

VALUES OF TRIGONOMETRIC RATIOS FOR DIFFERENT ANGLES

	15°	18°	$\left(22\frac{1}{2}\right)^\circ$	36°
Sin	$\frac{\sqrt{3}-1}{2\sqrt{2}}$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{2}\sqrt{2-\sqrt{2}}$	$\frac{\sqrt{10-2\sqrt{5}}}{4}$
Cos	$\frac{\sqrt{3}+1}{2\sqrt{2}}$	$\frac{\sqrt{10+2\sqrt{5}}}{4}$	$\frac{1}{2}\sqrt{2+\sqrt{2}}$	$\frac{\sqrt{5}+1}{4}$
tan	$2-\sqrt{3}$	$\frac{\sqrt{25-10\sqrt{5}}}{5}$	$\sqrt{2}-1$	$\sqrt{5-2\sqrt{5}}$

TRIGONOMETRIC IDENTITIES

$$\cot x - \tan x = 2 \cot 2x$$

$$\tan (A+B) = \tan A + \tan B + \tan A \cdot \tan B \cdot \tan(A+B)$$

$$\text{If } A+B = \frac{\pi}{4} \Rightarrow \tan A + \tan B + \tan A \cdot \tan B = 1$$

$$\cos \alpha + \cos \beta + \cos \gamma + \cos (\alpha + \beta + \gamma) = 4 \cos \left(\frac{\alpha + \beta}{2} \right) \cos \left(\frac{\beta + \gamma}{2} \right) \cos \left(\frac{\gamma + \alpha}{2} \right)$$

$$\sin \alpha + \sin \beta + \sin \gamma - \sin (\alpha + \beta + \gamma) = 4 \sin \left(\frac{\alpha + \beta}{2} \right) \sin \left(\frac{\beta + \gamma}{2} \right) \sin \left(\frac{\gamma + \alpha}{2} \right)$$

$$4 \sin x \sin(60-x) \sin(60+x) = \sin 3x$$

$$4 \cos x \cos(60-x) \cos(60+x) = \cos 3x$$

$$\tan x \tan(60-x) \tan(60+x) = \tan 3x$$

SUM OF SINE & COSINE SERIES

$$\sin \alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + \dots + \sin\{\alpha + (n-1)\beta\}$$

$$= \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \sin \left[\frac{\alpha + \{\alpha + (n-1)\beta\}}{2} \right]$$

$$\cos \alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \dots + \cos\{\alpha + (n-1)\beta\}$$

$$= \frac{\sin \frac{n\beta}{2}}{\sin \frac{\beta}{2}} \cos \left[\frac{\alpha + \{\alpha + (n-1)\beta\}}{2} \right]$$

$$\cos \theta \cdot \cos 2\theta \cdot \cos 2^2\theta \dots \cos 2^{n-1}\theta$$

$$= \prod_{r=1}^n \cos(2^{r-1}\theta) = \prod_{r=0}^{n-1} \cos 2^r\theta$$

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$$\frac{\sin 2^n \theta}{2^n \sin \theta}, \theta \neq k\pi$$

$$1, \theta = 2k\pi$$

$$-1, \theta = (2k+1)\pi$$

CONDITIONAL TRIGONOMETRIC IDENTITIES IN $\triangle ABC$

$$\sin^2 A + \sin^2 B + \sin^2 C = 4 \sin A \sin B \sin C$$

$$\cos^2 A + \cos^2 B + \cos^2 C = -1 - 4 \cos A \cos B \cos C$$

$$\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2 \cos A \cos B \cos C$$

$$\sin^2\left(\frac{A}{2}\right) + \sin^2\left(\frac{B}{2}\right) + \sin^2\left(\frac{C}{2}\right) = 1 - 2 \sin\left(\frac{A}{2}\right) \sin\left(\frac{B}{2}\right) \sin\left(\frac{C}{2}\right)$$

$$\tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

$$\tan \frac{A}{2} \tan \frac{B}{2} + \tan \frac{B}{2} \tan \frac{C}{2} + \tan \frac{C}{2} \tan \frac{A}{2} = 1$$

$$\cot\left(\frac{A}{2}\right) + \cot\left(\frac{B}{2}\right) + \cot\left(\frac{C}{2}\right) = \cot\left(\frac{A}{2}\right) \cot\left(\frac{B}{2}\right) \cot\left(\frac{C}{2}\right)$$

$$\sin A + \sin B + \sin C = 4 \cos\left(\frac{A}{2}\right) \cos\left(\frac{B}{2}\right) \cos\left(\frac{C}{2}\right)$$

$$\cos A + \cos B + \cos C = 1 + 4 \sin\left(\frac{A}{2}\right) \sin\left(\frac{B}{2}\right) \sin\left(\frac{C}{2}\right)$$

$$\sin^2 A + \sin^2 B + \sin^2 C = 2 (1 + \cos A \cos B \cos C)$$