Infinite Series - Class XI

Past Year JEE Questions

Questions

Quetion: 01

If x is so small that x^3 and higher powers of x may be neglected, then $\frac{(1+\frac{3}{2})^3-(1+\frac{1}{2}x^3)}{(1-\frac{3}{2})}$ may be

approximated as

A.
$$1 - \frac{3}{8}x^2$$

B.
$$3x + \frac{3}{8}x^2$$

$$C. -\frac{3}{8}x^2$$

B.
$$3x + \frac{3}{8}x^2$$

C. $-\frac{3}{8}x^2$
D. $\frac{x}{2} - \frac{3}{8}x^2$

Solutions

Solution: 01

Explanation

$$(1+x)^{\frac{3}{2}} = 1 + \frac{3}{2}x + \frac{\frac{3}{2} \cdot \frac{1}{2}}{1 \cdot 2}x^2 + \dots$$

= 1 + $\frac{3}{2}x$ + $\frac{3}{8}x^2$ (As x is so small, so x^3 and higher powers of x neglected)

$$\frac{(1+x^{2})-(1+\frac{1}{2}x^{2})}{(1-x^{2})}$$

$$=\frac{\left(1+\frac{3}{2}x+\frac{3}{8}x^2\right)-\left(1+\frac{3}{2}C_0\frac{x}{2}+\frac{3}{2}C_1\left(\frac{x}{2}\right)^2\right)}{(1-x)}$$

$$=\frac{\frac{3}{8}x^2-\frac{3}{4}x^2}{(1-x)}$$

$$=x^{2}\left(\frac{3}{8}-\frac{3}{4}\right)(1-x)^{-2}$$

$$=-\frac{3}{8}x^2(1-\frac{1}{2}(-x)+\ldots)$$

$$=-\frac{3}{8}x^2-\frac{3}{16}x^3$$

[As x^3 is so small we can ignore $-\frac{3}{16}x^3$]

$$=-\frac{3}{8}x^2$$