IME692A: Assignment 1

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Question-2

Part-a

Note: For random variable we have used X, Y symbols and for specific value of random variables in sample space($\chi, \mathcal{Y}$) we have used x, y symbols. All the other symbols and formulas have their usual meaning taken from class notes.

We have been given bi-variate joint mass function of (X, Y). Marginal p.m.f. for X:

$$p(X = 0) = \frac{2}{3}$$
$$p(X = 1) = \frac{1}{3}$$

Marginal p.m.f. for Y:

$$p(Y = 0) = \frac{1}{3}$$
$$p(Y = 1) = \frac{2}{3}$$

As we know conditional Entropy is defined by,

$$\mathbf{H}(Y|X) = \sum_{x \in \chi} p(X)\mathbf{H}(Y|X = x)$$

Using joint density, marginal densities in conditional entropy formula, we get $\mathbf{H}(Y|X) = 0.77$.

Part-b

As we know cross Entropy is defined by,

$$\mathbf{H}(X, Y) = - \sum_{x \in \chi, y \in \mathcal{Y}} p(X) log_2(p(Y))$$

Using joint density, marginal densities in cross entropy formula, we get $\mathbf{H}(X, Y) = 1.255$.

Part-c

As we know mutual information is defined by,

$$\mathbf{I}(X, Y) = \sum_{x \in \chi} \sum_{y \in \mathcal{Y}} p(X, Y) log_2 \frac{p(X, Y)}{p(X)p(Y)}$$

Using joint density, marginal densities in mutual information formula, we get $\mathbf{I}(X, Y) = 1.51$.