4 JEE Main 2021 (Online) 20th July Evening Shift

MCQ (Single Correct Answer)

The value of k $\in R$, for which the following system of linear equations

$3x - y + 4z = 3,$

$x + 2y - 3z = -2$

$6x + 5y + kz = -3,$

has infinitely many solutions, is :

A 3

B -5

C 5

D -3

Explanation

$$\begin{vmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & k \end{vmatrix} = 0$$

$\Rightarrow 3(2k + 15) + K + 18 - 28 = 0$

$\Rightarrow 7k + 35 = 0$

$\Rightarrow k = -5$

2 JEE Main 2021 (Online) 26th February Evening Shift
MCQ (Single Correct Answer)

Consider the following system of equations :

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$,

where a, b and c are real constants. Then the system of equations :

A has no solution for all a, b and c

B has a unique solution when 5a = 2b + c

C has infinite number of solutions when 5a = 2b + c

D has a unique solution for all a, b and c

Explanation

$$D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix}$$

$= 20 - 2(25) - 3(-10)$

$= 20 - 50 + 30 = 0$

$$D_1 = \begin{vmatrix} a & 2 & -3 \\ b & 6 & -11 \\ c & -2 & 7 \end{vmatrix}$$

$= 20a - 2(7b + 11c) - 3(-2b - 6c)$

$= 20a - 14b - 22c + 6b + 18c$

$= 20a - 8b - 4c$

$= 4(5a - 2b - c)$

$$D_2 = \begin{vmatrix} 1 & a & -3 \\ 2 & b & -11 \\ 1 & c & 7 \end{vmatrix}$$

$= 7b + 11c - a(25) - 3(2c - b)$

$= 7b + 11c - 25a - 6c + 3b$

$= -25a + 10b + 5c$

$= -5(5a - 2b - c)$

$$D_3 = \begin{vmatrix} 1 & 2 & a \\ 2 & 6 & b \\ 1 & -2 & c \end{vmatrix}$$

$= 6c + 2b - 2(2c - b) - 10a$

$= -10a + 4b + 2c$

$= -2(5a - 2b - c)$

for infinite solution

$D = D_1 = D_2 = D_3 = 0$

$\Rightarrow 5a = 2b + c$

JEE Main 2021 (Online) 24th February Evening Slot

MCQ (Single Correct Answer)

For the system of linear equations:

$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in R$,

consider the following statements :

(A) The system has unique solution if $k \neq 2, k \neq -2$.

(B) The system has unique solution if k = -2

(C) The system has unique solution if k = 2

(D) The system has no solution if k = 2

(E) The system has infinite number of solutions if k $\neq$ -2.

Which of the following statements are correct?

A (B) and (E) only

B (C) and (D) only

C (A) and (E) only

D (A) and (D) only

Explanation

$x - 2y + 0.z = 1$

$x - y + kz = -2$

$0.x + ky + 4z = 6$

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$$

For unique solution $4 - k^2 \neq 0$

$\Rightarrow$ k $\neq$ $\pm$ 2

For k = 2 :

$x - 2y + 0.z = 1$

$x - y + 2z = -2$

$0.x + 2y + 4z = 6$

$$\Delta x = \begin{vmatrix} 1 & -2 & 0 \\ 2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = (-8) + 2[-20]$$

$\Delta x = -48 \neq 0$

For k = 2, $\Delta x \neq 0$

$\therefore$ For K = 2; The system has no solution.

4 JEE Main 2020 (Online) 6th September Evening Slot

MCQ (Single Correct Answer)

Let $\theta = \frac{\pi}{5}$ and $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$.

If B = A + A^4 , then det (B) :

A lies in (1, 2)

B lies in (2, 3).

C is zero.

D is one.

Explanation

$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

Similarly, $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$

$\therefore$ B = A + A^4

$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$

$= \begin{bmatrix} \cos 4\theta + \cos\theta & \sin 4\theta + \sin\theta \\ -\sin 4\theta - \sin\theta & \cos 4\theta + \cos\theta \end{bmatrix}$

detB = $(\cos 4\theta + \cos\theta)^2 + (\sin 4\theta + \sin\theta)^2$

$= \cos^2 4\theta + \cos^2\theta + 2\cos 4\theta \cos\theta$
$+ \sin^2 4\theta + \sin 2\theta + 2\sin 4\theta - \sin\theta$

$= 2 + 2 (\cos 4\theta \cos\theta + \sin 4\theta \sin\theta)$

$\Rightarrow$ detB $= 2 + 2 \cos 3\theta$

at $\theta = \frac{\pi}{5}$

detB $= 2 + 2\cos \frac{3\pi}{5}$

= 2(1 - sin18)

$= 2(1 - \frac{\sqrt{5}-1}{4})$

$= 2\left(\frac{5-\sqrt{5}}{4}\right)$

$= \frac{5-\sqrt{5}}{2} \simeq 1.385$

$\therefore$ detB $\in (1, 2)$

4 JEE Main 2020 (Online) 5th September Evening Slot

MCQ (Single Correct Answer)

If a + x = b + y = c + z + 1, where a, b, c, x, y, z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to :

A y(b – a)

B y(a – b)

C y(a – c)

D 0

Explanation

$$\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$$

$C_3 \to C_3 - C_1$

$$= \begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix}$$

$C_2 \to C_2 - C_3$

$$= \begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix}$$

$R_3 \to R_3 - R_1$, $R_2 \to R_2 - R_1$

$$= \begin{vmatrix} x & y & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix}$$

= (–y)[(y – x) (c – a) – (b – a) (z – x)]

Given, a + x = b + y = c + z + 1

= (–y)[(a – b) (c – a) + (a – b) (a – c – 1)]

= (–y)[(a – b) (c – a) + (a – b) (a – c) + b – a)

= –y(b – a) = y(a – b)

JEE Main 2021 (Online) 25th July Morning Shift

MCQ (Single Correct Answer)

The values of a and b, for which the system of equations

2x + 3y + 6z = 8

x + 2y + az = 5

3x + 5y + 9z = b

has no solution, are :

A a = 3, b $\neq$ 3

B a $\neq$ 3, b $\neq$ 13

C a $\neq$ 3, b = 3

D a = 3, b = 13

Explanation

$$D = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 3 - a$$

$$D = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = b - 13$$

If a = 3, b $\neq$ 13, no solution.