Differential Equations - Class XII

Past Year JEE Questions

Questions

Quetion: 01

Let y = y(x) be the solution of the differential equation,

$$\frac{dy}{dx} + y \tan x = 2x + x^2 \tan x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$
, such that y(0) = 1. Then:

A.
$$y\left(\frac{\pi}{4}\right) - y\left(-\frac{\pi}{4}\right) = \sqrt{2}$$

B.
$$y'(\frac{\pi}{2}) - y'(-\frac{\pi}{2}) = \pi - \sqrt{2}$$

C.
$$y\left(\frac{\pi}{4}\right) + y\left(-\frac{\pi}{4}\right) = \frac{\pi^2}{2} + 2$$

D. $y'\left(\frac{\pi}{4}\right) + y'\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

D.
$$y'(\frac{\pi}{4}) + y'(-\frac{\pi}{4}) = -\sqrt{2}$$

Solutions

Solution: 01

Explanation

$$\frac{dy}{dx} + y(\tan x) = 2x + x^2 \tan x$$

I.F. =
$$e^{\int \tan x \, d\underline{x}} e^{\ln \sec \underline{x}} \sec x$$

y.
$$\sec x = \int (2x + x^2 \tan x) \sec x \, dx$$

$$\Rightarrow y \sec x = x^2 \sec x + \lambda$$

$$\Rightarrow$$
 y(0) = 0 + λ = 1 \Rightarrow λ = 1

$$\Rightarrow$$
 y = x² + cos x

$$\Rightarrow$$
 y' = 2x - sinx

$$\Rightarrow y'(\frac{\pi}{4}) = \frac{\pi}{2} - \frac{1}{\sqrt{2}}$$

$$\Rightarrow y'\left(-\frac{\pi}{4}\right) = -\frac{\pi}{2} + \frac{1}{\sqrt{2}}$$

$$\therefore y'\left(\frac{\pi}{4}\right) - y'\left(-\frac{\pi}{4}\right) = \pi - \sqrt{2}$$