3 JEE Main 2021 (Online) 27th August Evening Shift

MCQ (Single Correct Answer)

Let A(a, 0), B(b, 2b + 1) and C(0, b), b $\neq$ 0, |b| $\neq$ 1, be points such that the area of triangle ABC is 1 sq. unit, then the sum of all possible values of a is :

A $\frac{-2b}{b+1}$

B $\frac{2b}{b+1}$

C $\frac{2b^2}{b+1}$

D $\frac{-2b^2}{b+1}$

Explanation

$$\left|\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix}\right| = 1$$

$$\Rightarrow \begin{vmatrix} a & 0 & 1 \\ b & 2b+1 & 1 \\ 0 & b & 1 \end{vmatrix} = \pm 2$$

$$\Rightarrow a(2b+1-b) - 0 + 1(b^2 - 0) = \pm 2$$

$$\Rightarrow a = \frac{\pm 2 - b^2}{b+1}$$

$\therefore\ a = \frac{2-b^2}{b+1}$ and $a = \frac{-2-b^2}{b+1}$

Sum of possible values of 'a' is

$$= \frac{-2b^2}{a+1}$$

3 JEE Main 2021 (Online) 17th March Morning Shift

MCQ (Single Correct Answer)

If $A=\begin{pmatrix}0 & \sin\alpha \\ \sin\alpha & 0\end{pmatrix}$ and $\det\left(A^2-\frac{1}{2}I\right)=0$, then a possible value of α is :

A $\frac{\pi}{4}$

B $\frac{\pi}{6}$

C $\frac{\pi}{2}$

D $\frac{\pi}{3}$

Explanation

$$A^2=\begin{bmatrix}0 & \sin\alpha \\ \sin\alpha & 0\end{bmatrix}\begin{bmatrix}0 & \sin\alpha \\ \sin\alpha & 0\end{bmatrix}=\begin{bmatrix}\sin^2\alpha & 0 \\ 0 & \sin^2\alpha\end{bmatrix}$$

$$A^2-\frac{1}{2}I=\begin{bmatrix}\sin^2\alpha & 0 \\ 0 & \sin^2\alpha\end{bmatrix}-\begin{bmatrix}\frac{1}{2} & 0 \\ 0 & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}\sin^2\alpha-\frac{1}{2} & 0 \\ 0 & \sin^2\alpha-\frac{1}{2}\end{bmatrix}$$

Given, $\left|A^2-\frac{1}{2}I\right|=0$

$$\Rightarrow\begin{vmatrix}\sin^2\alpha-\frac{1}{2} & 0 \\ 0 & \sin^2\alpha-\frac{1}{2}\end{vmatrix}=0$$

$$\Rightarrow\left(\sin^2\alpha-\frac{1}{2}\right)^2=0$$

$$\Rightarrow\sin^2\alpha=\frac{1}{2}\Rightarrow\sin\alpha=\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}$$

$$\therefore\ \alpha=\frac{\pi}{4}$$

1 JEE Main 2021 (Online) 20th July Morning Shift

MCQ (Single Correct Answer)

Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, a∈R be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the um of all possible values of determinant of P is equal to :

A 36

B 24

C 45

D 18

4 JEE Main 2021 (Online) 31st August Morning Shift

MCQ (Single Correct Answer)

If $a_r = \cos\frac{2r\pi}{9} + i\sin\frac{2r\pi}{9}$, r = 1, 2, 3, ..., i = $\sqrt{-1}$, then the determinant $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to :

A $a_2a_6 - a_4a_8$

B a_9

C $a_1a_9 - a_3a_7$

D a_5

Explanation

$a_r = e^{\frac{i2\pi r}{9}}$, r = 1, 2, 3, ..., a_1, a_2, a_3, ... are in G.P.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_n & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_2^2 & a_1^3 \\ a_1^4 & a_1^5 & a_1^6 \\ a_1^7 & a_1^8 & a_1^9 \end{vmatrix}$$

$$= a_1 \,.\, a_1^4 \,.\, a_1^7 \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \end{vmatrix} = 0$$

Now, $a_1a_9 - a_3a_7 = {a_1}^{10} - {a_1}^{10} = 0$