Related Questions with Solutions

Questions

Quetion: 01

Suppose $0 < x < 1$ and $(1 - x)^{-5/2}$ is expanded as ascending powers of x.

A. then coefficient of x^r is $\dfrac{(2r+3)!}{3 \cdot 2^{2r+1} r!(r+1)!}$

B. if greatest term is t_5, then $8/11 < x < 10/13$

C. if greatest term is t_5, then $10/13 < x < 4/5$

D. none of these.

Solutions

Solution: 01

We have, $t_{r+1} = \dfrac{\left(\frac{-5}{2}\right)\left(\frac{-5}{2}-1\right)\left(\frac{-5}{2}-2\right)\ldots\left(\frac{-5}{2}-r+1\right)}{r!}(-x)^r$

$= \dfrac{5 \cdot 7 \cdot 9 \ldots (2r+3)}{2^r r!} x^r (-1)^{2r}$

$= \dfrac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \ldots (2r+3)}{1 \cdot 2 \cdot 3[4 \cdot 6 \cdot 8 \ldots (2r+2)]2^r(r!)} x^r$

$= \dfrac{(2r+3)!}{3 \cdot 2^{r+1}(r+1)!} \cdot \dfrac{1}{2^r r!} x^r = \dfrac{(2r+3)!}{3 \cdot 2^{2r+1} r!(r+1)!} x^r$

Since t_5 is the greatest term, $t_4 < t_5$ and $t_6 < t_5$.

$\Rightarrow \dfrac{t_5}{t_4} > 1$ and $\dfrac{t_6}{t_5} < 1.$

But $\dfrac{t_{r+1}}{t_r} = \dfrac{2r+3}{2r} x$

Thus, $\dfrac{t_5}{t_4} = \dfrac{11}{8} x$ and $\dfrac{t_6}{t_5} = \dfrac{13}{10} x$

Therefore, $\dfrac{11}{8} x > 1, \dfrac{13}{10} x < 1 \Rightarrow \dfrac{8}{11} < x < \dfrac{10}{13}.$

Correct Options

Answer:01

Correct Options: A, B