

Sequence and Series - Class XI

Related Questions with Solutions

Questions

Question: 01

If $x = 1 + a + a^2 + a^3 + \dots \infty$ and $y = 1 + b + b^2 + b^3 + \dots \infty$, the sum $1 + ab + a^2b^2 + a^3b^3 + \dots \infty$ is equal to, where $0 < a < 1$ and $0 < b < 1$.

- A. $\frac{xy}{x + y - 1}$
B. $\frac{x^2y^2}{x^2 + y^2 - 1}$
C. $\frac{2xy}{x - y + 1}$
D. $\frac{xy}{x - y + 1}$

Solutions

Solution: 01

$$\begin{aligned} x &= 1 + a + a^2 + a^3 + \dots \infty (\infty \text{ G.P.}) \\ \Rightarrow x &= \frac{1}{1-a} \Rightarrow 1-a = \frac{1}{x} \Rightarrow a = 1 - \frac{1}{x} = \frac{x-1}{x} \text{ Similarly,} \\ b &= \frac{y-1}{y} \therefore LHS = 1 + ab + (ab)^2 + (ab)^3 + \dots + \infty [\infty \text{ G.P.}] \\ &= \frac{1}{1-ab} = \frac{1}{1 - \left(\frac{x-1}{x}\right)\left(\frac{y-1}{y}\right)} \\ &= \frac{xy}{x+y-1} \end{aligned}$$

Correct Options

Answer:01

Correct Options: A