

## Practice Questions

**Q1.** Show that the point  $(x, y)$  given by  $x = \frac{2at}{1+t^2}$  and  $y = \frac{a(1-t^2)}{1+t^2}$  lies on a circle for all real values of  $t$  such that  $-1 < t < 1$  where  $a$  is any given real numbers.

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**Q2.** If the lines  $3x - 4y + 4 = 0$  and  $6x - 8y - 7 = 0$  are tangents to a circle, then find the radius of the circle.

Hint: Distance between given parallel lines gives the diameter of the circle

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**Q3.** Find the equation of a circle which touches both the axes and the line  $3x - 4y + 8 = 0$  and lies in the third quadrant.

Hint: Let  $a$  be the radius of the circle, then  $(-a, -a)$  will be centre and perpendicular distance from the centre to the given line gives the radius of the circle.

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**Q4.** If the line  $y = \sqrt{3}x + k$  touches the circle  $x^2 + y^2 = 16$ , then find the value of  $k$ .

Hint: Equate perpendicular distance from the centre of the circle to its radius.

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## Solution and Hints

**S1.** Given point is,

$$x = \frac{2at}{1+t^2}, y = \frac{a(1-t^2)}{1+t^2}$$

Put its coordinates in  $x^2 + y^2$  to see what it gives,

$$\begin{aligned} x^2 + y^2 &= \left( \frac{2at}{1+t^2} \right)^2 + \left( \frac{a(1-t^2)}{1+t^2} \right)^2 \\ &= \frac{4a^2t^2 + a^2(1-t^2)^2}{(1+t^2)^2} \\ &= \frac{a^2(1+t^4+2t^2)}{(1+t^2)^2} \\ &= \frac{a^2(1+t^2)^2}{(1+t^2)^2} \\ &= a^2 \end{aligned}$$

Now compare it with center-radius form  $(x-h)^2 + (y-k)^2 = r^2$  and we get,

$$\begin{aligned} \text{center} &= (0,0) \\ \text{radius} &= a \end{aligned}$$

**S2.** Just by looking at st. line equations we can see that they are — lines. They touch circle on ends of diameter line. So distance bwtween these tangents should give us diameter of the circle. Equate both lines coefficients,

$$\begin{aligned} 3x - 4y + 4 &= 0 \\ 3x - 4y - 3.5 &= 0 \end{aligned}$$

Recall from st. line chapter that distance bwtween two — lines is,

$$\text{dist} = \left| \frac{c-d}{\sqrt{a^2+b^2}} \right|$$

Compare given st. lines with  $ax + by + c = 0$  and  $ax + by + d = 0$  and we get,

$$\text{dist} = \left| \frac{4 - (-3.5)}{\sqrt{3^2 + (-4)^2}} \right|$$

By solving above, distance comes out = 1.5. Since this is the diameter of the circle. Radius is,

$$r = \frac{1.5}{2} = 0.75 \text{ units}$$

**S3.** Since circle touches both axes in 3rd quadrant, its center can be assumed to be  $(-a, -a)$ . We need to find **a**.

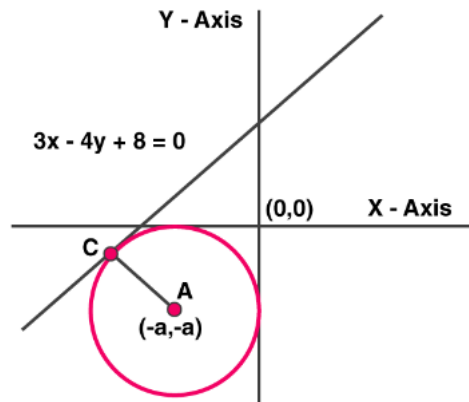


Figure 1: Circle in third quadrant

Also recall from st. lines chapter that perpendicular distance of st. line  $(ax + by + c = 0)$  from a point  $(x_1, y_1)$  is given by,

$$= \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

Since radius(=a) is the distance, we can compare both to get value of a.

$$\begin{aligned} a &= \left| \frac{3(-a) - 4(-a) + 8}{\sqrt{3^2 + (-4)^2}} \right| \\ &= \left| \frac{8 + a}{\sqrt{25}} \right| \\ &= \frac{8 + a}{5} \\ 5a &= a + 8 \\ a &= 2 \end{aligned}$$

Now put values in center radius form of circle,

$$(x - (-2))^2 + (y - (-2))^2 = a^2$$

General form after expanding is,

$$x^2 + y^2 + 4x + 4y + 4 = 0$$

**S4.** This problem uses similar concept to Prob-3. We need to compute tangent line's distance from center and equate it with known radius to get value of  $k$ . Comparing circle equation with center-radius form we get, center =  $(0,0)$  and radius = 4. After converting line equation in  $ax + by + c = 0$  form and using similar formula as prob-3,

$$\sqrt{3}x - y + k = 0$$

Compare distance from formula with radius,

$$\begin{aligned} 4 &= \left| \frac{\sqrt{3}(0) - 1(0) + k}{\sqrt{(\sqrt{3})^2 + (-1)^2}} \right| \\ 4 &= \frac{k}{\sqrt{4}} \\ k &= (4) \cdot (2) = 8 \end{aligned}$$

Hence the value of  $k = 8$ .