25. The value of determinant

$$\begin{vmatrix} a-b & b+c & a \\ b-a & c+a & b \\ c-a & a+b & c \end{vmatrix}$$

(A) $a^3 + b^3 + c^3$ (B) 3 bc (C) $a^3 + b^3 + c^3 - 3abc$ (D) none of these

Solution:

Option (C) $a^3 + b^3 + c^3 - 3abc$

Given,

$$\Delta = \begin{vmatrix} a-b & b+c & a \\ b-c & c+a & b \\ c-a & a+b & c \end{vmatrix}$$

$$[\text{Applying } C_1 \rightarrow C_1 - C_3]$$

$$= \begin{vmatrix} -b & b+c & a \\ -c & c+a & b \\ -a & a+b & c \end{vmatrix}$$

$$[\text{Applying } C_2 \rightarrow C_2 + C_1]$$

$$= \begin{vmatrix} -b & c & a \\ -c & a & b \\ -a & b & c \end{vmatrix} = - \begin{vmatrix} b & c & a \\ c & a & b \\ a & b & c \end{vmatrix}$$

$$= -[b(ac - b^2) - c(c^2 - ab) + a(bc - a^2)]$$
$$= a^3 + b^3 + c^3 - 3abc$$