

1 Position of a point wrt Circle

To know whether a point in 2-D plane (x_1, y_1) lies inside or outside of a circle, we can use general form of circle i.e. $x^2 + y^2 + 2gx + 2fy + c = 0$:

$$\text{if } x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c < 0 \implies \text{Inside}$$

$$\text{if } x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c > 0 \implies \text{Outside}$$

$$\text{if } x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0 \implies \text{On Circle}$$

Note that we just have to put value of (x_1, y_1) and by its sign we know whether point is inside or outside or on circle.

2 Equation of Circle

For equation of a circle that passes through 3 non-collinear points, simply put values of points in general form of the circle. Suppose that points are (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) . Since these points lie on the circle. So,

$$x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c = 0$$

$$x_2^2 + y_2^2 + 2gx_2 + 2fy_2 + c = 0$$

$$x_3^2 + y_3^2 + 2gx_3 + 2fy_3 + c = 0$$

There are three equations and three unknowns. Simple algebra gives us values of g, f , and c . Values are,

$$2g = \frac{[(x_1^2 - x_3^2)(y_1 - y_2) + (y_1^2 - y_3^2)(y_1 - y_2) + (x_2^2 - x_1^2)(y_1 - y_3) + (y_2^2 - y_1^2)(y_1 - y_3)]}{[(x_3 - x_1)(y_1 - y_2) - (x_2 - x_1)(y_1 - y_3)]}$$

$$2f = \frac{[(x_1^2 - x_3^2)(x_1 - x_2) + (y_1^2 - y_3^2)(x_1 - x_2) + (x_2^2 - x_1^2)(x_1 - x_3) + (y_2^2 - y_1^2)(x_1 - x_3)]}{[(y_3 - y_1)(x_1 - x_2) - (y_2 - y_1)(x_1 - x_3)]}$$

To get c simply put values of g and f in,

$$c = -x_1^2 - y_1^2 - 2gx_1 - 2fy_1$$

With the help of g, f, c we can get center of circle i.e. $(-g, -f)$ and radius i.e. $\sqrt{g^2 + f^2 - c}$. Note that one can try to remember above formulas using the pattern that is in them.