Tips and Tricks:

Tip-1

Area of a triangle

If $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are the vertices of a triangle

then its area is:

Area of
$$\triangle$$
 ABC = $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

OR

Area of
$$\triangle$$
 ABC = $\frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$

Tip-2

One can try to remember some standard determinants to solve question faster that uses them. After solving many problems these things start to automatically register in mind.

TIP

PRACTICE. PRACTICE. PRACTICE

Bonus from Calculas: these topics are covered in calculas.

(A)

Differentiation of Determinants

$$\text{Let } \Delta(x) = \left| \begin{array}{ccc} f_1(x) & & g_1(x) \\ f_2(x) & & g_2(x) \end{array} \right|, \quad where \quad f_1(x), f_2(x), g_1(x) \quad and \quad g_2(x)$$

are functions of x. Then,

$$\Delta^{\text{\tiny \prime}}(x) = \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1{}^{\text{\tiny \prime}}(x) \\ f_2(x) & g_2(x) \end{array} \right| + \left| \begin{array}{cc} f_1(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2{}^{\text{\tiny \prime}}(x) \end{array} \right| \quad Also, \quad \Delta^{\text{\tiny \prime}}(x) = \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_2(x) \end{array} \right| \cdot \left| \begin{array}{cc} f_1{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny \prime}}(x) & g_1(x) \\ f_2{}^{\text{\tiny $$$

·(B)

Integration of Determinants

If f(x), g(x) and h(x) are functions of x and a, b, c, α , β and γ are constants such that

$$\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ \alpha & \beta & \gamma \end{vmatrix}$$

then the integral of the determinants is given by i.e.

$$\int \Delta(x)dx = \begin{vmatrix} \int f(x)dx & \int g(x)dx & \int h(x)dx \\ a & b & c \\ \alpha & \beta & \gamma \end{vmatrix}$$

IMPORTANT DETERMINANTS

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

$$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^4 & b^4 & c^4 \end{vmatrix} = (a-b)(b-c)(c-a)(a^2+b^2+c^2-ab-bc-ca)$$

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$$

$$\begin{vmatrix} a & bc & abc \\ b & ca & abc \\ c & ab & abc \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = abc(a-b)(b-c)(c-a)$$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -a^3 - b^3 - c^3 + 3abc$$