Exemplar Problem

Matrix and Determinants

Example 14: The value of the determinant

$$\triangle = \begin{vmatrix} \sin^2 23^\circ & \sin^2 67^\circ & \cos 180^\circ \\ -\sin^2 67^\circ & -\sin^2 23^\circ & \cos^2 180^\circ \\ \cos 180^\circ & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix} = \dots$$

Ans: Here, we have
$$\triangle = \begin{vmatrix} sin^2 23^\circ & sin^2 67^\circ & cos 180^\circ \\ -sin^2 67^\circ & -sin^2 23^\circ & cos^2 180^\circ \\ cos 180^\circ & sin^2 23^\circ & sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \triangle = \begin{vmatrix} sin^2 23^\circ & sin^2 (90 - 23)^\circ & -1 \\ -sin^2 67^\circ & -sin^2 (90 - 67)^\circ & 1 \\ -1 & sin^2 23^\circ & sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \triangle = \begin{vmatrix} sin^2 23^\circ & cos^2 23^\circ & -1 \\ -sin^2 67^\circ & -cos^2 67^\circ & 1 \\ -1 & sin^2 23^\circ & sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \triangle = \begin{vmatrix} \sin^2 23^\circ & \sin^2 (90 - 23)^\circ & -1 \\ -\sin^2 67^\circ & -\sin^2 (90 - 67)^\circ & 1 \\ -1 & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \triangle = \begin{vmatrix} \sin^2 23^\circ & \cos^2 23^\circ & -1 \\ -\sin^2 67^\circ & -\cos^2 67^\circ & 1 \\ -1 & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

Applying
$$C_1 \to C_1 + C_2 + C_3$$

$$\Rightarrow \triangle = \begin{vmatrix} sin^2 23^\circ + cos^2 23^\circ - 1 & cos^2 23^\circ & -1 \\ -(sin^2 67^\circ + cos^2 67^\circ) + 1 & -cos^2 67^\circ & 1 \\ -1 + sin^2 23^\circ + sin^2 67^\circ & sin^2 23^\circ & sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \triangle = \begin{vmatrix} 1 - 1 & cos^2 23^\circ & -1 \\ -1 + 1 & -cos^2 67^\circ & 1 \\ -1 + 1 & sin^2 23^\circ & sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \triangle = \begin{vmatrix} 0 & cos^2 23^\circ & -1 \\ 0 & -cos^2 67^\circ & 1 \\ 0 & sin^2 23^\circ & sin^2 67^\circ \end{vmatrix}$$

$$\Rightarrow \triangle = 0 \text{ All the elements of a column is } 0$$

$$\Rightarrow \triangle = \begin{vmatrix} 1 - 1 & \cos^2 23^{\circ} & -1 \\ -1 + 1 & -\cos^2 67^{\circ} & 1 \\ -1 + 1 & \sin^2 23^{\circ} & \sin^2 67^{\circ} \end{vmatrix}$$

$$\Rightarrow \triangle = \begin{vmatrix} 0 & \cos^2 23^\circ & -1 \\ 0 & -\cos^2 67^\circ & 1 \\ 0 & \sin^2 23^\circ & \sin^2 67^\circ \end{vmatrix}$$

 $\Rightarrow \triangle = 0$ Alltheelements of a column is 0

Hence,
$$\triangle = \begin{vmatrix} sin^2 23^\circ & sin^2 67^\circ & cos 180^\circ \\ -sin^2 67^\circ & -sin^2 23^\circ & cos^2 180^\circ \\ cos 180^\circ & sin^2 23^\circ & sin^2 67^\circ \end{vmatrix} = 0.$$