Tips and Tricks to use circle equation quickly:

Lots of questions are asked in JEE Exams, that ask to find center and radius of a circle.
If one knows some formulas then it becomes quick and can save a lot of time.
Here are some of the standard forms worth remembering:

Center Radius Form:

$$(x-h)^2+(y-k)^2=r^2$$

Centre (h, k), Radius = r

General Form:

$$x^2+y^2+2gx+2fy+c=0$$

Where (-g, -f) centre

$r^2 = g^2 + f^2 - c$.

Radius = $\sqrt{g^2+f^2-c}$

→ Here important results are of center and radius formula.

Parametric Form:

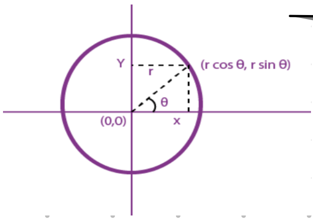
Equation of circle = $x^2 + y^2 = r^2$

X = r cos Θ

Y = r sin Θ

Squaring both side

$x^2 + y^2 = (r^2 \cos \Theta + r^2 \sin 2\Theta)$



→ When parametric form is mentioned in que, try to start with this diagram.

Bonus:

If in question, there is an assumption about a circle; it is a best practice to start with simplest circle. That is standard equation of circle. Most of the time a result that is claimed to be true in an option, ought to hold for simplest circle also. This way one can check particular option quickly and save time. One can practice this method in other concepts as well.

$$x^2+y^2=r^2$$

centre (0, 0) and Radius (r)

→ Standard Circle Equation

Illustration 1: Find the centre and the radius of the circle

$3x^2+3y^2-8x-10y+3=0.$

→ General form formulas are used to solve this one.

Solution:

We rewrite the given equation as $x^2+y^2-\frac{8}{3}x-\frac{10}{3}y+1=0$

$\Rightarrow g=-\frac{4}{3}, f=-\frac{5}{3}, c=1.$

Hence the centre is $\left(\frac{4}{3},\frac{5}{3}\right)$ and the radius is

$$\sqrt{\frac{16}{9}+\frac{25}{9}-1}=\sqrt{\frac{32}{9}}=\frac{4\sqrt{2}}{3}.$$

Tips and Tricks to solve questions quickly:

Trick-1

In lecture Prof, talked about equation of circle that passes through three non-collinear points, and lots of algebra is done to get it. It is helpful to understand the process. But for exam, one must have a trick to remember things that are bit complicated. Here is one,

Equation of circle through three non-collinear points $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$ is

$$\begin{vmatrix} x^2+y^2 & x & y & 1 \\ x_1^2+y_1^2 & x_1 & y_1 & 1 \\ x_2^2+y_2^2 & x_2 & y_2 & 1 \\ x_3^2+y_3^2 & x_3 & y_3 & 1 \end{vmatrix} = 0.$$

With coordinates putted in place, it is a simple determinats problem.

Area of circle = πr^2

Perimeter = $2\pi r$, where r is the radius.

Trick-2

In JEE exams many a times it is asked to find equation of circle whose diameter end points are given. One should understand how to solve it. But for exam one can remember a trick formula to get the job done quickly.

Equation of circle with points $P(x_1, y_1)$ and $Q(x_2, y_2)$ as extremities of diameter is

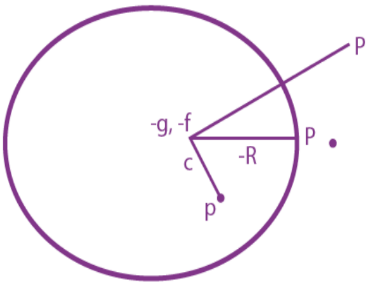
$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Focus on structure of the formula and devise a simple mental note to remember it.

Trick-3

Prof, also talked about how to know position of a point wrt a circle. Here is a simple diagram to show concept summary.

Let the circle be $x^2 + y^2 + 2gx + 2fy + c = 0$ and $p(x_1, y_1)$ be the point.



R – radius

cp > R , {Point lie outside}

cp = R , {on the curve}

cp < R , {inside the curve}

Illustration 3: Find the equation of the circle whose diameter is the line joining the points (-4, 3) and (12, -1). Find also the length of intercept made by it on the y-axis.

Solution:

The required equation of the circle is

Application of Trick-2

$$(x+4)(x-12)+(y-3)(y+1)=0.$$

NOTE: For second part of above question see next lecture tricks and notes.

Tips and Tricks to solve questions quickly:

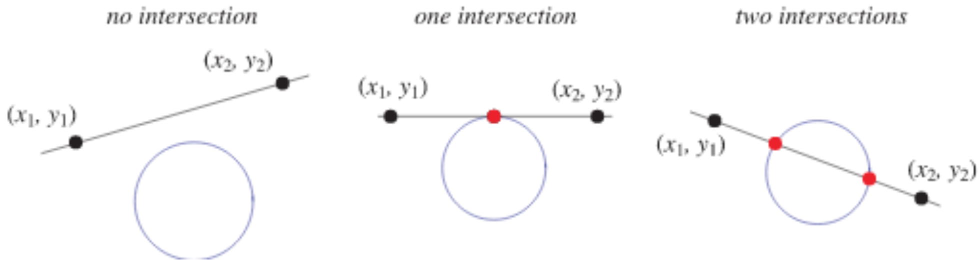
Trick-1

Intersection of a line and circle

To get point of intersection of two curves(in 2D), first get value of one coordinate from one curve, and then replace it in second curve. This way we get an equation in one variable. Just solve this to get point(s) of intersection.

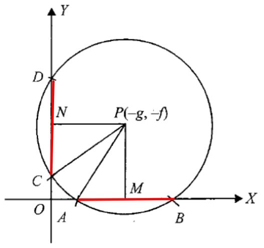
In case of circle that equation is a quadratic one. Analyse roots ot it to know points. There are three ways a line and a circle can be associated,

1. the line cuts the circle at two distinct points
2. the line is a tangent to the circle
3. the line misses the circle.



Trick-2

Intercept made by Circle on Axes



$$AB = 2AM = 2\sqrt{f^2 - c}$$

$$CD = 2CN = 2\sqrt{g^2 - c}$$

IMPORTANT FORMULAS

Illustration 3: Find the equation of the circle whose diameter is the line joining the points (-4, 3) and (12, -1). Find also the length of intercept made by it on the y-axis.

Solution:

The required equation of the circle is

$$(x+4)(x-12)+(y-3)(y+1)=0.$$

On the y-axis, $x=0 \Rightarrow -48+y^2-2y-3=0 \Rightarrow y^2-2y-51=0 \Rightarrow y=1\pm\sqrt{52}$

Hence the length of intercept on the y-axis $=2\sqrt{52}=4\sqrt{13}$.

Tips and Tricks to for circle tangent and normal:

Tangent and Normal Equation: A General Tip

Rather than trying to remember tangent and normal equation for specific curves like circle, one can simply learn to get such equations for any curve at a point; given that it is at least once differentiable. Practice this one on paper to understand the concept.

We know that the equation of the straight line that passes through the point (x_0, y_0) with finite slope "m" is given as

$y - y_0 = m (x - x_0)$

It is noted that the slope of the tangent line to the curve f(x)=y at the point (x_0, y_0) is given by

$\frac{dy}{dx}]_{(x_0,y_0)} (= f'(x_0))$

Therefore, the equation of the tangent (x_0, y_0) to the curve y=f(x) is

$$y - y_0 = f'(x_0)(x - x_0)$$

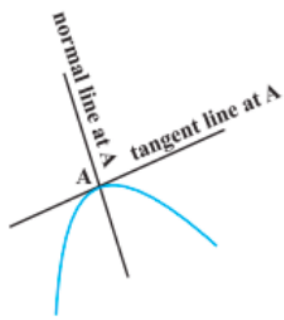
Also, we know that normal is the perpendicular to the tangent line. Hence, the slope of the normal to the curve f(x)=y at the point (x_0, y_0) is given by $-1/f'(x_0)$, if $f'(x_0) \neq 0$.

Hence, the equation of the normal to the curve y=f(x) at the point (x_0, y_0) is given as:

$y-y_0 = [-1/f'(x_0)]\ (x-x_0)$

The above expression can also be written as

$$(y-y_0)\, f'(x_0) + (x-x_0) = 0$$



Circle is a specific curve. Hence simply put f(x) to be circle equation, and proceed from there. Here is a solved example.

Example 3: Find the equation of normal to the circle $2x^2 + 2y^2 - 2x - 5y + 3 = 0$ at (1, 1).

Solution:

The centre of the circle is (1/2, 5/4)

Normal to circle at point (1, 1) is line passing through the points (1, 1) and (1/2, 5/4) which is $x + 2y = 3$.

Trick:

Prof, also talked about length of tengent from a point. Best tip is to remember the diagram shown on right side. Also remember the point that length from point P is square root of general form of circle with coordinates of point P, replaced in place.

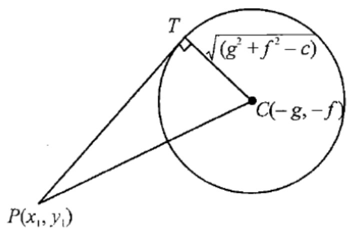
Length of PC,

$$PT = \sqrt{(PC)^2 - (CP)^2}$$

Put value of PC and CP, we get after solving

$$PT = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

Put point value in general form of circle, and take its sq. root to get length of tangent from a point.



NOTE: For detailed proof of formulas, please read class notes.

Useful forms for tangets on circle:

VARIOUS FORMS OF EQUATIONS OF TANGENTS IN CIRCLE

$$x^2 + y^2 = a^2$$

	Point	Equation of Tangent
Point Form	(x_1, y_1)	$xx_1 + yy_1 = a^2$
Slope Form	$\left(\mp \frac{ma}{\sqrt{1+m^2}}, \pm \frac{a}{\sqrt{1+m^2}}\right)$	$y = mx \pm a\sqrt{1+m^2}$
Parametric Form	$(a\cos\theta, a\sin\theta)$	$x\cos\theta + y\sin\theta = a$