

## 1 Intersection of Line and Circle

Suppose that a general line equation is  $y = mx + c$  and a circle with origin as its center is  $x^2 + y^2 = r^2$ . To know whether line intersects circle or not, simply put value of  $y$  from line in circle. This gives a simple quadratic equation in  $x$ . Now one can consider different cases of existence of roots of this quadratic equation to situation of Line and Circle.

$$x^2 + (mx + c)^2 = r^2$$

Three cases can be analysed,

**case-1:** If both roots are real and different then we have two intersection points.

**case-2:** If both roots are same and real then we have one intersection points and the line is a tangent of the circle.

**case-3:** If one root is imaginary then second one will be its complex-conjugate then we have no intersection points.

Try to visualize these scenarios.

## 2 Intercept made by Circle on Axes

Consider general form of a circle  $x^2 + y^2 + 2gx + 2fy + c = 0$ . To have a visual clarity of concept have a look at below diagram,

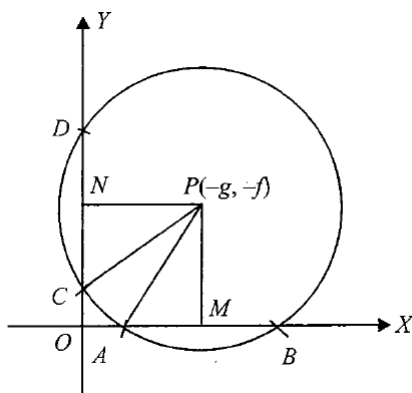


Figure 1: Intercept made by Circle on Axes

From diagram it is clear that,

$$\begin{aligned} PM &= |g| \\ PN &= |f| \\ AP = CP = r &= \sqrt{g^2 + f^2 - c} \end{aligned}$$

From triangles  $CPN$  and  $APM$  we have intercept terms as,

$$\begin{aligned} AB = 2AM &= 2\sqrt{f^2 - c} \\ CD = 2CN &= 2\sqrt{g^2 - c} \end{aligned}$$

Thing to note is that make a clear diagram of scenario that is under consideration and proceed from there.

**NOTE:**

1. Since intercepts are square roots, they are always positive.
2. When circle touches either axis, then that axis has intercept to be zero since  $f^2 = c$  or  $g^2 = c$ .
3. When circle touches both axis then both intercepts are zero since  $f^2 = c$  and  $g^2 = c$ .