Practice Questions

Q1.

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Example 6 Prove that $(A^{-1})' = (A')^{-1}$, where A is an invertible matrix.

Solution Since A is an invertible matrix, so it is non-singular.

We know that $|A| = |A'|$. But $|A| \neq 0$. So $|A'| \neq 0$ i.e. A' is invertible matrix.

Now we know that $AA^{-1} = A^{-1} A = I$.

Taking transpose on both sides, we get $(A^{-1})' A' = A' (A^{-1})' = (I)' = I$

Hence $(A^{-1})'$ is inverse of A', i.e., $(A')^{-1} = (A^{-1})'$

Q2.

Example 17 If $A = \begin{bmatrix} x & 5 & 2 \\ 2 & y & 3 \\ 1 & 1 & z \end{bmatrix}$, $xyz = 80$, $3x + 2y + 10z = 20$, then

$$A\ adj.\ A = \begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix}.$$

Solution : False.

Q3.

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34. If A and B are invertible matrices, then which of the following is not correct?

(a) adj. $A = |A| \cdot A^{-1}$ (b) $\det(A)^{-1} = [\det(A)]^{-1}$

(c) $(AB)^{-1} = B^{-1}A^{-1}$ (d) $(A+B)^{-1} = B^{-1} + A^{-1}$

Sol. (d) Given A and B are invertible matrices.

Now $(AB)B^{-1}A^{-1}$

$$= A(BB^{-1})A^{-1} = AIA^{-1} = (AI)A^{-1} = AA^{-1} = I$$

$\Rightarrow$ $(AB)^{-1} = B^{-1}A^{-1}$

Also $AA^{-1} = I$

$\Rightarrow$ $|AA^{-1}| = |I|$

$\Rightarrow$ $|A||A^{-1}| = 1$

$\Rightarrow$ $|A^{-1}| = \dfrac{1}{|A|}$

$\Rightarrow$ $\det(A)^{-1} = [\det(A)]^{-1}$

Also we know that $A^{-1} = \dfrac{\text{adj. } A}{|A|}$

$\Rightarrow$ adj. $A = |A| \cdot A^{-1}$

$$(A+B)^{-1} = \frac{1}{|A+B|}\text{adj.}(A+B)$$

But $B^{-1} + A^{-1} = \dfrac{1}{|B|}\text{adj. } B + \dfrac{1}{|A|}\text{adj. } A$

$\Rightarrow$ $(A+B)^{-1} \neq B^{-1} + A^{-1}$

Q4.

49. $(aA)^{-1} = \frac{1}{a} A^{-1}$, where a is any real number and A is a square matrix.

Sol. False

Since, we know that, if A is a non–singular square matrix, then for any scalar a (non–zero), aA is invertible such that

$$(aA)\left(\frac{1}{a}A^{-1}\right) = \left(a \cdot \frac{1}{z}\right)(A \cdot A^{-1}) = I$$

i.e, $\left(\frac{1}{a}A^{-1}\right)$ is inverse of (aA).

or $(aA)^{-1} = \frac{1}{a} A^{-1}$, where a is any non-zero scalar.

In the above statement it is not given that A is non-singular matrix. Hence, statement is false.

Q5.—Q6.—Q7.

50. $|A^{-1}| \neq |A|^{-1}$, where A is non-singular matrix.

Sol. False

We know that $|A^{-1}| = |A|^{-1}$, where A is a non-singular matrix.

51. If A and B are matrices of order 3 and $|A| = 5$, $|B| = 3$, then $|3AB| = 27 \times 5 \times 3 = 405$.

Sol. True.

We know that, $|AB| = |A| \cdot |B|$ and $|kA| = k^n|A|$, where k is scalar and n is order of matrix A

$\therefore \quad |3AB| = 3^3|AB| = 27|A| \cdot |B| = 27 \times 5 \times 3 = 405$

52. If the value of a third order determinant is 12, then the value of the determinant formed by replacing each element by its co-factor will be 144.

Sol. True

Let A is the determinant.

Given $|A| = 12$

Also, we know that, if A is a square matrix of order n, then $|\text{adj } A| = |A|^{n-1}$

For $n = 3$, $|\text{adj } A| = |A|^{3-1} = |A|^2 = (12)^2 = 144$