

**Project**  
**Non-Linear Regression Analysis**  
**Due date is 14.04.22, 7 p.m. NO EXTENSION. NO CONSULTATION WITH**  
**YOUR FRIENDS. IF I FIND SIMILAR COPIES BOTH WILL GET 0**

1. Consider the data set you have received in your e-mail, which is of the form  $(t, y(t))$ . Try to fit the following model to the given data set:

$$y(t) = \alpha_1 + \alpha_2 e^{\beta t} + \epsilon(t).$$

Let us assume that  $\{\epsilon(t)\}$  is a sequence of *i.i.d.* normal random variable with mean zero and finite variance. Analyze the data keeping the following points in mind, and write a report based on that.

1. Plot the data.
2. Plot the residual sum of squares as a function of  $\beta$ .
3. Find the least squares estimators of  $\alpha_1$ ,  $\alpha_2$  and  $\beta$  based on the Gauss-Newton method. Clearly mention which initial value you are taking? Does your result affect by the choice of the initial guess?
4. Find the estimate of  $\sigma^2$ .
5. Find the associated confidence intervals based on the Fisher information matrix.
6. Plot the residuals.
7. Test whether it satisfies the normality assumption or not?
8. Use any standard package (say R) and try to obtain the least squares estimators of the unknown parameters based on three dimensional optimization problem. Repeat all the above questions. Indicate which initial values you are taking and why?
9. What will you do to fit the following model

$$y(t) = \alpha_1 e^{\beta_1 t} + \alpha_2 e^{\beta_2 t} + \epsilon(t),$$

to the same data set? Let us assume that  $\epsilon(t)$  satisfies the same assumption as above.