Questions

Quetion: 01

Let $\vec{a}, \vec{b}, \vec{c}$ three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies.

$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0}$, then $\vec{r}$ is equal to :

A. $\frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$

B. $\frac{1}{3}(2\vec{a} + \vec{b} - \vec{c})$

C. $\frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$

D. $\frac{1}{2}(\vec{a} + \vec{b} + 2\vec{c})$

Solutions

Solution: 01

Explanation

Suppose $\vec{r} = x\vec{a} + y\vec{b} + 2\vec{c}$

and $|\vec{a}| = |\vec{b}| = |\vec{c}| = k$

$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0}$

$\Rightarrow k^2(\vec{r} - \vec{b}) - k^2 x\vec{a} + k^2(\vec{r} - \vec{c}) - k^2 y\vec{b} + k^2(\vec{r} - \vec{a}) - k^2 z\vec{c} = \vec{0}$

$\Rightarrow 3\vec{r} - (\vec{a} + \vec{b} + \vec{c}) - \vec{r} = \vec{0}$

$\Rightarrow \vec{r} = \frac{\vec{a} + \vec{b} + \vec{c}}{2}$