Concepts and Formulas
Infinite Series

The series which arises in the binomial theorem for negative integer $-n$,

$$(x+a)^{-n} = \sum_{k=0}^{\infty} \binom{-n}{k} x^k a^{-n-k} \tag{1}$$

$$= \sum_{k=0}^{\infty} (-1)^k \binom{n+k-1}{k} x^k a^{-n-k} \tag{2}$$

for $|x| < a$.

For $a = 1$, the negative binomial series simplifies to

$$(x+1)^{-n} = 1 - n\,x + \tfrac{1}{2}\,n\,(n+1)\,x^2 - \tfrac{1}{6}\,n\,(n+1)\,(n+2)\,x^3 + \ldots . \tag{3}$$