Amath481 HW3 presentation skills

HAOLOU SUN

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1 Problem 2b

1.1 Introduction

In this paper, we will discuss the time takes for solving PDE using Gaussian Elimination and LU decomposition.

The model we will use is the vorticity function ω(x,y,t) and streamfunction ψ(x,y,t) given by the following equations

$$\omega_t + [\psi, \omega] = \nu \nabla^2 \omega \tag{1}$$

and the streamfunction satisfies

$$\nabla^2 \psi = \omega \tag{2}$$

Thus, for solving the streamfunction ψ from vorticity ω, we need a linear systems solving method to solve

$$A = \nabla^2 \tag{3}$$

$$A\psi = \omega \tag{4}$$

in each time step.

1.2 Results

We set the matrix we want to solve to $64^2 \times 64^2$ and $128^2 \times 128^2$, and then we get the following time that Gaussian Elimination takes and LU decomposition takes.

	$64^2 \times 64^2$	$128^2 \times 128^2$
Gaussian Elimination	0.5956547260284424	3.064423084259033
LU decomposition	0.0583798885345459	0.2504310607910156
ratio	10.203080906466065	12.23659347438651

Table 1: The time takes for Gaussian Elimination and LU decomposition in solving $64^2 \times 64^2$ matrix and $128^2 \times 128^2$ matrix, and their ratio.

1.3 Results Analysis

From the table, we can see that for $64^2 \times 64^2$ matrix, Gaussian Elimination method takes 10 times longer than LU decomposition method. For $128^2 \times 128^2$ matrix, Gaussian Elimination method takes 12 times longer than LU decomposition.

We know that Gaussian Elimination method is $O(N^3)$, and LU decomposition method is $O(N^2)$ with $O(N^3)$ to find the L,U,P matrix. N means a $N \times N$ matrix. In our model, N is 64^2 or 128^2. Thus we can calculate the steps need for Gaussian Elimination method and LU decomposition method in 9 time steps.

For Gaussian Elimination method of $64^2 \times 64^2$ matrix, we have

$$64^2 \times 64^2 \times 64^2 \times 9$$

For LU decomposition method of $64^2 \times 64^2$ matrix, we have

$$64^2 \times 64^2 \times 64^2 + 64^2 \times 64^2 \times 9$$

Then the ratio is

$$\frac{64^2 \times 64^2 \times 64^2 \times 9}{64^2 \times 64^2 \times 64^2 + 64^2 \times 64^2 \times 9} = 8.98$$

For Gaussian Elimination method of $128^2 \times 128^2$ matrix, we have

$$128^2 \times 128^2 \times 128^2 \times 9$$

For LU decomposition method of $128^2 \times 128^2$ matrix, we have

$$128^2 \times 128^2 \times 128^2 + 128^2 \times 128^2 \times 9$$

Then the ratio is

$$\frac{128^2 \times 128^2 \times 128^2 \times 9}{128^2 \times 128^2 \times 128^2 + 128^2 \times 128^2 \times 9} = 8.995$$

Thus, by mathematical calculation, we find that while solving really large matrix, the time Gaussian Elimination takes is approximately 9 times of the time

LU decomposition method takes. And that is exactly the number of time steps we use. From the table we can see that, the experiment results generally fit our mathematical result.

1.4 Conclusion

By looking at the experiment data we got (Table 1), and the mathematical data we get, we can conclude that for really large matrix solving, Gaussian Elimination should take x times longer than LU decomposition, where x is the number of time steps we will use.

In future study, we can compare the time that Gaussian Elimination takes with the time LU decomposition takes exclude the time for finding the L,U,P matrix. The ratio we get should be $N \times x$, where N is the N of our $N \times N$ matrix, and x is the number of time steps we use. For this model, we didn't do that because our matrix reshape function will cost a lot of time and that will influence our judgement of this time comparision.