

1. Injection Principle(IP):

Let there be two sets A and B such that they are finite, meaning they have finite elements. If there exists a one-to-one mapping f from A to B, then

$$n(A) \leq n(B)$$

2. Bijection Principle(BP):

Let there be two sets A and B such that they are finite, meaning they have finite elements. If there exists a bijection mapping f between A and B, then

$$n(A) = n(B)$$

3. Fundamental theorem of arithmetic:

A composite number is expressed in the form of the product of primes and this factorization is unique apart from the order in which the prime factor occurs.

statement: a composite number "a" can be expressed as, $a = p_1 p_2 p_3 \dots p_n$, where $p_1, p_2, p_3 \dots p_n$ are the prime factors of a written in ascending order i.e. $p_1 \leq p_2 \leq p_3 \dots \leq p_n$.

4. Now combine FTA and bijection principle to get no. of divisors of any number. See examples in video and problem pdfs.

5. The Inclusion-Exclusion Principle:

Suppose two tasks A and B can be performed simultaneously. Let $n(A)$ and $n(B)$ represent the number of ways of performing the tasks A and B independent of each other. The principle says:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

6. Occupancy Problems:

The occupancy problem in probability theory is about the problem of randomly assigning a set of balls into a group of cells.

Result-1: the number of ways of distributing r identical balls in n distinct boxes is given by:

$$\binom{n+r-1}{n-1} = \binom{n+r-1}{r}$$

Result-2: the number of ways of distributing r identical balls in n distinct boxes so that no boxes remains empty, is given by:

$$\binom{r-1}{n-1} = \binom{r-1}{r-n} \quad \text{Here } r \geq n$$

Result-3: the number of ways of distributing r distinct balls in n distinct boxes so that each box hold at least one ball, is given by:

$${}_n P_r \quad \text{here } r \leq n$$

Result-4: the number of ways of distributing r distinct balls in n distinct boxes so that any box hold any number of balls, is given by:

$$n^r$$

Result-5: the number of ways of distributing r distinct balls in n distinct boxes such that ordering of balls matters in each box, is given by:

$$\binom{n+r-1}{r} r!$$

Try to understand proof of these results from video lecture.

Tips and Tricks

Helpful Permutation and Combination difference with examples.

Description	Permutation	Combination
What is a	Number of Arrangement or Listing of objects	Number of Selections or Grouping of objects
Where to use	If the ordering of objects matters	If the ordering of objects does not matter
Representation	${}^n P_r$	${}^n C_r$
Examples		
In a game of cricket	The number of batting line up of 11 players out of the 15 players	The number of teams consisting of 11 players out of 15 players
In a process of prize distribution	The number of ways of distributing 3 distinct prizes	The number of ways of distributing 3 identical prizes

Relation between permutation and combination:

$${}^n P_r = {}^n C_r \times r!.$$

In permutation and combination it is important to understand basic type problems and then being able to apply them on new problems.

To be able to master this you should first try to understand basic problems taught in video lectures in depth. Then try to solve as many problems as possible, with more problems you will get better idea of concepts

As always try to practice timed tests.

Other related tips are given in next two pages.