Online Appendix to Demand-Side Management using Deep Learning for Smart Charging of Electric Vehicles

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***Abstract*—The following is supplementary material to the paper *Demand-Side Management using Deep Learning for Smart Charging of Electric Vehicles*. It presents in detail the model used in the experimentations for batteries charging of the plug-in hybrid vehicles.**

APPENDIX

A. State of charge of Lithium-Ion battery

Since the battery is a nonlinear system, the modelling can be complex. We propose to use a simplified model with an equivalent electric circuit containing a voltage source in series with a resistor, which is described by Eq. 2 in order to estimate the State-of-Charge (SoC(t)). The model implements an ideal voltage source $V_{oc}[V]$ that represents the open-circuit voltage of the battery (voltage between the battery terminals with no load applied). The open circuit voltage V_{oc}, is known to be a function related to remaining SoC [1]. For the battery pack being considered herein, a full battery pack ($SoC = 100\%$) corresponds to a voltage of $N_s \cdot V_{max}$ volts. And, an empty battery pack ($SoC = 0\%$) corresponds to a voltage of $N_s \cdot V_{min}$ volts.

$R_B[\Omega] = \dfrac{N_s}{N_p} \cdot R_{cell}$ describes the internal resistance of the battery, N_s and N_p are the number of battery cells of the battery pack connected in serial and parallel form, respectively. $I_B[A]$ is the battery current. $V_B[V]$ is the battery terminal voltage. This model is very simple, but previous experiments indicate that the linear approximation is sufficient when considering the electric vehicle charging plan optimization [2]:

$$V_B(t) = V_{oc}(t) + R_B \cdot I_B(t), \tag{1}$$

$$V_{oc}(t) = SoC(t) \cdot V^* + N_s \cdot V_{min}, \tag{2}$$

where $V^* = N_s(V_{max} - V_{min})$, V_{min} is the minimum allowable cell voltage (Cut-off Voltage). V_{max} is the maximum cell voltage and I_B is the battery current.

The State of Charge $SoC(t)[\%]$ is the present battery capacity as a percentage of maximum capacity. $SoC(t)$ is not directly measurable, it is estimated based on the amount of charge that has been supplied or extracted from the battery and can be simply calculated as [3]:

$$SoC(t) = SoC(0) + \frac{1}{Q_{nom}} \int_0^t I_B(t)dt. \tag{3}$$

$Q_{nom}[Ah] = N_p \cdot Q_{cell}$ is the capacity of the battery pack, t_{dh} is the time of discharge at a constant current of discharge I_{dh}.

This online appendix provides extra material to paper TSG-01128-2017, *Demand-Side Management using Deep Learning for Smart Charging of Electric Vehicles*, accepted for publication in the journal *IEEE Transactions on Smart Grid. (Corresponding author: Christian Gagné.)*

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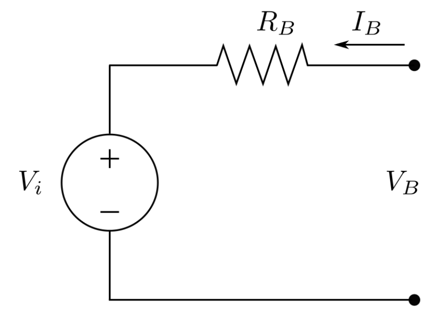

Fig. 1. Battery Model.

B. Charging the battery pack

The batteries should be charged in two phases. During the first phase, the current is constant at I_{Bmax} and the voltage increases linearly from V_{min} to V_{max}. During the second phase, the voltage remains constant at V_{max} and the current decreases exponentially. It is possible to calculate the charging time of each of the phases. However, if the initial charge is high, the charging process passes directly to the second phase without passing through the first phase.

Accordingly to this model, Eq. 2 can then be rewritten as follows:

$$\begin{aligned} V_B(t) = SoC(0) \cdot V^* + \frac{V^*}{Q_{nom}} \int_0^t I_B(t)dt \\ + N_s \cdot V_{min} + R_B \cdot I_B(t) \end{aligned} \tag{4}$$

There may be a first phase if the following condition is respected:

$$V_{oc}(0) < N_s \cdot V_{max} - R_B \cdot I_{Bmax} \tag{5}$$

We solve Eq. 2 for $t = 0$, so the condition (5) can be restated as:

$$SoC(0) < 1 - \frac{R_B \cdot I_{Bmax}}{V^*} \tag{6}$$

If the condition described by Eq. 6 is true, the current is constant at I_{Bmax}. Replacing $I_B(t)$ by I_{Bmax} in Eq. 4 we obtain:

$$V_B(t) = SoC(0)\cdot V^* + \frac{V^*}{Q_{nom}} I_{Bmax}\cdot t + N_s\cdot V_{min} + R_B\cdot I_{Bmax} \tag{7}$$

The first phase is completed when the voltage reaches the maximum value $V_B = Ns\cdot V_{max}$.

Then, we simply solve Eq. 7 for $t=\tau_1$:

$$\tau_1 = \frac{Q_{nom}}{I_{Bmax}}\left[1 - SoC(0) - \frac{R_B\cdot I_{Bmax}}{V^*}\right], \tag{8}$$

where τ_1 is the time of charge of the first phase.

Finally, the battery is charged when the voltage reaches the upper threshold voltage $V_B = Ns\cdot V_{max}$ and when the intensity of current decays exponentially and stabilizes at I_{Bmin} at about 5% of $1C_r$ [4] [1].

The derivative of Eq. 4 with respect to t is:

$$R_B\cdot\frac{dI_B(t)}{dt} + \frac{V^*}{Q_{nom}}\cdot I_B(t) = 0. \tag{9}$$

The integral of 9 in the interval $[0,t]$ is:

$$\int_0^t \frac{1}{I_B(t)}\cdot dI_B(t) = -\frac{V^*}{R_B\cdot Q_{nom}}\int_0^t dt. \tag{10}$$

Equation 10 is then solved for $t=\tau_2$:

$$\tau_2 = -\frac{R_B\cdot Q_{nom}}{V^*}\cdot\ln\left(\frac{I_{Bmin}}{I_{Bi}}\right), \tag{11}$$

where τ_2 is the time of charge of the second phase.

The current I_{Bi} is calculated from Eq. 2 before the second phase of the charging process begins, as follows:

$$I_{Bi} = \frac{(1-SoC(0))\cdot V^*}{R_B}. \tag{12}$$

If the first phase precedes the second phase, the initial current should be the same at the end of the first phase $I_{Bi} = I_{Bmax}$.

In summary, the charging time ($t_{ch} = \tau_1 + \tau_2$) is defined as follows:

- If Eq. 5 is respected:

$$t_{ch} = \frac{Q_{nom}}{V^*}\left[\frac{1}{I_{Bmax}}((1-SoC(0))\cdot V^* - R_B\cdot I_{Bmax}) - R_B\cdot\ln\left(\frac{I_{Bmin}}{I_{Bmax}}\right)\right]; \tag{13}$$

- else:

$$t_{ch} = -\frac{R_B\cdot Q_{nom}}{V^*}\cdot\ln\left(\frac{R_B\cdot I_{Bmin}}{(1-SoC)\cdot V^*}\right). \tag{14}$$

Algorithm 1 Time of charge t_{ch}, τ_1 and I_{Bi}

Require: Q_{nom}, V^*, I_{Bmax}, I_{Bmin}, R_B and $SoC(0)$
  if $SoC(0) < 1 - \left(\frac{R_B\cdot I_{Bmax}}{V^*}\right)$ **then**
    $I_{Bi} = I_{Bmax}$;
    $\tau_1 = \frac{Q_{nom}}{I_{Bmax}}\left[1 - SoC(0) - \frac{R_B\cdot I_{Bmax}}{V^*}\right]$;
  else
    $I_{Bi} = \frac{(1-SoC(0))\cdot V^*}{R_B}$
    $\tau_1 = 0$
  end if;
  $\tau_2 = -\frac{R_B\cdot Q_{nom}}{V^*}\cdot\ln\left(\frac{I_{Bmin}}{I_{Bi}}\right)$;
  $t_{ch} = \tau_1 + \tau_2$;

Algorithm 2 State of charge $SoC(t)$ after charging during an interval of time Δ_t

Require: Q_{nom}, t_{ch}, τ_1, I_{Bi} and $SoC(0)$
  if $\tau_1 = 0$ **then**
    $SoC(t) = SoC(0) + \frac{I_{Bi}\cdot min(t_{ch},\Delta_t))}{Q_{nom}}$
  else if $0 < \tau_1 < \Delta_t$ **then**
    $w_1 = \frac{I_{Bi}\cdot\tau_1}{Q_{nom}}$
    $w_2 = \frac{I_{Bi}\cdot min(\Delta_t - \tau_1, t_{ch} - \tau_1))}{Q_{nom}}$
    $SoC(t) = SoC(0) + w_1 + w_2$
  else if $\tau_1 >= \Delta_t$ **then**
    $SoC(t) = SoC(0) + \frac{I_{Bi}\cdot\Delta_t}{Q_{nom}}$;
  end if;

C. State of Charge Estimation

When we want to estimate $SoC(t)$ and the electric driving range in kilometers as a function of $SoC(t)$, we first calculate the time of charge t_{ch} with Algorithm 1 and then, we calculate the $SoC(t)$ with Algorithm 2.

For the electric driving range we use Eq. 15:

$$R_e\ [\text{km}] = \frac{m_e\cdot 0.0470\cdot SoC(t)\cdot E_{nom}}{\eta_d}, \tag{15}$$

where m_e [MPG_e] are the miles per gallon gasoline equivalent of vehicle (1 $\text{MPG}_\text{e} \approx 0.0470$ [km/kWh]), $\eta\in[0,1]$ is the discharging efficiency; and E_{nom} is the nominal energy.

Fig. 2 shows the charge characteristics calculated with our model of batteries assuming that the vehicle is a Plug-in Hybrid EV (PHEV) similar to a Chevrolet Volt 2013, with Panasonic Lithium-ion NRC18650 batteries, with specifications shown in Table I. The I_{Bmax} current is dependent on both the battery characteristics and the power of the battery charger. In particular, this current should be limited in order to avoid exceeding 15 A at the wall (120V, household current) for level 1 charging stations and 80 A for level 2.

[1] C_r is a measure of the rate at which a battery is discharged relative to its maximum capacity. A $1C$ rate means that the discharge current will discharge the entire battery in 1 hour. For a battery with a capacity of 100 Ah, a $2C$ rate would be 200 A and a $C/2$ rate would be 50 A.

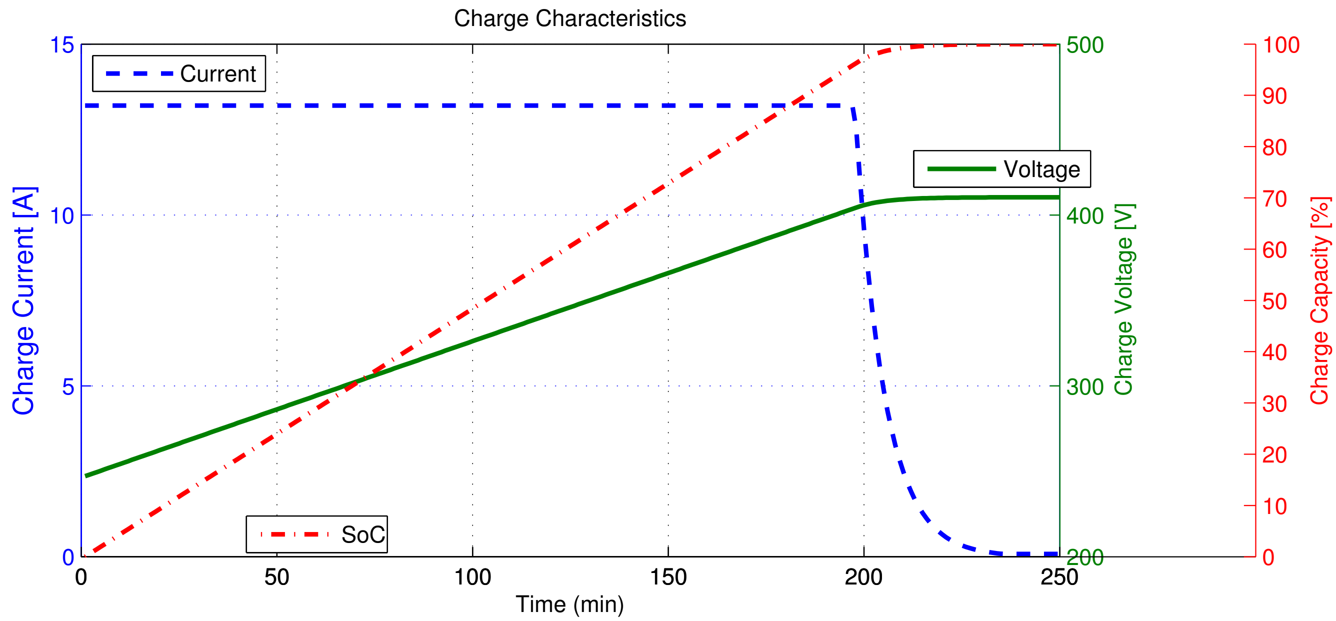

Fig. 2. Charge characteristics calculated with the model of batteries used for the experiments.

TABLE I
SPECIFICATIONS OF BATTERY PACKS, BASED ON CHEVROLET VOLT 2013 AND PANASONIC NRC18650 LI-ION BATTERIES.

Chevrolet Volt 2013 specifications	
Nominal System Voltage V_{nom}	355.2 V
Rated Pack Capacity Q_{nom}	45 Ah
Rated Pack Energy E_v	16.5 kWh
Fuel economy (electricity) m_e	98 mpg_e (35 kWh/100 miles)
Fuel economy (gasoline) m	37 mpg
Panasonic NRC18650 specifications	
Nominal Capacity Q_{nom}	2.75 Ah
Nominal Cell Voltage V_{cell}	3.6 V
Max. Cell Charge Voltage V_{max}	4.2 V
Cut-off Discharge Voltage V_{min}	2.5 V
Maximum Charge Current I_{max}	0.825 A
Constant Current Discharge I_{dh}	0.55 A
Cut-off Charge Current I_{min}	0.05 A

In the case of PHEV, the distance of the trip (D_{trip}) may be greater that the driving range when using only power from its electric battery (R_e), because the gas engine works as a backup when the batteries are depleted. Once the battery is fully depleted, the fuel consumed (E_{gas}) is:

$$E_{gas}\ [\text{l}] = k_d \cdot (D_{trip} - R_e), \tag{16}$$

where $D_{trip} - R_e$ is the distance travelled using gasoline when $D_{trip} > R_e$ and $k_d = C_d/m$ is a constant [l/km] allowing distance to be converted into volume of gasoline. m [MPG] is the miles per gallon specification of the vehicle and $C_d = 0.621371 \cdot 3.785$ (1 km $=$ 0.621371 miles and 1 gallon $=$ 3.785 L).

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