Aim

- Understanding the forward pass
- Manually calculating gradients
- Implementing automated backpropagation
- Visualizing the computation graph
- Perform a backpropagation using PyTorch

In [18]:
```
# A Value object represents a variable in a computation and tracks two
# 1. The current value of the variable
# 2. How this variable affects the final output (its gradient)

from graphviz import Digraph

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import numpy as np

class Value:
    def __init__(self, data, label="", prev=[], op=""):
        self.data = data
        self.grad = 0
        self.label = label
        self.prev = prev
        self._backwards = lambda: None
        self.op = op

    def backwards(self):
        topo = []
        visited = set()
        def build_topo(v):
            if v not in visited:
                visited.add(v)
                for child in v.prev:
                    build_topo(child)
                topo.append(v)
        build_topo(self)
        self.grad = 1
        for v in reversed(topo):
            v._backwards()

    def __add__(self, other):
        other = other if isinstance(other, Value) else Value(other)
        out = Value(self.data + other.data, prev=[self, other], op="+"

        def backwards():
```

```
            self.grad += 1 * out.grad
            other.grad += 1 * out.grad
        out._backwards = backwards
        return out

    def pow(self, other):
        assert isinstance(other, (int, float)), "Exponent must be a sc
        out = Value(self.data ** other, prev=[self], op=f"**{other}")
        def backwards():
            self.grad += other * self.data ** (other - 1) * out.grad
        out._backwards = backwards
        return out

    def __neg__(self):
        return self * -1

    def __sub__(self, other):
        return self + (-other)

    def __truediv__(self, other):
        return self * other.pow(-1)


    def __radd__(self, other):
        return self + other

    def __mul__(self, other):
        other = other if isinstance(other, Value) else Value(other)
        out = Value(self.data * other.data, prev=[self, other], op="*"
        def backwards():
            self.grad += other.data * out.grad
            other.grad += self.data * out.grad
        out._backwards = backwards
        return out

    def __rmul__(self, other):
        return self * other

    def tanh(self):
        tanh_value = (np.exp(self.data) - np.exp(-1 * self.data)) / (n
        out = Value(tanh_value , prev=[self], op="tanh")
        def backwards():
            self.grad += (1 - tanh_value ** 2) * out.grad
        out._backwards = backwards
        return out

    def exp(self):
        out = Value(np.exp(self.data), prev=[self], op="exp")
        def backwards():
            self.grad += np.exp(self.data) * out.grad
        out._backwards = backwards
        return out
```

```
    def __repr__(self):
        return f"Value(label={self.label}, data={self.data}, grad={sel

    def _build(self):
        """builds a set of all nodes and edges in a graph"""
        nodes, edges = set(), set()

        def build(v):
            if v not in nodes:
                nodes.add(v)
                for child in v.prev:
                    edges.add((child, v))
                    build(child)
        build(self)
        return nodes, edges

    def draw_dot(self):
        """Creates a visualization of the computation graph"""
        dot = Digraph(format='svg', graph_attr={'rankdir': 'LR'})

        nodes, edges = self._build()

        # Add all nodes to graph
        for n in nodes:
            uid = str(id(n))
            # Create a node label with data and optional label
            node_label = f"data {n.data:.4f}"
            if n.label:
                node_label += f" | label {n.label}"

            if n.grad:
                node_label += f" | grad {n.grad}"

            # Add the node as a box
            dot.node(name=uid,
                    label=node_label,
                    shape='record')

            # If it's an operation result, add the operation node
            if n.op:
                op_id = uid + n.op
                dot.node(name=op_id, label=n.op, shape='circle')
                dot.edge(op_id, uid)

        # Add edges between nodes
        for n1, n2 in edges:
            dot.edge(str(id(n1)), str(id(n2)) + n2.op)

        return dot
```

Manual backpropagation

A neuron can be expressed as

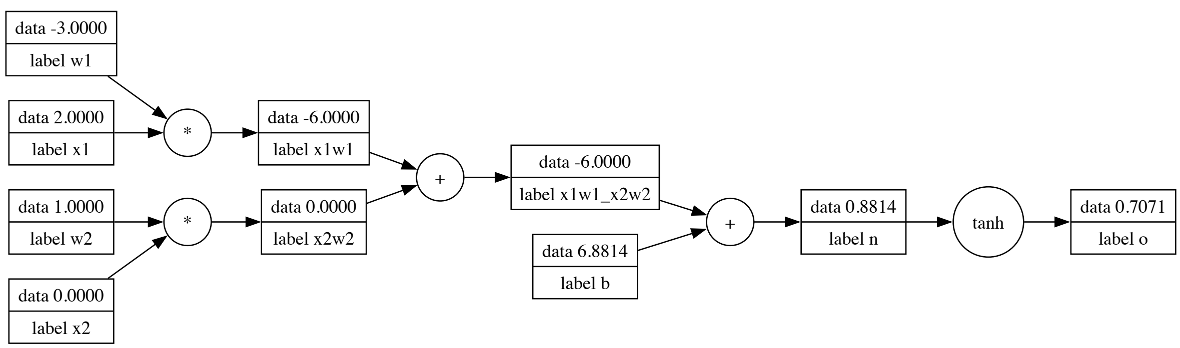
$$o = \tanh(x_1 w_1 + x_2 w_2 + b)$$

We will back propagate the neuron at x1 = 2, x2 = 0, w1= -3, w2 =1, b = 6.8814

In [19]:
```
#  x1 = 2, x2 = 0, w1= -3, w2 =1, b = 6.8814
x1 = Value(2.0, label="x1")
x2 = Value(0, label="x2")
b = Value(6.8814, label="b")
w1 = Value(-3.0, label="w1")
w2 = Value(1.0, label="w2")

x1w1 = x1 * w1; x1w1.label = "x1w1"
x2w2 = x2 * w2; x2w2.label = "x2w2"
x1w1_x2w2 = x1w1 + x2w2 ; x1w1_x2w2.label = "x1w1_x2w2"
n = x1w1_x2w2 + b; n.label = "n"
o = n.tanh(); o.label = "o"
o.draw_dot()
```

Out[19]:



Output Gradient ∂o/∂o

By definition ∂o/∂o = 1

∂o/∂n

```
o = tanh(n)
∂o/∂n = 1 - tanh²(n) = 1 - 0.7071² = 0.5
```

∂o/∂x1w1_x2w2 and ∂o/∂b

```
n = x1w1_x2w2 + b
∂o/∂(x1w1_x2w2) = ∂o/∂n ∗ ∂n/∂(x1w1_x2w2)
                = 0.5 ∗ 1 = 0.5    [since ∂n/
∂(x1w1_x2w2) = 1 for addition]

∂o/∂b = ∂o/∂n ∗ ∂n/∂b
      = 0.5 ∗ 1 = 0.5      [since ∂n/∂b = 1 for
addition]
```

∂o/∂x1w1 and ∂o/∂x2w2

```
x1w1_x2w2 = x1w1 + x2w2

∂o/∂(x1w1) = ∂o/∂(x1w1_x2w2) ∗ ∂(x1w1_x2w2)/∂(x1w1)
           = 0.5 ∗ 1 = 0.5    [since ∂(x1w1_x2w2)/
∂(x1w1) = 1 for addition]

∂o/∂(x2w2) = ∂o/∂(x1w1_x2w2) ∗ ∂(x1w1_x2w2)/∂(x2w2)
           = 0.5 ∗ 1 = 0.5    [since ∂(x1w1_x2w2)/
∂(x2w2) = 1 for addition]
```

∂o/∂w1, ∂o/∂x1

```
∂o/∂w1 = ∂o/∂(x1w1) ∗ ∂(x1w1)/∂w1
       = 0.5 ∗ x1               [since ∂(x1w1)/∂w1 =
x1]
       = 0.5 ∗ 2 = 1           [given x1 = 2]

∂o/∂x1 = ∂o/∂(x1w1) ∗ ∂(x1w1)/∂x1
       = 0.5 ∗ w1              [since ∂(x1w1)/∂x1 = w1]
       = 0.5 ∗ −3 = −1.5       [given w1 = −3]
```

∂o/∂w2, ∂o/∂x2

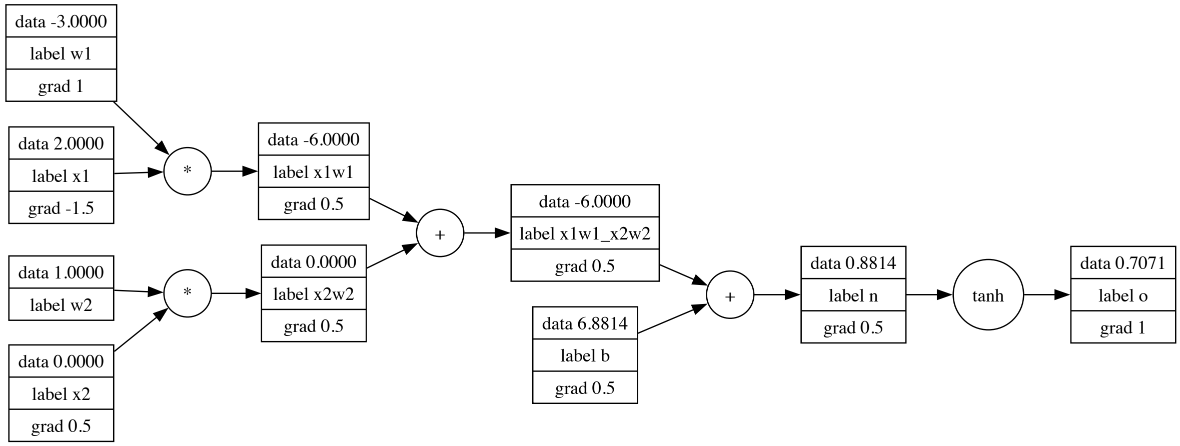
```
x2w2 = x2 ∗ w2

∂o/∂w2 = ∂o/∂(x2w2) ∗ ∂(x2w2)/∂w2
       = 0.5 ∗ x2               [since ∂(x2w2)/∂w2 =
x2]
       = 0.5 ∗ 0 = 0           [given x2 = 0]

∂o/∂x2 = ∂o/∂(x2w2) ∗ ∂(x2w2)/∂x2
       = 0.5 ∗ w2              [since ∂(x2w2)/∂x2 = w2]
```

In [20]:
```
x2.grad = 0.5; w2.grad = 0; w1.grad = 1; x1.grad = -1.5; x1w1.grad = 0
o.draw_dot()
```

Out[20]:



Automated backpropagation

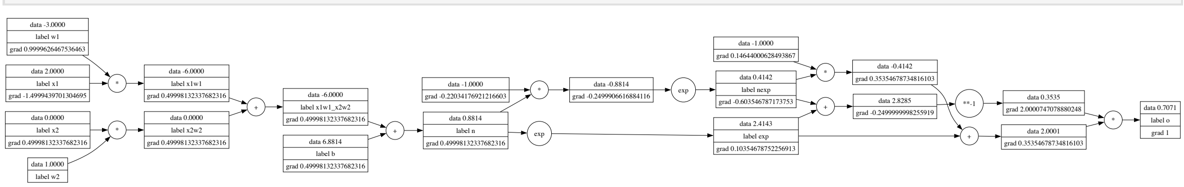
We can recursively backpropagate from `o` to derive gradient of above equation tree.

In [21]:
```
x1 = Value(2.0, label="x1")
x2 = Value(0, label="x2")
b = Value(6.8814, label="b")
w1 = Value(-3.0, label="w1")
w2 = Value(1.0, label="w2")

x1w1 = x1 * w1; x1w1.label = "x1w1"
x2w2 = x2 * w2; x2w2.label = "x2w2"
x1w1_x2w2 = x1w1 + x2w2 ; x1w1_x2w2.label = "x1w1_x2w2"
n = x1w1_x2w2 + b; n.label = "n"
# exp = (2*n).exp(); exp.label = "exp"
# o = (exp - 1)/(exp + 1); o.label = "o"
# o.backwards()
# o.draw_dot()

exp = n.exp(); exp.label = "exp"
nexp = (-1*n).exp(); nexp.label = "nexp"
o = (exp - nexp)/(exp + nexp); o.label = "o"
o.backwards()
o.draw_dot()
```

Out[21]:



Backpropagation using PyTorch

In [22]:
```
import torch
x1  = torch.Tensor([2.0]).double() ; x1.requires_grad = True
x2  = torch.Tensor([0.0]).double() ; x2.requires_grad = True
b  = torch.Tensor([6.8814]).double() ; b.requires_grad = True
w1  = torch.Tensor([-3.0]).double() ; w1.requires_grad = True
w2  = torch.Tensor([1.0]).double() ; w2.requires_grad = True

n = x1*w1 + x2*w2 + b
o = torch.tanh(n)
o.backward()
print(o.data.item())
print(x1.grad.item(), x2.grad.item(), w1.grad.item(), w2.grad.item(),
```

```
0.7071200415967962
-1.4999437403164355 0.49998124677214517 0.9999624935442903 0.0 0.49998
124677214517
```