Example 1: A circular wire of radius

$$3cm$$

is cut and bent so as to lie along the circumference of a hoop whose radius is $48cm$ **. Find the angle in degrees which is subtended at the centre of hoop.**

Ans: Given that, radius of circular wire = $3cm$

When it is cut then its length becomes $2\pi \times 3 = 6\pi$

Again, it is being placed along a circular hoop of radius $48cm$.

The length (s) of the arc = 6π

Radius of circle, $r = 48cm$

Therefore, the angle θ (in radian) subtended by the arc at the centre of circle is given by

$\Rightarrow \theta = \frac{Arc}{Radius}$

$\Rightarrow \theta = \frac{6\pi}{48}$

$\Rightarrow \theta = \frac{\pi}{8}$

$\Rightarrow \theta = 22.5^{\circ}$

Example 15: If $\tan\theta = \frac{-4}{3}$, then $\sin\theta$ is

a) $\frac{-4}{5}$ but not $\frac{4}{5}$

b) $\frac{-4}{5}$ or $\frac{4}{5}$

c) $\frac{4}{5}$ but not $\frac{-4}{5}$

d) None of these

Ans: The correct answer is option (b) $\frac{-4}{5}$ or $\frac{4}{5}$

Given that, $\tan\theta = \frac{-4}{3} = \frac{P}{B}$.

By Pythagoras theorem, we have

$\Rightarrow H^2 = P^2 + B^2$

$\Rightarrow H^2 = 4^2 + 3^2$

(Here, we have taken positive value of perpendicular because length can't be negative)

$\Rightarrow H^2 = 16 + 9$

$\Rightarrow H^2 = 25$

$\Rightarrow H = 5$

Since $\tan\theta = \frac{-4}{3}$ is negative, θ lies either in second quadrant or in fourth quadrant.

We know that $\sin\theta = \frac{P}{H}$. Therefore, we get

If θ lies in second quadrant, $\sin\theta = \frac{4}{5}$ and if θ lies in fourth quadrant, $\sin\theta = -\frac{4}{5}$.

Hence, the required answer is (b) $\frac{-4}{5}$ or $\frac{4}{5}$

Example 16: If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then a, b and c satisfy the relation.

a) $a^2 + b^2 + 2ac = 0$

b) $a^2 - b^2 + 2ac = 0$

c) $a^2 + c^2 + 2ab = 0$

d)

$$a^2 - b^2 - 2ac = 0$$

Ans: The correct answer is option (b) $a^2 - b^2 + 2ac = 0$

Given that, $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$.

We know that if the roots of the quadratic equation $ax^2 + bx + c = 0$ are α and β. Then we have, $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$. Therefore, we get

$\Rightarrow \sin\theta + \cos\theta = \frac{b}{a} \ldots (i)$ and $\sin\theta\cos\theta = \frac{c}{a} \ldots\ldots (ii)$

On squaring both the sides in equation (i), we get

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \frac{b^2}{a^2}$

We have, $\sin\theta\cos\theta = \frac{c}{a}$ and we know that $\sin^2\theta + \cos^2\theta = 1$. Therefore, we get

$\Rightarrow 1 + \frac{2c}{a} = \frac{b^2}{a^2}$

$\Rightarrow \frac{a + 2c}{a} = \frac{b^2}{a^2}$

$\Rightarrow \frac{a + 2c}{1} = \frac{b^2}{a}$

On cross multiplication, we get

$\Rightarrow a^2 + 2ac = b^2$

$\Rightarrow a^2 - b^2 + 2ac = 0$

Exemplar Problem

Trigonometry Functions

Example 17: The greatest value of $\sin x \cos x$ is

a) 1

b) 2

c) $\sqrt{2}$

d)

$$\frac{1}{2}$$

Ans: The correct answer is option (d) $\frac{1}{2}$

We have, $\sin x \cos x$

Multiply and divide the expression by 2

$\Rightarrow \frac{1}{2} \times 2\sin x \cos x$

We know that $2\sin x \cos x = \sin 2x$. Therefore, we get

$$\Rightarrow \frac{1}{2} \times \sin 2x$$

We know that,

$\Rightarrow -1 \leqslant \sin 2x \leqslant 1$

Divide the expression by 2

$\Rightarrow -\frac{1}{2} \leqslant \frac{\sin 2x}{2} \leqslant \frac{1}{2}$

Hence, the greatest is $\frac{1}{2}$.

70. $\sin 10^\circ$ **is greater than** $\cos 10^\circ$**.**

Ans: Given, $\sin 10^\circ > \cos 10^\circ$

$\Rightarrow \sin 10^\circ > \cos\left(90^\circ - 80^\circ\right)$

We know that $\cos\left(90^\circ - \theta\right) = \sin\theta$. Therefore, we get

$\Rightarrow \sin 10^\circ > \sin 80^\circ$

It is incorrect because value of $\sin e$ is in increasing order.

Thus, the given statement is false.

72. One value of θ which satisfies the equation $\sin^4\theta - 2\sin^2\theta - 1$ lies between 0 and 2π.

Ans: We have, $\sin^4\theta - 2\sin^2\theta - 1$

Let $y = \sin^2\theta$. Therefore, we get

$$\Rightarrow y^2 - 2y - 1 = 0$$

We know that for quadratic equation $ax^2 + bx + c = 0$, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Therefore, we get

$$\Rightarrow y = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times -1}}{2 \times 1}$$

$$\Rightarrow y = \frac{2 \pm \sqrt{4+4}}{2}$$

$$\Rightarrow y = \frac{2 \pm 2\sqrt{2}}{2} = \frac{2(1 \pm \sqrt{2})}{2}$$

On canceling common terms, we get

$$\Rightarrow y = 1 \pm \sqrt{2}$$

$$\Rightarrow \sin^2\theta = 1 \pm \sqrt{2}$$

$$\Rightarrow \sin^2\theta = 1 + \sqrt{2} \text{ or } \sin^2\theta = 1 - \sqrt{2}$$

We know that $-1 \leqslant \sin\theta \leqslant 1$. Therefore, we say that $\sin^2\theta \leqslant 1$.

But we have,

$$\Rightarrow \sin^2\theta = 1 + \sqrt{2} \text{ or } \sin^2\theta = 1 - \sqrt{2}$$

Which is not possible.

Thus, the given statement is false.

73. If $\cos ecx = 1 + \cot x$ then $x = 2n\pi, 2n\pi + \frac{\pi}{2}$.

Ans: Given, $\cos ecx = 1 + \cot x$

$\Rightarrow \frac{1}{\sin x} = 1 + \frac{\cos x}{\sin x}$

$\Rightarrow \frac{1}{\sin x} = \frac{\sin x + \cos x}{\sin x}$

$\Rightarrow \sin x + \cos x = 1$

Divide whole equation by $\sqrt{2}$.

$\Rightarrow \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}}$

We know that $\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$. Therefore, we can write above written equation as,

$\Rightarrow \sin \frac{\pi}{4} \sin x + \cos \frac{\pi}{4} \cos x = \frac{1}{\sqrt{2}}$

Or

$\Rightarrow \cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

We know that $\cos (x - y) = \cos x \cos y + \sin x \sin y$. Therefore, we get

$\Rightarrow \cos \left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos \left(x - \frac{\pi}{4}\right) = \cos \frac{\pi}{4}$

We know that if $\cos \theta = \cos \alpha$, then $\theta = 2n\pi \pm \alpha$. Therefore, we get

$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$

$\Rightarrow x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$

$\Rightarrow x = 2n\pi + \frac{\pi}{4} + \frac{\pi}{4}$ or $\Rightarrow x = 2n\pi - \frac{\pi}{4} + \frac{\pi}{4}$

$\Rightarrow x = 2n\pi + \frac{\pi}{2}$ or $\Rightarrow x = 2n\pi$

Thus, the given statement is true.