Practice Questions

Q1.

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Example 1 If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$, then find x.

Solution We have $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & 5 \\ 8 & 3 \end{vmatrix}$. This gives

$$2x^2 - 40 = 18 - 40 \quad \Rightarrow x^2 = 9 \quad \Rightarrow \quad x = \pm 3.$$

Q2.

1. $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

Sol. We have, $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

[Applying $C_1 \to C_1 - C_2$]

$$= \begin{vmatrix} x^2-2x+2 & x-1 \\ 0 & x+1 \end{vmatrix}$$

$$= (x^2-2x+2)\cdot(x+1)-(x-1)\cdot 0$$

$$= x^3-2x^2+2x+x^2-2x+2 = x^3-x^2+2$$

Q3.

11. If the co-ordinates of the vertices of an equilateral triangle with sides of length 'a' are (x_1, y_1), (x_2, y_2), (x_3, y_3), then prove that $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \dfrac{3a^4}{4}$.

Sol. The area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Also, area of an equilateral triangle with side a is given by

$$\Delta = \frac{\sqrt{3}}{4}a^2$$

$$\therefore \qquad \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{\sqrt{3}}{4}a^2$$

Squaring both sides, we get

$$\Rightarrow \qquad \Delta^2 = \frac{1}{4}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3}{16}a^4$$

$$\text{or} \qquad \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3a^4}{4}$$

Q4.

15. Show that the points $(a + 5, a - 4)$, $(a - 2, a + 3)$ and (a, a) do not lie on a straight line for any value of a.

Sol. Given, the points are $(a + 5, a - 4)$, $(a - 2, a + 3)$ and (a, a).
We have to prove that these points do not lie on a straight line.
So, we have to prove that these points form a triangle.

$$\text{Area, } \Delta = \frac{1}{2}\begin{vmatrix} a+5 & a-4 & 1 \\ a-2 & a+3 & 1 \\ a & a & 1 \end{vmatrix}$$

[Applying $R_1 \to R_1 - R_3$ and $R_2 \to R_2 - R_3$]

$$= \frac{1}{2}\begin{vmatrix} 5 & -4 & 0 \\ -2 & 3 & 0 \\ a & a & 1 \end{vmatrix} = \frac{1}{2}[(1 \cdot (15 - 8)] = \frac{7}{2} \neq 0$$

Hence, given points from a triangle i.e., points do not lie on a straight line.

Q5.

24. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} \begin{vmatrix} 6 & 2 \\ 7 & 3 \end{vmatrix}$, then value of x is

(a) 3 (b) ± 3 (c) ± 6 (d) 6

Sol. **(c)** We have $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$

$\Rightarrow \quad 2x^2 - 40 = 18 + 14$

$\Rightarrow \quad 2x^2 = 72$

$\Rightarrow \quad x^2 = 36$

$\therefore \quad x = \pm 6$