Maths Skills in Science GCSE (ITTECF 3, 4, 6)

'Schools should tackle in-school inconsistency of teaching, making more good or outstanding, so that every pupil receives a good mathematics education' (OFSTED 2022)

Session Leader:- Pietro Tozzi

GUMLEY HOUSE
Convent School F.C.J.

- 40 years' experience of teaching/training in London comprehensive schools
- Head of Department Experience
- Mentored over 150 ITT trainees and ECTs
- Senior Lecturer & Course Lead (PGCE Maths) at St Mary's University, Twickenham
- Acted as a case study for an awarding body
- Assisted in the writing of the GCSE 2010 Modular, Linked Pair Pilot & A-Level (2017) Schemes of Work
- Nominated for a National Teaching Award

Maths Skills in Science

What the examiners said…

Maths Questions

Overall:

- Questions with a mathematical component were generally answered well.
- Maths rarely resulted in candidates failing to successfully answer a question
- When given an equation, candidates generally scored highly on calculations

Where candidates 'lost' marks:

- Standard form - often well answered, but this is where marks if any were dropped
- Significant figures
- Rounding
- Units
- Conversions

Maths Skills in Science

- Generally Maths skills were applied well

- Some specific challenges included:
  - Calculating concentration
  - Mass - mass calculations using balanced equations
  - Empirical formula
  - Calculating kinetic energy
  - SI units e.g. Mass, kg, g and N and how to handle these in calculations

Examiner report comment

- “Always show mathematical workings when doing a calculation, as (method) marks will be awarded for errors carried forward”

Maths skills in Science

- Key challenges for students include:
  - Changing the subject of an equation
  - Significant figures
  - Standard form
  - Reading, describing and interpreting graphs
  - Physics Acceleration due to gravity 9.81 ms^{-2} and for Mechanics A Level g = 9.8 ms^{-2}

Any other areas of weakness?

Maths skills in Science

Significant figures

- Examiner report comments indicate students did not always attempt to give an answer to the number of significant figures asked for OR they rounded incorrectly.

- It is useful to highlight the advantages of significant figures over decimal places by showing that the accuracy does not change when changing units, with significant figures.

  i.e. 4.2 mm is 0.42 cm is 0.0042 m all to 2 s.f. but,

  0.42 cm is **0.00 m** to 2 d.p. (!)

Maths skills in Science

Standard Form

- Examiner report comments indicate that students either missed the instruction to give the answer in standard form OR they did not know how to show their answer in this way.

- Which of the following numbers are written in standard form?
  - 16×10^3 N
  - 0.16×10^5 N
  - 1.6×10^4 N

Maths skills in Science

Standard Form

- Which of the following numbers are written in standard form?
  - 16×10^3 N is not in standard form
  - 0.16×10^5 N is not in standard form
  - **1.6×10^4 N is in standard form**

Writing large numbers in standard form e.g. 16 000 N

Write the number as a number between 1 and 10 multiplied by a power of 10

= 1.6 x 10 000 N

Write the power of 10 using indices

= 1.6 x 10^4 N

Activity

Question from Physics foundation paper involving standard form and significant figures.

(d) The mass of a proton is 1.6726×10^{-27} kg.
The mass of an electron is 9.1094×10^{-31} kg.

Calculate how many times the mass of a proton is greater than the mass of an electron.

Give your answer to two significant figures.

(3)

Discuss how you would teach students to access this question.

Guess the Success Rate?

9 $T = \sqrt{\frac{w}{d^3}}$

$w = 5.6 \times 10^{-5}$

$d = 1.4 \times 10^{-4}$

37%

(a) Work out the value of T.

Give your answer in standard form correct to 3 significant figures.

Guess the Success Rate?

9 Emily drives 186 miles in 3 hours.

(a) What is her average speed?

86%

..... mph

(2)

Sarah drives at an average speed of 58 mph for 4 hours.

(b) How many miles does Sarah drive?

25 A force of 70 newtons acts on an area of $20\,cm^2$

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

The force is increased by 10 newtons.
The area is increased by $10\,cm^2$

Helen says,

"The pressure decreases by less than 20%"

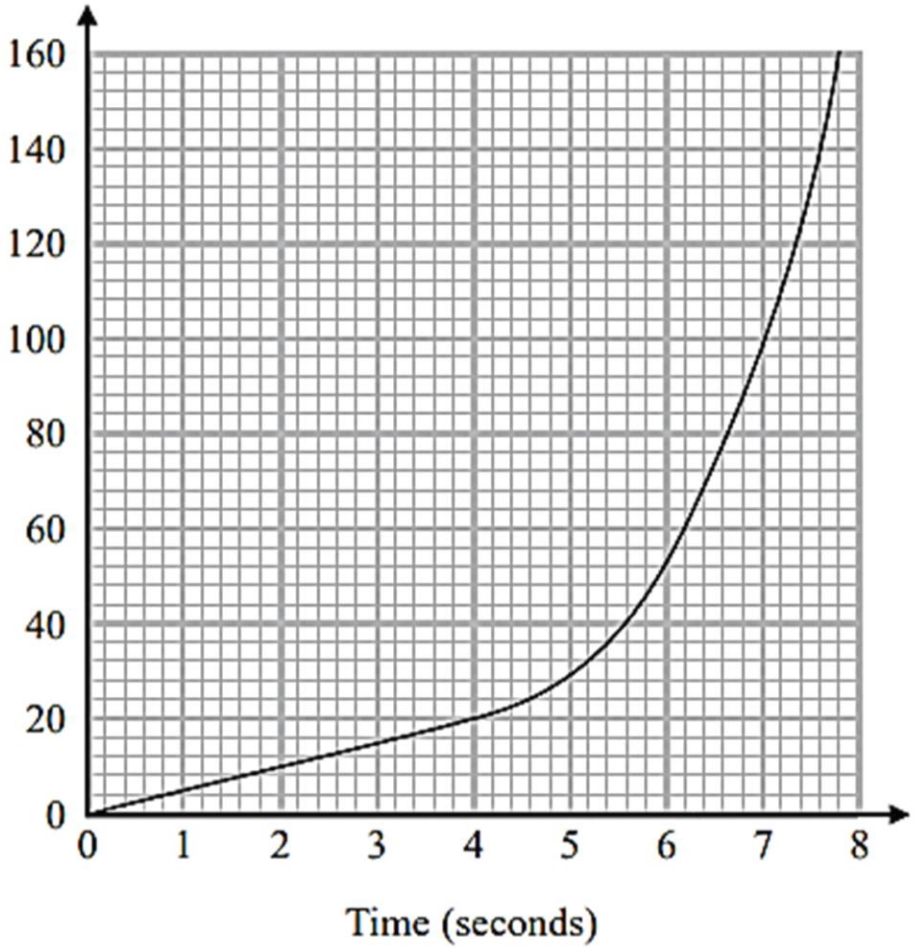
Foundation

Is Helen correct?
You must show how you get your answer.

32%

Guess the Success Rate?

14 The distance-time graph shows information about part of a car journey.



21%

Use the graph to estimate the speed of the car at time 5 seconds.

Guess the Success Rate?

21 Jackson is trying to find the density, in g/cm^3, of a block of wood.
The block of wood is in the shape of a cuboid.

He measures

the length as 13.2 cm, correct to the nearest mm
the width as 16.0 cm, correct to the nearest mm
the height as 21.7 cm, correct to the nearest mm

He measures the mass as 1970 g, correct to the nearest 5 g.

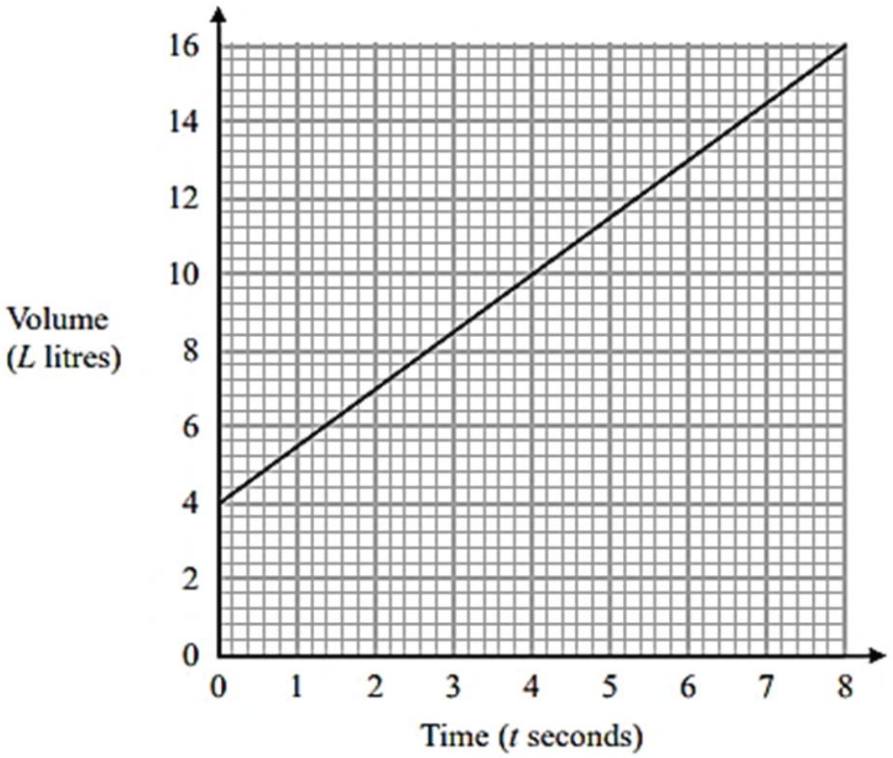
By considering bounds, work out the density of the wood.
Give your answer to a suitable degree of accuracy.

28%

You must show all your working and give a reason for your final answer.

Guess the Success Rate?

12 The graph shows the volume of liquid (L litres) in a container at time t seconds.



(a) Find the gradient of the graph.

56%

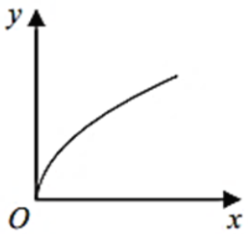
.....

(b) Explain what this gradient represents.

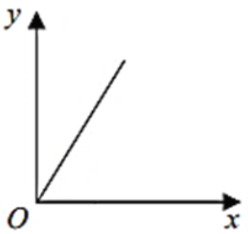
Guess the Success Rate?

12

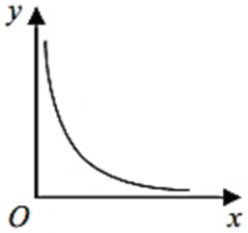
Graph A



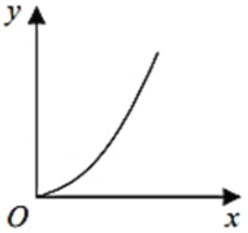
Graph B



Graph C



Graph D



The graphs of y against x represent four different types of proportionality.

Match each type of proportionality in the table to the correct graph.

56%

Type of proportionality	Graph letter
$y \propto x$	
$y \propto x^2$	
$y \propto \sqrt{x}$	
$y \propto \frac{1}{x}$	

(Total for Question 12 is 2 marks)

Guess the Success Rate?

18 Work out the value of $\dfrac{2.645 \times 10^9}{1.15 \times 10^3}$

Give your answer in standard form.

Foundation

52%

Foundation

28 Make g the subject of the formula $T = \sqrt{\dfrac{g+6}{2}}$

4%

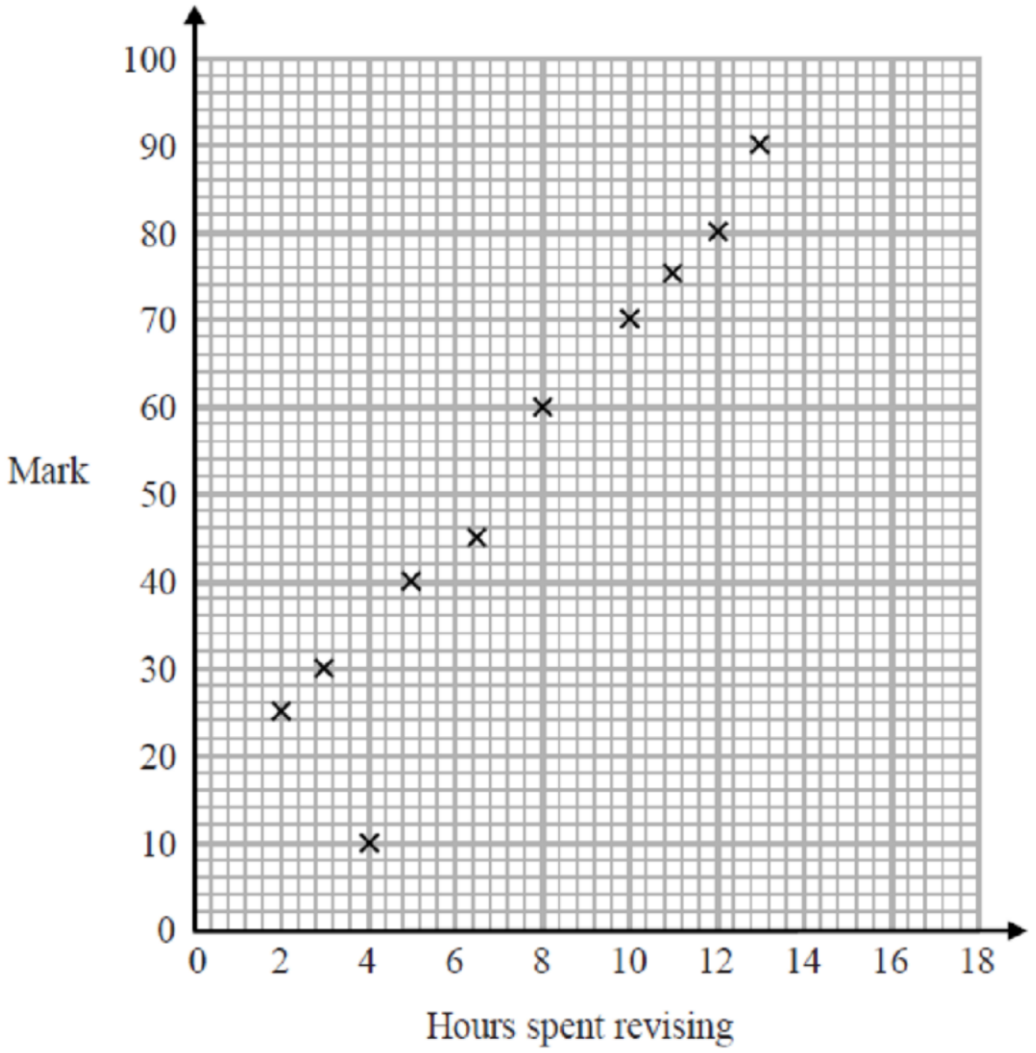
Maths requirements of GCSE Science

Mathematical Skill
1. Arithmetic and numerical computation
a) Recognise and use expressions in decimal form
b) Recognise and use expressions in standard form
c) Use ratios, fractions and percentages
d) Make estimates of the results of simple calculations
2. Handling data
a) Use an appropriate number of significant figures
b) Find arithmetic means
c) Construct and interpret frequency tables and diagrams, bar charts and histograms
d) Understand the principles of sampling as applied to scientific data (biology questions only)
e) Understand simple probability
f) Understand the terms mean, mode and median
g) Use a scatter diagram to identify a correlation between two variables
h) Make order of magnitude calculations
3. Algebra
a) Understand and use the symbols: =, <>, >, ∝ , ~
b) Change the subject of an equation
c) Substitute numerical values into algebraic equations using appropriate units for physical quantities
d) Solve simple algebraic equations
4. Graphs
a) Translate information between graphical and numeric form
b) Understand that y = mx + c represents a linear relationship
c) Plot two variables from experimental or other data
d) Determine the slope and intercept of a linear graph
e) Draw and use the slope of a tangent to a curve as a measure of rate of change
f) Understand the physical significance of area between a curve and the x-axis and measure it by counting squares as appropriate
5. Geometry and trigonometry
a) Use angular measures in degrees
b) Visualise and represent 2D and 3D forms including two dimensional representations of 3D objects
c) Calculate areas of triangles and rectangles, surface areas and volumes of cubes

Maths

3. The scatter diagram shows information about 10 students.

For each student, it shows the number of hours spent revising and the mark the student achieved in a Spanish test.



One of the points is an outlier.

(a) Write down the coordinates of the outlier.

.....

[1 mark]

(b) For all the other points

(i) draw the line of best fit,

(ii) describe the correlation.

..... [2 marks]

Maths - Answers

3. The scatter diagram shows information about 10 students.

For each student, it shows the number of hours spent revising and the mark the student achieved in a Spanish test.

One of the points is an outlier.

(a) Write down the coordinates of the outlier.

.....(4, 10).....

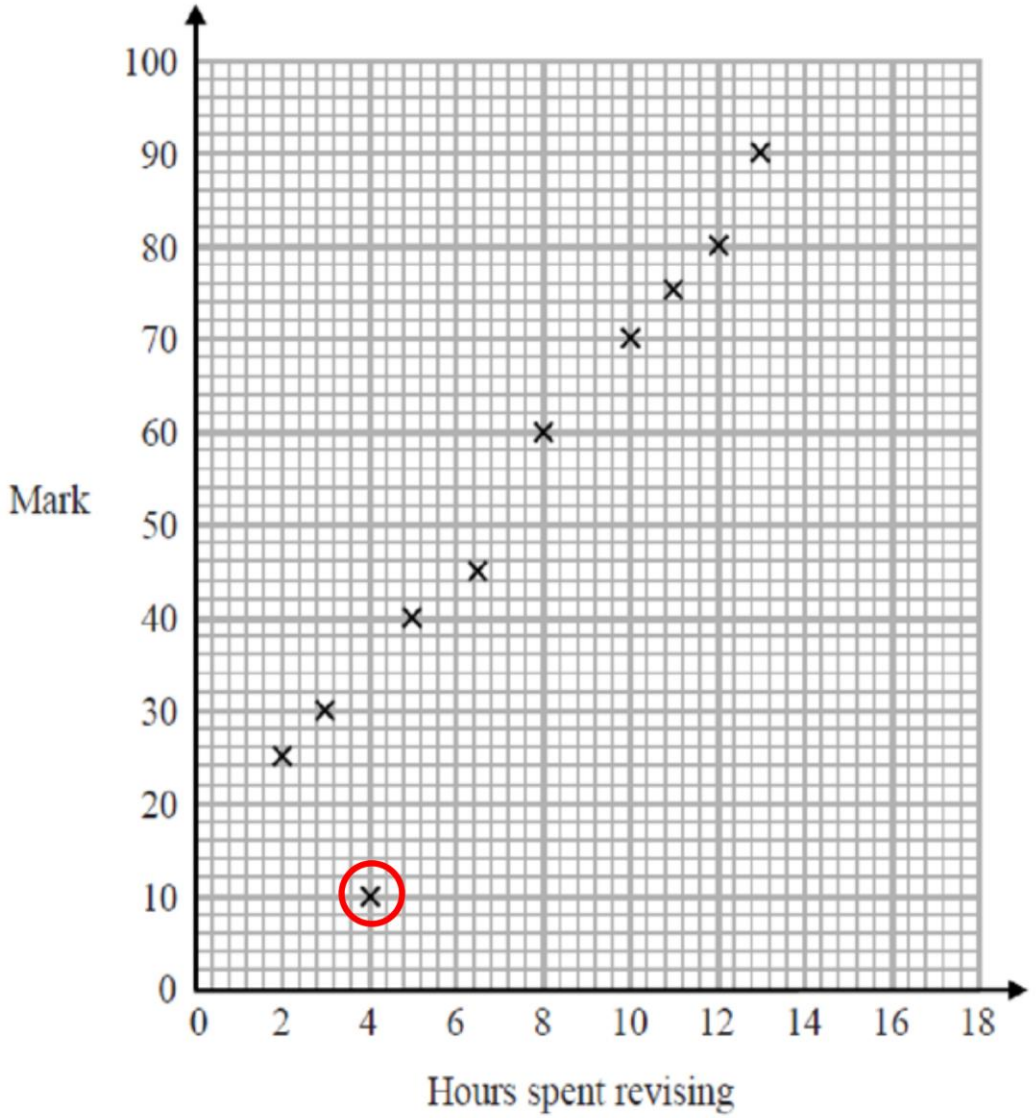
[1 mark]

(b) For all the other points

(i) draw the line of best fit,

(ii) describe the correlation.

.....positive..... [2 marks]



Scatter graphs and correlation

Relationship/correlation

- Describe the relationship:
  - As … increases … increases
  - As … increases … decreases
- State the type of correlation: positive or negative

Line of best fit

A line of best fit does not have to pass through the origin

Maths – points should be evenly distributed along the line

Science – the line should pass through as many points as possible. Science also use curves of best fit.

Outliers

- In science outliers are sometimes referred to as anomalies
- In maths students may be asked to provide a reason for the outlier e.g. an error in the measurement

4. Calculate the percentage (%) decrease in the number of seals caught from 2004 to 2010.

Year	Number of seals caught in thousands
2004	362
2005	316
2006	348
2007	224
2008	215
2009	91
2010	67

Decrease in seals = ...%

[2 marks]

4. In 1999 the minimum wage for adults was £3.60 per hour. In 2013 it was £6.31 per hour.

Work out the percentage increase in the minimum wage.

_____% [3 marks]

Maths & Science - Answers

4. Calculate the percentage (%) decrease in the number of seals caught from 2004 to 2010.

$$\frac{362 - 67}{362} \times 100 = 81.49\%$$

Decrease in seals = ...%

[2 marks]

Year	Number of seals caught in thousands
2004	362
2005	316
2006	348
2007	224
2008	215
2009	91
2010	67

4. In 1999 the minimum wage for adults was £3.60 per hour. In 2013 it was £6.31 per hour.

Work out the percentage increase in the minimum wage.

$$\frac{6.31-3.60}{3.60} \times 100 = 75\%$$

_____% [3 marks]

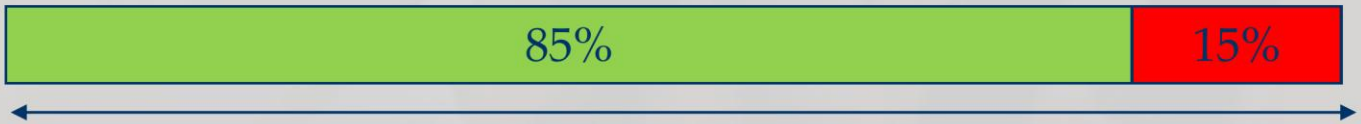
Percentage increase

- Really important in science. Regularly assessed.
- In maths more able students are encouraged to use % multipliers.

Increase by 10%

$110\% = \frac{110}{100} = 1.1$ Percentage multiplier is 1.1

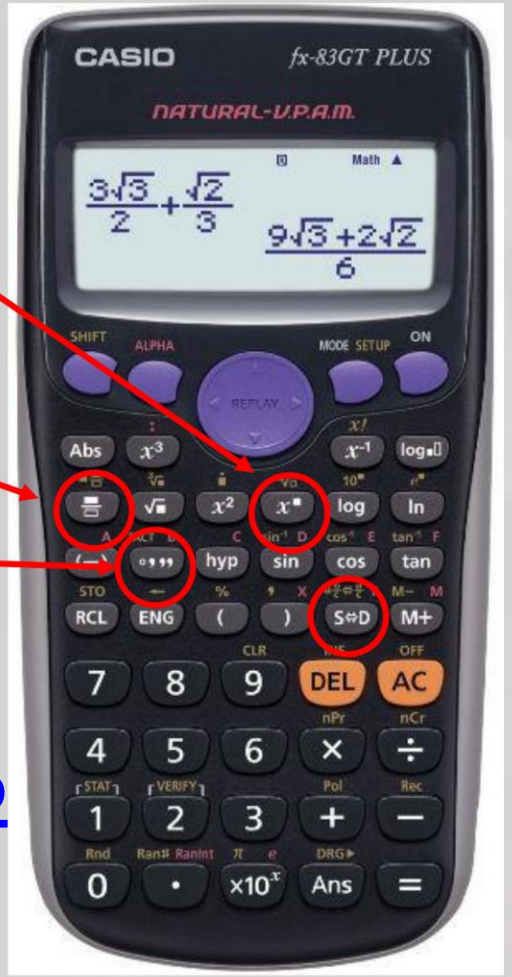
Decrease by 15%



$85\% = \frac{85}{100} = 0.85$ Percentage multiplier is 0.85

Standard form

- Students should use the power key when inputting numbers in standard form into their calculators.
- The fraction key is also useful.
- S-D key
- ° ’ ” key for time



Click here for a ‘Time’ Video

Click here for a ‘Standard Form’ Video

- The calculation on the calculator should look exactly the same as the calculation on the exam paper.

Science & Maths

5. A coarse particle has a diameter of 1×10^{-6} m.
A nanoparticle has a diameter of 1.6×10^{-9} m.
Calculate how many times bigger the diameter of the coarse particle is than the diameter of the nanoparticle.

[2 marks]

5. During an experiment, a scientist notices that the number of bacteria halves every second.

There were $2{\cdot}3 \times 10^{30}$ bacteria at the start of the experiment.

Calculate how many bacteria were left after 5 seconds.

Give your answer in standard form correct to two significant figures.

[2 marks]

Maths & Science - Answers

5. A coarse particle has a diameter of 1×10^{-6} m.
A nanoparticle has a diameter of 1.6×10^{-9} m.
Calculate how many times bigger the diameter of the coarse particle is than the diameter of the nanoparticle.

$$\frac{1 \times 10^{-6}}{1.6 \times 10^{-9}} = 625$$

[2 marks]

5. During an experiment, a scientist notices that the number of bacteria halves every second.

There were $2{\cdot}3 \times 10^{30}$ bacteria at the start of the experiment.

Calculate how many bacteria were left after 5 seconds.

Give your answer in standard form correct to two significant figures.

$2{\cdot}3 \times 10^{30} \times 2^5 = 7.36 \times 10^{31} = 7.4 \times 10^{31}$ *to* $2sf$

[2 marks]

Science

6. Figure 15 shows how the velocity of the train changes with time as the train travels along a straight section of the journey.

   Estimate the distance travelled by the train along the section of the journey shown in Figure 15.
   To gain full marks you must show how you worked out your answer.

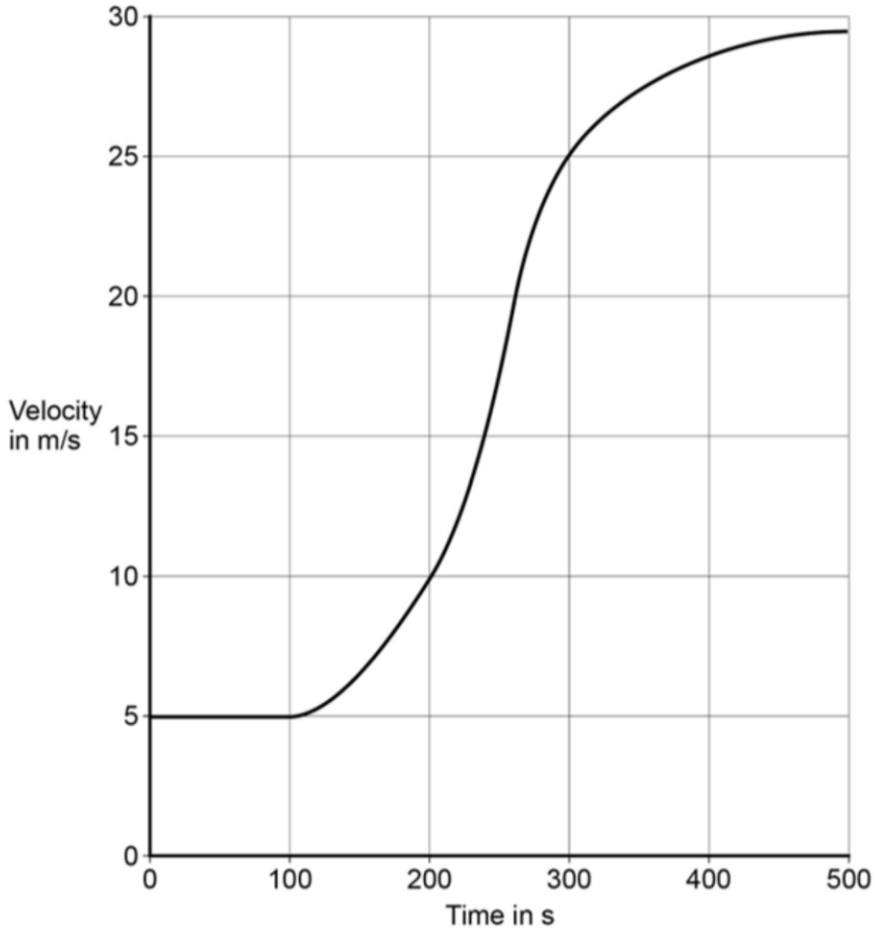


Figure 15

[3 marks]

Science - Answers

6. Figure 15 shows how the velocity of the train changes with time as the train travels along a straight section of the journey.

Estimate the distance travelled by the train along the section of the journey shown in Figure 15.
To gain full marks you must show how you worked out your answer.

Number of squares below the curve = 17

Distance = 17 ×500 = 8500m

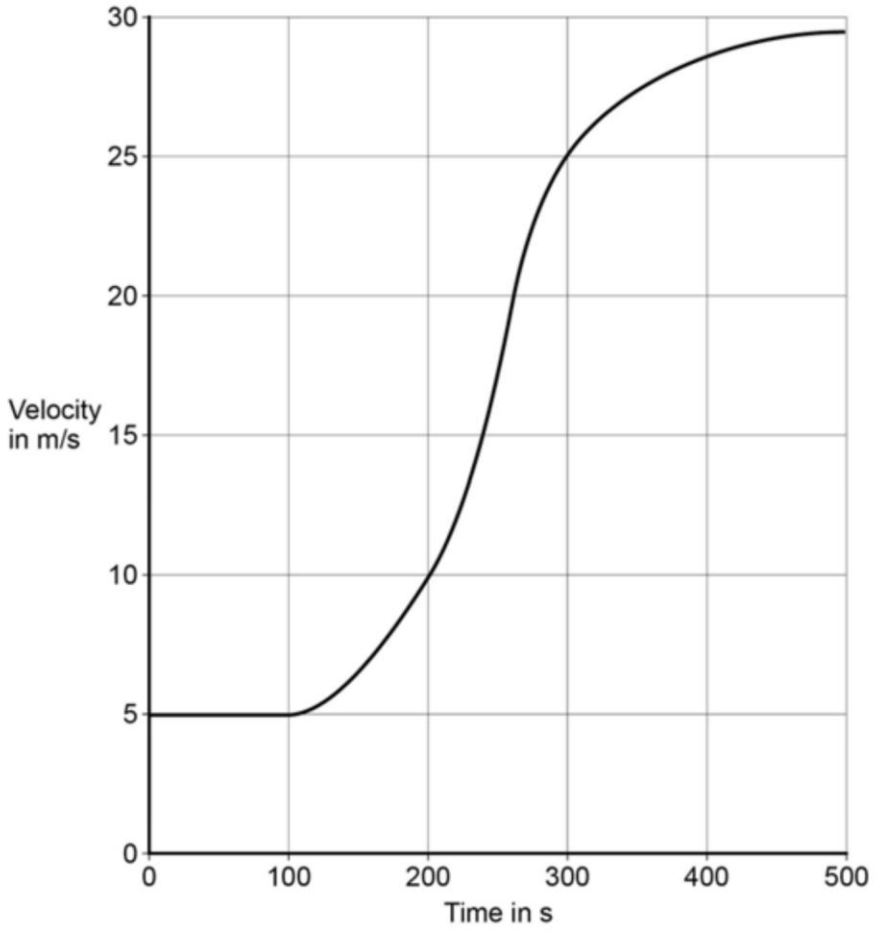
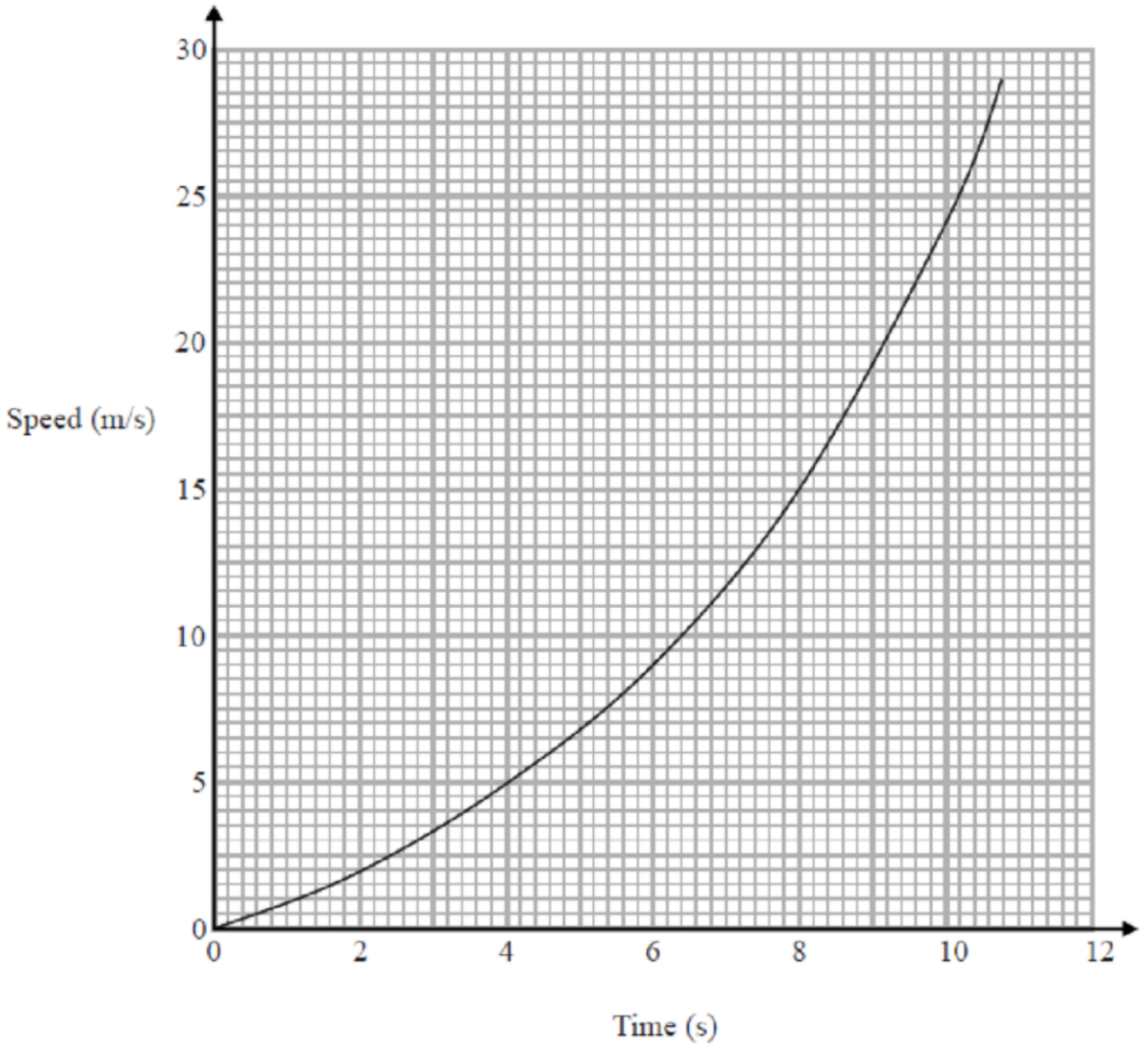


Figure 15

[3 marks]

Maths

6. Here is a speed-time graph for a car.



a) Work out an estimate for the distance the car travelled in the first 10 seconds.
Use 5 strips of equal width.

.....m

[3 marks]

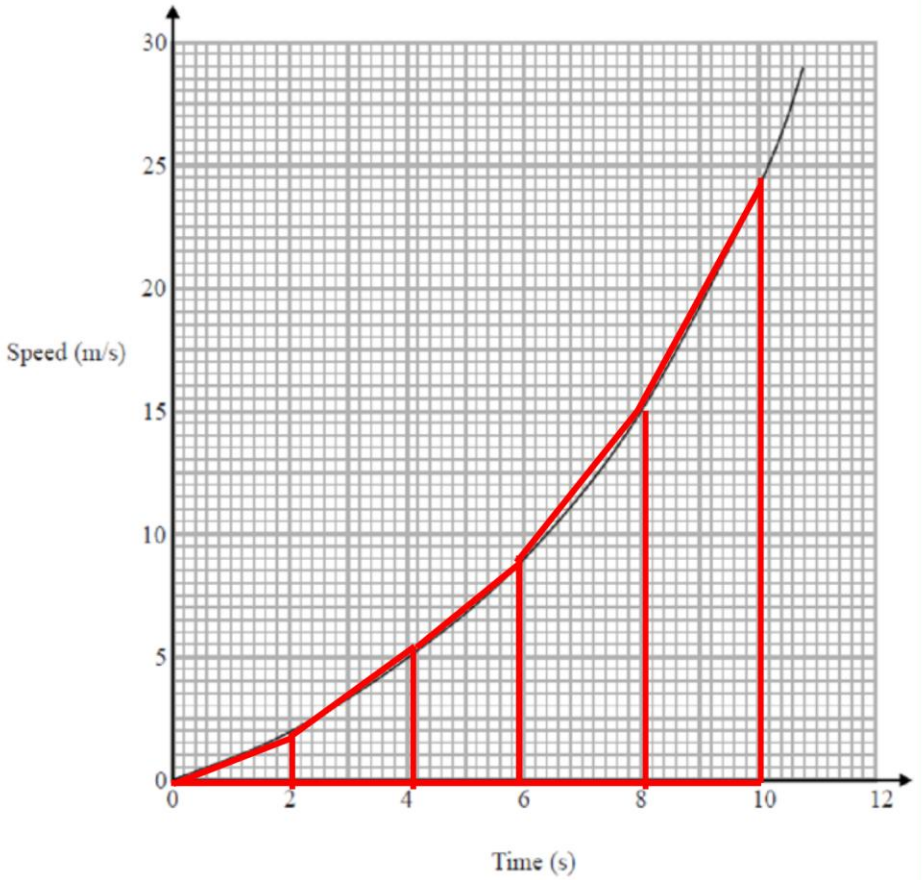
b) Is your answer to (a) an underestimate or an overestimate of the actual distance?

Give a reason for your answer.

[1 mark]

Maths - Answers

6. Here is a speed-time graph for a car.



a) Work out an estimate for the distance the car travelled in the first 10 seconds.
Use 5 strips of equal width.

Trapezium rule

$$\frac{1}{2} \times height \times (ends + 2 \times middles)$$

$0.5\times 2(0 + 24 + 2 \times (2 + 5 + 9 + 15))$

= 86m

.....m

[3 marks]

b) Is your answer to (a) an underestimate or an overestimate of the actual distance?

Give a reason for your answer.

Overestimate, as each trapezium used to calculate the area is drawn above the curve.

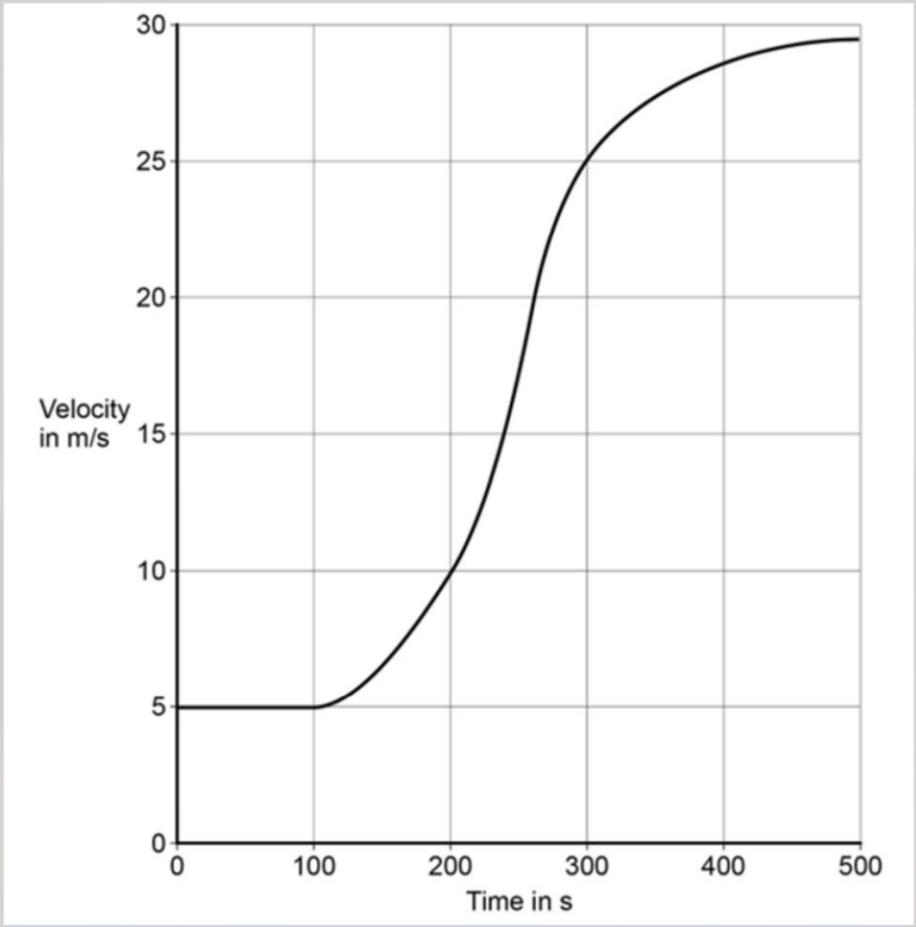
[1 mark]

Velocity/time graphs

- The distance travelled is the area under the curve.
- Consider the units:

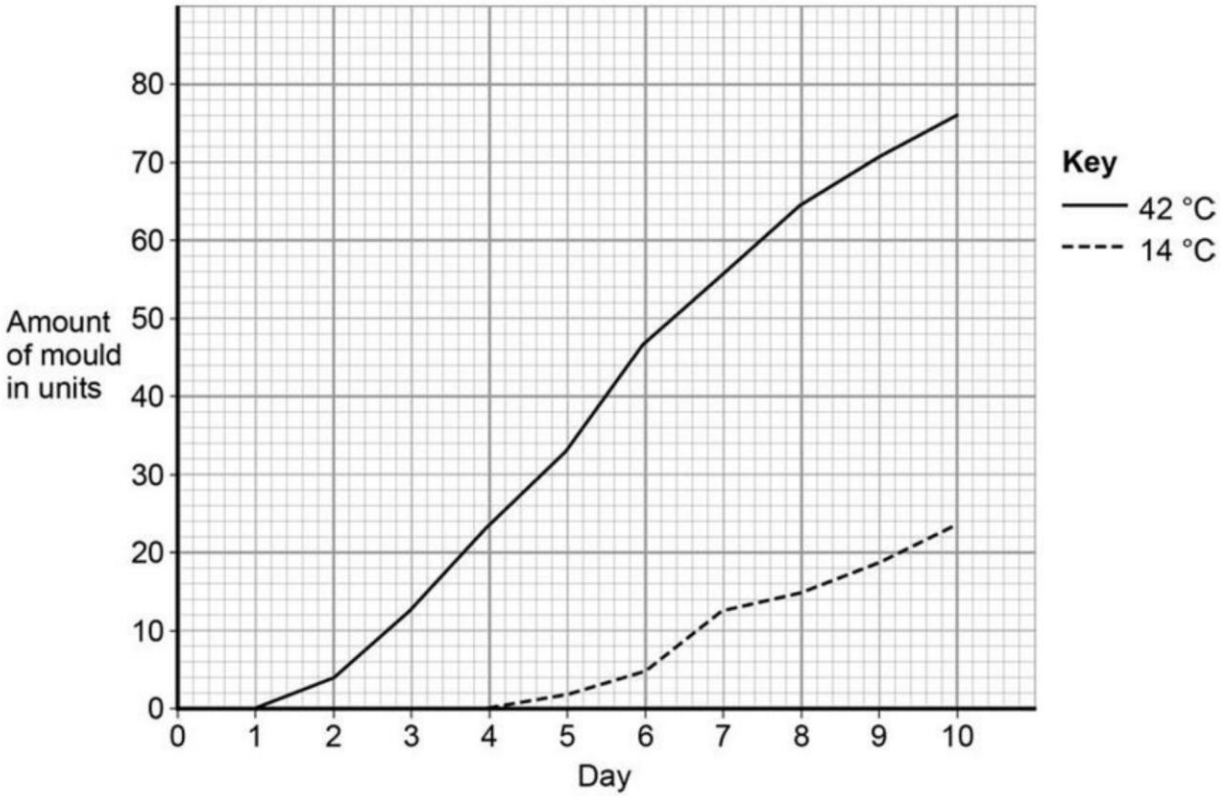
$$\frac{m}{s} \times s = m$$

- Science – count the number of squares
- Maths – students may use the trapezium rule.



Science

7. Determine the rate of mould growth at 42 °C between day 2 and day 7.



Rate of mould growth = ...units per day

[2 marks]

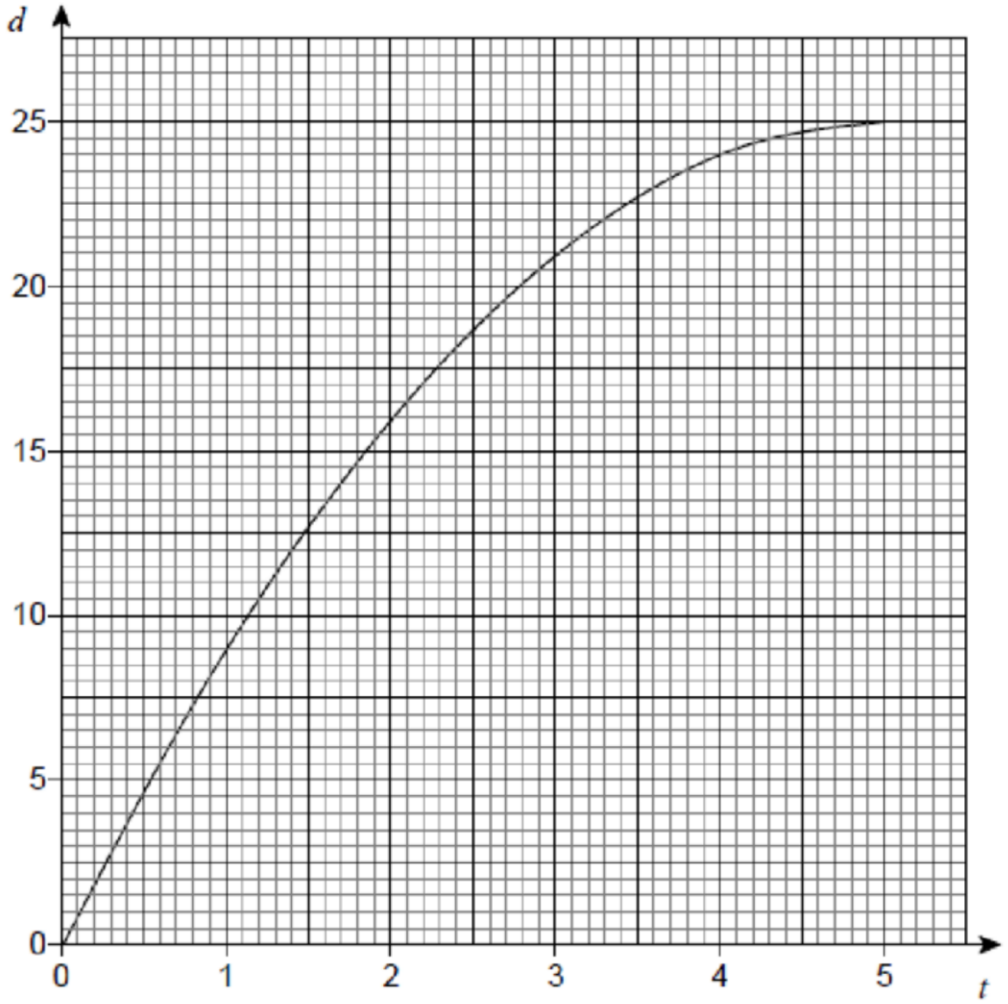
Maths

7. A container is filled with water in 5 seconds.

The graph shows the depth of water, d cm, at time t seconds.

Use the graph to estimate the rate at which the depth of water is increasing at 3 seconds.

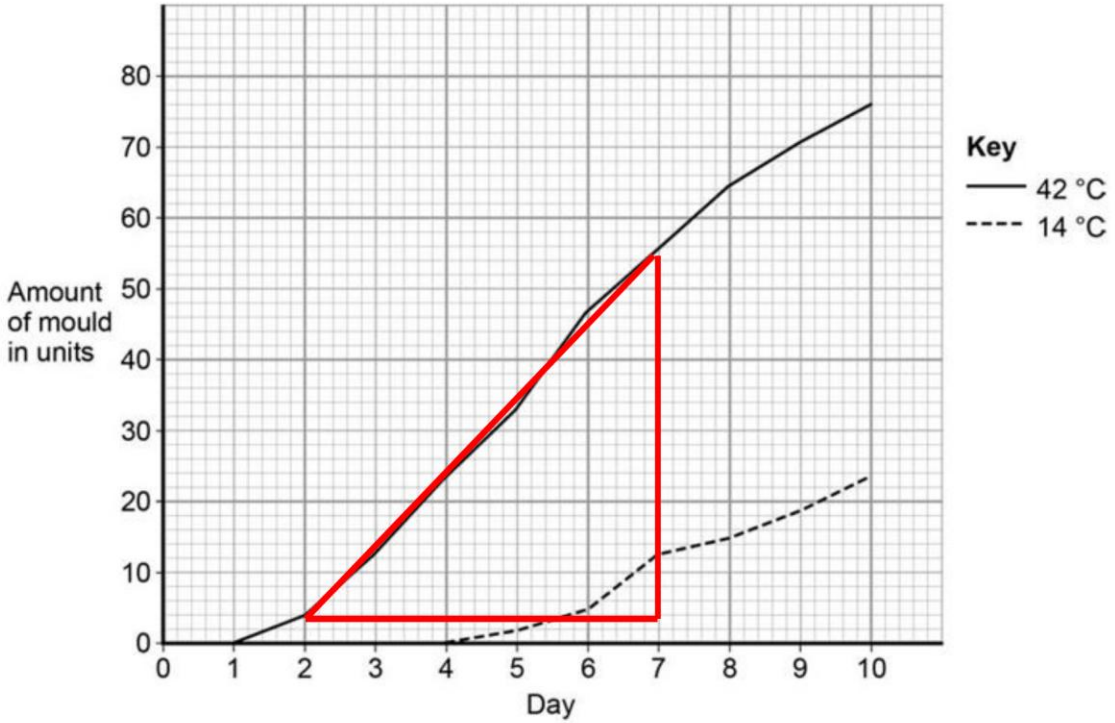
You must show your working.



[2 marks]

Science - Answers

7. Determine the rate of mould growth at 42 °C between day 2 and day 7.



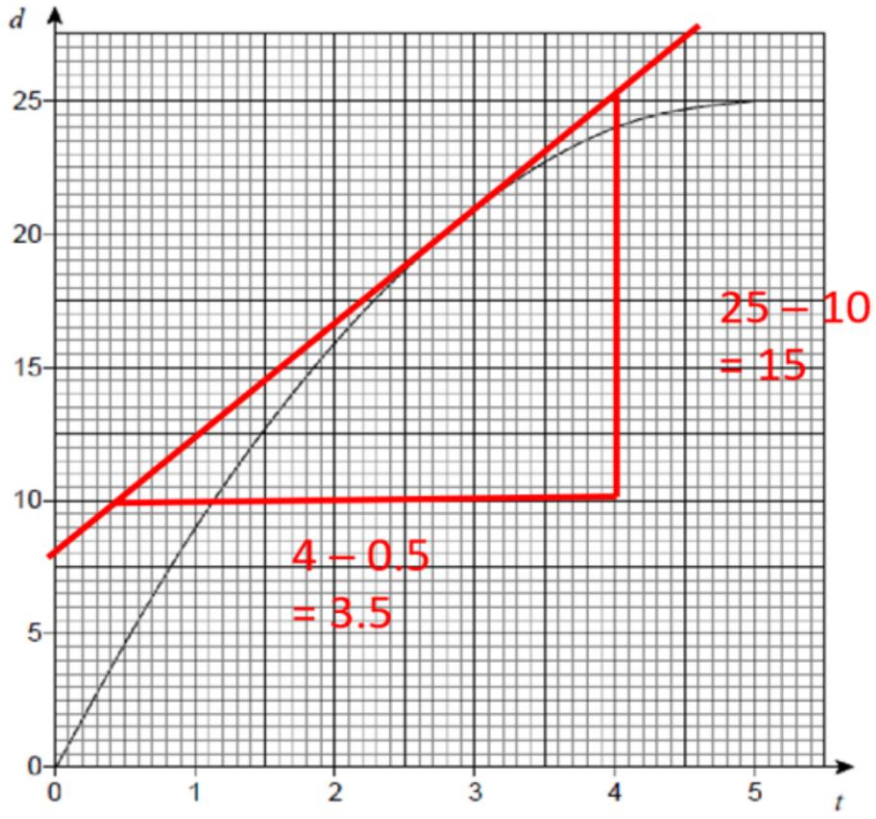
Rate of mould growth = ... $\frac{56-4}{5} = 10.4$ .....units per day

[2 marks]

Maths - Answers

7. A container is filled with water in 5 seconds.

The graph shows the depth of water, d cm, at time t seconds.



Use the graph to estimate the rate at which the depth of water is increasing at 3 seconds.

You must show your working.

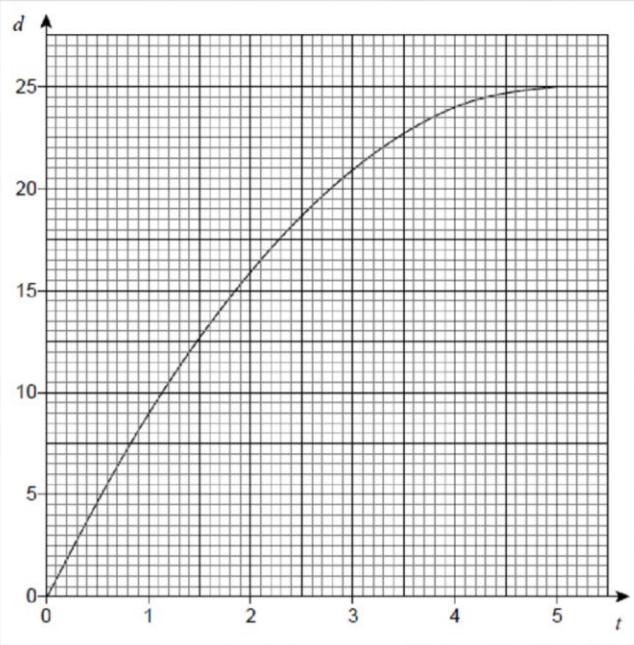
$\frac{15}{3.5} = 4.29\ cm/s$

[3.9-4.5]

[2 marks]

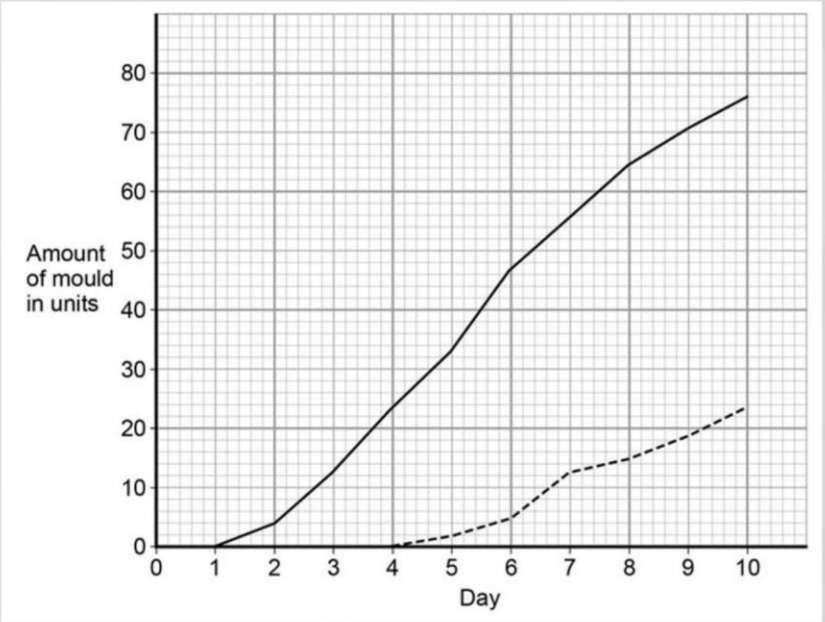
Rates of change

- There is a greater focus on rates of change in the new maths and science GCSE
- Consider the units of the gradient of the tangent



Units of gradient = $\frac{amount\ of\ mould}{day}$

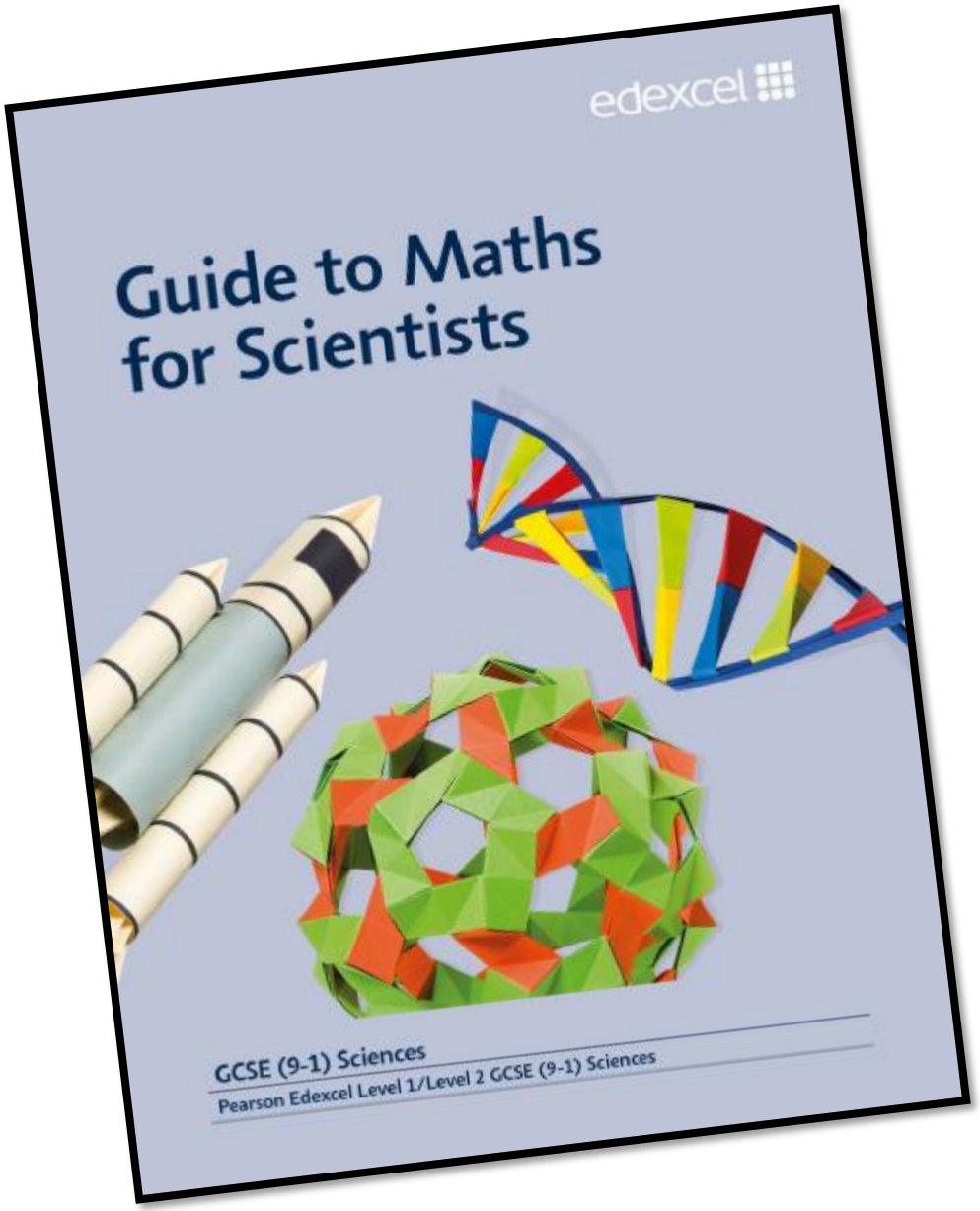
i.e. amount of mould per day



Units of gradient = $\frac{cm}{seconds}$

i.e. change in depth per second

Sharing Resources



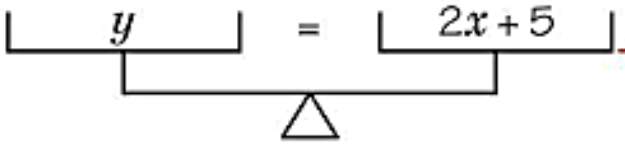
Changing the Subject

Changing the subject of a formula using inverse operations – balancing

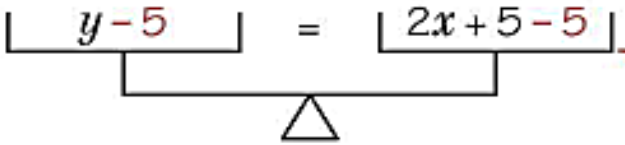
Using the same approach as in learning how to solve equations, as above.

Example 6

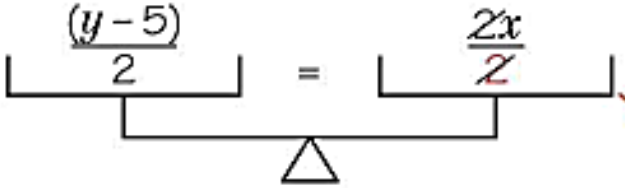
Rearrange $y = 2x + 5$ to make x the **subject** of the formula.



Use the balancing method.



The inverse of +5 is −5. Do this to both sides.



The inverse of ×2 is ÷2. Do this to both sides.

$x = \frac{y-5}{2}$ ✓

Communication hint

The subject of a formula is the letter on its own, on one side of the equals sign.

Changing the Subject

In maths we show them formulae triangles for speed and density formulae. But we use a more general approach to solving equations and rearranging formulae, using inverse operations.

Inconsistency: In maths, to solve equations and change the subject of a formula we use inverse operations and the idea of balancing equations by doing the same to both sides.

We do not say 'cross multiply' or 'move through the equals and change the sign', or similar explanations.

Inconsistency: The list of mathematical skills required for science includes: change the subject of an equation. In maths we distinguish between equations and formulae, and only formulae have subjects.

The subject of a formula is the letter on its own, on one side of the equals sign.

A formula is a rule that shows the relationship between two or more variables (letters). You can use substitution to find an unknown value. So $v = \frac{x}{t}$ or $F = ma$ or $v = u + at$ are formulae.

Further Reading

Deans for Impact (2015) The Science of Learning [Online] Accessible from: https://deansforimpact.org/resources/the-science-oflearning/ [retrieved 10 October 2018].

Education Endowment Foundation (2017) Improving Mathematics in Key Stages Two and Three Guidance Report. [Online] Accessible from: https://educationendowmentfoundation.org.uk/tools/guidance-reports/ [retrieved 10 October 2018].

Hodgen, J., Foster, C., Marks, R. & Brown, M. (2018) Improving Mathematics in Key Stages Two and Three: Evidence Review. [Online] Accessible from https://educationendowmentfoundation.org.uk/evidence-summaries/evidence-reviews/improvingmathematics-in-key-stages-two-and-three/ [retrieved 22 October 2018], 149-157.

Resources

0-Maths Skills in Science GCSE.pptx

Guide Maths for Scientists.pdf

Maths and Science GCSE Questions.pdf

Maths GCSE Questions (with Data).pdf

https://tinyurl.com/yc5efvvs

Link to ITTECF Statements

CCF3: Learn that...

- 3.1. A school's curriculum enables it to set out its vision for the knowledge, skills and values that its pupils will learn, encompassing the national curriculum within a coherent wider vision for successful learning.

- 3.2. Secure subject knowledge helps teachers to motivate pupils and teach effectively.

- 3.4. Anticipating common misconceptions within particular subjects is also an important aspect of curricular knowledge; working closely with colleagues to develop an understanding of likely misconceptions is valuable.

- 3.8. Pupils are likely to struggle to transfer what has been learnt in one discipline to a new or unfamiliar context.

CCF4: Learn that…

- 4.8. Practice is an integral part of effective teaching; ensuring pupils have repeated opportunities to practise, with appropriate guidance and support, increases success.

CCF6: Learn that...

- Learn that ...
- 6.1. Effective assessment is critical to teaching because it provides teachers with information about pupils' understanding and needs.

- 6.2. Good assessment (data) helps teachers avoid being over-influenced by potentially misleading factors…..