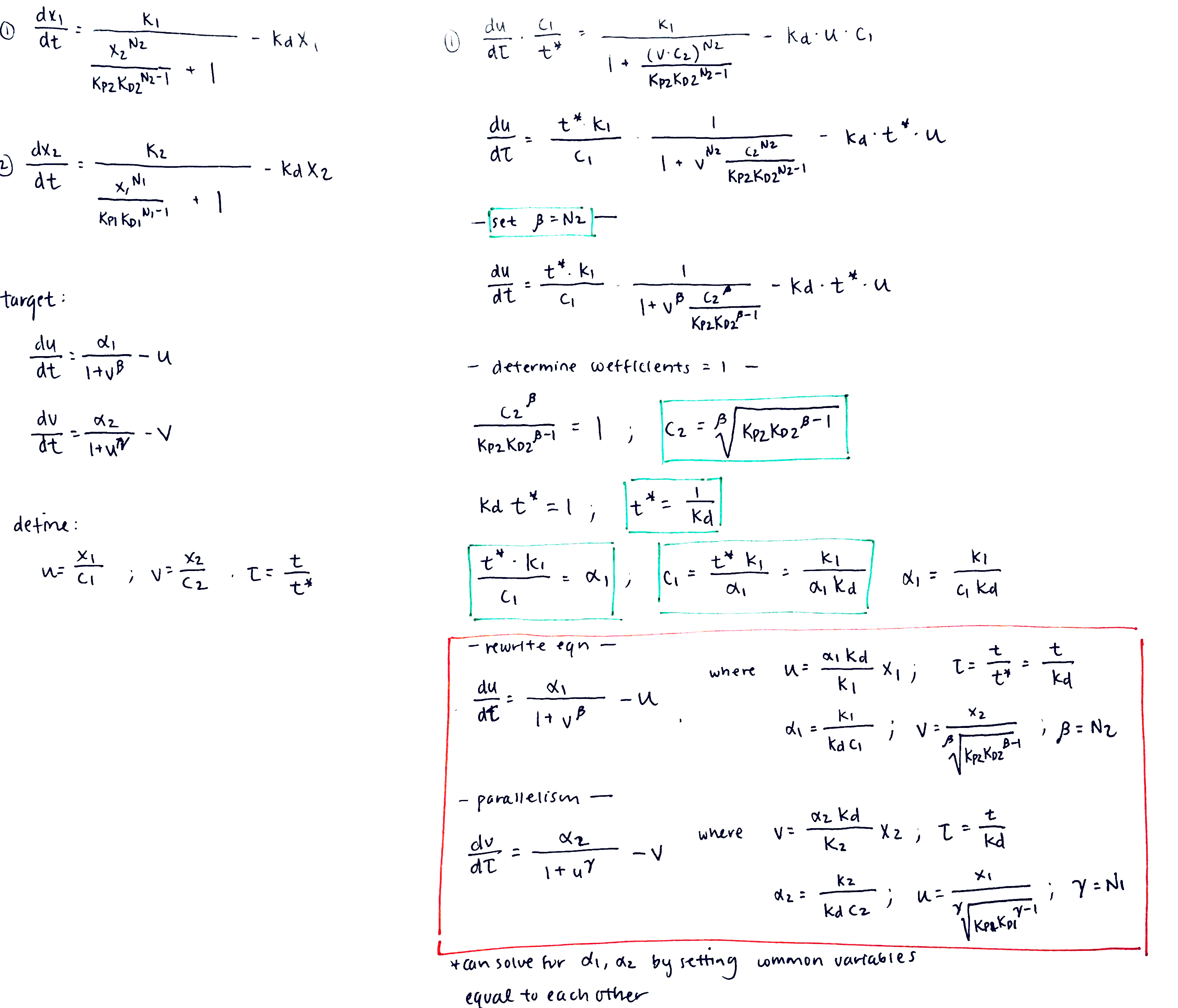
1. Reproduce Gardner Figure 2



This figure is a reproduction of the Gardner paper, Figure 2, not including panel d (lower right). The first plot shows the nullcline polynomials for u and v if the promoter strengths were equal (alpha 1 = alpha 2) and the second plot shows the same plot when alpha 2 is smaller than alpha 1. The third plot shows a bifurcation plot, where we selected values of alpha 1, alpha 2 from 0 – 1000 and calculated the stability with those promoter strengths. The red dots resulted in bistable systems and the black points represent monostable systems, and we can see the clear lines separating the bistable regions from the two monostable regions.

This bifurcation is a pitchfork bifurcation. We know that the alpha values cannot be less than zero, but as you increase the values, there are two stable states that the system could fall into. If we held one of the alphas constant and changed the value of the other one, there would always be range of values where there are two steady states.

# Question 4: Model Building



# Appendix: MATLAB Code

\*note: I have the ode\_uv function but saved it on the lab computer + can’t access it before I turn it in. I can turn this in later/email it to you if it is necessary—please let me know!

## Lab 3: Non-linear systems

Samantha Sun BIOEN 585 20190417

clear all; close all; clc  
plotOn = 0;

## Question 1: Stability Analysis

% solve for nullclines + equilibrium points - plot on phase plane plot  
a1 = 3.5;  
a2 = 4;  
beta = 2;  
  
% polynomial eqn - solving for u, v  
u\_co = [2, -2\*a1, 4, -4\*a1, 2\*(a2^2+1), -2\*a1]; % polynomial coefficients (found analytically)  
null\_u = roots(u\_co);  
null\_u = null\_u (null\_u > 0);  
null\_v = a2 ./ (1 + null\_u.^2);  
  
% plotting polynomial  
u = 0:0.01:5;  
v = 0:0.01:5;  
u\_null = a1./(1+v.^2);  
v\_null = a2./(1+u.^2);  
  
figure;  
plot(u,v\_null)  
hold on  
plot(u\_null,v)  
plot(null\_u, null\_v, 'o')  
xlabel('u')  
ylabel('v')  
xlim([0 5])  
ylim([0 5])  
title('Phase Plane Plot of u, v, nullclines + equilibrium points')  
  
% stability analysis - get eigenvalues @ each eq point  
J1 = [-1, -2\*null\_v(1)\*a1/((1+null\_v(1)^2)^2);  
 -2\*null\_u(1)\*a2/((1+null\_u(1)^2)^2), -1];  
[V1,D1] = eig(J1);  
  
J2 = [-1, -2\*null\_v(2)\*a1/(1+null\_v(2)^2)^2;  
 -2\*null\_u(2)\*a2/(1+null\_u(2)^2)^2, -1];  
[V2,D2] = eig(J2);  
  
J3 = [-1, -2\*null\_v(3)\*a1/(1+null\_v(3)^2)^2;  
 -2\*null\_u(3)\*a2/(1+null\_u(3)^2)^2, -1];  
[V3,D3] = eig(J3);

## Question 2: Phase Plane Analysis

clearvars -except null\_u null\_v a1 a2 V1 D1 V2 D2 V3 D3  
  
% define eq points  
eq1 = [null\_u(1), null\_v(1)];  
eq2 = [null\_u(2), null\_v(2)];  
eq3 = [null\_u(3), null\_v(3)];  
  
% solve trajectories for array of initial conditions  
  
% real-time plotting magic  
figure;  
plot(null\_u, null\_v, 'o')  
hold on  
xlabel('u')  
ylabel('v')  
xlim([0 5])  
ylim([0 5])  
title('Phase Plane Plot of u, v trajectories')  
  
tspan = [0 50];  
for i = 0:5  
 u0 = i;  
 for j = 0:5  
 v0 = j;  
 [t,y] = ode45(@ode\_uv,tspan,[u0 v0]);  
 u = y(:,1);  
 v = y(:,2);  
  
 % colors + add to plot  
 if [round(u(end),2), round(v(end),2)] == round(eq1,2)  
 plot(u,v,'Color','[0.929 0.694 0.098]') % gold  
 end  
  
 if [round(u(end),2),round(v(end),2)] == round(eq2,2)  
 plot(u,v,'c') % cyan  
 end  
  
 if [round(u(end),2),round(v(end),2)] == round(eq3,2)  
 plot(u,v,'Color','[0.494 0.184 0.556]') % PURPLE  
 end  
 %pause(0.1)  
  
 end  
end  
  
% plotting eigenvalues + eigenvectors (EXTRA CREDIT)  
% line centered @ each EQ point, length of eigenvalue + direction  
% of eigenvector  
% color black or red depending on sign of eigenvalue  
% black = pos, red = neg  
Ds = [diag(D1); diag(D2); diag(D3)];  
EQs = [eq1; eq2; eq3];  
Vs = [V1; V2; V3];  
  
for idx = 1:3  
 D = Ds(2\*idx-1:2\*idx);  
 eq = EQs(idx,:);  
 V = Vs(2\*idx-1:2\*idx,:);  
 x\_eq1 = [eq(1) - V(1,1)\*sqrt(2)\*D(1),eq(1) + V(1,1)\*sqrt(2)\*D(1)];  
 y\_eq1 = [eq(2) - V(2,1)\*sqrt(2)\*D(1),eq(2) + V(2,1)\*sqrt(2)\*D(1)];  
 if D(1) > 0  
 plot(x\_eq1,y\_eq1,'-k')  
 else  
 plot(x\_eq1,y\_eq1,'-r')  
 end  
  
 x\_eq1b = [eq(1) - V(1,2)\*sqrt(2)\*D(2),eq(1) + V(1,2)\*sqrt(2)\*D(2)];  
 y\_eq1b = [eq(2) - V(2,2)\*sqrt(2)\*D(2),eq(2) + V(2,2)\*sqrt(2)\*D(2)];  
 % plot(x\_eq1b,y\_eq1b,'-r')  
  
 if D(2) > 0  
 plot(x\_eq1b,y\_eq1b,'-k')  
 else  
 plot(x\_eq1b,y\_eq1b,'-r')  
 end  
end

## Question 3: Bifurcation Analysis

reproduce Gardner Fig 2, panels a, b, c

clearvars  
  
a1 = 7;  
a2 = 7;  
a\_mod = 3;  
beta = 2;  
  
% plotting polynomial  
u = 0:0.01:10;  
v = 0:0.01:10;  
u\_null = a1./(1+v.^2);  
v\_null = a2./(1+u.^2);  
v\_null\_mod = a\_mod./(1+u.^2);  
  
figure;  
  
subplot(2,2,1) % panel a  
plot(u,v\_null)  
hold on  
plot(u\_null,v)  
plot([0 10],[0 10],'k--')  
xlabel('u')  
ylabel('v')  
xlim([0 8])  
ylim([0 8])  
title('Balanced promoter strengths')  
set(gca,'XTickLabel',[],'YTickLabel',[]);  
  
subplot(2,2,2) % panel b  
plot(u,v\_null\_mod)  
hold on  
plot(u\_null,v)  
xlabel('u')  
ylabel('v')  
xlim([0 8])  
ylim([0 8])  
title('Unbalanced promoter strengths')  
set(gca,'XTickLabel',[],'YTickLabel',[]);  
  
subplot(2,2,3) % panel c  
ln\_a1 = [1:10, 10:10:100, 100:100:1000];  
ln\_a2 = [1:10, 10:10:100, 100:100:1000];  
  
% for each alpha value, find the bifurcation lines  
% for each alpha pair - does it produce a bistable switch  
% or does it have 0, 1, 2, 3 points of intersection  
% 0 - no stability  
% 1 - monostability  
% 2+ bistable  
  
for i = ln\_a1  
 for j = ln\_a2  
 % find number of intersection pts  
 u\_co = [2, -2\*i, 4, -4\*i, (j^2+1)\*2, -2\*i]; % polynomial coefficients (found analytically)  
 null\_u = roots(u\_co);  
 null\_u = null\_u (null\_u > 0);  
 insx = sum(null\_u == real(null\_u));  
 if (insx == 1)  
 % monostable - black  
 plot(log(i),log(j),'ko'); hold on  
 elseif insx == 3  
 % bistable - red  
 plot(log(i),log(j),'ro'); hold on  
 end  
 xlim([0 log(ln\_a1(end))]);  
 ylim([0 log(ln\_a2(end))]);  
 xlabel('log(a1)')  
 ylabel('log(a2)')  
 title('Bifurcation plot')  
 set(gca,'XTickLabel',[],'YTickLabel',[]);  
 end  
end