DYNAMIC PROGRAMMING

All Pair Shortest Path (APSP)

FLOYD-WARSHALL ALGORITHM.

ALL pair shortest path is to find shortest paths between all pair of vertices in a graph.

One way to accomplish this would be to simply run SSSP algorithm like Bellman Ford / Dijkstra's algorithm for each vertex in the graph.

FLOYD-WARSHALL ALGORITHM is a ~~all~~ APSP algorithm using dynamic programming for dense graph.
— Directed graph, negative edges.

Step I: The structure of a shortest path.

- It ~~works~~ is based on a property of intermediate vertices of a shortest path.

- An intermediate vertex of a simple path $P = \{v_1, v_2 \ldots v_l\}$ is any vertex of p other than v_1 or v_l i.e any vertex in the set $\{v_2, v_3 \ldots v_{l-1}\}$.

- If the vertices of graph G are indexed as $\{1, 2 \ldots n\}$ then consider a subset of vertices $\{1, 2 \ldots k\}$ for some k.

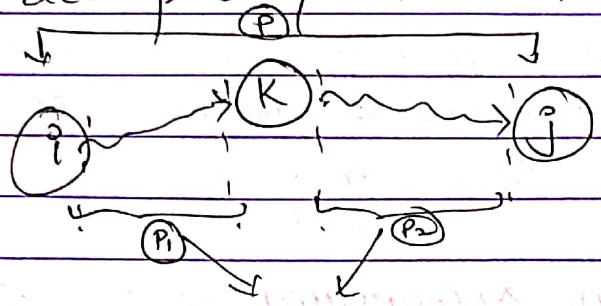
- Assume 'p' is the minimum weight path from vertex i to j whose intermediate vertices are drawn from subset $\{1, 2 \ldots k\}$

- There are 2 possibilities

(1) 'k' is not an intermediate vertex of p i.e

not present in minimum path. Thus, all intermediate vertices are from subset $\{1, 2 \ldots k-1\}$.

(2) 'K' is an intermediate vertex of p, then we can decompose p into $i \overset{p_1}{\rightsquigarrow} k \overset{p_2}{\rightsquigarrow} j$.



All intermediate vertices in $\{1, 2 \ldots k-1\}$

- There is a property of shortest path which says, "Subpaths of shortest paths are also shortest paths".
- Thus, p_1 & p_2 are shortest paths from i to k & k to j respectively with the intermediate vertices in the set $\{1, 2 \ldots k-1\}$.

Step II: A Recursive solution to APSP problem.

- Let $d_{ij}^{(k)}$ be the weight of shortest path from vertex i to j for which all intermediate vertices are in the set $\{1, 2 \ldots k\}$.

- When $k=0$, a path from vertex i to j has no intermediate vertex i.e there is a direct edge from vertex i to j. Such a path has atmost one edge, hence $d_{ij}^{(0)} = w_{ij}$ ($w_{ij} \rightarrow$ cost of edge i to j).

- We define $d_{ij}^{(k)}$ recursively by

$$d_{ij}^{(k)} = \begin{cases} w_{ij} & \text{if } k=0 \\ \min\left(d_{ij}^{(k-1)},\ d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right) & \text{if } k \geq 1 \end{cases}$$

– Thus, we can represent the optimal values (when $k=n$) in a matrix as.

$$D^{(n)} = (d_{ij}^{(n)}) = |\delta(i,j)| \rightarrow (\delta(i,j) \rightarrow \text{shortest path from } i \text{ to } j)$$

Step III : Computing the shortest path weights bottom up.

Input :- $n \times n$ matrix W define as

$$w_{ij} = \begin{cases} 0 & \text{if } i=j \\ \text{the weight of edge } (i,j) & \text{if } i \neq j \ \& \ (i,j) \in E \\ \infty & \text{if } i \neq j \ \& \ (i,j) \notin E \end{cases}$$

Output :- Returns matrix of shortest path $D^{(n)}$, $\Pi^{(n)}$

ALGORITHM :-

```
          FLOYD-WARSHALL (W)
θ(1)        n = W.rows                  // no. of vertices in G
n×n         D(0) = W                    // input Graph G
            ~~InitializePredecessor(π(0), W)~~
n           for k = 1 to n
                let D(k) ~~and d_ij(k)~~ be a new n×n matrix
n×n             for i = 1 to n
n×n×n               for j = 1 to n
n×n×n×1                 d_ij(k) = min(d_ij(k-1), d_ik(k-1) + d_kj(k-1))
                        if d_ij(k-1) ≤ d_ik(k-1) + d_kj(k-1)
                            π_ij(k) = π_ij(k-1)
                        else π_ij(k) = π_kj(k-1)
            return D(n), π(n)
```

Analysis:

Because of triply nested for loop from lines 3 to 7 algorithm takes $\Theta(n^3)$ or $\Theta(|V^3|)$

Step IV : Constructing a shortest path

- For constructing solution, predecessor matrix π is computed while algorithm computes $D^{(k)}$.
- specifically, sequence of matrices are computed ie $\pi^{(0)}, \pi^{(1)} \ldots \pi^{(n)}$

- For $k = 0$, i.e no intermediate vertex from i to j,

$$\pi_{ij}^{(0)} = \begin{cases} NIL & \text{if } i = j \text{ or } w_{ij} = \infty \\ i & \text{if } i \neq j \text{ \& } w_{ij} < \infty \end{cases}$$

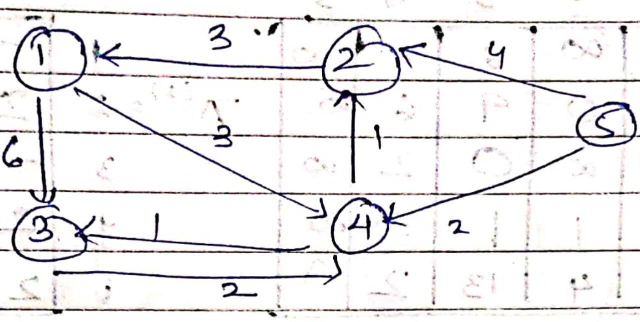
- For $k \geq 1$,

$$\pi_{ij}^{(k)} = \begin{cases} \pi_{ij}^{(k-1)} & \text{if } d_{ij}^{(k-1)} \leq d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \\ \pi_{kj}^{(k-1)} & \text{if } d_{ij}^{(k-1)} > d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \end{cases}$$

```
InitializePredecessor (π(0), W)
1.  for i = 1 to n
2.      for j = 1 to n
3.          If (i==j || wij ==∞)
4.              πij = NIL
5.          else
6.              πij = i
}
```

```
Printpath (π, i, j)
1.  if i==j
        print i
2.  elseif πij ==NIL
        print "No path"
3.  else
    {
        printpath (π, i, πij)
        print j
    }
```

Q APSP using Floyd-Warshall algo:



I] Initialization : $K = 0$. $D^{(0)} = |w|$

$D^{(0)} =$

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	∞	∞	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

$\pi^{(0)} =$

	1	2	3	4	5
1	–	–	1	1	–
2	2	–	–	–	–
3	–	–	–	3	–
4	–	4	4	–	–
5	–	5	–	5	–

II] Iterations :

1] $K = 1$ ⟹ subset = {1}

$D^{(1)} =$

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	9	6	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

$\pi^{(1)} =$

	1	2	3	4	5
1	–	–	1	1	–
2	2	–	1	1	–
3	–	–	–	3	–
4	–	4	4	–	–
5	–	5	–	5	–

$d_{23}^{(1)} = \min(d_{23}^{(0)}, d_{21}^{(0)} + d_{13}^{(0)}) = \min(\infty, 3+6) = 9$

$\pi_{23}^{(1)} = \pi_{13}^{(0)} = 1$

$d_{24}^{(1)} = \min(d_{24}^{(0)}, d_{21}^{(0)} + d_{14}^{(0)}) = \min(\infty, 3+3) = 6$

$\pi_{24}^{(1)} = \pi_{14}^{(0)} = 1$

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2] $k = 2 \Rightarrow$ subset $= \{1, 2\}$

$D^{(2)} =$

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	9	6	∞
3	∞	∞	0	2	∞
4	4	1	1	0	∞
5	7	4	13	2	0

$\pi^{(2)} =$

	1	2	3	4	5
1	–	–	1	1	–
2	2	–	1	1	–
3	–	–	–	3	–
4	2	4	4	–	–
5	2	5	1	5	–

$d_{41}^{(2)} = \min(d_{41}^{(1)}, d_{42}^{(1)} + d_{21}^{(1)}) = \min(\infty, 1+3) = 4$

$\pi_{42}^{(2)} = \pi_{21}^{(1)} = 2$

$d_{51}^{(2)} = \min(d_{51}^{(1)}, d_{52}^{(1)} + d_{21}^{(1)}) = \min(\infty, 4+3) = 7$

$\pi_{51}^{(2)} = \pi_{21}^{(1)} = 2$

$d_{53}^{(2)} = \min(d_{53}^{(1)}, d_{52}^{(1)} + d_{23}^{(1)}) = \min(\infty, 4+9) = 13$

$\pi_{53}^{(2)} = \pi_{23}^{(1)} = 1$

3] $k = 3 \Rightarrow$ subset $= \{1, 2, 3\}$

$D^{(3)} =$

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	9	6	∞
3	∞	∞	0	2	∞
4	4	1	1	0	∞
5	7	4	13	2	0

$\pi^{(3)} =$

	1	2	3	4	5
1	–	–	1	1	–
2	2	–	1	1	–
3	–	–	–	3	–
4	2	4	4	–	–
5	2	5	1	5	–

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4] $k=4$ $\Rightarrow \{1,2,3,4\}$

$D^{(4)}$ =

	1	2	3	4	5
1	0	4	4	3	∞
2	3	0	7	6	∞
3	6	3	0	2	∞
4	4	1	1	0	∞
5	6	3	3	2	0

$\pi^{(4)}$ =

	1	2	3	4	5
1	—	4	4	1	—
2	2	—	4	1	—
3	2	4	—	3	—
4	2	4	4	—	—
5	2	4	4	5	—

5] $k=5$ $\Rightarrow \{1,2,3,4,5\}$

$D^{(5)}$ =

	1	2	3	4	5
1	0	4	4	3	∞
2	3	0	7	6	∞
3	6	3	0	2	∞
4	4	1	1	0	∞
5	6	3	3	2	0

$\pi^{(5)}$ =

	1	2	3	4	5
1	—	4	4	1	—
2	2	—	4	1	—
3	2	4	—	3	—
4	2	4	4	—	—
5	2	4	4	5	—

III Printing Path

Vertex 1

1 4 3

1) 1 to 2 : $1 \rightarrow 4 \rightarrow 2$
2) 1 to 3 : $1 \rightarrow 4 \rightarrow 3$
3) 1 to 4 : $1 \rightarrow 4$
4) 1 to 5 : No path exist

2 1 4 3

Vertex 2

1) 2 to 1 : $2 \rightarrow 1$
2) 2 to 3 : $2 \rightarrow 1 \rightarrow 4 \rightarrow 3$
3) 2 to 4 : $2 \rightarrow 1 \rightarrow 4$
4) 2 to 5 : No path exist

8] Vertex 3

1) 3 to 1 : 3 → 4 → 2 → 1
2) 3 to 2 : 3 → 4 → 2
3) 3 to 4 : 3 → 4
4) 3 to 5 : No path exist.

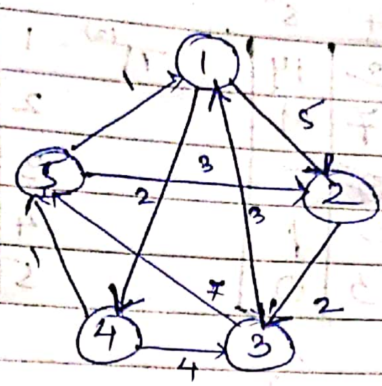
Vertex 4

1) 4 to 1 : 4 → 2 → 1
2) 4 to 2 : 4 → 2
3) 4 to 3 : 4 → 3
4) 4 to 5 : No path exist

Vertex 5

1) 5 to 1 : 5 → 4 → 2 → 1
2) 5 to 2 : 5 → 4 → 2
3) 5 to 3 : 5 → 4 → 3
4) 5 to 4 : 5 → 4

Q. Run floyd warshall on following graph



⇒

I] Initialization, $k = 0$, $W = D^{(0)}$

$D^{(0)} =$

	1	2	3	4	5
1	0	5	∞	2	∞
2	∞	0	2	∞	∞
3	3	∞	0	∞	7
4	∞	∞	4	0	1
5	1	3	∞	∞	0

$\pi^{(0)} =$

	1	2	3	4	5
1	–	1	–	1	–
2	–	–	2	–	–
3	3	–	–	–	3
4	–	–	4	–	4
5	5	5	–	–	–

II] Iteration:-

1) $k = 1$ ⇒ subset = {1}

$D^{(1)} =$

	1	2	3	4	5
1	0	5	∞	2	∞
2	∞	0	2	∞	∞
3	3	8	0	5	7
4	∞	∞	4	0	1
5	1	3	∞	3	0

$\pi^{(1)} =$

	1	2	3	4	5
1	–	1	–	1	–
2	–	–	2	–	–
3	3	1	–	1	3
4	–	–	4	–	4
5	5	5	–	1	–

2) $k = 2$ ⇒ Subset = {1, 2}

$D^{(2)} =$

	1	2	3	4	5
1	0	5	7	2	∞
2	∞	0	2	∞	∞
3	3	8	0	5	7
4	∞	∞	4	0	1
5	1	3	5	3	0

$\pi^{(2)} =$

	1	2	3	4	5
1	–	1	2	1	–
2	–	–	2	–	–
3	3	1	–	1	3
4	–	–	4	–	4
5	5	5	2	1	–

3) $k = 3 \Rightarrow$ subset $= \{1, 2, 3\}$

$D^{(3)} =$

	1	2	3	4	5
1	0	5	7	2	14
2	5	0	2	7	9
3	3	8	0	5	7
4	7	12	4	0	1
5	1	3	5	3	0

$\pi^{(3)} =$

	1	2	3	4	5
1	—	1	2	1	3
2	3	—	2	1	3
3	3	1	—	1	3
4	3	1	4	—	4
5	5	5	2	1	—

4) $k = 4 \Rightarrow$ Subset $= \{1, 2, 3, 4\}$

$D^{(4)} =$

	1	2	3	4	5
1	0	5	6	2	3
2	5	0	2	7	8
3	3	8	0	5	6
4	7	12	4	0	1
5	1	3	5	3	0

$\pi^{(4)} =$

	1	2	3	4	5
1	—	1	4	1	4
2	3	—	2	1	4
3	3	1	—	1	4
4	3	1	4	—	4
5	5	5	2	1	—

5) $k = 5 \rightarrow$ Subset $= \{1, 2, 3, 4, 5\}$

$D^{(5)} =$

	1	2	3	4	5
1	0	5	6	2	3
2	5	0	2	7	8
3	3	8	0	5	6
4	2	4	4	0	1
5	1	3	5	3	0

$\pi^{(5)} =$

	1	2	3	4	5
1	[illegible]	1	4	1	4
2	3	—	2	1	4
3	3	1	—	1	4
4	5	5	4	—	4
5	5	5	2	1	—

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III] Printing Path

Vertex 1 :

1) 1 to 2 : $1 \rightarrow 2$
2) 1 to 3 : $1 \rightarrow 4 \rightarrow 3$
3) 1 to 4 : $1 \rightarrow 4$
4) 1 to 5 : $1 \rightarrow 4 - 5$

Vertex 2 :

1) 2 to 1 : $2 \rightarrow 3 \rightarrow 1$
2) 2 to 3 : $2 \rightarrow 3$
3) 2 to 4 : $2 \rightarrow 3 \rightarrow 1 \rightarrow 4$
4) 2 to 5 : $2 \rightarrow 3 \rightarrow 1 \rightarrow 4 \rightarrow 5$

Vertex 3

1) 3 to 1 : $3 \rightarrow 1$
2) 3 to 2 : $3 \rightarrow 1 \rightarrow 2$
3) 3 to 4 : $3 \rightarrow 1 \rightarrow 4$
4) 3 to 5 : $3 \rightarrow 1 \rightarrow 4 \rightarrow 5$

Vertex 4

1) 4 to 1 : $4 \rightarrow 5 \rightarrow 1$
2) 4 to 2 : $4 \rightarrow 5 \rightarrow 2$
3) 4 to 3 : $4 \rightarrow 3$
4) 4 to 5 $4 \rightarrow 5$

Vertex 5

1) 5 to 1 : $5 \rightarrow 1$
2) 5 to 2 : $5 \rightarrow 2$
3) 5 to 3 : $5 \rightarrow 2 \rightarrow 3$
4) 5 to 4 : $5 \rightarrow 1 \rightarrow 4$

Problem on Floyd Warshall

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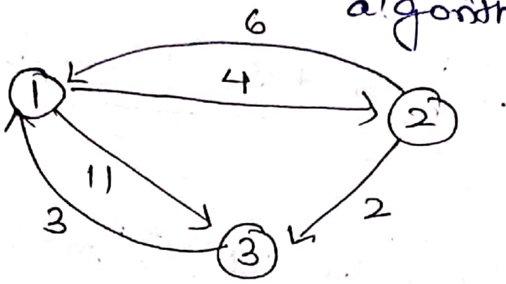
Q. Find all pair shortest path for the graph given below using Floyd warshall algorithm.



⇒ I) Initialization : $(k=0)$

$W = D^{(0)}$

$\cancel{\pi^{(0)}}$

$D^{(0)} =$

	1	2	3
1	0	4	11
2	6	0	2
3	3	∞	0

$\pi^{(0)} =$

	1	2	3
1	N	1	1
2	2	N	2
3	3	N	N

II) Iterations :

$\boxed{I}$ $(k=1)$

$\cancel{D^{(1)}}$

$D^{(1)} =$

	1	2	3
1	0	4	11
2	6	0	2
3	3	7	0

$\pi^{(1)} =$

	1	2	3
1	N	1	1
2	2	N	2
3	3	1	N

(i) $d_{12}^{(1)} = \min(d_{12}^{(0)}, d_{11}^{(0)} + d_{12}^{(0)}) = \min(4, 0+4) = 4$

$\pi_{12}^{(1)} = \pi_{12}^{(0)} = 1$

(ii) $d_{13}^{(1)} = \min(d_{13}^{(0)}, d_{11}^{(0)} + d_{13}^{(0)}) = \min(11, 0+11) = 11$

$\pi_{13}^{(1)} = \pi_{12}^{(0)} = 1$

(iii) $d_{21}^{(1)} = \min(d_{21}^{(0)}, d_{21}^{(0)} + d_{11}^{(0)}) = \min(6, 6+0) = 6$

$\pi_{21}^{(1)} = \pi_{21}^{(0)} = 2$

(iv) $d_{23}^{(1)} = \min(d_{23}^{(0)}, d_{21}^{(0)} + d_{13}^{(0)}) = \min(2, 6+11) = 2$

$\pi_{23}^{(1)} = \pi_{23}^{(0)} = 2$

(v) $d_{31}^{(1)} = \min(d_{31}^{(0)}, d_{31}^{(0)} + d_{11}^{(0)}) = \min(3, 3+0) = 3$

$\pi_{31}^{(1)} = \pi_{31}^{(0)} = 3$

(vi) $d_{32}^{(1)} = \min(d_{32}^{(0)}, d_{31}^{(0)} + d_{12}^{(0)}) = \min(\infty, 3+4) = 7$

$\pi_{32}^{(1)} = \pi_{12}^{(0)} = 1$

2] $k = 2$

$\Delta^{(2)} =$

	1	2	3
1	0	4	6
2	6	0	2
3	3	7	0

$\pi^{(2)} =$

	1	2	3
1	N	1	2
2	2	N	2
3	3	1	N

(i) $d_{13}^{(2)} = \min(d_{13}^{(1)}, d_{12}^{(1)} + d_{23}^{(1)}) = \min(11, 4+2) = 6$

$\pi_{13}^{(2)} = \pi_{23}^{(1)} = 2$

(ii) $d_{21}^{(2)} = \min(d_{21}^{(1)}, d_{22}^{(1)} + d_{21}^{(1)}) = \min(6, 0+6) = 6$

$\pi_{21}^{(2)} = \pi_{21}^{(1)} = 2$ (same)

$K = 3$

$$D^{(3)} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 6 \\ 2 & 5 & 0 & 2 \\ 3 & 3 & 7 & 0 \end{array} \qquad \pi^{(3)} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & N & 1 & 2 \\ 2 & 2 & N & 2 \\ 3 & 3 & 1 & N \end{array}$$

(i) $d_{21}^{(3)} = \min\left(d_{21}^{(2)},\ d_{23}^{(2)} + d_{13}^{(2)}\right) = \min(6, 2+3) = 5$

$\pi_{21}^{(3)} = \pi_{23}^{(2)} = 2$

[III] constructing minimum path from $\pi^{(3)}$

1] For vertex 1,

path <1,2> : $1 \rightarrow 2$

path <1,3> : $1 \rightarrow 2 \rightarrow 3$

path <2,1> $2 \rightarrow 1$

2] For vertex 2,

path <2,1> : $2 \rightarrow 3 \rightarrow 1$

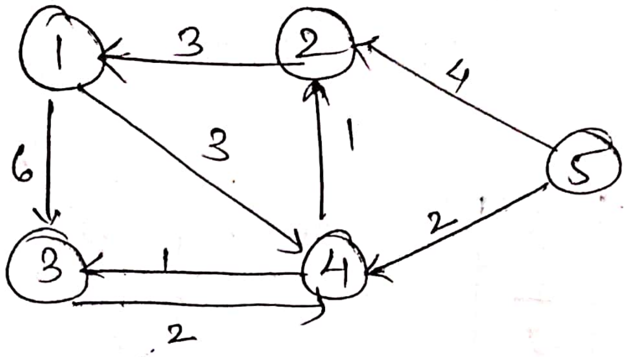
path <2,3> : $2 \rightarrow 3$

3] For vertex 3,

path <3,1> : $3 \rightarrow 1$

path <3,2> : $3 \rightarrow 1 \rightarrow 2$

Q. Find APSP using Floyd Warshall on following graph.



⇒ I] <u>Initialization</u> : <u>K = 0</u> , <u>W = $D^{(0)}$</u>

$D^{(0)}$ =

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	∞	∞	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

$\pi^{(0)}$ =

	1	2	3	4	5
1	N	N	1	1	N
2	2	N	N	N	N
3	N	N	N	3	N
4	N	4	4	N	N
5	N	5	N	5	N

II) <u>Iterations</u>

1] K = 1

$D^{(1)}$ =

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	9	6	∞
3	∞	∞	0	2	∞
4	∞	1	1	0	∞
5	∞	4	∞	2	0

$\pi^{(1)}$ =

	1	2	3	4	5
1	N	N	1	1	N
2	2	N	1	1	N
3	N	N	N	3	N
4	N	4	4	N	N
5	N	5	N	5	N

(i) $d_{23}^{(1)} = \min(d_{23}^{(0)}, d_{21}^{(0)} + d_{13}^{(0)}) = \min(\infty, 3+6) = 9$.

$\pi_{23}^{(1)} = \pi_{13}^{(0)} = 1$

(ii) $d_{24}^{(1)} = \min(d_{24}^{(0)}, d_{21}^{(0)} + d_{14}^{(0)}) = \min(\infty, 3+3) = 6$

$\pi_{24}^{(1)} = \pi_{14}^{(0)} = 1$

$k = 2$

$D^{(2)} =$

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	9	6	∞
3	∞	∞	0	2	∞
4	4	1	1	0	∞
5	7	4	13	2	0

$\pi^{(2)} =$

	1	2	3	4	5
1	N	N	1	1	N
2	2	N	1	1	N
3	N	N	N	3	N
4	2	4	4	N	N
5	2	5	1	5	N

$\cancel{d_{31}^{(2)} = \min(d_{31}^{(1)}, d_{32}^{(1)} + d_{21}^{(1)}) = \min(\infty, \infty + 3) = \infty}$

(i) $d_{41}^{(2)} = \min(d_{41}^{(1)}, d_{42}^{(1)} + d_{21}^{(1)}) = \min(\infty, 1+3) = 4$

$\pi_{41}^{(2)} = \pi_{21}^{(1)} =$

(ii) $d_{51}^{(2)} = \min(d_{51}^{(1)}, d_{52}^{(1)} + d_{21}^{(1)}) = \min(\infty, 4+3) = 7$.

$\pi_{51}^{(2)} = \pi_{21}^{(1)} = 3$

(iii) $d_{53}^{(2)} = \min(d_{53}^{(1)}, d_{52}^{(1)} + d_{23}^{(1)}) = \min(\infty, 4+9) = 13$

$\pi_{53}^{(2)} = \pi_{23}^{(1)} = 1$

3] $k = 3$ (No shorter path)

$D^{(3)} =$

	1	2	3	4	5
1	0	∞	6	3	∞
2	3	0	9	6	∞
3	∞	∞	0	2	∞
4	4	1	1	0	∞
5	7	4	13	2	0

$\pi^{(3)} =$

	1	2	3	4	5
1	N	N	1	1	N
2	2	N	1	1	N
3	N	N	N	3	N
4	2	4	4	N	N
5	2	5	1	5	N

4] $K=4$

$$D^{(4)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 4 & 4 & 3 & \infty \\ 2 & 3 & 0 & 7 & 6 & \infty \\ 3 & 6 & 3 & 0 & 2 & \infty \\ 4 & 4 & 1 & 1 & 0 & \infty \\ 5 & 6 & 3 & 3 & 2 & 0 \end{array}$$

$$\pi^{(4)} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & N & 4 & 4 & 1 & N \\ 2 & 2 & N & 4 & 1 & N \\ 3 & 2 & 4 & N & 3 & N \\ 4 & 2 & 4 & 4 & N & N \\ 5 & 2 & 4 & 4 & 5 & N \end{array}$$

(i) $d_{12}^{(4)} = \min(d_{12}^{(3)}, d_{14}^{(3)} + d_{42}^{(3)}) = \min(\infty, 3+1) = 4$

$\pi_{12}^{(4)} = \pi_{42}^{(3)} = 4$

(ii) $d_{13}^{(4)} = \min(d_{13}^{(3)}, d_{14}^{(3)} + d_{43}^{(3)}) = \min(6, 3+1) = 4$

$d_{13}^{(4)} = \pi_{43}^{(3)} = 4$

(iii) $d_{23}^{(4)} = \min(d_{23}^{(3)}, d_{24}^{(3)} + d_{43}^{(3)}) = \min(9, 6+1) = 7$

$\pi_{23}^{(4)} = \pi_{43}^{(3)} = 4$

(iv) $d_{31}^{(4)} = \min(d_{31}^{(3)}, d_{34}^{(3)} + d_{41}^{(3)}) = \min(\infty, 2+4) = 6$

$\pi_{31}^{(4)} = \pi_{41}^{(3)} =$

(v) $d_{32}^{(4)} = \min(d_{32}^{(3)}, d_{34}^{(2)} + d_{42}^{(3)}) = \min(\infty, 2+1) = 3$

$\pi_{32}^{(4)} = \pi_{42}^{(3)} = 4$

(vi) $d_{51}^{(4)} = \min(d_{51}^{(3)}, d_{54}^{(3)} + d_{41}^{(3)}) = \min(7, 2+4) = 6$

$\pi_{51}^{(4)} = \pi_{41}^{(2)} = 2$

(vii) $d_{52}^{(4)} = \min(d_{52}^{(3)}, d_{54}^{(3)} + d_{42}^{(3)}) = \min(4, 2+1) = 3$

$\pi_{52}^{(4)} = \pi_{42}^{(3)} = 4$

(viii) $d_{53}^{(4)} = \min(d_{53}^{(3)}, d_{54}^{(3)} + d_{43}^{(3)}) = \min(13, 2+1) = 3$

$\pi_{53}^{(4)} = \pi_{43}^{(3)} = 4$

$k = 5$ (No shorter path)

$D^{(5)} =$

	1	2	3	4	5
1	0	4	4	3	∞
2	3	0	7	6	∞
3	6	3	0	2	∞
4	4	1	1	0	∞
5	6	3	3	2	0

$\pi^{(5)} =$

	1	2	3	4	5
1	N	4	4	1	N
2	2	N	4	1	N
3	2	4	N	3	N
4	2	4	4	N	N
5	2	4	4	5	N

III] Constructing minimum path from $\pi^{(5)}$

1] For vertex 1,

path <1,2> : $1 \to 4 \to 2$

path <1,3> : $1 \to 4 \to 3$

path <1,4> : $1 \to 4$

path <1,5> : Not reachable.

2] For vertex 2,

path <2,1> : $2 \to 1$

path <2,3> : $2 \to 1 \to 4 \to 3$

path <2,4> : $2 \to 1 \to 4$

path <2,5> : Not reachable.

3] For vertex 3,

path <3,1> : $3 \to 4 \to 2 \to 1$

path <3,2> : $3 \to 4 \to 2$

path <3,4> : $3 \to 4$

path <3,5> : Not reachable.

4th pract

4] for vertex 4,

path <4, 1> : $4 \to 2 \to 1$

path <4, 2> : $4 \to 2$

path <4, 3> : $4 \to 3$

path <4, 5> : Not reachable.

5] For vertex 5,

path <5, 1> : $5 \to 4 \to 2 \to 1$

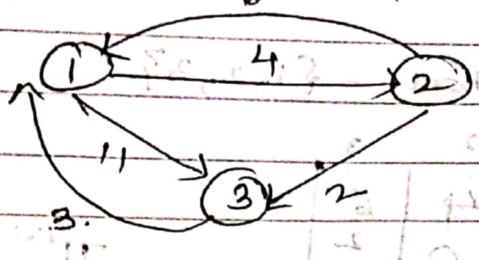
path <5, 2> : $5 \to 4 \to 2$

path <5, 3> : $5 \to 4 \to 3$

path <5, 4> : $5 \to 4$

New Doc 2017-11-28_1.jpg

Q. Find all pair shortest path for the graph given below using Floyd Warshall Algorithms.



I] Initialization : k = 0

$D^{(0)} = W$

$D^{(0)} =$

	1	2	3
1	0	4	11
2	6	0	2
3	3	∞	0

$\pi^{(0)} =$

	1	2	3
1	θ	1	1
2	2	θ	2
3	3	–	θ

II] Iterations :-

1] k = 1 $\Rightarrow$ subset = {1}

$D^{(1)} =$

	1	2	3
1	0	4	11
2	6	0	2
3	3	7	0

$\pi^{(1)} =$

	1	2	3
1	–	1	1
2	2	–	2
3	3	1	–

$d_{32}^{(1)} = \min(d_{32}^{(0)}, d_{31}^{(0)} + d_{12}^{(0)}) = \min(\infty, 3+4) = 7$

$\pi_{32}^{(1)} = \pi_{12}^{(0)} = 1$

2] k = 2 $\Rightarrow$ subset = {1, 2}

$D^{(2)} =$

	1	2	3
1	6	4	6
2	6	0	2
3	3	7	0

$\pi^{(2)} =$

	1	2	3
1	–	1	2
2	2	–	2
3	3	1	–

$$d_{13}^{(2)} = \min(d_{13}^{(1)}, d_{12}^{(1)} + d_{23}^{(1)}) = \min(11, 4+2) = 6$$

$$\pi_{13}^{(2)} = \pi_{23}^{(1)} =$$

3] $k = 3 \Rightarrow$ subset $= \{1, 2, 3\}$

$$D^{(3)} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 4 & 6 \\ 2 & 5 & 0 & 2 \\ 3 & 3 & 7 & 0 \end{array} \qquad \Pi^{(3)} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & - & 1 & 2 \\ 2 & 2 & - & 2 \\ 3 & 3 & 1 & - \end{array}$$

$$d_{21}^{(3)} = \min(d_{21}^{(2)}, d_{23}^{(2)} + d_{31}^{(2)}) = \min(6, 2+3) = 5$$

$$\pi_{21}^{(3)} = \pi_{31}^{(2)} = 2$$

III] ~~to~~ Printing paths

For vertex 1

1] 1 to 2 : $1 \to 2$

2] 1 to 3 : $1 \to 2 \to 3$

For vertex 2 :

1] 2 to 1 : $2 \to 1$

2] 2 to 3 : $2 \to 3$