**Example 1** Let A = {1, 2, 3, 4} and B = {5, 7, 9}. Determine

(i) A × B (ii) B × A

(iii) Is A × B = B × A ? (iv) Is $n$ (A × B) = $n$ (B × A) ?

**Solution** Since A = {1, 2, 3, 4} and B = {5, 7, 9}. Therefore,

(i) A × B = {(1, 5), (1, 7), (1, 9), (2, 5), (2, 7), (2, 9), (3, 5), (3, 7), (3, 9), (4, 5), (4, 7), (4, 9)}

(ii) B × A = {(5, 1), (5, 2), (5, 3), (5, 4), (7, 1), (7, 2), (7, 3), (7, 4), (9, 1), (9, 2), (9, 3), (9, 4)}

(iii) No, A × B ≠ B × A. Since A × B and B × A do not have exactly the same ordered pairs.

(iv) $n$ (A × B) = $n$ (A) × $n$ (B) = 4 × 3 = 12

$n$ (B × A) = $n$ (B) × $n$ (A) = 4 × 3 = 12

Hence $n$ (A × B) = $n$ (B × A)



**Example 2** Find $x$ and $y$ if:

(i) $(4x + 3, y) = (3x + 5, -2)$ (ii) $(x - y, x + y) = (6, 10)$

**Solution**

(i) Since $(4x + 3, y) = (3x + 5, -2)$, so

$4x + 3 = 3x + 5$

or $x = 2$

and $y = -2$

(ii) $x - y = 6$

$x + y = 10$

$\therefore$ $2x = 16$

or $x = 8$

$8 - y = 6$

$\therefore$ $y = 2$

**Example 3** If A = {2, 4, 6, 9} and B = {4, 6, 18, 27, 54}, $a \in$ A, $b \in$ B, find the set of ordered pairs such that '$a$' is factor of '$b$' and $a < b$.

**Solution** Since A = {2, 4, 6, 9}

B = {4, 6, 18, 27, 54},

we have to find $a$ set of ordered pairs $(a, b)$ such that $a$ is factor of $b$ and $a < b$.

Since 2 is a factor of 4 and 2 < 4.

So (2, 4) is one such ordered pair.

Similarly, (2, 6), (2, 18), (2, 54) are other such ordered pairs. Thus the required set of ordered pairs is

{(2, 4), (2, 6), (2, 18), (2, 54), (6, 18), (6, 54,), (9, 18), (9, 27), (9, 54)}.



1. Let A = {-1, 2, 3} and B = {1, 3}. Determine
(i) A × B

(ii) B × A
(iii) B × B

(iv) A × A

Solution:

According to the question,

A = {-1, 2, 3} and B = {1, 3}

(i) A × B

{-1, 2, 3} × {1, 3}

So, A × B = {(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)}

Hence, the Cartesian product = {(-1, 1), (-1, 3), (2, 1), (2, 3), (3, 1), (3, 3)}

(ii) B × A.

{1, 3} × {-1, 2, 3}

So, B × A = {(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)}

Hence, the Cartesian product = {(1, -1), (1, 2), (1, 3), (3, -1), (3, 2), (3, 3)}

(iii) B × B

{1, 3} ×{1, 3}

So, B × B = {(1, 1), (1, 3), (3, 1), (3, 3)}

Hence, the Cartesian product = {(1, 1), (1, 3), (3, 1), (3, 3)}

(iv) A × A

{-1, 2, 3} × {-1, 2, 3}

So, A × A = {(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)}

Hence,

the Cartesian product ={(-1, -1), (-1, 2), (-1, 3), (2, -1), (2, 2), (2, 3), (3, -1), (3, 2), (3, 3)}

39.If

$$P = \{1, 2\},$$

then

$$P \times P \times P = \{(1,1,1), (2,2,2), (1,2,2), (2,1,1)\}.$$

Ans: Given:

$$P = \{1, 2\}.$$

First, find the total number of elements

$$n(P \times P \times P).$$

Then, compare.

$P = \{1, 2\}$ and $n(P) = 2$

$\therefore n(P \times P \times P) = 8$

But there are 4 elements

Therefore, False



40.If

$$A = \{1, 2, 3\}, B = \{3, 4\} \text{ and } C = \{4, 5, 6\},$$

then

$$(A \times B) \cup (A \times C) = \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,3), (3,4), (3,5), (3,6)\}$$

Ans: Given:

$A = \{1, 2, 3\}$,

$B = \{3, 4\}$,

$C = \{4, 5, 6\}$.

First, find

$$A \times B \text{ and } A \times C,$$

then find

$$(A \times B) \cup (A \times C).$$

$A \times B = \{(1,3), (1,4), (2,3), (2,4), (3,3), (3,4)\}$,

$A \times C = \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$,

$(A \times B) \cup (A \times C) = \{(1,3), (1,4), (1,5), (1,6), (2,3), (2,4), (2,5), (2,6), (3,3),$

$(3,4), (3,5), (3,6)\}$.

Therefore, True.

4. Find the domain and range of the relation

$$R$$

given by $R = \{(x, y) : y = x + \frac{6}{x};$ where $x, y \in N$ and $x < 6\}$.

Ans: Given: A relation

$$R$$

.

Domain and range are values of x and y for which relation is defined.

R is defined only for $x = \{1, 2, 3\}, y \in N$

$\therefore$ Domain of $R = \{1, 2, 3\}$

for, $x = 1, y = 7$,

$x = 2, y = 5$,

$x = 3, y = 5$.

$\therefore$ Range of $R = \{7, 5\}$.



8. If

$$R_2 = \{(x, y) | x \text{ and } y \text{ are integers and } x^2 + y^2 = 64\}$$

is a relation, then find the value of

$$R_2.$$

Ans: Given: A relation

$$R_2 = \{(x, y) || \text{ x and y are integers and } x^2 + y^2 = 64\}$$

.

Use the given condition in a relation and then write the set in roster form.

Since,

$$64$$

is the sum of square of

$$0 \text{ and } \pm 8.$$

$\Rightarrow$ when $x = 0$, then $y^2 = 64$,

$\Rightarrow y = \pm 8$

$\Rightarrow x = 8$, then $y^2 = 64 - (8)^2 = 0$

$\Rightarrow x = -8$, then $y^2 = 64 - (-8)^2 = 0$

$\therefore R_2 = \{(0, 8), (0, -8), (8, 0), (-8, 0)\}$

9.If

$$R_3 = \{(x, |x|) | x\}$$

is a real number is a relation, then find domain and range of

$$R_3.$$

Ans: Given: A relation

$$R_3 = \{(x, |x|) | x\}$$

is a real number.

The value of

$$x$$

represents the domain and the values of

y for all x.

Domain of

$$R_3 = \text{real number} \ .$$

Since, the image of any real number under

$$R_3$$

is a positive real number or zero.

Range of

$$R_3 = R^+ \cup \{0\} or (0, \infty).$$



38.The ordered pair

$$(5, 2)$$

belongs to the relation

$$R = \{(x, y) : y = x - 5, x, y \in Z\}.$$

Ans: Given: Ordered pair

$$(5, 2).$$

Relation

$$R = \{(x, y) : y = x - 5, x, y \in Z\}.$$

The ordered pair must satisfy the relation.

$R = \{(x, y) : y = x - 5, x, y \in Z\}$

If $x = 5$, then

$y = 5 - 5 = 0$.

Hence,

$$(5, 2)$$

does not belong to

$$R.$$

24.Let

$$n(A) = m \text{ and } n(B) = n.$$

Then, total number of non-empty relations that can be defined from

$$A \text{ to } B$$

is

(A).

$$m^n$$

(B).

$$n^m - 1$$

(C).

$$mn - 1$$

(D).

$$2^{mn} - 1$$

Ans: Given:

$$n(A) = m \text{ and } n(B) = n.$$

First, find the number of elements in

$$A \times B$$

and then find the number of relation by using

$$2^{m(A \times B)} - 1.$$

We have,

$n(A) = m$ and $n(B) = n$

$n(A \times B) = n(A) . n(B)$

$n(A \times B) = mn$

Total number of relations from

$$A \text{ to } B = 2^{mn} - 1.$$

Correct Answer: D

Past Year Questions from Relations

4 JEE Main 2021 (Online) 31st August Morning Shift
MCQ (Single Correct Answer)

Which of the following is not correct for relation R on the set of real numbers ?

A $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric.

B $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive.

C $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric.

D $(x, y) \in R \Leftrightarrow |x - y| \leq 1$ is reflexive nd symmetric.

## Explanation

Note that (a, b) and (b, c) satisfy $0 < |x - y| \leq 1$ but (a, c) does not satisfy it so $0 \leq |x - y| \leq 1$ is symmetric but not transitive.

For example,

$x = 0.2, y = 0.9, z = 1.5$

$0 \leq |x - y| = 0.7 \leq 1$

$0 \leq |y - z| = 0.6 \leq 1$

But $|x - z| = 1.3 > 1$

So, (b) is correct.

Concept of symmetric relation is used in this question.

4 JEE Main 2021 (Online) 16th March Evening Shift

MCQ (Single Correct Answer)

Let $A = \{2, 3, 4, 5, ....., 30\}$ and '$\simeq$' be an equivalence relation on $A \times A$, defined by $(a, b) \simeq (c, d)$, if and only if $ad = bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair $(4, 3)$ is equal to :

A 5

B 6

C 8

D 7

## Explanation

$ad = bc$

$(a, b)\ R\ (4, 3) \Rightarrow 3a = 4b$

$a = \frac{4}{3}b$

b must be multiple of 3

$b = \{3, 6, 9 \ ..... \ 30\}$

$(a, b) = \{(4, 3), (8, 16), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)\}$

$\Rightarrow$ 7 ordered pair

1 JEE Main 2020 (Online) 3rd September Evening Slot

MCQ (Single Correct Answer)

Let $R_1$ and $R_2$ be two relation defined as follows :

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$ and

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$,

where Q is the set of all rational numbers. Then :

A Neither $R_1$ nor $R_2$ is transitive.

B $R_2$ is transitive but $R_1$ is not transitive.

C $R_1$ and $R_2$ are both transitive.

D $R_1$ is transitive but $R_2$ is not transitive.

## Explanation

For $R_1$ :

Let $a = 1 + \sqrt{2}$, $b = 1 - \sqrt{2}$, $c = 8^{\frac{1}{4}}$

$aR_1b$ : $a^2 + b^2 = 6 \in Q$

$bR_1c$ : $b^2 + c^2 = 3 - 2\sqrt{2} + 2\sqrt{2} = 3 \in Q$

$aR_1c$ : $a^2 + c^2 = 3 + 2\sqrt{2} + 2\sqrt{2} \notin Q$

$\therefore R_1$ is not transitive.

For $R_2$ :

Let $a = 1 + \sqrt{2}$, $b = \sqrt{2}$, $c = 1 - \sqrt{2}$

$aR_2b$ : $a^2 + b^2 = 5 + 2\sqrt{2} \notin Q$

$bR_2c$ : $b^2 + c^2 = 5 - 2\sqrt{2} \notin Q$

$aR_2c$ : $a^2 + c^2 = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6 \in Q$

$\therefore R_2$ is not transitive.

Again diferent types of relations definition is used to solve this question.

1 JEE Main 2020 (Online) 2nd September Morning Slot

MCQ (Single Correct Answer)

If R = $\{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of $R^{-1}$ is :

A $\{0, 1\}$

B $\{-2, -1, 1, 2\}$

C $\{-1, 0, 1\}$

D $\{-2, -1, 0, 1, 2\}$

## Explanation

Given R = $\{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$

So R = $\{(0,1), (0,-1), (1,0), (-1,0), (1,1), (1,-1)$
$(-1,1), (-1,-1), (2,0), (-2,0), (-2,0), (2,1), (2,-1), (-2,1), (-2,-1)\}$

$\Rightarrow$ R : $\{ -2, -1, 0, 1, 2\} \rightarrow \{-1, 0, 1\}$

$\therefore R^{-1}$ : $\{-1, 0, 1\} \rightarrow \{ -2, -1, 0, 1, 2\}$

$\therefore$ Domain of $R^{-1} = \{-1, 0, 1\}$

4 JEE Main 2018 (Online) 16th April Morning Slot
MCQ (Single Correct Answer)

Let N denote the set of all natural numbers. Define two binary relations on N as $R = \{(x, y) \in N \times N : 2x + y = 10\}$ and $R_2 = \{(x, y) \in N \times N : x + 2y = 10\}$. Then :

A Range of $R_1$ is {2, 4, 8}.

B Range of $R_2$ is {1, 2, 3, 4}.

C Both $R_1$ and $R_2$ are symmetric relations.

D Both $R_1$ and $R_2$ are transitive relations.

## Explanation

For $R_1$; $2x + y = 10$ and $x, y \in N$ possible values for x and y are :

$x = 1, y = 8$ i.e. (1, 8);

$x = 2, y = 6$ i.e (2, 6);

$x = 3, y = 4$ i.e (3, 4);

$x = 4, y = 2$ i.e (4, 2)

$\therefore R_1 = \{ (1, 8), (2, 6), (3, 4), (4, 2) \}$

$\therefore$ Range of $R_1$ is {2, 4, 6, 8}

$R_1$ is not symmetric.

$R_1$ is not transitive also as

$(3, 4), (4, 2) \in R$ , but $(3, 2) \notin R_1$

For $R_2$ : $x + 2y = 10$ and $x, y \in N$

Possible values of x, and y are :

$x = 8, y= 1$ i.e (8, 1)

$x = 6, y = 2$ i.e (6, 2)

$x = 4, y = 3$ i.e (4, 3) and

$x = 2, y = 4$ i.e (2, 4)

$\therefore R_2 = \{(8, 1) (6, 2) (4, 3) (2, 4)\}$

$\therefore$ Range of $R_2 = \{1,2,3,4\}$

$R_2$ is not symmetric and transitive

Past year questions form Sets and Cartesian Products

3 JEE Main 2021 (Online) 26th August Morning Shift
MCQ (Single Correct Answer)

Out of all patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set :

A $\{80, 83, 86, 89\}$

B $\{84, 86, 88, 90\}$

C $\{79, 81, 83, 85\}$

D $\{84, 87, 90, 93\}$

## Explanation

$n(A \cup B) \geq n(A) + n(B) - n(A \cap B)$

$100 \geq 89 + 98 - n(A \cup B)$

$n(A \cap B) \geq 87$

$87 \leq n(A \cap B) \leq 89$

Concepts used here are simple set theory results.

NOTE: Although there are almost no question specifically from cartisian products, but it is a concept that helps in more complex problems from relations and functions. So do learn it.

4 JEE Main 2021 (Online) 16th March Morning Shift

MCQ (Single Correct Answer)

The number of elements in the set $\{x \in R : (|x| - 3)\,|x + 4| = 6\}$ is equal to :

A 4

B 2

C 3

D 1

## Explanation

Case 1 :

$x \leq -4$

$(-x - 3)(-x - 4) = 6$

$\Rightarrow (x + 3)(x + 4) = 6$

$\Rightarrow x^2 + 7x + 6 = 0$

$\Rightarrow x = -1$ or $-6$

but $x \leq -4$

$x = -6$

Case 2 :

$x \in (-4, 0)$

$(-x - 3)(x + 4) = 6$

$\Rightarrow -x^2 - 7x - 12 - 6 = 0$

$\Rightarrow x^2 + 7x + 18 = 0$

$D < 0$ No solution

Case 3 :

$x \geq 0$

$(x - 3)(x + 4) = 6$

$\Rightarrow x^2 + x - 12 - 6 = 0$

$\Rightarrow x^2 + x - 18 = 0$

$x = \frac{-1 \pm \sqrt{1 + 72}}{2}$

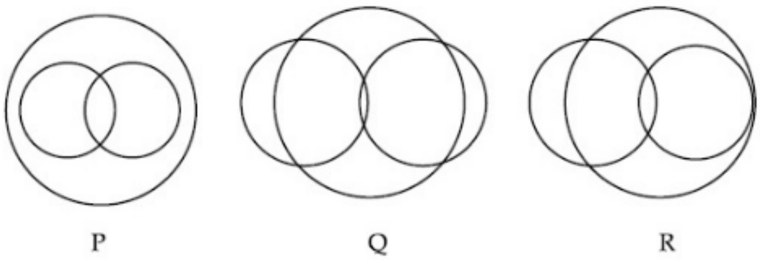
$\therefore x = \frac{\sqrt{73} - 1}{2}$ only

This problem does seem lengthy but the concept used here is very basic.
It is just an repetative application quadratic equations and set theory.

2 JEE Main 2021 (Online) 17th March Morning Shift

MCQ (Single Correct Answer)

In a school, there are three types of games to be played. Some of the students play two types of games, but none play all the three games. Which Venn diagrams can justify the above statement?




A Q and R

B None of these

C P and R

D P and Q

## Explanation

As none play all three games the intersection of all three circles must be zero.

Hence none of P, Q, R justify the given statement

It is an application of Venn diagrams concept from set theory.

1 JEE Main 2021 (Online) 31st August Evening Shift

Numerical

The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is ____________.

## Answer

Correct Answer is **5143**

## Explanation

A = 4-digit numbers divisible by 3

A = 1002, 1005, ....., 9999.

$9999 = 1002 + (n - 1)3$

$\Rightarrow (n - 1)3 = 8997 \Rightarrow n = 3000$

B = 4-digit numbers divisible by 7

B = 1001, 1008, ......., 9996

$\Rightarrow 9996 = 1001 + (n - 1)7$

$\Rightarrow n = 1286$

$A \cap B = 1008, 1029, \ldots\ldots, 9996$

$9996 = 1008 + (n - 1)21$

$\Rightarrow n = 429$

So, no divisible by either 3 or 7

$= 3000 + 1286 - 429 = 3857$

total 4-digits numbers = 9000

required numbers = $9000 - 3857 = 5143$

2 JEE Main 2020 (Online) 5th September Morning Slot

MCQ (Single Correct Answer)

A survey shows that 73% of the persons working in an office like coffee, whereas 65% like tea. If x denotes the percentage of them, who like both coffee and tea, then x cannot be :

A 63

B 36

C 54

D 38

## Explanation

$C \rightarrow$ person like coffee

$T \rightarrow$ person like Tea

$n(C) = 73$

$n(T) = 65$

$n(C \cup T) \leq 100$

$n(C) + n(T) - n(C \cap T) \leq 100$

$73 + 65 - x \leq 100$

$x \geq 38$

$73 - x \geq 0 \Rightarrow x \leq 73$

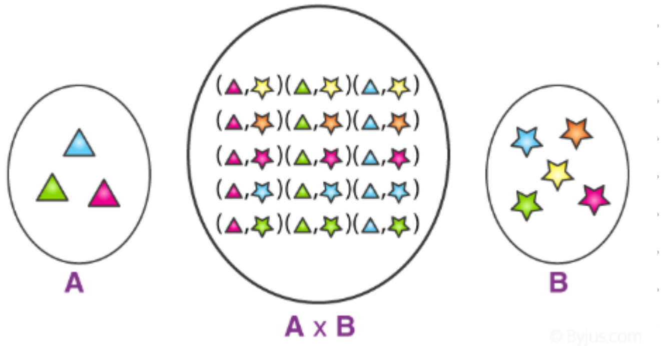
$65 - x \geq 0 \Rightarrow x \leq 65$

$\therefore 38 \leq x \leq 65$

Once again basic set theory concepts are used here.

Tips and Tricks

Visual Tip: If you really don't understand cartesian product then this might help.




The Cartesian product of given sets A and B is given as a combination of distinct colours of triangles and stars. Thus, a total of 15(=3 x 5) pairs are formed in A × B from the given sets.

Formula Tip-1: To quickly get number of elements in the final cartesian product.

If there are m elements in A and n elements in B, then there will be mn elements in A × B. That means:

$n(A) = m$ and $n(B) = n \rightarrow \boxed{n(A \times B) = mn}$

Formula Tip-2: Cartesian product with intersection or union of sets

Let A, B and C be three non-empty sets, then,

- $A \times (B \cap C) = (A \times B) \cap (A \times C)$
- $A \times (B \cup C) = (A \times B) \cup (A \times C)$
- $(A \cap B) \times C = (A \times C) \cap (B \times C)$
- $(A \cup B) \times C = (A \times C) \cup (B \times C)$

In JEE Exams and NCERT, questions have been asked that takes help of these formulas.

Bonus: As cartesian products use sets as their basic block. It is useful to know your basic results from Set chapter.

Tips and Tricks

**Formula Tip:** Total number of relations

The total number of relations that can be defined from a set A to a set B is the number of possible subsets of A × B.

n(A)=p and n(B)=q

$n(A \times B) = pq$

total # of relations $= 2^{pq}$

**Visual Tip:** In JEE Exams lots of questions are based on whether a relations is a function or not. To solve such questions, difference between them might help. It is given in the visual.