Exemplar Problem

Trigonometric Functions

22. Find the value of the expression

$$3\left[\sin^4\left(\frac{3\pi}{2}-\alpha\right)+\sin^4\left(3\pi+\alpha\right)\right]-2\left[\sin^6\left(\frac{\pi}{2}+\alpha\right)+\sin^6\left(5\pi-\alpha\right)\right]$$

Solution:

According to the question,

Let,
$$y = 3[\sin^4 (3\pi/2 - \alpha) + \sin^4 (3\pi + \alpha)] - 2[\sin^6 (\pi/2 + \alpha) + \sin^6 (5\pi - \alpha)]$$

We know that,

$$\sin(3\pi/2 - \alpha) = -\cos \alpha$$

$$sin(3\pi + \alpha) = -sin \alpha$$

$$sin(\pi/2 + \alpha) = cos \alpha$$

$$sin(5\pi - \alpha) = sin \alpha$$

Therefore,

$$y = 3[(-\cos \alpha)^4 + (-\sin \alpha)^4] - 2[\cos^6 \alpha + \sin^6 \alpha]$$

$$\Rightarrow$$
 y = 3 [cos 4 α + sin 4 α] - 2[sin 6 α + cos 6 α]

$$\Rightarrow$$
 y = 3[(sin 2 a + cos 2 a) 2 - 2sin 2 a cos 2 al - 2[(sin 2 a) 3 + (cos 2 a) 3]

Since, we know that,

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

Also, we know that,

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$\Rightarrow$$
 y = 3[1 - 2sin 2 a cos 2 a] - 2[(sin 2 a + cos 2 a)(cos 4 a + sin 4 a - sin 2 a cos 2 a)]

$$\Rightarrow$$
 y = 3[1 - 2sin 2 a cos 2 a] - 2[cos 4 a + sin 4 a- sin 2 a cos 2 a]

$$\Rightarrow$$
 y = 3[1 - 2sin 2 a cos 2 al - 2[(sin 2 a + cos 2 a) 2 - 2sin 2 a cos 2 a - sin 2 a cos 2 al

$$\Rightarrow$$
 y = 3[1 - 2sin 2 a cos 2 a] - 2[1 - 3sin 2 a cos 2 a]

$$\Rightarrow$$
 y = 3 - 6sin² a cos² a - 2 + 6 sin² a cos² a