11. Find the sum of the series $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots$ to

(i) n terms

(ii) 10 terms

Solution:

Given $(3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots$

Let the series be $S = (3^3 - 2^3) + (5^3 - 4^3) + (7^3 - 6^3) + \ldots$

i) Generalizing the series in terms of i

$$S = \sum_{i=1}^{n} [(2i+1)^3 - (2i)^3]$$

Using the formula $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$\Rightarrow S = \sum_{i=1}^{n} (2i + 1 - 2i)((2i+1)^2 + (2i+1)(2i) + (2i)^2)$$

$$\Rightarrow S = \sum_{i=1}^{n} (4i^2 + 4i + 1 + 4i^2 + 2i + 4i^2)$$

On simplifying and computing we get

$$\Rightarrow S = \sum_{i=1}^{n} (12i^2 + 6i + 1)$$

Now by splitting the summation we get

$$\Rightarrow S = 12\sum_{i=1}^{n} i^2 + 6\sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$$

We know that $\sum n^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum n = \frac{n(n+1)}{2}$

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Using the above formula we get

$$\Rightarrow S = 12\frac{n(n+1)(2n+1)}{6} + 6\frac{n(n+1)}{2} + n$$

Simplifying we get

$\Rightarrow S = 2n(n+1)(2n+1) + 3n(n+1) + n$

$\Rightarrow S = 2n(2n^2 + 2n + n + 1) + 3n^2 + 3n + n$

$\Rightarrow S = 4n^3 + 6n^2 + 2n + 3n^2 + 4n$

$\Rightarrow S = 4n^3 + 9n^2 + 6n$

Hence sum up to n terms is $4n^3 + 9n^2 + 6n$

ii) Sum of first 10 terms or up to 10 terms

To find sum up to 10 terms put n = 10 in S

$\Rightarrow S = 4(10)^3 + 9(10)^2 + 6(10)$

$\Rightarrow S = 4000 + 900 + 60$

$\Rightarrow S = 4960$

Hence sum of series up to 10 terms is 4960