Concepts and Formulas

Properties of Binomial Coefficients:

1. General Term: (r+1)th term of the binomial expansion is called general term.

$$T_{r+1} = {}^{n}C_{r}\, a^{n-r} b^{r}$$

2. Middle Term: in Binomial expansion of $(a+b)^n$

Case-1: **n is even**

middle term is: $\left(\frac{n+1+1}{2}\right)^{th} = \left(\frac{n}{2}+1\right)^{th}$

Case-2: **n is odd**

middle terms are: $\left(\frac{n+1}{2}\right)^{th}$ **and** $\left(\frac{n+1}{2}+1\right)^{th}$

3. Constant Term: also called term that is independent of x in expansion of

$$\left(x+\frac{1}{x}\right)^{2n}$$

In this case middle term is (n+1)th term as 2n is even.

$$^{2n}C_{n} x^{n} \left(\frac{1}{x}\right)^{n} = {}^{2n}C_{n} \text{ (constant)}$$

4. Important results:

1. $C_0 + C_1 + C_2 + \ldots + C_n = 2^n$
2. $C_0 + C_2 + C_4 + \ldots = C_1 + C_3 + C_5 + \ldots = 2^{n-1}$
3. $C_0 - C_1 + C_2 - C_3 + \ldots + (-1)^n \cdot nC_n = 0$
4. $nC_1 + 2.nC_2 + 3.nC_3 + \ldots + n.nC_n = n.2^{n-1}$
5. $C_1 - 2C_2 + 3C_3 - 4C_4 + \ldots + (-1)^{n-1} C_n = 0$ for $n > 1$
6. $C_0^2 + C_1^2 + C_2^2 + \ldots C_n^2 = [(2n)!/ (n!)^2]$