## **Exemplar Problem**

## Trigonometric Functions

12. If  $\cos\alpha + \cos\beta = 0 = \sin\alpha + \sin\beta$ , then prove that  $\cos 2\alpha + \cos 2\beta = -2\cos(\alpha + \beta)$ . [Hint:  $\cos\alpha + \cos\beta$ )  $\frac{2}{3} - (\sin\alpha + \sin\beta) = 0$ ]

## Solution:

According to the question,

$$\cos \alpha + \cos \beta = 0 = \sin \alpha + \sin \beta$$
 ...(i)

Since, LHS = 
$$\cos 2\alpha + \cos 2\beta$$

We know that,

$$\cos 2x = \cos^2 x - \sin^2 x$$

Therefore,

LHS = 
$$\cos^2 \alpha - \sin^2 \alpha + (\cos^2 \beta - \sin^2 \beta)$$

$$\Rightarrow$$
 LHS =  $\cos^2 \alpha + \cos^2 \beta - (\sin^2 \alpha + \sin^2 \beta)$ 

Also, since,

$$a^2 + b^2 = (a+b)^2 - 2ab$$

$$\Rightarrow$$
 LHS = (cosα + cosβ)<sup>2</sup> - 2cosα cosβ -(sinα + sinβ)<sup>2</sup> +2sinα sinβ

From equation (i),

$$\Rightarrow$$
 LHS = 0 - 2cosa cos $\beta$  -0 + 2sina sin $\beta$ 

$$\Rightarrow$$
 LHS = -2(cosα cosβ - sinα sinβ)

$$\because \cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$$

Therefore, LHS = 
$$-2 \cos (\alpha + \beta)$$
 = RHS

Hence,  $\cos 2\alpha + \cos 2\beta = -2\cos (\alpha + \beta)$