Concepts and Formulas

Dot Product of Two Vectors:

$$\boxed{\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta}$$

Properties:

(i) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ [i.e. dot product is commutative].

(ii) $(\vec{a}\cdot\vec{b})\cdot\vec{c}$ is not defined.

(iii) $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$ [distributive property]

(iv) If $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then $\vec{a}\cdot\vec{b}=0$, converse is also true.

(v) Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ and projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$.

(vi) If $\theta=0$, then the projection vector of $\overrightarrow{AB}$ will be $\overrightarrow{AB}$ itself and if $\theta=\pi$, then the projection vector of $\overrightarrow{AB}$ will be $\overrightarrow{BA}$.

(vii) If $\theta=\dfrac{\pi}{2}$ or $\theta=\dfrac{3\pi}{2}$, then the projection vector of $\overrightarrow{AB}$ will be zero vector.

(viii) Angle between two vectors $\vec{a}$ and $\vec{b}$ is

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} \quad \text{or} \quad \theta = \cos^{-1}\left[\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}\right]$$

(ix) $\vec{a}\cdot\vec{a} = |\vec{a}|^2$

(x) $\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = 1$

(xi) $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

(xii) If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then $\vec{a}\cdot\vec{b} = a_1b_1+a_2b_2+a_3b_3$.

(xiii) $(\lambda\cdot\vec{a})\cdot\vec{b} = \lambda(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(\lambda\cdot\vec{b})$, where λ is any scalar.

(xiv) If $\theta=\pi$, then $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$; If $\theta=\pi$, then $\vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$

Vector (or Cross) Product of Vectors:

$$\boxed{\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}, \text{ such that } 0\le\theta\le\pi}$$

here n makes right handed system with two vectors a and b.

Concepts and Formulas

Properties:

(i) Angle between two vectors is $\sin\theta = \dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$ or $\theta = \sin^{-1}\left[\dfrac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}\right]$

(ii) $\vec{a}\times\vec{a} = 0$

(iii) $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

(iv) In general, $\vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})\times\vec{c}$

(v) $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$ [distributive property]

(vi) $\lambda(\vec{a}\times\vec{b}) = (\lambda\vec{a})\times\vec{b} = \vec{a}\times(\lambda\vec{b})$

(vii) If $\vec{a}$ is parallel to $\vec{b}$, then $\vec{a}\times\vec{b} = \vec{0}$ and converse is also true.

(viii) If $\theta = \dfrac{\pi}{2}$, then $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|$.

(ix) Area of parallelogram whose adjacent sides are along $\vec{a}$ and $\vec{b} = |\vec{a}\times\vec{b}|$.

(x) Area of triangle, whose adjacent sides are along $\vec{a}$ and $\vec{b} = \dfrac{1}{2}|\vec{a}\times\vec{b}|$.

(xi) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$ and $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$

(xii) $\hat{j}\times\hat{i} = -\hat{k},\ \hat{k}\times\hat{j} = -\hat{i}$ and $\hat{i}\times\hat{k} = -\hat{j}$

(xiii) If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$\Rightarrow \quad (a_2b_3-a_3b_2)\hat{i}+(a_3b_1-a_1b_3)\hat{j}+(a_1b_2-a_2b_1)\hat{k}$

(xiv) Unit vector $\hat{n}$, which is perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, is given by

$$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

(xv) For vectors $\vec{a}$ and $\vec{b}$, if $\vec{a}\times\vec{b} = \vec{0}$, then either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ or $\vec{a}||\vec{b}$.

Tips and Tricks

1. I strongly advice to go through your class notes and make some cheat notes that you can look through before seating for a test. Or one can look into the concepts and formulas section for such cheat notes.

2. With practice you should be able to remember each and every formula easily. It is all about knowing your formulas and when to apply them.

3. Since vectors are elements in 2D/3D, try to visualize them. In questions try to make a neat visual image of the question if possible. Sometimes there is no other way then to apply formulas.

4. Same thing can be applied to fundamental concepts of vectors like dot/cross product. Even more complex concepts are easy to understand if you visualize them.

5. Again practice is a must thing in vectors. Even if you know formulas you would not be able to apply them quikly if you don't practice.

6. This chapter is all about practice and memory. So make a cheat sheet and PRACTICE LOADS OF QUESTIONS.

7. For practice questions are given in problem sections.