1. Solve the following recurrence relation

a) $x(n) = x(n-1) \neq 5$ for $n > k$ with $x(1) = 0$

1) write down the first two terms to identify the pattern

$$x(1) = 0$$
$$x(2) = x(1) + 5 = 5$$
$$x(3) = x(2) + 5 = 0$$
$$x(4) = x(3) + 5 = 15$$

2) identify the pattern (or) the general term

→ The first term $x(1) = 0$

The common difference $d = 5$

The general formula for the n^{th} term of an AP is

$$x(n) = x(1) + x(n-1)\, d$$

Substuting the given values

$$x(n) = 0 + (n-1) \cdot 5 = 5(n-1)$$

The solution is $x(n) = 5(n-1)$

b) $x(n) = 3x(n-1)$. for $n > 1$ with $x(1) = 4$

1) write down the first two terms to identify to the pattern

$$x(1) = 4$$
$$x(2) = 3x(1) = 3 \cdot 4 = 12$$
$$x(3) = 3x(2) = 36$$
$$x(4) = 3x(3) = 108$$

2) Identify the general term

→ the first term $x(1) = 4$

→ The common ratio $r = 3$

The general formula for the n^{th} term of a gp is

$$x(n) = x(1) \cdot r^{n-1}$$

Substuting the given values

$$x(n) = 4 \cdot 3^{n-1}$$

The solution is $x(n) = 4 \cdot 3^{n-1}$

c) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (solve for $n = 2^k$)

For $n = 2^k$, we can write recurrence in terms of k

1) substitute $n = 2^k$ in the recurrence

$$x(2^k) = x(2^{k-1}) + 2^k$$

2) write down the first few terms to identify the pattern

$$x(1) = 1$$

$$x(2) = x(2^1) = x(1) + 2 = 1+2 = 3$$

$$x(4) = x(2^2) = x(2) + 4 = 3+4 = 7$$

$$x(8) = x(2^3) = x(4) = 7+8 = 15$$

3) Identify the general term by finding the pattern

we observe that

$$x(2^k) = x(2^{k-1}) + 2^k$$

we sum the series:

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \cdots$$

since $x(1) = 1$

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} + \cdots$$

The geometric series with the term $a=2$ and the last term 2^k except the additional term

The sum of a geometric series S with ratio $r=2$ is given by

$$S = a\frac{r^n - 1}{r-1}$$

were $a=2$, $r=2$ and $n=k$:

$S = 2\,\frac{2^k - 1}{2-1} = 2(2^k - 1) = 2^{k+1}$

Adding the +1 term

$x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$

Solution is $x(2^k) = 2^{k+1} - 1$

(d) $x(n) = x(n/3) + 1$ for $n > 1$ with $x(1) = 1$ (solve for $n = 3^k$)

For $n = 3^k$, we can write the recurrence in terms of k

1) substitute $n = 3^k$ in the recurrence

$x(3^k) = x(3^{k-1}) + 1$

2) write down the first few terms to identify the pattern

$x(1) = 1$

$x(3) = x(3^1) = x(1) + 1 = 1 + 1 = 2$

$x(9) = x(3^2) = x(3) + 1 = 2 + 1 = 3$

$x(27) = x(3^3) = x(9) + 1 = 3 + 1 = 4$

3) identify the general term:

we observe that

$x(3^k) = x(3^{k-1}) + 1$

summing up the series

$x(3^k) = 1 + 1 + 1 + \dots + 1$

$\therefore x(3^k) = k + 1$

The solution is

$x(3^k) = k + 1$

(2) Evalute the following recurrences complexity

i) $T(n) = T(n/2) + 1$, where $n = 2k$ for all $k \geq 0$

The recurrence relation can be solved using iteration method.

i) Substitute $n = 2^k$ in the recurrence

ii). Iterate the recurrence

For $k=0$ $T(2^0) = T(1) = T(1)$

$k=1$ $T(2^1) = T(1)+1$

$k=2$ $T(2^2) = T(8) = T(n)+1 = T(1)+2)+1 =$

$k=3$ $T(2^3) = T(8) = T(n)+1 = T(1)+2) = T(1)+3$ $T(1)+2$

3). generalize the pattern

$T(2^k) = T(1)+k$

Since $n = 2^k$, $k = \log_2 n$

$T(n) = T(2^k) = T(1) + \log_2 n$

4) Assume $T(1)$ is a constant c

$T(n) = c + \log_2 n$

The solution is $T(n) = O(\log n)$

(ii) $T(n) = T(n/3) + n(2n/3) + cn$ where c is constant)

The recurrence can be solved using the masters thereom for divide and conquer recurrence of the form

$T(n) = a \cdot (n/b) + f(n)$

where $a=2$, $b=3$ and $f(n) = cn$

lets determine the value of $\log_b a$

$\log_b a = \log_3 2$

using the properties of logorithimics

$$\log_3 2 = \frac{\log 2}{\log 3}$$

Now we compare $f(n) = Cn$ with $n^{\log_3 2}$

$f(n) = O(n)$

$n = n^1$

since $\log_3 2$ we are in the third case of the master's thereom

$f(n) = O(n^c)$ with $c > \log_b a$

The solution is:

$T(n) = O(f(n) = O(n] = O(n)$

(5) Consider the Follwing recurrence algorithm

min [A[0, ... n-2]]

if n=1 return A[0]

Else temp=min ([A|0 .... n-2]]

if temp < A(n-1) return temp

else

return A[n-1]

a). what does this algorithm compute?

The given algorithm, min [A[0.. ...n--1]] computes the minimum value in the array 'A' from index '0' For value if does this recurrences finding the minimum value in the sub array [A. [0....n-2] and then, comparing it with the last element 'A[n-3] to determine the overall maximum value.

b) setup a recurrence relation for the algorithmic basic operation count and solve it

The solution is

$T(n) = n$

This means the algorithm performs basic operations for input array of size n

(4) Analyze the order of growth

i) $f(n) = 2n^2+5$ and $g(n) = 7n$ use the $g(n)$ notation

To analyze the order of growth and use the Ω notation, we need to compare the given function $f(n)$ and $g(n)$

given functions

$F(n) = 2n^2+5$

$g(n) = 7n$

order of growth using $g(n)$ notation

The notation Ω $g(n)$ describes a lower bound on the growth rate that for successively large $f(n)$ grows at least as for $g(n)$

$F(n) \geq c.g(n)$

lets analyze $F(n) = 2n^2+5$ with respect to $g(n) = 7n$

1) identify Dominant terms:

The dominant terms in $f(n)$ is $2n^2$ since it grows faster then the constant terms as n increases.

-) the dominant term in $g(n)$ is $7n$

2) establish the inequality

-) we want to find constants c and n_0 such that

$2n^2+5 \geq c.7n$ for all $n \geq n_0$

3) simplify the inequality

→ ignore the lower order term 5 For integer

$$2n^2 = 7cn$$

→ Divide both sides by n

$$2n \geq 7c$$

→ Solve For n:-

$$n \geq n/2$$

4). Choose constants

let $c = 1$

$$n \geq 1 \frac{7-1}{2} = 3.5$$

∴ For $n \geq n$ the inequality holds:

$$2n^2 + 5 \geq 7n \text{ For all } n \geq n$$

We have shown that there exist Constants c=1 and $n_0 = n$ such that For all $n \geq n_0$:-

$$2n^2 + 5 \geq 7n$$

Thus, we can conclude that:

$$n = 2n^2 + 5 = 2(7n)$$

"Ω" notation the dominant term $2n^2$ in $f(n)$ clearly grows Faster than. Hence

$$f(n) = \Omega(n^2)$$

However For the specific Comparision asked $F(n) = \Omega(7n)$ is also correct

Showing that $F(n)$ grows at least as part as $7n$.