Shortcuts for Permutation and Combination

In how many ways can a word be arranged.

Tricks and Tips on type 1 Question

This is Permutation Question.

Let us take this ahead as an example –

In how many ways can the letters of the word 'LEADER' be arranged?

Count number of Occurances

- L – 1
- E – 2
- A – 1
- D – 1
- R – 1

Total Unique Occurrences – 6(as E repeated 2 times)

Direct Formula = (Unique Occurrences)!/(Each Individual Unique Occurrences)

so = 6!/(1!)(2!)(1!)(1!)(1!) = 360

In how many ways x objects out of a and y objects out of b can be arranged.

Tips and Tricks type 2 problems

Let us take this as well with an example –

Out of 7 consonants and 4 vowels, how many words of 3 consonants and 2 vowels can be formed?

Number of ways of selecting (3 consonants out of 7) and (2 vowels out of 4)

$= ({}^{7}C_3 \times {}^{4}C_2)$

$= \left(\frac{7 \times 6 \times 5}{3 \times 2 \times 1} \times \frac{4 \times 3}{2 \times 1}\right)$

= 210.Number of groups, each having 3 consonants and 2 vowels = 210.

Each group contains 5 letters.

Number of ways of arranging
5 letters among themselves = 5!
= 5 x 4 x 3 x 2 x 1
= 120.

Required number of ways = (210 x 120) = 25200.

There are x objects and y objects, a from x has be selected and b from y. How many ways can it be done when N Number of objects from x should always be selected

Tricks and Tips Type 3 Problems

In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?

We may have (1 boy and 3 girls) or (2 boys and 2 girls) or (3 boys and 1 girl) or (4 boys).

Required number of ways $= ({}^6C_1 \times {}^4C_3) + ({}^6C_2 \times {}^4C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_4)$

$= ({}^6C_1 \times {}^4C_1) + ({}^6C_2 \times {}^4C_2) + ({}^6C_3 \times {}^4C_1) + ({}^6C_2)$

$= (6 \times 4) + \left(\frac{6 \times 5}{2 \times 1} \times \frac{4 \times 3}{2 \times 1}\right) + \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times 4\right) + \left(\frac{6 \times 5}{2 \times 1}\right)$

$= (24 + 90 + 80 + 15)$

$= 209.$

Coloured Ball Questions

Tricks and Tips Type 4 Problems

A box contains 2 white balls, 3 black balls and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

Required number of ways $= ({}^3C_1 \times {}^6C_2) + ({}^3C_2 \times {}^6C_1) + ({}^3C_3)$

$= \left(\frac{3 \times 6 \times 5}{2 \times 1}\right) + \left(\frac{3 \times 2}{2 \times 1} \times 6\right) + 1$

$= (45 + 18 + 1)$

$= 64.$

Circular Combinations Problems

If 6 people are going to sitting at a round table, but Sam will not sit next to Suzie, how many different ways can the group of 6 sit?

Couple of of ways of doing this:

First:
a. Total circular permutations = (6-1)! = 5! = 120.
b. Ways in which Sam and Suzie sit together = 2! * 4! = 2*24 = 48
Required ways = Total – Together = 120 – 48 = 72.

Second:
a. We have total of 6 places. Fix Suzie. Now Sam can't sit at either seat beside her. So number of places where Sam can sit = 5-2 = 3.
For the other 4 people we can arrange them in 4! ways in 4 seats.
So total ways = 3 * 4! = 72.