Aim

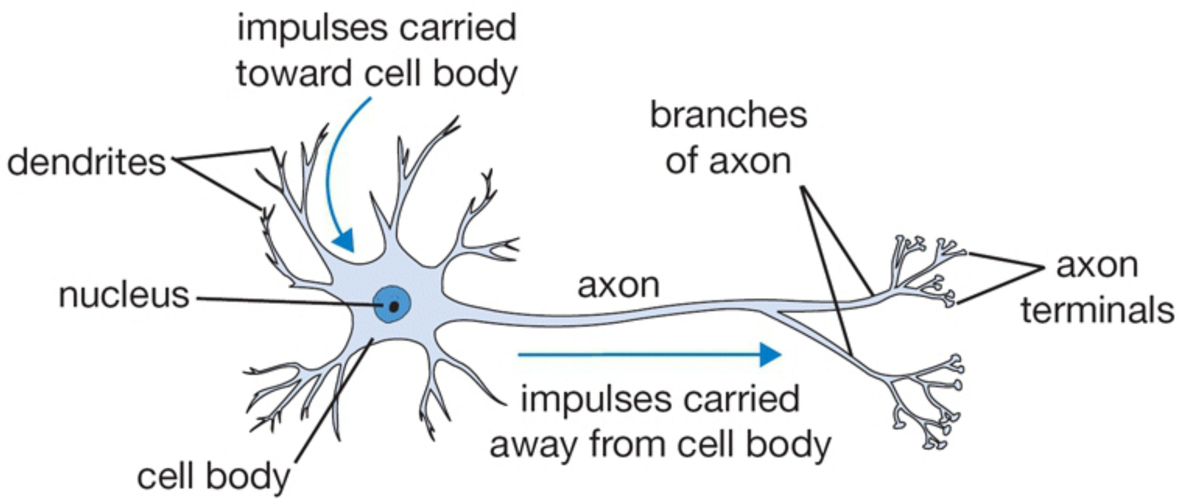
- Understand a neuron
- Manually backpropagate a neuron to minimize a loss function

Modeling one neuron / Biological motivations and connections

- Neuron is a basic computational unit of a brain
- Each neuron receives input signal from dendrites and produces output signal via axon
- Axon of a neuron connects to dendrites via synapse (weight, w)
- Signals travel along axons (x) and interact multiplicatively with synapse (wx) with the dendrite of another neuron
- A neuron sums wx from all the dendrites-synapse and adds a bias b. If the sum is above a threshold it will fire sending a spike via axon to the connected neuron.
- The synaptic strengths (i.e., w) and the bias can be learned and we can control the influence of one neuron on another
- Activation function is used to bound the signal on axon within a specific range (e.g., -1 to 1). Commonly used activation functions are sigmoid, tanh, ReLU, Leaky ReLU, Maxout

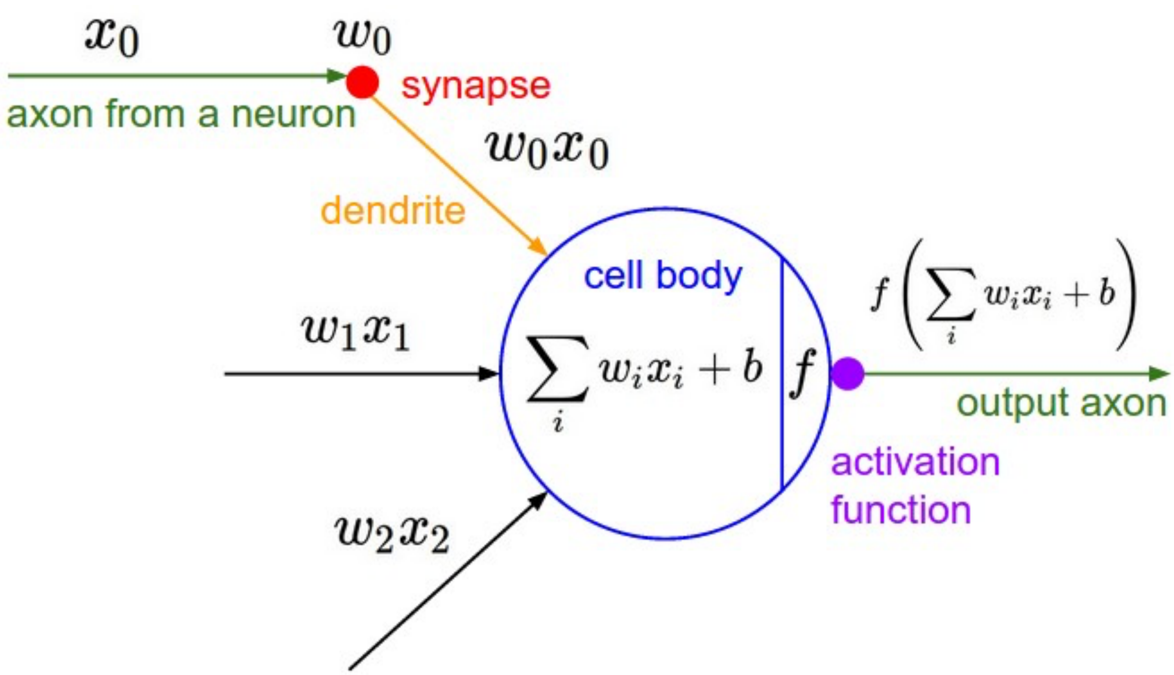
In [6]:
```
from IPython.display import Image
from IPython.core.display import HTML
PATH_NEURON = "/Users/sunil/Yashvi/img/neuron.png"
Image(filename = PATH_NEURON )
```

Out[6]:



In [7]:
```
PATH_NEURON_MODEL = "/Users/sunil/Yashvi/img/neuron_model.jpeg"
Image(filename = PATH_NEURON_MODEL )
```

Out[7]:



A basic neuron is expressed as $f(\sum_i w_i x_i + b)$

Where:

- The inputs x_i (signals from connected neuron's axon) are multiplied by their corresponding weights w_i (synapse)
- All weighted inputs are summed together ($\sum_i w_i x_i$)
- A bias term b is added
- The result is passed through an activation function f

Activation Function

Sigmoid

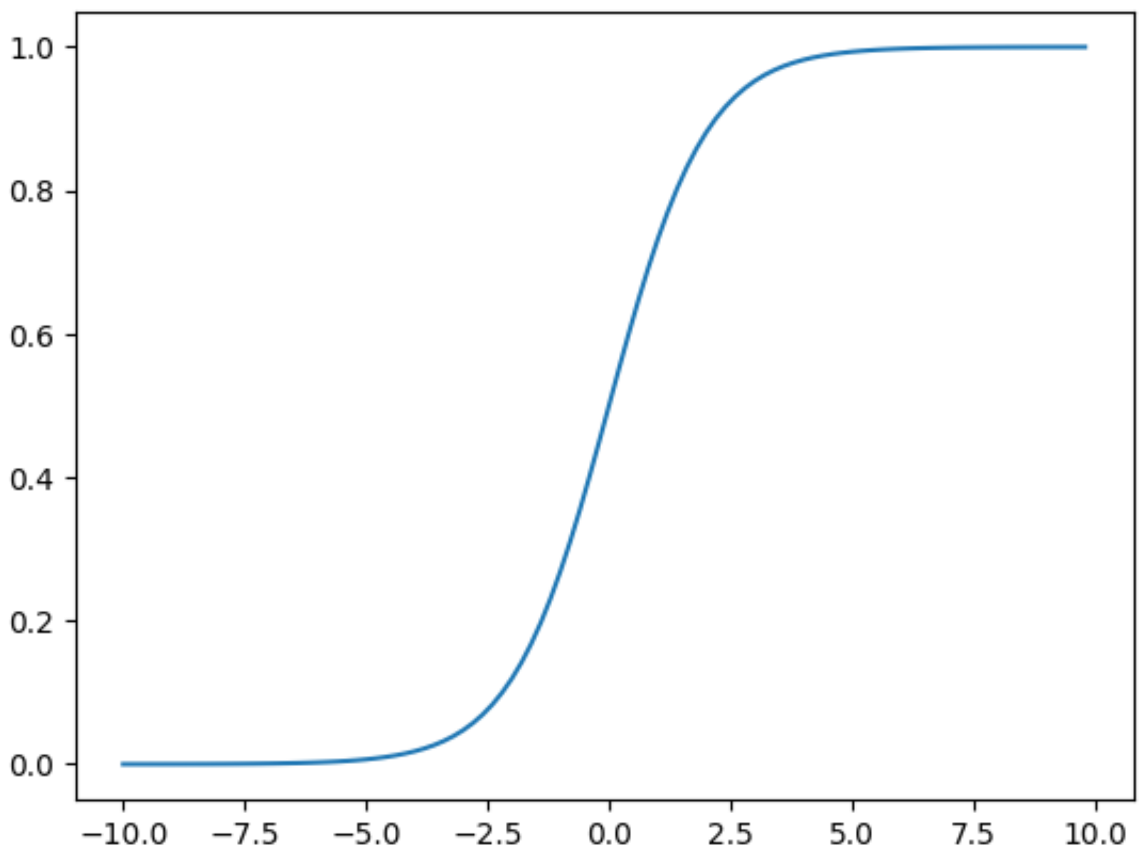
The sigmoid function is defined as: $\sigma(x) = \frac{1}{1+e^{-x}}$

In [ ]:
```
import math
import numpy as np
import matplotlib.pyplot as plt

def sigmoid(x):
    return 1 / (1 + np.exp(-x))

# Example usage
xs = np.arange(-10, 10, 0.2)
ys = sigmoid(xs)
plt.plot(xs, ys)
```

Out[ ]:
```
[<matplotlib.lines.Line2D at 0x10cc17f40>]
```



- When x is a large negative number → output approaches 0
- When x is a large positive number → output approaches 1
- When x = 0 → output = 0.5

Derivative of sigmoid

$$\sigma'(x) = \sigma(x)(1 - \sigma(x))$$

Drawbacks of sigmoid function

The Saturation

When x is very large positive or negative, the derivative of the sigmoid function is 0. As a result, while doing gradient descent, optimization takes very small steps and learning becomes extremely slow.

Sigmoid outputs are not zero-centered

TODO: Understand why it is a drawback

Tanh

The tanh function is defined as:

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

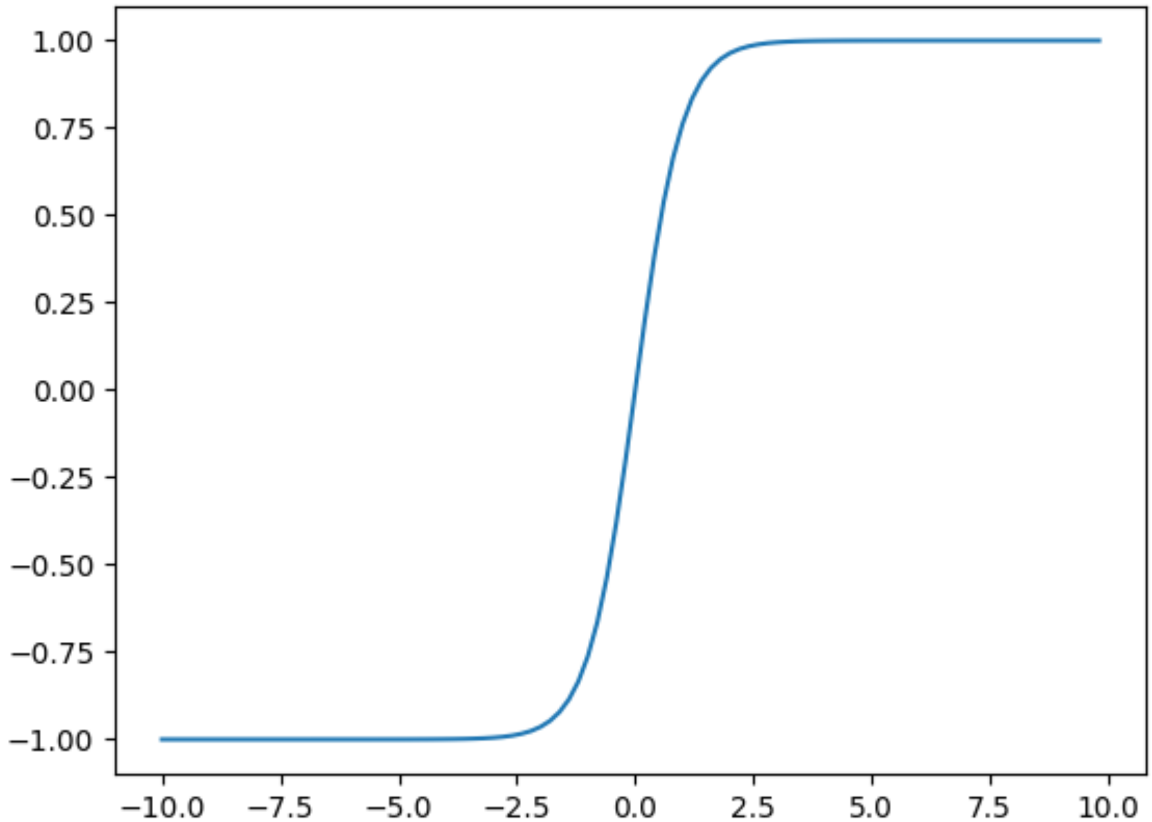
It squashes a real-valued number to the range [-1, 1].

- Unlike sigmoid, tanh is zero-centered.

In [9]:
```
def tanh(x):
    return (np.exp(x) - np.exp(-x)) / (np.exp(x) + np.exp(-x))

xs = np.arange(-10, 10, 0.2)
tanhy = tanh(xs)
plt.plot(xs, tanhy)
```

Out[9]:
```
[<matplotlib.lines.Line2D at 0x10cd23c10>]
```



Derivative of tanh

$\tanh'(x) = 1 - \tanh^2(x)$

ReLU

Rectified Linear Unit is defined as:

$f(x) = \max(0, x)$

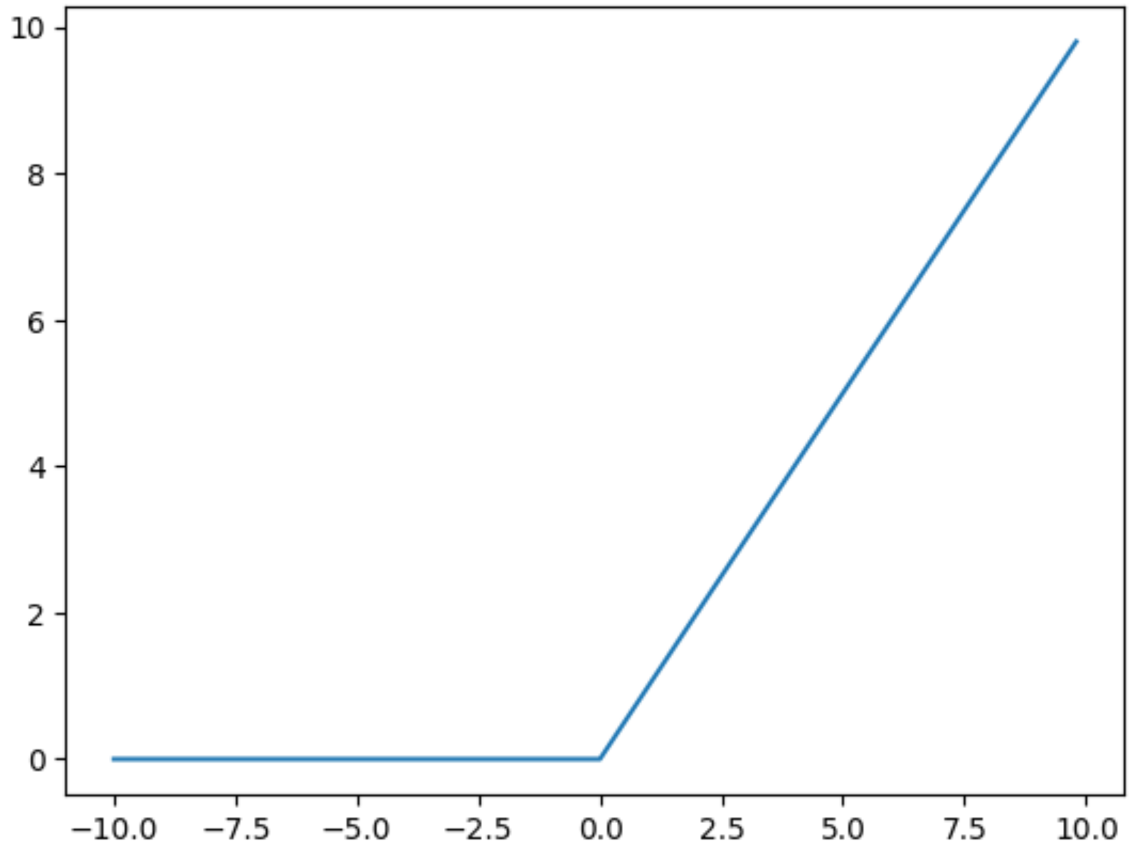
Derivative of ReLU

The derivative of ReLU: $f'(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \\ \text{undefined} & \text{if } x = 0 \end{cases}$

ReLU is computationally very simple.

In [10]:
```
def draw_relu():
    xs = np.arange(-10, 10, 0.2)
    ys = np.maximum(xs, 0)
    plt.plot(xs, ys)

draw_relu()
```



Refereces

- Neuron : https://cs231n.github.io/neural-networks-1
- Sigmoind funcation : https://youtu.be/3nmAA30MDFg?si=Z3oeZXm8NjN2FQg0