

Binomial Theorem - Class XI

Past Year JEE Questions

Questions

Question: 01

If the coefficient of x^7 in $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ equals the coefficient of x^{-7} in $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$, then a and b satisfy the relation

- A. $a - b = 1$
- B. $a + b = 1$
- C. $\frac{a}{b} = 1$
- D. $ab = 1$

Solutions

Solution: 01

Explanation

General term of $\left[ax^2 + \left(\frac{1}{bx}\right)\right]^{11}$ is T_{r+1} .

$$\begin{aligned} T_{r+1} &= {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx}\right)^r \\ &= {}^{11}C_r (a)^{11-r} (b)^{-r} (x)^{22-3r} \end{aligned}$$

For the coefficient of x^7 ,

$$\Rightarrow 22 - 3r = 7$$

$$\Rightarrow r = 5$$

$$\text{So coefficient of } x^7 = {}^{11}C_5 (a)^6 (b)^{-5} \dots (1)$$

Now General term of $\left[ax - \left(\frac{1}{bx^2}\right)\right]^{11}$ is T_{r+1} .

$$\begin{aligned} T_{r+1} &= {}^{11}C_r (ax)^{11-r} \left(-\frac{1}{bx^2}\right)^r \\ &= {}^{11}C_r (a)^{11-r} (-1)^r (b)^{-r} (x)^{11-2r} \end{aligned}$$

For the coefficient of x^{-7} ,

$$11 - 3r = -7$$

$$\Rightarrow r = 6$$

$$\therefore \text{Coefficient of } x^{-7} = {}^{11}C_6 (a)^5 (-1)^6 (b)^{-6}$$

According to question,

$$\text{Coefficient of } x^7 = \text{Coefficient of } x^{-7}$$

$$\Rightarrow {}^{11}C_{5(a)}{}^{6(b)-5} = {}^{11}C_{6(a)5}(-1)^{6(b)-6}$$

$$\Rightarrow ab = 1$$