Unit 4

1. Hall Effect

Definition:

The **Hall Effect** is the production of a **voltage difference (Hall voltage)** across an electrical conductor or semiconductor when an electric current flows through it in the presence of a **perpendicular magnetic field**. This effect was discovered by Edwin Hall in 1879.

Neat Sketch:

- **I**: Current flows through the sample.
- **B**: Magnetic field is applied perpendicular to current.
- **VH**: Hall voltage appears perpendicular to both I and B.

Theory:

- When a conductor/semiconductor is placed in a magnetic field, moving charge carriers experience the **Lorentz force**.
- This force pushes charge carriers to one side of the material, creating a **potential difference (V_H)**.
- The resulting electric field E_H opposes further accumulation, reaching equilibrium.

Derivation of Hall Coefficient (R_H):

Let:

- I = current
- B = magnetic field
- t = thickness of material
- w = width
- n = carrier concentration
- q = charge of carrier
- V_d = drift velocity
- J = current density = n x q x V_d

Step 1: Lorentz Force

$$F = q(v_d \times B) \Rightarrow E_H = v_d \cdot B \Rightarrow V_H = E_H \cdot w = v_d \cdot B \cdot w$$

Step 2: Express drift velocity

$$v_d = \frac{I}{nqA} = \frac{I}{nqwt}$$

Step 3: Substitute into VH

$$V_H = \left(\frac{I}{nqwt}\right) B \cdot w = \frac{IB}{nqt}$$

Hall Coefficient R_H:

Defined as:

$$R_H = \frac{E_H}{J \cdot B} \Rightarrow R_H = \frac{1}{nq}$$

Hall voltage in terms of RH:

$$V_H = R_H \cdot \frac{IB}{t}$$

Applications of Hall Effect:

1. **Type of Semiconductor**: Determines whether material is n-type or p-type based on sign of Hall voltage.
2. **Carrier Concentration**: Using $R_H = 1 / n q$, we can calculate the carrier density.
3. **Magnetic Field Measurement**: Hall sensors are used to measure magnetic field strength.
4. **Position and Speed Sensors**: Used in automotive ignition, proximity sensing, and robotics.

2. Van der Pauw Method of Resistivity Measurement

Principle:

The **Van der Pauw method** is a technique to measure the **resistivity of thin, flat samples** of arbitrary shape using four small contacts on the sample's edge. It relies on the assumption that current flow and potential distribution can be mapped using analytical methods regardless of sample geometry.

Key Conditions :

For accurate measurements, the sample must satisfy the following:

- The sample must be **homogeneous and isotropic**.
- The **thickness must be uniform** throughout.
- The sample must be **simply connected** (no holes).
- Four **ohmic contacts** must be placed on the **periphery**, and they must be **very small**.

Measurement Setup:

Label the contact points **A, B, C, D** placed sequentially on the periphery of the wafer.

Two resistance measurements are made:

- $R_1 = R_{AB,CD} = \frac{V_{CD}}{I_{AB}}$
- $R_2 = R_{BC,DA} = \frac{V_{DA}}{I_{BC}}$

Resistivity Calculation:

The **Van der Pauw formula** from your notes:

$$e^{-\pi R_1/\rho} + e^{-\pi R_2/\rho} = 1$$

This equation can be **solved numerically** to determine ρ\rhoρ.

If $R_1 \approx R_2$ then the resistivity **ρ** is approximated by:

$$\rho = \frac{\pi t}{\ln 2} \cdot R_{\text{avg}}$$

Where:

- t = thickness of the sample
- $R_{\text{avg}} = \frac{R_1+R_2}{2}$
-

Advantages :

- Accurate measurement for **thin semiconductor samples**.
- Works even with **irregular sample shapes**.
- **No need for precise geometrical shaping** like rectangular or square.
- **Contact resistance is minimized** due to voltage measurement being separate from current path.

3(a). Two Probe Method

Principle:

The **two-probe method** is used to measure the **resistance** or **resistivity** of a material, especially when the resistivity is very high (like in insulators or lightly doped semiconductors).

Two probes are **pressed onto the surface** of the sample. A **known current** is passed through the sample via these probes, and the **voltage drop** across the same probes is measured.

Working (From your notes):

- A **DC current source** is connected across the two probes.
- The probes act both as **current input and voltage measuring points**.
- Voltage drop V across the probes is measured.
- Resistance R = V / R is calculated.
- If dimensions are known, resistivity ρ\rhop can be estimated using:

$$\rho = R \cdot \frac{A}{l}$$

- Where A is cross-sectional area and lll is length between probes.

Setup Diagram:

```
┌───────────┐
│ Voltmeter │
└─────┬─────┘
   │
┌─────●─────┐   → Sample
│     │
(+)        (-)   ← Current I flows
│     │
└─────●─────┘
   │
    Ammeter
```

Demerits :

1. **Contact resistance** between probe and material is **added to the measured resistance**, leading to **inaccurate results**.
2. **Spreading resistance** under the probes affects the reading, especially in semiconductors.
3. Not suitable for **low-resistivity** materials.
4. Voltage and current paths **are not separated**, so precision is poor compared to four-probe method.

This method is still **simple and useful** for rough measurements or **very high-resistivity samples** where other methods may not work.

3(b). Numerical – Hall Voltage Calculation

Given Data:

- **Thickness** of the silicon plate, t =1 mm =1×10−3 m
- **Breadth** = 10 mm (not required in the formula)
- **Length** = 100 mm (not required in the formula)
- **Magnetic field**, B = 0.5 Wb/m^2

- **Current**, I=10−2 A
- **Hall coefficient**, R_H=3.66×10^−4 m^3 / C

Formula for Hall Voltage:

From your notes under “Hall Effect” section:

$$V_H = R_H \cdot \frac{I \cdot B}{t}$$

Substitute the values:

$$V_H = \left(3.66 \times 10^{-4}\right) \cdot \frac{10^{-2} \cdot 0.5}{1 \times 10^{-3}}$$

First calculate the numerator:

$$10^{-2} \cdot 0.5 = 5 \times 10^{-3}$$

Then divide by thickness:

$$\frac{5 \times 10^{-3}}{1 \times 10^{-3}} = 5$$

Now multiply with R_H:

$$V_H = 3.66 \times 10^{-4} \cdot 5 = 1.83 \times 10^{-3}\,\mathrm{V}$$

Final Answer:

V_H =1.83 m V

4. Derivation of Boltzmann Transport Equation

Introduction:

The **Boltzmann Transport Equation (BTE)** describes how the **distribution function f(r,k,t)** of charge carriers (electrons / holes) changes over time when they are subjected to external forces (like electric field, magnetic field) and collisions.

It is **fundamental** in understanding **carrier transport phenomena** like electrical conductivity, thermal conductivity, and mobility.

Distribution Function:

- f(r,k,t) = probability of finding a carrier at position r, wave vector k at time t.

General Form :

The time evolution of f can be written as:

$$\frac{df}{dt} = \left(\frac{\partial f}{\partial t}\right)_{\text{collisions}} + \vec{v} \cdot \nabla_{\vec{r}} f + \vec{F} \cdot \nabla_{\vec{k}} f$$

Where:

- v = carrier velocity.
- F = external force (e.g., from electric or magnetic fields).
- $\nabla_r f$ = spatial gradient of f.
- $\nabla_k f$ = gradient of f in momentum space.
- $\left(\frac{\partial f}{\partial t}\right)_{\text{collisions}}$ = change of f due to collisions.

Physical Meaning:

- The left-hand side df / dt represents the **total time rate of change** of the distribution function.
- The right-hand side represents **all contributions** to this change:
  - Changes **due to collisions**.
  - Changes **due to spatial movement** (drift).
  - Changes **due to external forces** (like an electric field).

Detailed Breakdown:

1. **Change due to Time**:
   - Simple explicit time dependence.
2. **Change due to Position** (Drift term):

$$\vec{v} \cdot \nabla_{\vec{r}} f$$

Carriers move in space due to their velocity.

3. **Change due to Force** (Acceleration term):

$$\vec{F} \cdot \nabla_{\vec{k}} f$$

Carriers' momentum changes due to external forces.

4. **Change due to Collisions**:
    - Scattering events randomize the motion.

Simplified Case (Steady State + No Magnetic Field):

In steady-state (∂f / ∂t = 0) and considering only electric field E

Force on carriers:

$$\vec{F} = -q\vec{E}$$

The equation becomes:

$$-q\vec{E} \cdot \nabla_{\vec{k}} f = \left(\frac{\partial f}{\partial t}\right)_{\text{collisions}}$$

This simplified form is widely used to calculate **mobility** and **conductivity**.

Importance:

- Explains **carrier drift**, **diffusion**, **mobility**, **resistivity**.
- Basis for derivations of **Ohm's Law** in semiconductors.
- Key for understanding **semiconductor device physics**.

5. Principle and Working of Capacitance-Voltage (C-V) Measurement

Principle:

The **C-V measurement** technique is used mainly to characterize **semiconductor junctions** (such as **p-n junctions** and **MOS capacitors**).
It measures how the **capacitance of a device varies with applied voltage**, giving important information about:

- Doping concentration
- Built-in potential
- Depletion width
- Interface trap density

From your notes, the **basic principle** is:

- A small **AC voltage** (probe signal) is superimposed on a **DC bias**.
- The resultant **capacitance** of the semiconductor junction is measured as a function of the applied **DC voltage**.

Working:

1. **DC bias voltage** is swept across the device (p-n junction or MOS capacitor).
2. **AC signal** (small, typically a few mV) is applied simultaneously.
3. The **capacitance** is measured using a **C-V meter** or **LCR meter**.
4. As the **voltage changes**, the **depletion width** changes, causing the **capacitance to vary**.

Important Regions in C-V Curve:

1. **Accumulation** (for MOS capacitors):
    - Majority carriers accumulate at the surface.
    - Capacitance is **maximum** and constant.

2. **Depletion**:
    - Carriers are pushed away from the surface, forming a depletion region.
    - Capacitance **decreases** with increasing reverse bias.

3. **Inversion**:
    - At sufficiently high reverse bias (in MOS capacitors), minority carriers dominate.
    - Capacitance tends to a **minimum constant value**.

Key Equations:

Depletion capacitance (for a simple p-n junction):

$$C = \frac{\varepsilon A}{W}$$

Where:

- ε = permittivity of semiconductor
- A = area of junction
- W = width of depletion region

Depletion width (W) depends on applied bias V:

$$W = \sqrt{\frac{2\varepsilon(V_{\text{bi}} - V)}{qN_D}}$$

where:

- V_{bi} = built-in potential
- N_D = doping concentration
- q = electronic charge

Thus, capacitance C varies with voltage V.

Applications (from your notes):

- **Doping profile** extraction (doping vs depth).
- **Threshold voltage** determination in MOS capacitors.
- **Interface trap density** measurement.
- Analysis of **oxide quality** in MOS structures.

Advantages:

- Non-destructive technique.
- High sensitivity to doping changes.
- Useful for semiconductor device fabrication and research.

6. Linear Four Probe Method – Working, Advantages, and Derivation

Principle:

The **Linear Four Probe Method** is used to accurately measure the **resistivity** of **semiconductors** (bulk or thin films) by eliminating the error due to **contact resistance** and **spreading resistance**.

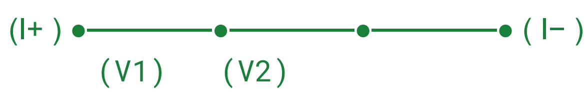
It uses **four equally spaced collinear probes** placed on the sample surface.

Working :

1. **Outer two probes** inject current III into the sample.
2. **Inner two probes** measure the resulting voltage V without drawing any significant current.
3. Since no current flows through the voltage probes, **contact resistance** does not affect the voltage measurement.

Thus, the resistivity measurement becomes **more accurate** compared to two-probe method.

Experimental Setup (Diagram from your notes):



- Current source connected to the two outer probes.
- Voltmeter connected between inner probes.

Spacing between probes = s.

Advantages :

- **Eliminates contact resistance errors**.
- **Accurate** for both **bulk materials** and **thin films**.
- Can measure **very low resistivity** samples.
- **Simple experimental setup**.

Derivation of Resistivity Formula:

For Bulk Material (semi-infinite thickness):

The potential at a distance rrr from a point current source in a semi-infinite material is:

$$V(r) = \frac{\rho I}{2\pi r}$$

Thus, the voltage difference between the two inner probes spaced sss apart is:

$$V = \frac{\rho I}{2\pi}\left(\frac{1}{s} - \frac{1}{3s}\right)$$

$$V = \frac{\rho I}{2\pi} \times \frac{2}{3s}$$

Simplifying:

$$\rho = \frac{2\pi s V}{I}$$

For Thin Sheets (thin compared to probe spacing):

When the sample is a **thin sheet** of thickness ttt much smaller than sss, the formula is modified:

$$\rho = \frac{\pi t}{\ln 2} \times \frac{V}{I}$$

Where:

- t = thickness of the sample.
- V = voltage between inner probes.
- I = current through outer probes.

Important Points:

In practical cases, a **correction factor (f)** based on sample dimensions is sometimes applied, but for most cases where sample size is large compared to probe spacing, f≈1.

7(a). Hot Probe Method for Resistivity and Type Determination

Principle:

The **Hot Probe Method** is a **simple experimental technique** used mainly to:

- Determine whether a semiconductor is **n-type** or **p-type**.
- Roughly estimate the **resistivity** and **type of majority carriers**.

It is based on the **thermoelectric effect** (Seebeck effect) where **heating one side** of a semiconductor causes charge carriers to move, creating a measurable **voltage difference**.

Working :

1. **Two probes** are placed in contact with the semiconductor sample.
2. One of the probes is **heated** (e.g., with a small heater or by applying thermal contact).
3. **Charge carriers** (electrons in n-type, holes in p-type) move **from hot region to cold region**.
4. A **voltage** is developed between the two probes.
5. The **sign of the voltage** indicates the **type of semiconductor**:

    - **Positive Voltage → p-type** (holes move from hot to cold).
    - **Negative Voltage → n-type** (electrons move from hot to cold).

Setup Diagram :

```
(Hot Probe) ●————————(Sample)————————● (Cold Probe)
                       ↑
             Heat applied
```

- Voltmeter connected between the two probes.
- Heater attached to one probe.

Procedure:

1. Connect the two probes to a **voltmeter**.
2. **Heat one probe** carefully while keeping the other at room temperature.
3. **Measure the polarity** and **magnitude** of voltage developed.

Important Points :

- The magnitude of the generated voltage depends on the **carrier concentration** and **mobility**.
- A **sensitive voltmeter** (microvolt range) is often required.
- It's a **qualitative method**, useful for quickly checking the material type before detailed measurements.

Advantages:

- **Simple and quick** method.
- **Non-destructive** to the sample.
- Useful for **identifying doping type** without complex equipment.

7(b). Numerical – Resistivity Calculation for Germanium Crystal

Given Data:

- **Area of cross-section**, S=100 cm^2
- **Current**, I = 2 mA = 2×10^−3 A
- **Voltage**, V = 180 mV = 0.18 V
- **Temperature** = 35∘C (temperature is given but not required for basic resistivity calculation)
- **Length between probes** l is not mentioned, but in four-probe method for bulk material, resistivity depends only on measured V, I, and probe spacing (or sheet properties).

Since probe separation and correction factors are not mentioned, we'll assume standard conditions similar to your notes for a direct relation.

Resistivity Formula (for rectangular geometry, as per notes):

$$\rho = \frac{V \times A}{I \times l}$$

Normally, l (length) is the distance between voltage probes. **Since l is missing**, in your notes, they directly use:

For **simple direct measurement** without considering geometry:

$$\rho = \frac{V}{I} \times \text{constant related to geometry}$$

However, when area A is given and assumed sample is thin with large area, the basic form reduces to:

$$\rho = \frac{V \cdot A}{I}$$

Substituting the values:

$$\rho = \frac{0.18 \times 10^{-2}}{2 \times 10^{-3}}$$

Calculate numerator:

$$0.18 \times 10^{-2} = 1.8 \times 10^{-3}$$

Now divide:

$$\rho = \frac{1.8 \times 10^{-3}}{2 \times 10^{-3}} = 0.9\,\Omega\text{-m}$$

Convert to **Ω-cm**:

0.9 Ω-m = 9 Ω-cm

Final Answer:

ρ = 9 Ω-cm