4 JEE Main 2021 (Online) 16th March Evening Shift

MCQ (Single Correct Answer)

Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to :

A $\sqrt{10}$

B $\sqrt{6}$

C $\sqrt{11}$

D $\sqrt{7}$

Explanation

$2\sqrt{\frac{a^2}{4} - c} = 2\sqrt{2}$

$\sqrt{a^2 - 4c} = 2\sqrt{2}$

$a^2 - 4c = 8$ .... (1)

$2\sqrt{a^2 - c} = 2\sqrt{5}$

$a^2 - c = 5$ .... (2)

$(2) - (1)$

$3c = -3a \Rightarrow c = -1$

$a^2 = 4 \Rightarrow a = -2$ (Given $a < 0$)

Equation of circle

$x^2 + y^2 - 2x - 4y - 1 = 0$

Equation of tangent which is perpendicular to the line $x + 2y = 0$ is

$2x - y + \lambda = 0$

$\therefore$ p = r

$\left|\frac{2 - 2 + \lambda}{\sqrt{5}}\right| = \sqrt{6}$

$\Rightarrow \lambda = \pm\sqrt{30}$

$\therefore$ Tangent $2x - y \pm \sqrt{30} = 0$

Distance from origin = $\frac{\sqrt{30}}{\sqrt{5}} = \sqrt{6}$

4 JEE Main 2020 (Online) 9th January Morning Slot

MCQ (Single Correct Answer)

A circle touches the y-axis at the point (0, 4) and passes through the point (2, 0). Which of the following lines is not a tangent to this circle ?

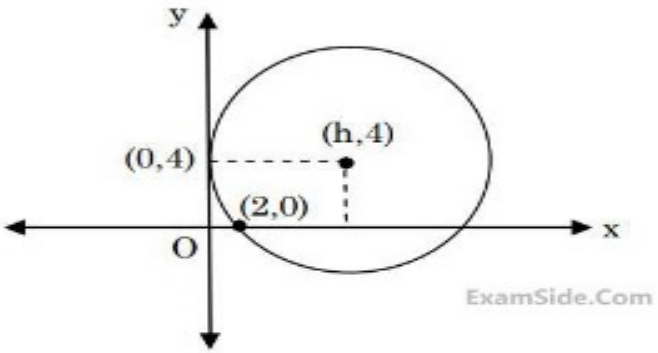
A $3x - 4y - 24 = 0$

B $4x + 3y - 8 = 0$

C $3x + 4y - 6 = 0$

D $4x - 3y + 17 = 0$

Explanation



Equation of family of circle touching y-axis at

$(0, 4)$ is given by $(x - 0)^2 + (y - 4)^2 + \lambda x = 0$.

$\because$ It passes through $(2, 0)$

$\Rightarrow \lambda = -10$.

$\Rightarrow$ Required circle is $(x - 0)^2 + (y - 4)^2 - 10x = 0$

$\Rightarrow x^2 + y^2 - 10x - 8y + 16 = 0$

$\therefore$ center of circle $(5, 4)$ and radius $= 5$

By checking all options you can see $4x + 3y - 8 = 0$ is not a tangent to the circle.

As distance of $4x + 3y - 8 = 0$ from $(5, 4)$

$= \left|\frac{24}{5}\right| \neq$ radius

3 JEE Main 2021 (Online) 17th March Morning Shift

MCQ (Single Correct Answer)

The line 2x − y + 1 = 0 is a tangent to the circle at the point (2, 5) and the centre of the circle lies on x − 2y = 4. Then, the radius of the circle is :

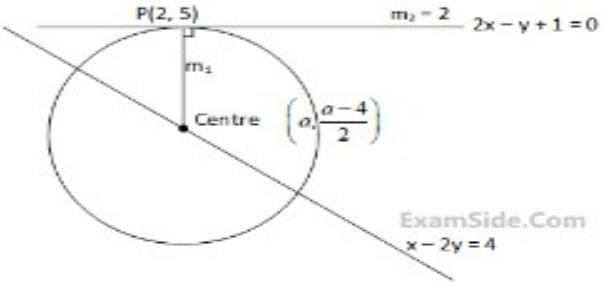
A $5\sqrt{3}$

B $4\sqrt{5}$

C $3\sqrt{5}$

D $5\sqrt{4}$

Explanation



$m_1 \times m_2 = -1$

$\frac{\frac{a-4}{2}-5}{a-2} \times 2 = -1$

$\frac{a-14}{a-2} = -1$

$a - 14 = 2 - a$

$2a = 16$

a = 8

∴ Centre (8, 2)

Radius = $\sqrt{36+9}$

$= \sqrt{45}$

$= 3\sqrt{5}$