A.M. – G.M. – H.M. INEQUALITY

$x_1, x_2, x_3, \ldots\ldots\ldots, x_n \rightarrow$ All positive numbers

$$\text{Arithmetic Mean (A.M.)} = \frac{x_1 + x_2 + x_3 + \ldots\ldots\ldots + x_n}{n}$$

$$\text{Geometric Mean (G.M.)} = (x_1 \cdot x_2 \cdot x_3 \cdot \ldots\ldots\ldots \cdot x_n)^{1/n}$$

$$\text{Harmonic Mean (H.M.)} = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots\ldots\ldots + \frac{1}{x_n}}$$

$$\text{A.M.} \geq \text{G.M.} \geq \text{H.M.}$$

Equality holds if all numbers are equal i.e. A.M. = G.M. = H.M.

If at least two numbers are unequal then A.M. > G.M. > H.M.

ARITHMETIC MEANS

$a, A_1, A_2, A_3, \ldots\ldots\ldots, A_n, b \rightarrow$ in A.P.

$A_i \rightarrow$ Arithmetic Means

$\Rightarrow A_1+A_2+\ldots\ldots\ldots+A_n = \frac{a+b}{2} \times n$

$\Rightarrow A_K = a+\left(\frac{b-a}{n+1}\right) K$

GEOMETRIC MEANS

$a, G_1, G_2, G_3, \ldots\ldots, G_n, b \rightarrow$ in G.P.

$G_i \rightarrow$ Geometric Means

$\Rightarrow G_1 \cdot G_2 \cdot G_3 \ldots\ldots\ldots G_n = \left(\sqrt{ab}\right)^n$

$\Rightarrow G_K = a.\left(\frac{b}{a}\right)^{\frac{K}{n+1}}$ $\quad G_K = K^{th}$ Geometric Mean

n^{th} TERM AND SUM OF N TERMS OF A.P., G.P., H.P.

	Arithmetic Progression (A.P.)	Geometric Progression (G.P.)	Harmonic Progression (H.P.)
	$a, a + d, a + 2d, \ldots$	$a, ar, ar^2, \ldots$	$a, b, \ldots$
n^{th} term (t_n)	$a + (n - 1)d$	$a \cdot r^{n-1}$	$\frac{1}{\frac{1}{a} + (n-1)\left(\frac{1}{b} - \frac{1}{a}\right)}$
Sum of n terms (S_n)	$\frac{n}{2}[2a + (n-1)d]$ OR $\frac{n}{2}[a + t_n]$	$\frac{a(1 - r^n)}{(1 - r)}$ $(r \neq 1)$; $S_\infty = \frac{a}{1-r}$ $(\lvert r \rvert < 1)$	Formula does not exist

Recognition of A.P., G.P., H.P.

If a, b, c are three consecutive terms

$\frac{a-b}{b-c} = \frac{a}{a}$	$\frac{a-b}{b-c} = \frac{a}{b}$	$\frac{a-b}{b-c} = \frac{a}{c}$
a, b, c are in A.P.	a, b, c are in G.P.	a, b, c are in H.P.

Tips and Tricks

- Create a cheat sheet from concepts and formulas section of this chapter from website. Writing them down in your words will help in keeping things organized and easy to remember during an exam.

- Try to be an active reader and problem solver to grasp any concept of any subject. By active I mean consistency. Consistency is everything.

- I have given similar types of problems in problem sections, try to solve them in timed manner. Timing your problem solving can make your study more efficient.

- Solve of lots of problems from other reference books and coaching notes that are given below each video.

- When you solve questions try to not use cheat sheets at first, because you will not have them in exam. Try the question at least for 5-10 minutes (not in an exam) and still you don't get it, then can look through cheatsheet for hints.