Sem IV comp ①

(Time : 3 Hours) (Total Marks : 80)

Note :

1) **Q. No. 01 is compulsory.**
2) **Solve any three from Q. No. 02 to 06.**
3) **Numbers to the right indicate full marks.**
4) **Use of statistical tables is allowed.**

Q. 1. Solve.

a) If $A = \begin{bmatrix} -1 & 2 & 38 \\ 0 & 2 & 37 \\ 0 & 0 & -2 \end{bmatrix}$ find the Eigen values of $A^3 + 5A + 8I$. 05

b) Integrate the function $f(z) = x^2 + i\,xy$ from A(1, 1) to B(2, 4) along $y = x^2$ 05

c) Find the Z-Transform of $f(k) = a^{-k}$, $k \geq 0$. 05

d) If a random variable X follows Poisson distribution such that P (x = 1) = 2 P (x = 2). Find mean and variance of the distribution. 05

Q. 2.

a) Find the Eigenvalues and Eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. 06

b) Find the Z-Transform of $\cos\left(\frac{\pi}{4} + ka\right)$ $k \geq 0$. 06

c) Use the dual simplex method to solve the LPP 08
Min. $Z = 2X_1 - X_2 - 3X_3$.
$3X_1 - X_2 + 3X_3 \leq 7,\ 2X_1 - 4X_2 \geq 12,\ X_1, X_2, X_3 \geq 0$

Q. 3.

a) Evaluate $\int_C \frac{z+8}{z^2+5z+6} dz$ Where C is a circle $|z|=5$. 06

b) Verify Caley-Hamilton theorem and hence find A^{-1} and A^4 where $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. 06

c) Solve the LPP by Big -M method 08
Max. $Z = X_1 + 2X_2 - 3X_3 - X_4$
$X_1 + 2X_2 + 3X_3 = 15,\ 2X_1 + X_2 + 5X_3 = 20,\ X_1 + 2X_2 + X_3 + X_4 = 10\ \ X_1, X_2, X_3, X_4 \geq 0$

Q. 4.

a) Find inverse Z transform of $F(z) = \frac{1}{(z-2)(z-3)}$ for i) $|z| < 2$, ii) $|z| > 3$. 06

b) A certain drug administered to 12 patients resulted in the following change in their blood pressure. 5, 2, 8, - 1, 3, 0, 6, - 2, 1, 0, 4,5 Can we conclude that the drug increases the blood pressure? 06

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c) Find all possible Laurent's series expansions of the function $f(z) = \frac{1}{(z+1)(z-2)}$ about $z = 0$ indicating the region of convergence in each case. 08

Q. 5.

a) Determine all basic solutions to the following problem 06

Max $= x_1 - 2x_2 + 4x_3$,

$x_1 + 2x_2 + 3x_3 = 7,\ 3x_1 + 4x_2 + 6x_3 = 15\ , x_1\ , x_2, x_3 \geq 0$.

b) If X is a Normal variate with mean 10 & s.d. 4, find i) $P(5 \leq X \leq 18)$, ii) $P(X \leq 12)$. 06

c) Solve the NLPP 08

Optimize $Z = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$

Subject to $x_1 + x_2 + x_3 = 10\ , x_1\ , x_2\ ,\ x_3 \geq 0$.

Q. 6.

a) Show that the given matrix is diagonalizable and hence find diagonal form and transforming matrix where $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. 06

b) Based on the following data if there is a relation between literacy and smoking. 06

	Smoking	Non-smoking
Literacy	83	57
Illiteracy	45	68

c) Max. $Z = 12x_1x_2 + 2x_1^2 - 7x_2^2$, Subject to $2x_1 + 5x_2 \leq 98,\ x_1\ , x_2 \geq 0$ by K-T condition. 08

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SE (computer)/Sem IV / 'C' scheme / EM – IV

(Time: 3 hours) Max. Marks: 80

N.B. (1) Question No. 1 is compulsory.
(2) Answer any three questions from Q.2 to Q.6.
(3) Use of Statistical Tables permitted.
(4) Figures to the right indicate full marks

Q1. (a) If matrix A = $\begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ find Eigen values of $A^3 + 5A + 8I$. [5]

(b) Evaluate the integral $\int_0^{1+i} (x - y + i x^2)\, dz$ along the parabola $y^2 = x$. [5]

(c) Find the z-transform of $f(k) = a^k,\ k \geq 0$. [5]

(d) Maximise $z = x_1 + 3x_2 + 3x_3$

Subject to $x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7$ find all basic solutions. Which

of them are basic feasible, And optimal basic feasible solutions. [5]

Q2 (a) Verify Cayley- Hamilton theorem for the matrix A where A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$

And hence find A^{-1} and A^{-2}. [6]

(b) The means of two random samples of size 9 and 7 are 196.42 and 198.82 respectively The sum of the squares of the deviations from the means are 26.94 and 18.73 respectively. Can The samples be considered to have been drawn from same population? [6]

(c) Solve the L.P.P by using simplex method.

Maximise $z = 3x_1 + 2x_2$

Subject to $3x_1 + 2x_2 \leq 18$;

$0 \leq x_1 \leq 4$;

$0 \leq x_2 \leq 6$;

$x_1, x_2 \geq 0$. [8]

Q3 (a) Find the Laurent's series for

$F(z) = \frac{4z+3}{(z-3)\, z\,(z+2)}$ valid for $2 < |z| < 3$. [6]

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(b) Using the method of Lagrange's multiplier solve the N.L.P.

Optimise $z = 12x_1 + 8x_2 + 6x_3 - x_1^2 - x_2^2 - x_3^2 - 23$.

Subject to $x_1 + x_2 + x_3 = 10$. $x_1, x_2, x_3 \geq 0$. [6]

(c) Marks obtained by students in an examination follow normal distribution. If 30 % Of the students got below 35 marks and 10 % got above 60 marks. Find the mean and standard deviation. [8]

Q4 (a) Find the Eigen values and Eigen vectors of matrix $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$ [6]

(b) Find inverse z- transform of $F(z) = \frac{3z^2 - 18z + 26}{(z-2)(z-3)(z-4)}$ $3 < |z| < 4$. [6]

(c) Using the Kuhn –Tucker conditions solve the N.L.P.P [8]

Maximise $z = 12x_1x_2 + 2x_1^2 - 7x_2^2$

Subject to $2x_1 + 5x_2 \leq 98$;

$x_1, x_2 \geq 0$.

Q5 (a) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalisable. Find the diagonal form D and the Diagonal zing matrix M. [6]

Find the relative maximum or minimum of the function

$z = x_1^2 + x_2^2 + x_3^2 - 4x_1 - 8x_2 - 12x_3 + 100$. [6]

Evaluate $\oint \frac{2z-1}{(2z+1)\, z\,(z+2)}\, dz$ using Cauchy's residue theorem, where C is the circle $|z| = 1$. [8]

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Q6 (a) The number of car accidents in a metropolitan city was found to be 20, 17, 12, 6, 7, 15, 8, 5, 16 and 14 per month respectively. Use χ^2–test To check whether these frequencies are in agreement with that occurrence was The same during 10 months period. Test at 5 % level of significance. [6]

(b) Find z – transform of [$2^k \cos(3k+2)$], $k \geq 0$. [6]

(c) Use the dual simplex method to solve the L.P.P. [8]

Minimise $z = 2x_1 + x_2$

Subject to
$$3x_1 + x_2 \geq 3;$$
$$4x_1 + 3x_2 \geq 6;$$
$$x_1 + 2x_2 \leq 3;$$
$$x_1, x_2 \geq 0;$$

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