Practice Questions

Q1. Find the equation of the circle which touches x-axis and whose centre is (1, 2).

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Q2. True/False: The point (1, 2) lies inside the circle $x^2 + y^2 - 2x + 6y + 1 = 0$.

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Q3. Find the equation of the circle having (1, -2) as its centre and passing through

$$3x + y = 14, 2x + 5y = 18$$

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Q4. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinate of the other end of the diameter.

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Q5. True/False: The line $x + 3y = 0$ is a diameter of the circle $x^2 + y^2 + 6x + 2y = 0$.

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Solution and Hints

S1. Since the circle has a centre (1, 2) and also touches x-axis.
Radius of the circle is, $r = 2$
The equation of a circle having centre (h, k), having radius as r units, is

$$(x-h)^2 + (y-k)^2 = r^2$$

So, the equation of the required circle is:

$$(x-1)^2 + (y-2)^2 = 2^2$$

$$x^2 - 2x + 1 + y^2 - 4y + 4 = 4$$
$$x^2 + y^2 - 2x - 4y + 1 = 0$$

The equation of the circle is

$$x^2 + y^2 - 2x - 4y + 1 = 0$$

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S2. Recall from lec-2 that we just simply have to put substitute coordinates in the circle equation. So,

$$x^2 + y^2 - 2x + 6y + 1 = 1^2 + 2^2 - 2 + 6x2 + 1 = 16 > 0$$

Since value is positive, we can say that point is lies outside of the Circle. So it is a FALSE statement.

S3. First don't get stressed that you have solved this problem in less page, I have just tried to give a detailed step wise solution; that is why it seems too lengthy. Solving the given equations,

$$3x + y = 14$$
$$2x + 5y = 18$$

Multiplying the first equation by 5, we get

$$15x + 5y = 70$$
$$2x + 5y = 18$$

Subtract equations, we get $13x = 52$. Therefore $x = 4$
Substituting $x = 4$, in first equation, we get

$$3(4) + y = 14$$

$$y = 14 - 12 = 2$$

So, the point of intersection is (4, 2).
Since, the equation of a circle having centre (h, k), having radius as r units, is

$$(x-h)^2 + (y-k)^2 = r^2$$

Putting the values of (4, 2) and centre co-ordinates (1,-2) in the above expression, we get

$$(4-1)^2 + (2-(-2))^2 = r^2$$

$$3^2 + 4^2 = r^2$$

$$r^2 = 9 + 16 = 25$$

$$r = 5$$

units

So, the expression is

$$(x-1)^2 + (y-(-2))^2 = 5^2$$

Expanding the above equation we get genreal form of circle

$$x^2 - 2x + 1 + (y+2)^2 = 25$$

$$x^2 - 2x + 1 + y^2 + 4y + 4 = 25$$

$$x^2 - 2x + y^2 + 4y - 20 = 0$$

Hence the required expression is

$$x^2 - 2x + y^2 + 4y - 20 = 0.$$

S4. Given equation of the circle, we first convert it into center-radius form to get center of the circle, or one can simply use center result from genreal form of circle.

$$x^2 - 4x + y^2 - 6y + 11 = 0$$

$$x^2 - 4x + 4 + y^2 - 6y + 9 + 11 - 13 = 0$$

the above equation can be written as

$$x^2 - 2(2)x + 2^2 + y^2 - 2(3)y + 3^2 + 11 - 13 = 0$$

on simplifying we get

$$(x-2)^2 + (y-3)^2 = 2$$

$$(x-2)^2 + (y-3)^2 = (\sqrt{2})^2$$

Since, the equation of a circle having centre (h, k), having radius as r units, is

$$(x-h)^2 + (y-k)^2 = r^2$$

We have centre = (2, 3)

The centre point is the mid-point of the two ends of the diameter of a circle.

Let the points be (p, q). So,

$$\frac{p+3}{2} = 2$$
$$\frac{q+4}{2} = 3$$

by solveing above we get, $p = 1$ and $q = 2$

Hence, the other ends of the diameter are (1, 2).

S5. For given line to be a diameter of the circle, it has to have intersection on two points and those points should have distance equal to $2r$. Recall from notes that to have two intersection point, quadratic equation in one variable has to have two real roots. Put $x = -3y$ in circle equation,

$$\begin{aligned} 9y^2 + y^2 - 18y + 2y &= 0 \\ 10y^2 - 16y &= 0 \\ y(y - \frac{8}{5}) &= 0 \end{aligned}$$

Put values of y in $x = -3y$, and we get two intersection points as: $(0, 0)$ and $(\frac{-24}{5}, \frac{8}{5})$. And distance between them,

$$2r = \sqrt{\frac{(24)^2 + (8)^2}{5^2}} = \sqrt{25.6} = 5.05$$

And from genreal form of circle we get $2r = 2\sqrt{g^2 + f^2 - c} = 2\sqrt{10}2x3.16 = 6.32$. It is clear that intersection points of line with circle makes a secant rather than a diameter. So it a FALSE statement.