Questions

Quetion: 01

Let the number 2,b,c be in an A.P. and

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$. If det(A) ∈ [2, 16], then c lies in the interval :

A. [2, 3)

B. [4, 6]

C. $(2 + 2^{3/4}, 4)$

D. $[3, 2 + 2^{3/4}]$

Solutions

Solution: 01

Explanation

2, b, c are in AP.

Let common difference = d

∴ b = 2 + d and c = 2 + 2d

$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{bmatrix}$

$C_2 = C_2 - C_1$

$C_3 = C_3 - C_1$

$= \begin{vmatrix} 1 & 0 & 0 \\ 2 & b-2 & c-2 \\ 4 & b^2-4 & c^2-4 \end{vmatrix}$

$= (b-2)(c-2) \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 4 & b+2 & c+2 \end{vmatrix}$

$= (b-2)(c-2)[c+2-b-2]$

$= (b-2)(c-2)(c-b)$

[As b = 2 + d and c = 2 + 2d, then b - 2 = 4, c - 2 = 2d and c - b = d]

= (d) (2d) (d)

= $2d^3$

Given $|A| \in [2, 16]$

$\therefore 2d^3 \in [2, 16]$

$\Rightarrow d^3 \in [1, 8]$

$\Rightarrow d \in [1, 2]$

As c = 2 + 2d

then $c \in [4, 6]$