Dijkstra’s Shortest Path Algorithm in Concept

Let distance of start vertex from start vertex = 0

Let distance of all other vertices from start = ∞

Repeat

    Visit the unvisited vertex with the smallest known distance from the start vertex

    For the current vertex, examine its unvisited neighbours

    For the current vertex, calculate distance of each neighbour from start vertex

    If the calculated distance of the vertex is less than the known distance, update the shortest distance

    Update the previous vertex for each of the updated distances

    Add the current vertex to the list of updated vertices

Until all vertices finished

Dijkstra’s Shortest Path Algorithm Formalism

Let G be a directed network with vertices V = {1, . . . ,n} such that, for each arc ij, its length $d_{ij} > 0$.

The essence of Dijkstra’s algorithm is a labelling procedure.

At a typical stage of the algorithm we have V = P ∪ S, where

- P is the set of permanently labelled vertices, and
- S is the set of temporarily (or tentatively) labelled vertices.

To begin with, S = V , then as the algorithm proceeds, S gets smaller, while P gets bigger until eventually

all vertices have received permanent labels and the algorithm terminates.

In fact, each vertex u will have two labels:

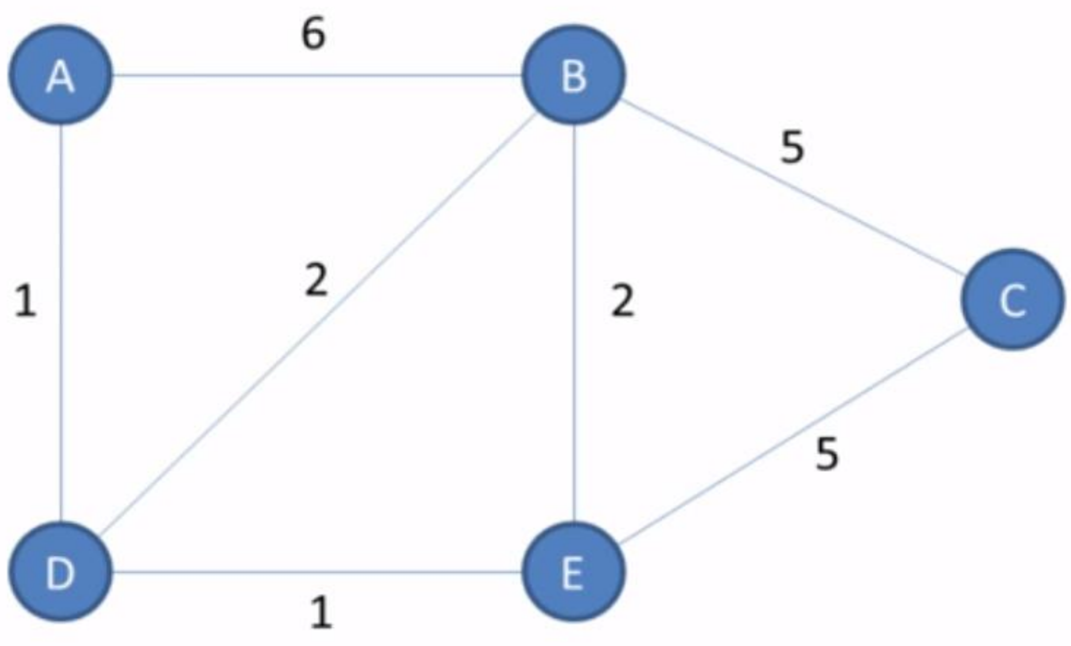
- D_u, our current guess at the distance from 1 to u;
- p_u, our current guess at the parent of u

Dijkstra’s Shortest Path Algorithm Pseudo Code

Algorithm 5.2: *Dijkstra*(r, G, T)

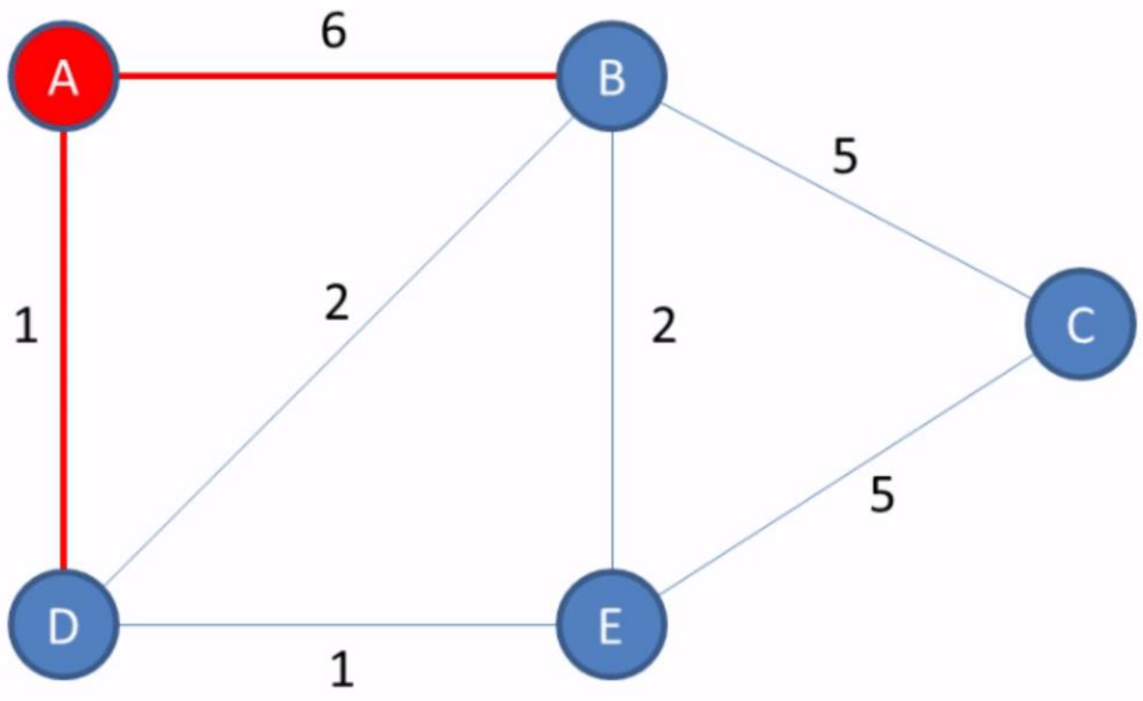
```
P := {1};   S := V ∖ P = {2, ..., n};   A := ∅;          // initialise P, S and A
D_1 := 0;
for each vertex u ∈ S do
    D_u := d_1u;   p_u := 1;                            // initialise D and p labels
end for
while S ≠ ∅ do
    Select u ∈ S with D_u = min_{v∈S} D_v;
    P := P ∪ {u};   S := S ∖ {u};                       // move u from S to P
    A := A ∪ {(p_u, u)};
    for all v ∈ S do                                    // note u is no longer in S
        if D_v > D_u + d_uv then
            D_v := D_u + d_uv;                          // relax inequality
            p_v := u;                                   // set the parent of v to be u
        end if
    end for
end while
```

Dijkstra’s Shortest Path Algorithm Solution



Vertex	Shortest distance from A	Previous vertex
A	0	
B	3	D
C	7	E
D	1	A
E	2	D

Dijkstra’s Shortest Path Algorithm Worked Example

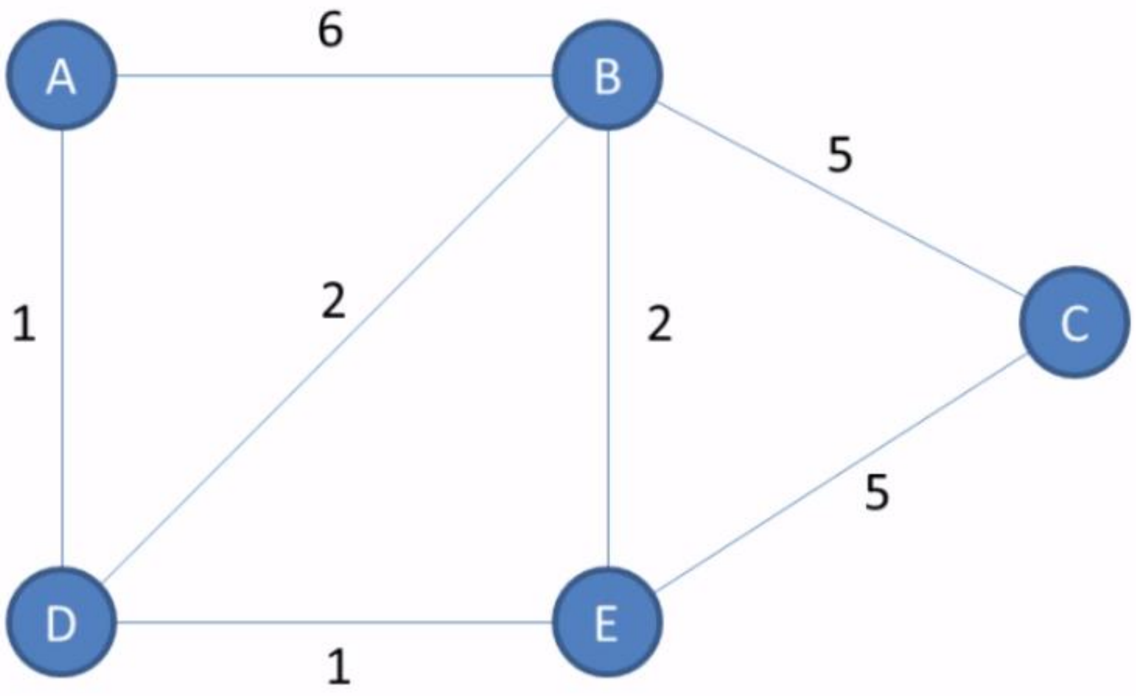


Vertex	Shortest distance from A	Previous vertex
A	0	
B	∞	
C	∞	
D	∞	
E	∞	

Visited = [ ]

Unvisited = [A, B, C, D, E]

Dijkstra’s Shortest Path Algorithm Worked Example

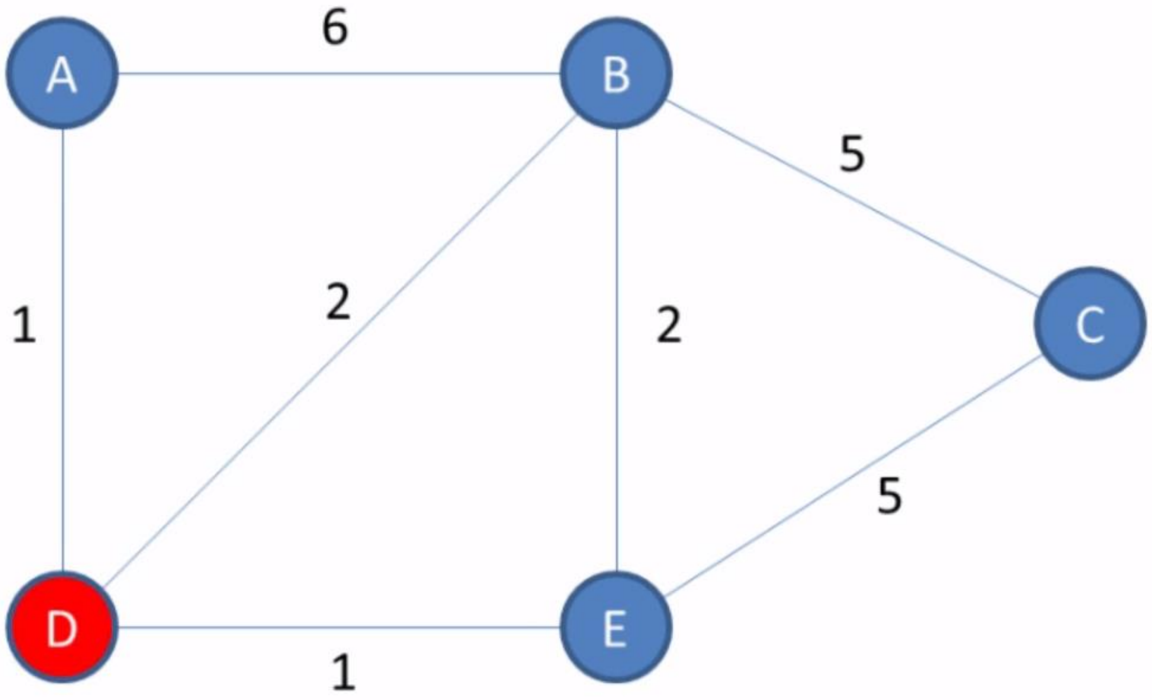


Vertex	Shortest distance from A	Previous vertex
A	0	
B	6	A
C	∞	
D	1	A
E	∞	

Visited = [A]

Unvisited = [B, C, D, E]

Dijkstra's Shortest Path Algorithm Worked Example

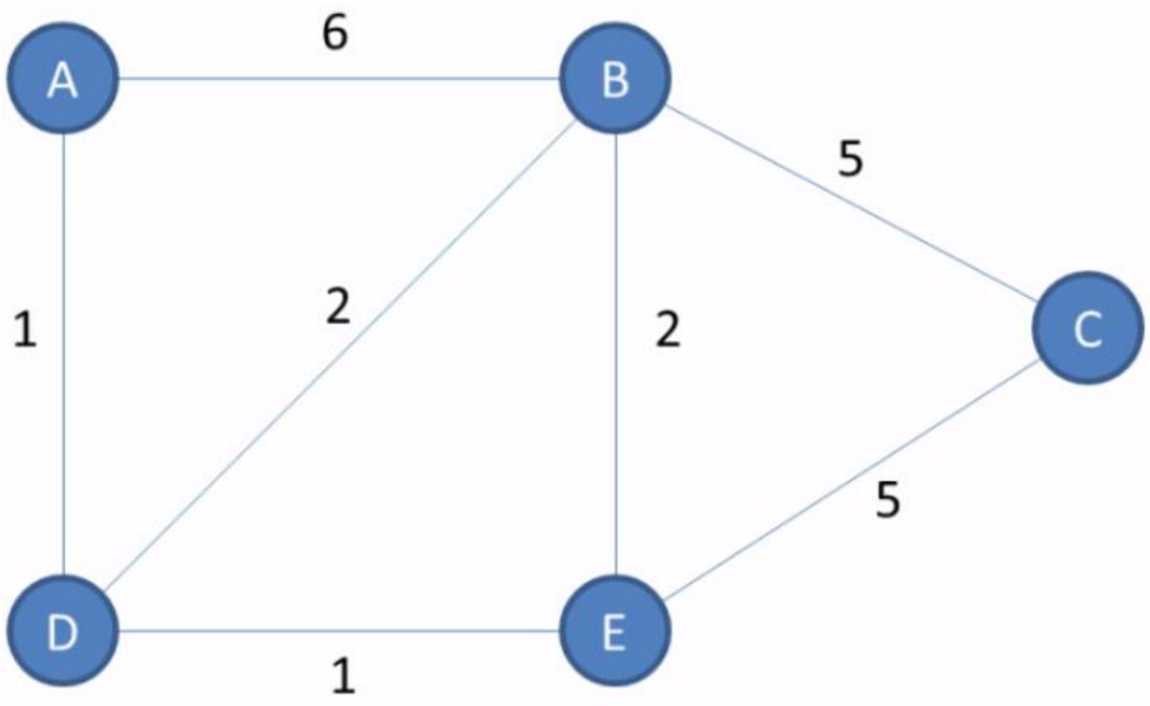


Vertex	Shortest distance from A	Previous vertex
A	0	
B	6	A
C	∞	
D	$\boxed{1}$	A
E	∞	

Visited = [A]

Unvisited = [B, C, D, E]

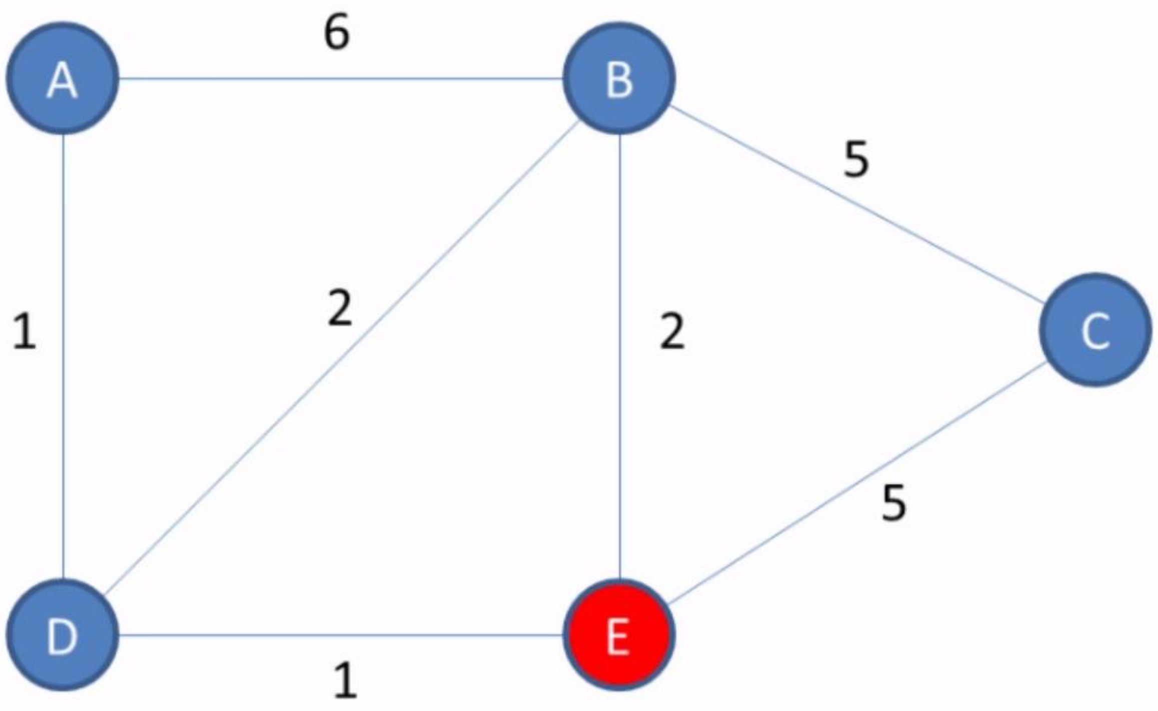
Dijkstra’s Shortest Path Algorithm Worked Example



Vertex	Shortest distance from A	Previous vertex
A	0	
B	3	D
C	∞	
D	1	A
E	2	D

Visited = [A, D]   Unvisited = [B, C, E]

Dijkstra’s Shortest Path Algorithm Worked Example

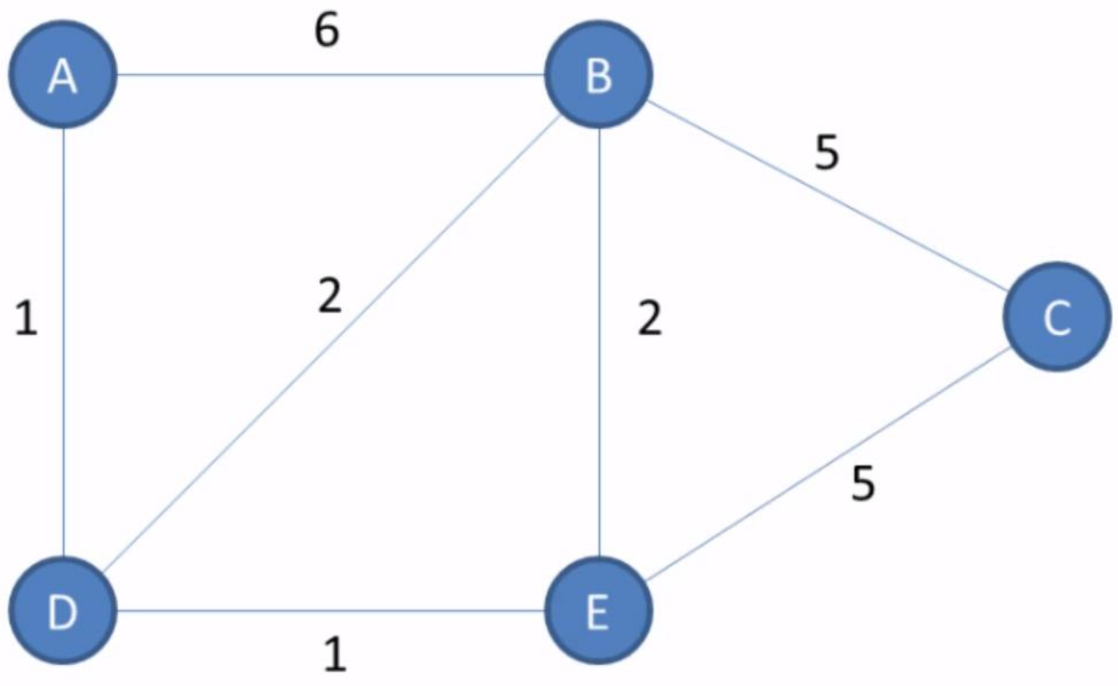


Vertex	Shortest distance from A	Previous vertex
A	0	
B	3	D
C	∞	
D	1	A
E	$\boxed{2}$	D

Visited = [A, D]

Unvisited = [B, C, E]

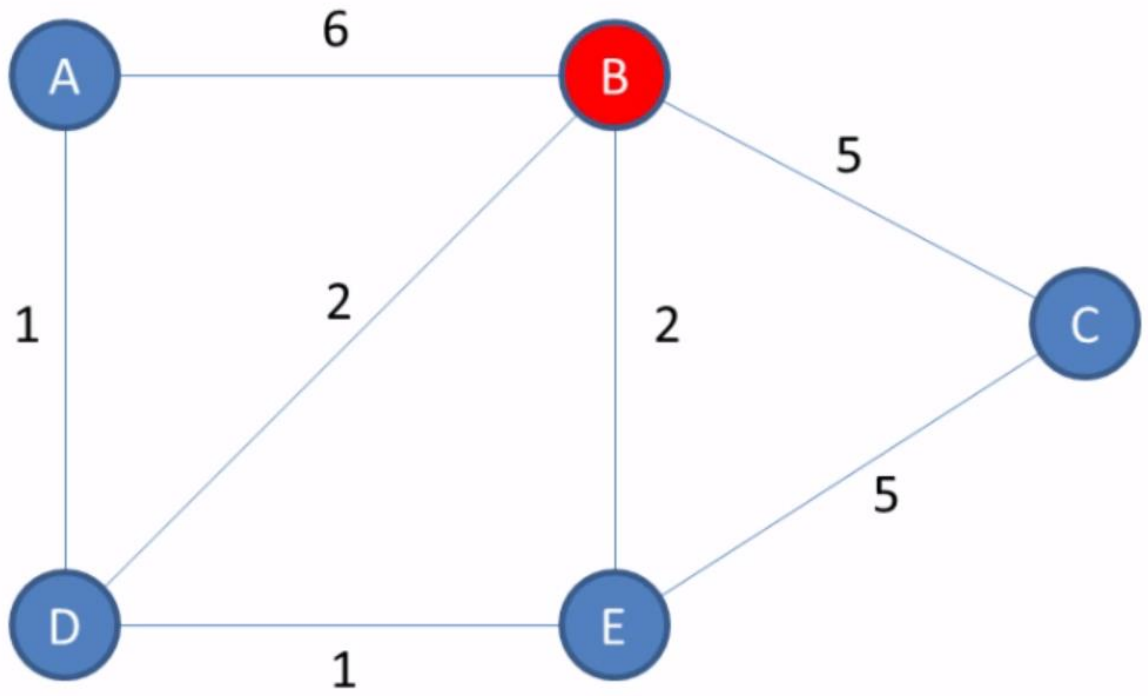
Dijkstra’s Shortest Path Algorithm Worked Example



Vertex	Shortest distance from A	Previous vertex
A	0	
B	3	D
C	7	E
D	1	A
E	2	D

Visited = [A, D, E]   Unvisited = [B, C]

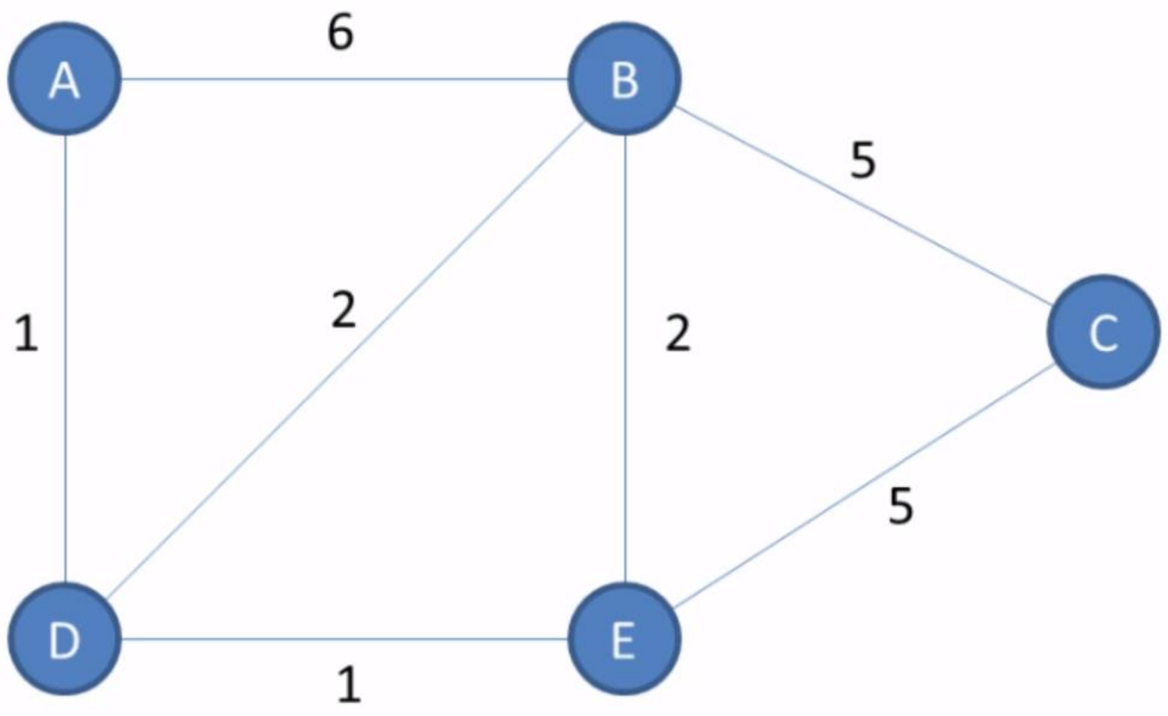
Dijkstra’s Shortest Path Algorithm Worked Example



Vertex	Shortest distance from A	Previous vertex
A	0	
B	$\boxed{3}$	D
C	7	E
D	1	A
E	2	D

Visited = [A, D, E]   Unvisited = [B, C]

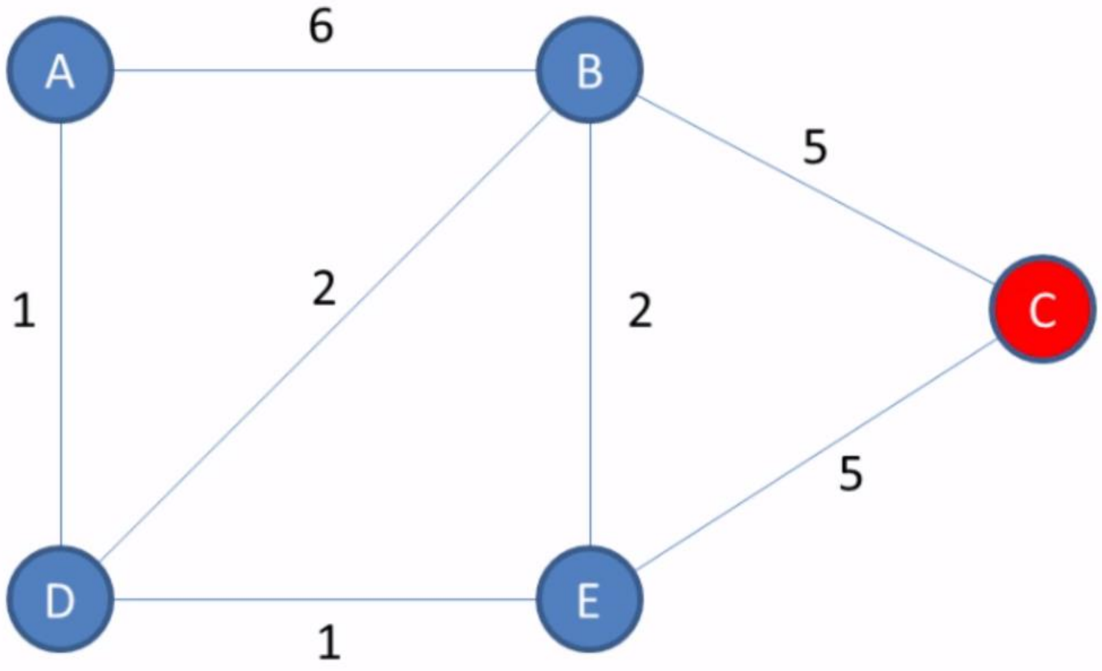
Dijkstra’s Shortest Path Algorithm Worked Example



Vertex	Shortest distance from A	Previous vertex
A	0	
B	3	D
C	7	E
D	1	A
E	2	D

Visited = [A, D, E, B]   Unvisited = [C]

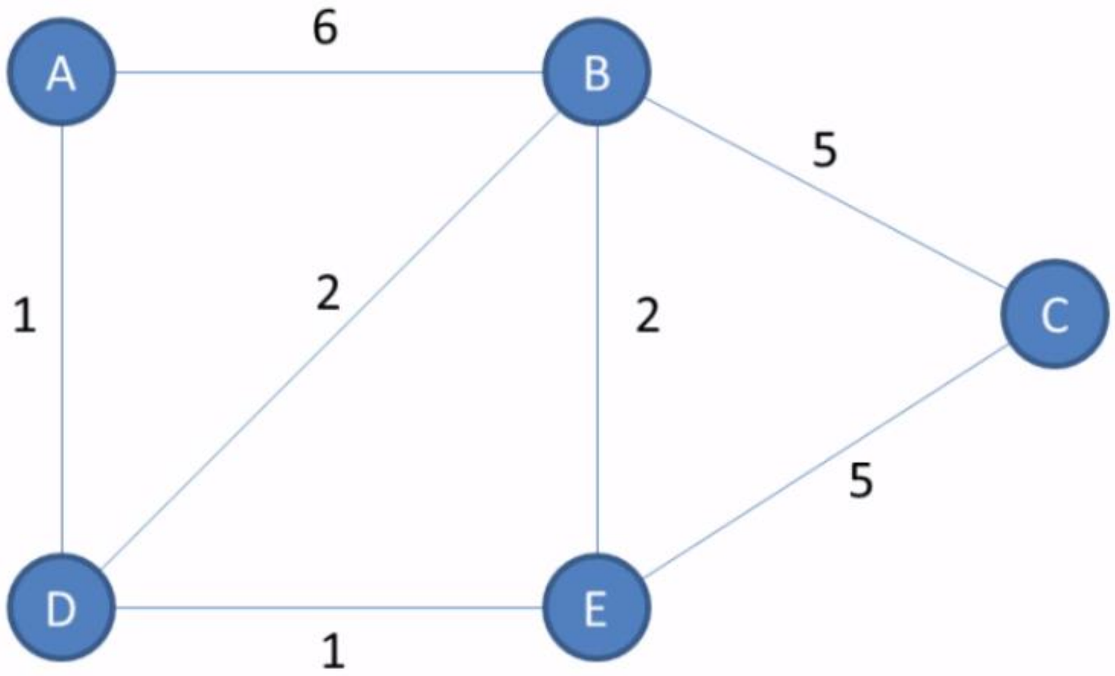
Dijkstra’s Shortest Path Algorithm Worked Example



Vertex	Shortest distance from A	Previous vertex
A	0	
B	3	D
C	7	E
D	1	A
E	2	D

Visited = [A, D, E, B]   Unvisited = [C]

Dijkstra’s Shortest Path Algorithm Worked Example



Vertex	Shortest distance from A	Previous vertex
A	0	
B	3	D
C	7	E
D	1	A
E	2	D

Visited = [A, D, E, B, C]   Unvisited = []