$\left(\frac{q^p}{p^q}\right)^{\frac{1}{p-q}}$. **4. If the p th and q th terms of a G.P. are q and p respectively, show that its (p + q) th term is**

Solution:

The n^{th} term of GP is given by $t_n = ar^{n-1}$ where a is the first term and r is the common difference

p^{th} term is given as q

$\Rightarrow t_p = ar^{p-1}$

The above equation can be written as

$\Rightarrow q = ar^{p-1}$

$\Rightarrow q = \frac{ar^p}{r}$

On rearranging the above equation we get

$\Rightarrow \frac{a}{r} = \frac{q}{r^p} \ldots (a)$

q^{th} term is given as p

$\Rightarrow t_q = ar^{q-1}$

$\Rightarrow p = ar^{q-1}$

The above equation can be written as

$\Rightarrow p = \frac{ar^q}{r}$

On rearranging the above equation we get

$\Rightarrow \frac{a}{r} = \frac{p}{r^q} \ldots (b)$

From equation (a) and (b) we have

$\Rightarrow \frac{q}{r^p} = \frac{p}{r^q}$

On rearranging we get

$\Rightarrow \frac{q}{p} = \frac{r^p}{r^q}$

$\Rightarrow r^{p-q} = \frac{q}{p}$

$\Rightarrow r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$

$(p + q)^{th}$ term is given by

$\Rightarrow t_{p+q} = a\, r^{p+q-1}$

$\Rightarrow t_{p+q} = (ar^{p-1})\, r^q$

But $t_p = ar^{p-1}$ and the p^{th} term is q

$\Rightarrow t_{p+q} = q\, r^q$

But

But

$$r = \left(\frac{q}{p}\right)^{\frac{1}{p-q}}$$

$$\Rightarrow t_{p+q} = q\left(\left(\frac{q}{p}\right)^{\frac{1}{p-q}}\right)^{q}$$

Using laws of exponents we get

$$= q\left(\frac{q^{\frac{1}{p-q}}}{p^{\frac{1}{p-q}}}\right)^{q}$$

$$= q\left(\frac{q^{\frac{q}{p-q}}}{p^{\frac{q}{p-q}}}\right)$$

On rearranging

$$= \frac{q^{\frac{q}{p-q}+1}}{p^{q\left(\frac{1}{p-q}\right)}}$$

Taking LCM and simplifying we get

$$= \frac{q^{\frac{q+p-q}{p-q}}}{p^{q\left(\frac{1}{p-q}\right)}}$$

$$= \frac{q^{p\left(\frac{1}{p-q}\right)}}{p^{q\left(\frac{1}{p-q}\right)}}$$

$$\Rightarrow t_{p+q} = \left(\frac{q^{p}}{p^{q}}\right)^{\left(\frac{1}{p-q}\right)}$$

Hence the proof.