## Exemplar Problem

**This problem gives nice step by step idea for solving NGT problems. Please try to do it by yourself before looking at the solution.**

**Example 11** Find numerically the greatest term in the expansion of $(2 + 3x)^9$, where $x = \frac{3}{2}$.

**Solution** We have $(2 + 3x)^9 = 2^9\left(1+\frac{3x}{2}\right)^9$

Now,
$$\frac{T_{r+1}}{T_r} = \frac{2^9\left[{}^9C_r\left(\frac{3x}{2}\right)^r\right]}{2^9\left[{}^9C_{r-1}\left(\frac{3x}{2}\right)^{r-1}\right]}$$

$$= \frac{{}^9C_r}{{}^9C_{r-1}}\left|\frac{3x}{2}\right| = \frac{\underline{|9}}{\underline{|r}\,\underline{|9-r}}\cdot\frac{\underline{|r-1}\,\underline{|10-r}}{\underline{|9}}\left|\frac{3x}{2}\right|$$

$$= \frac{10-r}{r}\left|\frac{3x}{2}\right| = \frac{10-r}{r}\left(\frac{9}{4}\right) \qquad \text{Since } x = \frac{3}{2}$$

Therefore,
$$\frac{T_{r+1}}{T_r} \geq 1 \Rightarrow \frac{90-9r}{4r} \geq 1$$

$$\Rightarrow 90 - 9r \geq 4r \qquad \text{(Why?)}$$

$$\Rightarrow r \leq \frac{90}{13}$$

$$\Rightarrow r \leq 6\,\frac{12}{13}$$

Thus the maximum value of $r$ is 6. Therefore, the greatest term is $T_{r+1} = T_7$.

Hence,
$$T_7 = 2^9\left[{}^9C_6\left(\frac{3x}{2}\right)^6\right], \qquad \text{where } x = \frac{3}{2}$$

$$= 2^9\cdot{}^9C_6\left(\frac{9}{4}\right)^6 = 2^9\cdot\frac{9\times8\times7}{3\times2\times1}\left(\frac{3^{12}}{2^{12}}\right) = \frac{7\times3^{13}}{2}$$

**Trick here is to just do exactly the thing that is being asked in question.**

**Example 15** If $a_1$, $a_2$, $a_3$ and $a_4$ are the coefficient of any four consecutive terms in the expansion of $(1 + x)^n$, prove that

$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$$

**Solution** Let $a_1$, $a_2$, $a_3$ and $a_4$ be the coefficient of four consecutive terms $T_{r+1}$, $T_{r+2}$, $T_{r+3}$, and $T_{r+4}$ respectively. Then

$$a_1 = \text{coefficient of } T_{r+1} = {}^nC_r$$

$$a_2 = \text{coefficient of } T_{r+2} = {}^nC_{r+1}$$

$$a_3 = \text{coefficient of } T_{r+3} = {}^nC_{r+2}$$

and

$$a_4 = \text{coefficient of } T_{r+4} = {}^nC_{r+3}$$

Thus

$$\frac{a_1}{a_1 + a_2} = \frac{{}^nC_r}{{}^nC_r + {}^nC_{r+1}}$$

$$= \frac{{}^nC_r}{{}^{n+1}C_{r+1}} \quad (\because \ {}^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1})$$

$$= \frac{\lfloor n}{\lfloor r \lfloor n-r} \times \frac{\lfloor r+1 \lfloor n-r}{\lfloor n+1} = \frac{r+1}{n+1}$$

Similarly,

$$\frac{a_3}{a_3 + a_4} = \frac{{}^nC_{r+2}}{{}^nC_{r+2} + {}^nC_{r+3}}$$

$$= \frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}} = \frac{r+3}{n+1}$$

Hence,

$$\text{L.H.S.} = \frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2r+4}{n+1}$$

and

$$\text{R.H.S.} = \frac{2a_2}{a_2 + a_3} = \frac{2\left({}^nC_{r+1}\right)}{{}^nC_{r+1} + {}^nC_{r+2}} = \frac{2\left({}^nC_{r+1}\right)}{{}^{n+1}C_{r+2}}$$

$$= 2\frac{\lfloor n}{\lfloor r+1 \lfloor n-r-1} \times \frac{\lfloor r+2 \lfloor n-r-1}{\lfloor n+1} = \frac{2(r+2)}{n+1}$$

**Two similar kind of problems based on the concept of independent term.**

Q1. Find the term independent of x, where x≠0, in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$

**Sol.** Given expansion is $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$

$$\therefore \quad T_{r+1} = {}^{15}C_r \left(\frac{3x^2}{2}\right)^{15-r} \left(-\frac{1}{3x}\right)^r$$

$$\text{or} \quad T_{r+1} = {}^{15}C_r (-1)^r 3^{15-2r} 2^{r-15} x^{30-3r} \qquad \text{(i)}$$

For the term independent of $x$, $30 - 3r = 0 \quad \Rightarrow r = 10$

$\therefore$ The term independent of $x$ is

$$T_{10+1} = {}^{15}C_{10}\, 3^{-5}\, 2^{-5} \qquad \text{(Putting } r = 10 \text{ in (i))}$$

$$= {}^{15}C_{10}\left(\frac{1}{6}\right)^5$$

**Q2. If the term free from x is the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then find the value of k.**

**Sol:** Given expansion is $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$

$$\therefore \quad T_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r} \left(\frac{-k}{x^2}\right)^r = {}^{10}C_r (x)^{\frac{1}{2}(10-r)} (-k)^r x^{-2r}$$

$$= {}^{10}C_r (x)^{5-\frac{r}{2}-2r} (-k)^r = {}^{10}C_r x^{\frac{10-5r}{2}} (-k)^r$$

For the term free from $x$, $\frac{10-5r}{2} = 0 \quad \Rightarrow \quad r = 2$

So, the term free from $x$ is $T_{2+1} = {}^{10}C_2 (-k)^2$.

$$\Rightarrow \quad {}^{10}C_2 (-k)^2 = 405$$

$$\Rightarrow \quad \frac{10 \times 9 \times 8!}{2! \times 8!} (-k)^2 = 405$$

$$\Rightarrow \quad 45k^2 = 405 \quad \Rightarrow \quad k^2 = 9 \quad \therefore k = \pm 3$$

Problem might seem a bit out of place by notation and all, but it is quite easy, give it a go.

**Q15. In the expansion of $(x + a)^n$, if the sum of odd term is denoted by 0 and the sum of even term by Then, prove that**

(i) $O^2 - E^2 = (x^2 - a^2)^n$

(ii) $4OE = (x + a)^{2n} - (x - a)^{2n}$

**Sol.** (i) We have $(x + a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a^1 + {}^nC_2x^{n-2}a^2 + {}^nC_3x^{n-3}a^3 + \dots + {}^nC_n a^n$

Sum of odd terms, $O = {}^nC_0x^n + {}^nC_2x^{n-2}a^2 + \dots$

And sum of even terms, $E = {}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + \dots$

Since $(x + a)^n = O + E$ (i)

$(x - a)^n = O - E$ (ii)

$\therefore$ $(O + E)(O - E) = (x + a)^n(x - a)^n$

$\Rightarrow$ $O^2 - E^2 = (x^2 - a^2)^n$

(ii) $4OE = (O + E)^2 - (O - E)^2 = [(x + a)^n]^2 - [(x - a)^n]^2 = (x + a)^{2n} - (x - a)^{2n}$

Finding middle term, or NGC.

**Q12. If p is a real number and the middle term in the expansion $\left(\frac{p}{2}+2\quad\right)^{o}$ is 1120, then find the value of p.**

**Sol.** Given expansion is $\left(\frac{p}{2}+2\right)^8$

Since index is $n = 8$, there is only one middle term, i.e., $\left(\frac{8}{2}+1\right)$th $= 5^{\text{th}}$ term

$$T_5 = T_{4+1} = {}^8C_4\left(\frac{p}{2}\right)^{8-4}\cdot 2^4$$

$\Rightarrow \quad 1120 = {}^8C_4\, p^4 \qquad \Rightarrow \quad 1120 = \dfrac{8\times7\times6\times5\times4!}{4!\times4\times3\times2\times1}p^4$

$\Rightarrow \quad 1120 = 7\times2\times5\times p^4 \qquad \Rightarrow \quad p^4 = \dfrac{1120}{70}$

$\Rightarrow \quad p^4 = 16 \qquad \Rightarrow \quad p^2 = 4 \quad \Rightarrow \quad p = \pm2$