1. Always Start from the More Complex Side:

To prove a trigonometric identity, we always start from either the left hand side (LHS) or the right hand side (RHS) and apply the identities step by step until we reach the other side. However, smart students always start from the more complex side. This is because it is a lot easier to eliminate terms to make a complex function simple than to find ways to introduce terms to make a simple function complex.

Example: (1) Prove the identity $tan^4x = sec^2x (tan^2x-1)+1$

Approach: It would be wise to start proving this from the right hand side (RHS) since it is more complex.

2. Express everything into Sine and Cosine:

To both sides of the equation, express all tan, cosec, sec and cot in terms of sin and cos. This is to standardize both sides of the trigonometric identity such that it is easier to compare one side to another.

Example Q2) Show that
$$\sec A \left(\frac{1}{\cot A} + \cot A \right) = \frac{1}{\sin A - \sin^3 A}$$

Approach: We should transform the sec A and cot A into sin A and cos A.

$$LHS = \sec A \left(\frac{1}{\cot A} + \cot A \right)$$
$$= \frac{1}{\cos A} \left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} \right)$$

=...

(convert everything into sin A and cos A)

3. Make a formulas and identities cheatsheet and solve questions in timed manner.

1. Combine Terms into a Single Fraction

When there are 2 terms on one side and 1 term on the other side, combine the side with 2 terms into 1 fraction after making their denominators the same.

Example Q3) Prove the identity
$$1 - \frac{\sin^2 x}{1 + \cos x} = \cos x$$

Notice that LHS has 2 terms: 1 and
$$\frac{\sin^2 x}{1+\cos x}$$

Notice that RHS has 1 term: $\cos x$

Approach: Start by combining the 2 terms on the LHS.

$$LHS = 1 - \frac{\sin^2 x}{1 + \cos x}$$

$$= \frac{1 + \cos x}{1 + \cos x} - \frac{\sin^2 x}{1 + \cos x}$$

$$= \frac{1 + \cos x - \sin^2 x}{1 + \cos x}$$
 (combine 2 terms into 1 fraction)
$$= \dots$$

2. Practice! Practice! Practice!

Proving trigonometric function becomes a piece of cake after you have conquered a massive number questions and expose yourself to all the different varieties of questions. There are no hard and fast rule to handling JEE-level trigonometry proving questions since every question is like a puzzle. But once you have solved a puzzle before, it becomes easier to solve the same puzzle again.

3. Make nice cheat sheets for quick revisions.

Tips and Tricks

1. Use Pythagorean Identities to transform between sin2x and cos2x

Pay special attention to addition of squared trigonometry terms. Apply the Pythagorean identities when necessary. Especially $\sin^2 x + \cos^2 x = 1$ since all the other trigo terms have been converted into sine and cosine. This identity can be used to convert into and vice versa. It can also be used to remove both by turning it into 1.

Example Q4) Prove that
$$\frac{1}{\sin x + 1} - \frac{1}{\sin x - 1} = \frac{2}{\cos^2 x}$$

$$LHS = \frac{1}{\sin x + 1} - \frac{1}{\sin x - 1}$$

$$= \frac{\sin x - 1}{(\sin x + 1)(\sin x - 1)} - \frac{\sin x + 1}{(\sin x + 1)(\sin x - 1)}$$

$$= \frac{\sin x - 1 - \sin x - 1}{(\sin x + 1)(\sin x - 1)}$$

$$= \frac{-2}{\sin^2 x - 1}$$

$$= \frac{2}{1 - \sin^2 x}$$

$$= \frac{2}{1 - \sin^2 x}$$
(Using Pythagorean identity to transform $\sin^2 x$ into $\cos^2 x$)

2. Know when to Apply Double Angle Formula (DAF)

Observe every trigonometric term in the question. Are there terms with angles that are 2 times of another? If there are, be ready to use DAF to transform them into the same angle. For example, if you see $\sin\theta$ and $\cot(\theta/2)$ in the same question, you have to use DAF since θ is 2 times of $(\theta/2)$.

Example Q5) Prove the identity
$$\frac{2\cos\theta-\sec\theta}{2\sin\theta}=\cot2\theta$$

Approach: Since RHS angle is 2θ and LHS has terms with angle θ , DAF has to be used.

$$LHS = \frac{2\cos\theta - \sec\theta}{2\sin\theta}$$

$$= \frac{2\cos\theta - \frac{1}{\cos\theta}}{2\sin\theta}$$

$$= \frac{2\cos^2\theta - 1}{2\sin\theta\cos\theta}$$

$$= \frac{\cos 2\theta}{\sin 2\theta}$$
(Apply DAF)
$$= \cot 2\theta = RHS (Proven)$$

1. Know when to Apply Addition Formula (AF)

Observe the angles in the trigonometric functions. Are there summations of 2 different terms in the same Trigonometric term? If the answer is yes, apply the addition formula (AF).

Example Q6) Prove that
$$\frac{\sin(A+B)-\sin(A-B)}{\cos(A+B)-\cos(A-B)} = -\cot A$$

Approach: It is very obvious from all the (A+B) and (A-B) that AF has to be applied.

$$LHS = \frac{\sin(A+B) - \sin(A-B)}{\cos(A+B) - \cos(A-B)}$$

$$= \frac{\sin A \cos B + \cos A \sin B - (\sin A \cos B - \cos A \sin B)}{\cos A \cos B - \sin A \sin B - (\cos A \cos B + \sin A \sin B)}$$

$$= \frac{2 \cos A \sin B}{-2 \sin A \sin B}$$

$$= -\cot A = RHS (Proven)$$
(Apply AF)

2. Good Old Expand/ Factorize/ Simplify/ Cancelling

Many students hold on to the false belief that every single trigonometry proving question require the use of trigonometric identities from the formula sheet. Whenever they get stuck, they resort to staring blindly at the formula sheet and praying that the answer will magically "jump out" at them. More often than not, the miracle does not happen. This is because most proving questions revolve majorly around good old expansion, factorization, simplification and cancelling of like terms.

3. Practice! Practice! Practice!

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Tips and Tricks

1. PRACTICE!!!

Proving trigonometric function becomes a piece of cake after you have conquered a massive number questions and expose yourself to all the different varieties of questions. There are no hard and fast rule to handling JEE-level trigonometry proving questions since every question is like a puzzle. But once you have solved a puzzle before, it becomes easier to solve the same puzzle again.

- 2. Read Tips and Tricks from previous videos and try to practice them when doing problems related to trigonometry.
- 3. Make nice cheat sheets for each video. Some important formulas that are used most of the times are given in concepts sections below each video. Make sure to read them.
- 4. Each video has about 15-20 specific related problem, so solve them then you can move on to do problems related to whole chapter.