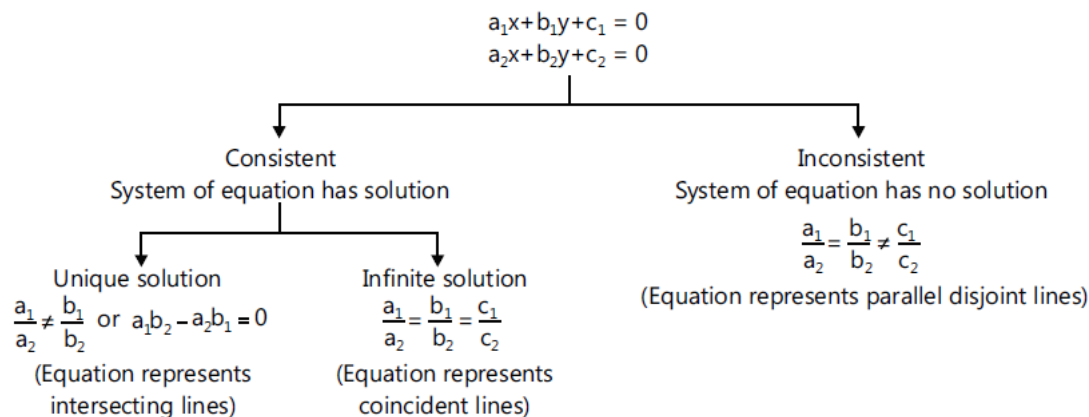


1 Consistency and Inconsistency

A system of linear equations having two and three variables can be easily solved using determinants. When system of equations has at least one solution it is called consistent, and if there are no solutions at all then it is called inconsistent. Below diagram clarifies,



2 Cramer's Rule

For 3 linear equations system,

$$\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_1 \\ a_3x + b_3y + c_3z &= d_1 \end{aligned}$$

Define following determinants,

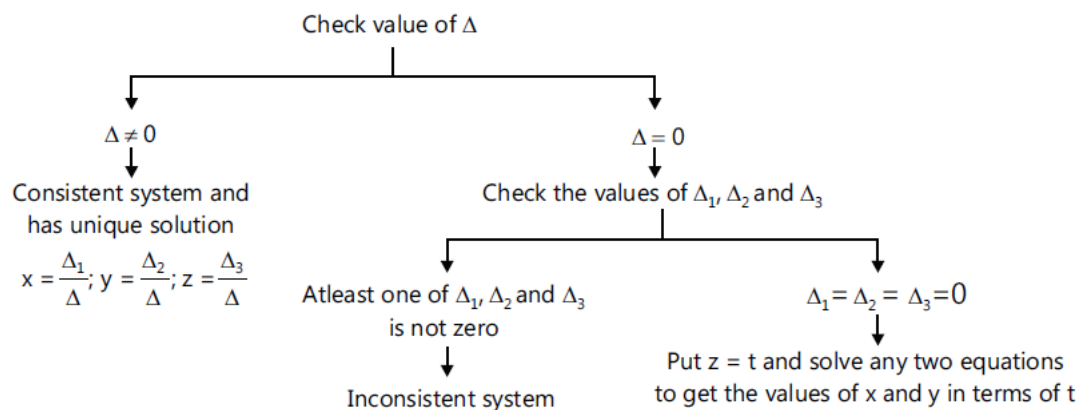
$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}, \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}, \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Now Cramer's rule gives solution when $\Delta \neq 0$

$$x = \frac{\Delta_1}{\Delta}, y = \frac{\Delta_2}{\Delta}, z = \frac{\Delta_3}{\Delta}$$

3 Conditions for Infinite and No Solutions



Points

1. If $\Delta \neq 0$ then the given system of equations are consistent and have unique solution.
2. If $\Delta = 0$ but at least one of $\Delta_1, \Delta_2, \Delta_3$ is not zero then the equations are inconsistent and have no solution.
3. If $\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 0$ then the given system of equations are consistent and have infinite solution except the case of parallel planes when there is no solution.

4 Homogeneous Linear Equations

If $d_1 = d_2 = d_3 = 0$, then system of linear equations is known as Homogeneous linear equations, which always possess at least one solution i.e. $(0, 0, 0)$. This is called a trivial solution for homogeneous linear equations.

1. Solution of Homogenous Equations is always consistent, as $x = 0 = y = z$ is always a solution. This is known as TRIVIAL solution.
2. For Homogenous Equations, if $\Delta \neq 0$. Then $x = 0 = y = z$ is only solution.
3. For Homogenous Equations, if $\Delta = 0$, then there exists non zero solutions (**NON TRIVIAL SOLUTIONS**) also.