Tips and Tricks:

Tip-1

As much as possible, avoid direct calculation of determinats using lengthy expansions. Be calm and try to use determinats properties to first simplify and then expand. Some

Tip-2

Minors and cofactors are important concept to understand inverse of a matrix in further lectures.

Note: (a) A determinant of order 3 will have 9 minors and each minor will be a determinant of order 2 and a determinant of order 4 will have 16 minors and each minor will be determinant of order 3.

(b) $a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0,$ i.e. cofactor multiplied to different row/column elements results in zero value.

Tip-3

Row and Column Operations of Determinants

(a) $R_i \leftrightarrow R_j \quad or \quad C_i \leftrightarrow C_j, \quad when \quad i \neq j$; This notation is used when we interchange ith row (or column) and jth row (or column).

(b) $R_i \leftrightarrow C_i$; This converts the row into the corresponding column.

(c) $R_i \to Rk_i \quad or \quad C_i \to kC_i; \ k \in R$; This represents multiplication of ith row (or column) by k.

(d) $R_i \to R_ik + R_j \quad or \quad Ci \to C_ik + C_j; (i \neq j)$; This symbol is used to multiply ith row (or column) by k and adding the jth row (or column) to it.

These operations are VERY USEFUL in simplifying complex determinants.

Tips and Tricks:

NOTE: After analysing many JEE Exam questions, I can surly say that determinants properties are used in every question that comes from this chapter. So go to NCERT book chapter and learn these properties. If you know these properties, then det. questions can be solved pretty fast; otherwise these can waste precious exam time.

Do remember these properties:

Important Properties of Determinants

1. Reflection Property:

The determinant remains unaltered if its rows are changed into columns and the columns into rows. This is known as the property of reflection.

2. All-zero Property:

If all the elements of a row (or column) are zero, then the determinant is zero.

3. Proportionality (Repetition) Property:

If the all elements of a row (or column) are proportional (identical) to the elements of some other row (or column), then the determinant is zero.

4. Switching Property:

The interchange of any two rows (or columns) of the determinant changes its sign.

5. Scalar Multiple Property:

If all the elements of a row (or column) of a determinant are multiplied by a non-zero constant, then the determinant gets multiplied by the same constant.

6. Sum Property:

$$\begin{vmatrix} a_1+b_1 & c_1 & d_1 \\ a_2+b_2 & c_2 & d_2 \\ a_3+b_3 & c_3 & d_3 \end{vmatrix} = \begin{vmatrix} a_1 & c_1 & d_1 \\ a_2 & c_2 & d_2 \\ a_3 & c_3 & d_3 \end{vmatrix} + \begin{vmatrix} b_1 & c_1 & d_1 \\ b_2 & c_2 & d_2 \\ b_3 & c_3 & d_3 \end{vmatrix}$$

7. Property of Invariance:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+\alpha b_1+\beta c_1 & b_1 & c_1 \\ a_2+\alpha b_2+\beta c_2 & b_2 & c_2 \\ a_3+\alpha b_3+\beta c_3 & b_3 & c_3 \end{vmatrix}$$

That is, a determinant remains unaltered under an operation of the form $C_i \to C_i + \alpha C_j + \beta C_k$, where $j, k \neq i$, or an operation of the form

$R_i \to R_i + \alpha R_j + \beta R_k$, where $j, k \neq i$

8. Factor Property:

If a determinant Δ becomes zero when we put $x = \alpha$, then $(x - \alpha)$ is a factor of Δ.

9. Triangle Property:

If all the elements of a determinant above or below the main diagonal consist of zeros, then the determinant is equal to the product of diagonal elements. That is,

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ 0 & b_2 & b_3 \\ 0 & 0 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 \\ a_2 & b_2 & 0 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 b_2 c_3$$

Tips and Tricks:

Tip-1

Inverse of a matrix using determinants:

The **product of a matrix A and its adjoint** is equal to unit matrix multiplied by the determinant A.

Let A be a square matrix, then (Adjoint A). A = A. (Adjoint A) = | A |. I

We know that, $A.(Adj\,A) = |A|I \quad or \quad \frac{A.(Adj\,A)}{|A|} = I \quad (Provided|A| \neq 0)$

And $A.A^{-1} = I;$ $\boxed{A^{-1} = \frac{1}{|A|}(Adj.A)}$

Illustration 2: If the product of a matrix A and $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ *is the matrix* $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$,

then A^{-1} is given by:

(a) $\begin{bmatrix} 0 & -1 \\ 2 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ -2 & -4 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$

(d) None of these

This is an example of above trick.

Solution:

(a) We know if AB = C, then $B^{-1}A^{-1} = C^{-1} \Rightarrow A^{-1} = BC^{-1}$ by using this formula

we will get value of A^{-1} in the above problem.

Here, $A\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$

Tip-2

Determinant of an Adjoint matrix

$det(AB) = det(A)det(B)$ → This is a useful property of det. worth remembering.

$$\boxed{det(Adj(A)) = det(A)^{(N-1)}}$$

A is N x N matrix

Tip-3

Theorem: Inverse of A exists If and only if det(A) is non zero.

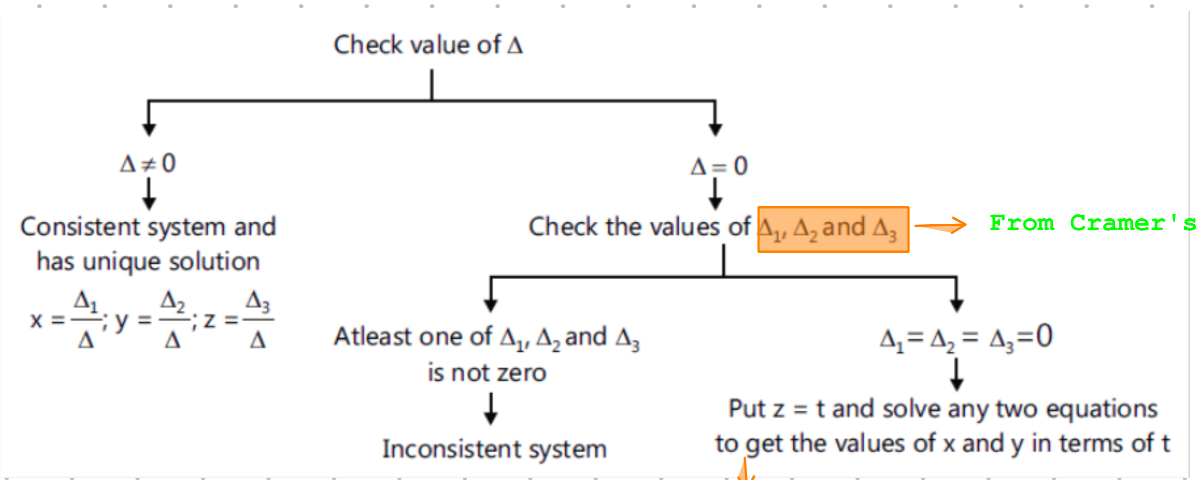
Tips and Tricks:

Note: Concept of solving System of Linear Equations using Determinants is the most important one from this chapter. I am almost sure there will be one question from this concept in JEE Exam. So do learn this one.

Tip-1

Homogeneous linear equations, always possess at least one solution i.e. (0, 0, 0). It is also called TRIVIAL solution. And if det is zero then there are infinte solutions.

Tip-2



From Cramer's Rule

For this case one has to work extra.

Tip-3

(a) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

Tip-4

(b) Area of a triangle whose vertices are

$$(x_r, y_r);\ r = 1, 2, 3\ \ is:\quad D = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

If D = 0 then the three points are collinear.

Tips and Tricks:

Tip-1

Area of a triangle

If $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are the vertices of a triangle then its area is :

$$\text{Area of } \Delta ABC = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

OR

$$\text{Area of } \Delta ABC = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$$

Tip-2

One can try to remember some standard determinants to solve question faster that uses them. After solving many problems these things start to automatically register in mind.

TIP

PRACTICE. PRACTICE. PRACTICE.

Bonus from Calculas: these topics are covered in calculas.

(A)

Differentiation of Determinants

Let $\Delta(x) = \begin{vmatrix} f_1(x) & g_1(x) \\ f_2(x) & g_2(x) \end{vmatrix}$, *where* $f_1(x), f_2(x), g_1(x)$ *and* $g_2(x)$

are functions of x. Then,

$$\Delta'(x) = \begin{vmatrix} f_1'(x) & g_1'(x) \\ f_2(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1(x) \\ f_2'(x) & g_2'(x) \end{vmatrix} \quad Also, \quad \Delta'(x) = \begin{vmatrix} f_1'(x) & g_1(x) \\ f_2'(x) & g_2(x) \end{vmatrix}$$

(B)

Integration of Determinants

If f(x), g(x) and h(x) are functions of x and a, b, c, α, β and γ are constants such that

$$\Delta(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ a & b & c \\ \alpha & \beta & \gamma \end{vmatrix},$$

then the **integral of the determinants** is given by i.e.

$$\int \Delta(x)dx = \begin{vmatrix} \int f(x)dx & \int g(x)dx & \int h(x)dx \\ a & b & c \\ \alpha & \beta & \gamma \end{vmatrix}$$