Infinite Series - Class XI

Past Year JEE Questions

Questions

Quetion: 01

For natural numbers m , n, if $(1-y)^m(1+y)^n = 1 + a_1y + a_2y^2 + \ldots\ldots\ldots$ and $a_1 = a_2 = 10$, then $(m,\ n)$ is

A. $(20,\ 45)$

B. $(35,\ 20)$

C. $(45,\ 35)$

D. $(35,\ 45)$

Solutions

Solution: 01

Explanation

$(1-y)^m(1+y)^n$

$= \left({}^mC_0 - {}^mC_1y + {}^mC_2y^2 + \ldots\right)$ -
$\left({}^nC_0 + {}^nC_1y + {}^nC_2y^2 + \ldots\right)$

a_1 = Coefficient of y = ${}^nC_1$ - ${}^mC_1$ = 10

$\Rightarrow$ n - m = 10

a_2 = Coefficient of y^2

$= {}^nC_2 + {}^nC_1 \times {}^mC_1 + {}^mC_2 = 10$

$\Rightarrow \frac{n(n-1)}{2} - nm + \frac{m(m-1)}{2} = 10$

$\Rightarrow n(n-1) - 2nm + m(m-1) = 20$

$\Rightarrow$ (m + 10)(m + 9) - 2(m + 10)m + m(m - 1) = 20

$\Rightarrow$ 90 + 19m + m^2 - $2m^2$ - 20m + m^2 - m - 20 = 0

$\Rightarrow$ 70 - 2m = 0

$\Rightarrow$ m = 35

$\therefore$ n = 10 + 35 = 45