

## Practice Questions

Q1.

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19. Using matrix method, solve the system of equations

$$3x + 2y - 2z = 3, x + 2y + 3z = 6, 2x - y + z = 2.$$

Sol. Given system of equations is:

$$3x + 2y - 2z = 3$$

$$x + 2y + 3z = 6$$

and  $2x - y + z = 2$

or  $AX = B$

i.e. 
$$\begin{bmatrix} 3 & 2 & -2 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$$

$$\therefore X = A^{-1}B$$

For  $A^{-1}$ ,

Cofactors are

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$$A_{11} = 5, A_{12} = 5, A_{13} = -5,$$

$$A_{21} = 0, A_{22} = 7, A_{23} = 7,$$

$$A_{31} = 10, A_{32} = -11 \text{ and } A_{33} = 4$$

$$\therefore \text{adj } A = \begin{bmatrix} 5 & 5 & -5 \\ 0 & 7 & 7 \\ 10 & -11 & 4 \end{bmatrix}^T = \begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}$$

$$|A| = 3(5) + 2(5) + (-2)(-5) = 35$$

$$\therefore A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{35} \begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}$$

$$\text{Now } X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{35} \begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix} = \frac{1}{35} \begin{bmatrix} 15 + 20 \\ 15 + 42 - 22 \\ -15 + 42 + 8 \end{bmatrix} = \frac{1}{35} \begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore x = 1, y = 1 \text{ and } z = 1$$

Q2.

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18. If  $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ , find  $A^{-1}$ .

Using  $A^{-1}$ , solve the system of linear equations  $x - 2y = 10$ ,  $2x - y - z = 8$ ,  $-2y + z = 7$ .

Sol. We have,  $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$  (i)

Cofactors are:

$$A_{11} = -3, A_{12} = 2, A_{13} = 2,$$

$$A_{21} = -2, A_{22} = 1, A_{23} = 1,$$

$$A_{31} = -4, A_{32} = 2, A_{33} = 3$$

$$\therefore \text{adj}A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$|A| = 1(-3) - 2(-2) + 0 = 1$$

$$\therefore A^{-1} = \frac{\text{adj}A}{|A|} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Now the system of linear equations is

$$x - 2y = 10,$$

$$2x - y - z = 8$$

and  $-2y + z = 7$

or  $AX = B$

$$\text{i.e., } \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\text{where, } A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\therefore X = A^{-1} B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -30 + 16 + 14 \\ -20 + 8 + 7 \\ -40 + 16 + 21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$$\therefore x = 0, y = -5 \text{ and } z = -3$$

Q3.

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20. Given  $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ , then find  $BA$  and use this to

solve the system of equations  $y + 2z = 7$ ,  $x - y = 3$ ,  $2x + 3y + 4z = 17$ .

Sol. We have,  $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$$\therefore BA = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$

$$\therefore B^{-1} = \frac{A}{6} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \quad (i)$$

Given system of equations is:

$$x - y = 3, 2x + 3y + 4z = 17 \text{ and } y + 2z = 7$$

$$\text{or } \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\begin{aligned} \therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} &= \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \\ &= \frac{1}{6} \begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} \end{aligned}$$

$$\therefore x = 2, y = -1 \text{ and } z = 4$$