Centrality Algorithms

PageRank Formula

PageRank is defined in the original Google paper as follows:

$$PR(u) = (1 - d) + d\left(\frac{PR(T1)}{C(T1)} + \cdots + \frac{PR(Tn)}{C(Tn)}\right)$$

where:
- We assume that a page u has citations from pages $T1$ to Tn
- d is a damping factor which is set between 0 and 1. It is usually set to 0.85. You
- can think of this as the probability that a user will continue clicking. This helps
- minimize rank sink, explained in the next section.
- $1 - d$ is the probability that a node is reached directly without following any relationships.
- $C(Tn)$ is defined as the out-degree of a node T.

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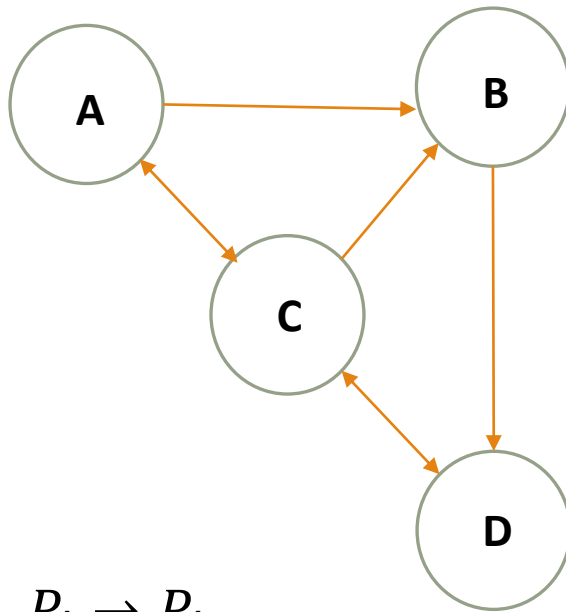
PageRank Formula – Worked Example – Iteration 1

	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4			
B	1/4			
C	1/4			
D	1/4			

$$P_i \rightarrow P_j$$

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$

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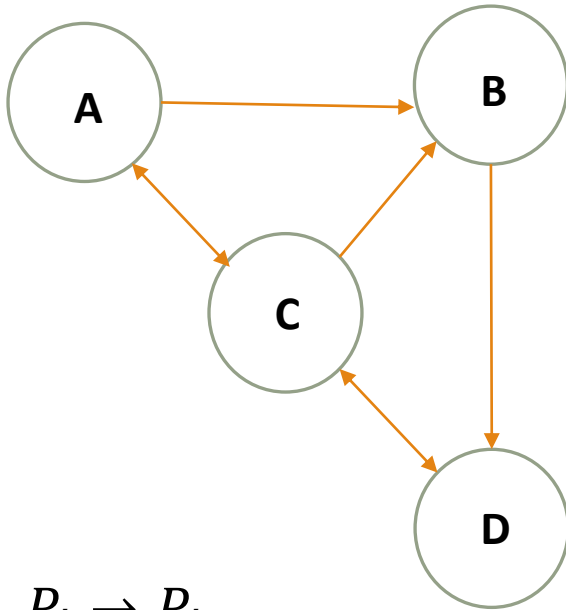
PageRank Formula – Worked Example – Iteration 1
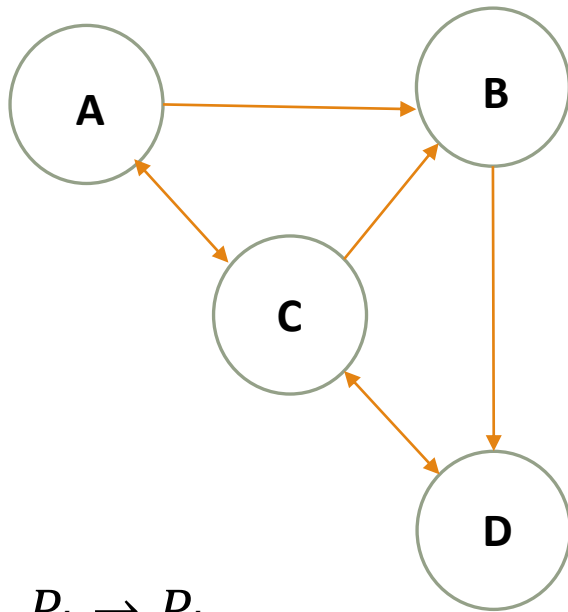
$$P_i \rightarrow P_j$$

	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4			
B	1/4			
C	1/4			
D	1/4			

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$

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PageRank Formula – Worked Example – Iteration 1

$$P_i \rightarrow P_j$$

$$PR(A) = \frac{1/4}{3} = \frac{1}{12}$$

	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4	1/12		
B	1/4			
C	1/4			
D	1/4			

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$

Centrality Algorithms

PageRank Formula – Worked Example – Iteration 1
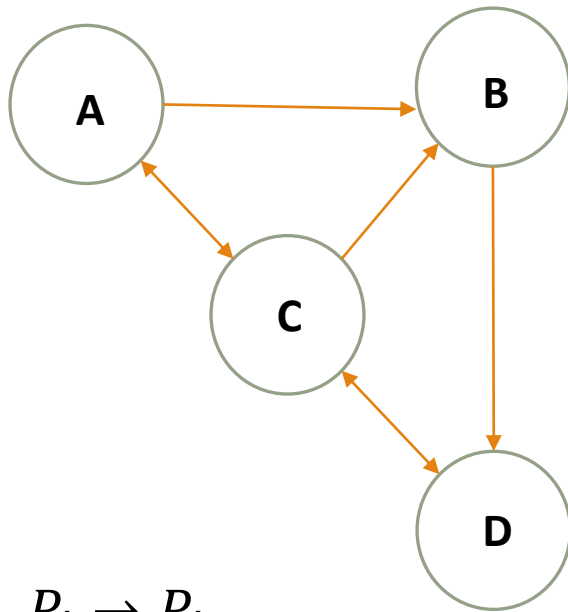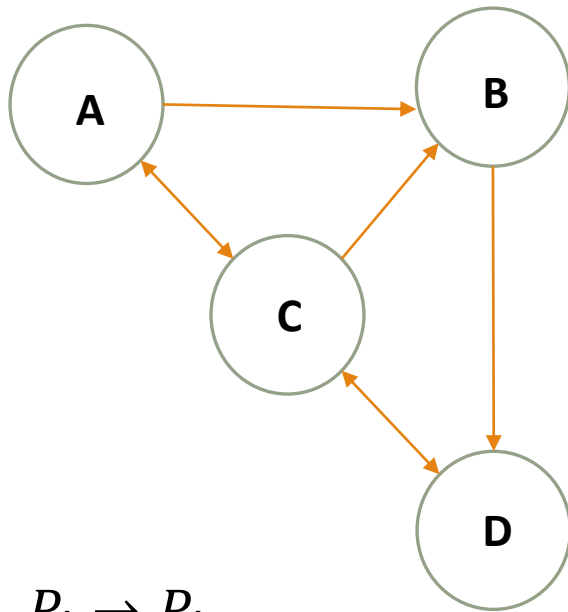

$$P_i \rightarrow P_j$$

$$PR(B) = \frac{1/4}{2} + \frac{1/4}{3} = \frac{2.5}{12}$$

	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4	1/12		
B	1/4	2.5/12		
C	1/4			
D	1/4			

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$

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PageRank Formula – Worked Example – Iteration 1

	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4	1/12		
B	1/4	2.5/12		
C	1/4	4.5/12		
D	1/4			

$$P_i \rightarrow P_j$$

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$

$$PR(C) = \frac{1/4}{2} + \frac{1/4}{1} = \frac{4.5}{12}$$

Centrality Algorithms

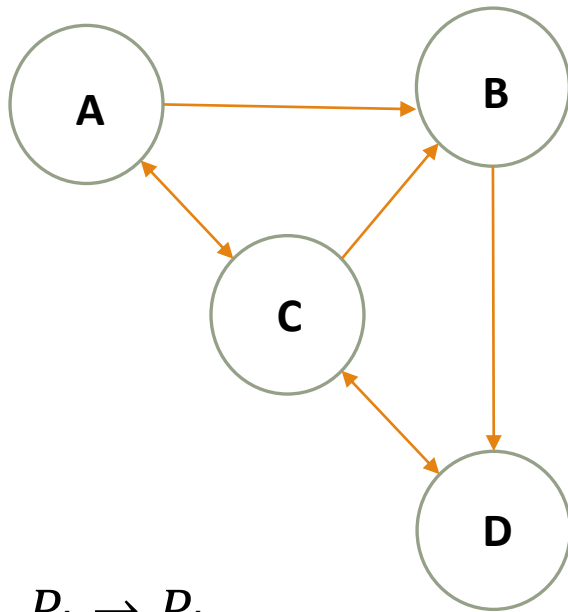## PageRank Formula – Worked Example – Iteration 1

	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4	1/12		
B	1/4	2.5/12		
C	1/4	4.5/12		
D	1/4	4/12		

$$P_i \rightarrow P_j$$

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$

$$PR(D) = \frac{1/4}{3} + \frac{1/4}{1} = \frac{4}{12}$$

Centrality Algorithms

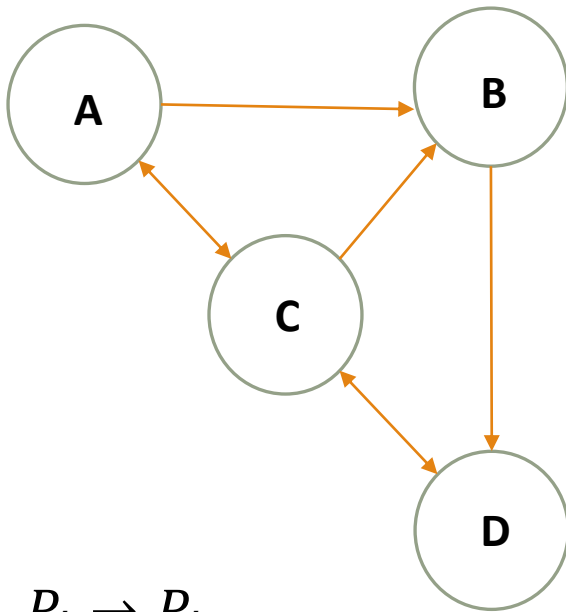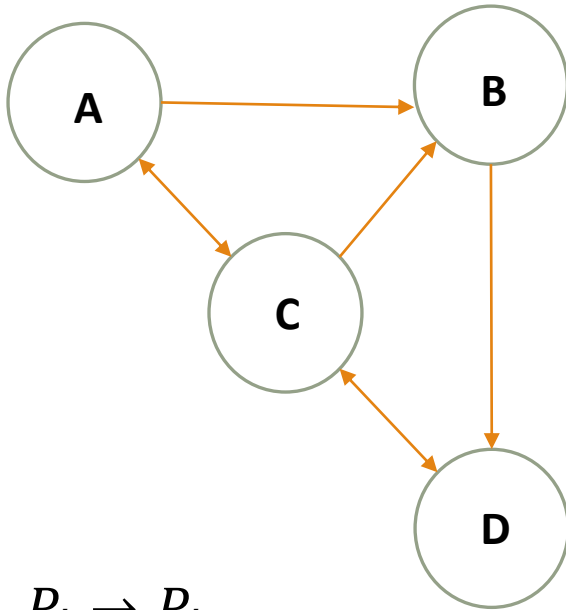## PageRank Formula – Worked Example – Iteration 2

	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4	1/12	1.5/12	
B	1/4	2.5/12		
C	1/4	4.5/12		
D	1/4	4/12		

$$P_i \rightarrow P_j$$

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$

$$PR(A) = \frac{4.5/12}{3} = \frac{1.5}{12}$$

Centrality Algorithms

PageRank Formula – Worked Example – Iteration 2

	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4	1/12	1.5/12	
B	1/4	2.5/12	2/12	
C	1/4	4.5/12		
D	1/4	4/12		

$$P_i \rightarrow P_j$$

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$

$$\boldsymbol{PR(B)} = \frac{1/12}{2} + \frac{4.5/12}{3} = \frac{2}{12}$$

Centrality Algorithms

PageRank Formula – Worked Example – Iteration 2

	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4	1/12	1.5/12	
B	1/4	2.5/12	2/12	
C	1/4	4.5/12	4.5/12	
D	1/4	4/12		

$$P_i \rightarrow P_j$$

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$

$$PR(C) = \frac{1/12}{2} + \frac{4/12}{1} = \frac{4.5}{12}$$

Centrality Algorithms

PageRank Formula – Worked Example – Iteration 2

	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4	1/12	1.5/12	
B	1/4	2.5/12	2/12	
C	1/4	4.5/12	4.5/12	
D	1/4	4/12	4/12	

$$P_i \rightarrow P_j$$

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$

$$PR(D) = \frac{4.5/12}{3} + \frac{2.5/12}{1} = \frac{4}{12}$$

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PageRank Formula – Worked Example – PageRank
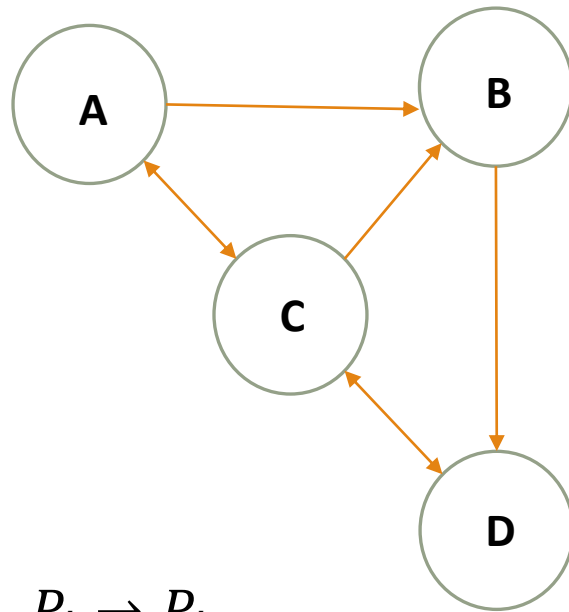
	Iteration 0	Iteration 1	Iteration 2	PageRank
A	1/4	1/12	1.5/12	1
B	1/4	2.5/12	2/12	2
C	1/4	4.5/12	4.5/12	4
D	1/4	4/12	4/12	3

$$P_i \rightarrow P_j$$

$$PR_{t+1}(P_i) = \frac{\sum_j PR_t(P_j)}{C(P_j)}$$