MTH215 ⋆⋆ 2019-20 ⋆⋆ ASSIGNMENT – 6

1. Show that $a_{n+1}^2 + a_{n-2}^2 = 2a_{2n-1}$ for $n \geq 3$.
2. Show that $a_{n+2}^2 + a_{n-1}^2 = 2(a_n^2 + a_{n-1}^2)$ for $n \geq 2$.
3. Show that $a_{n+1}^2 - 4a_n a_{n-1} = a_{n-2}^2$ for $n \geq 3$.
4. Show that $a_{n+1}a_{n-1} - a_{n+2}a_{n-2} = 2(-1)^n$ for $n \geq 3$.
5. Show that $a_n^2 - a_{n+2}a_{n-2} = (-1)^n$ for $n \geq 3$.
6. Show that $a_n^2 - a_{n+3}a_{n-3} = 4(-1)^{n+1}$ for $n \geq 4$.
7. Show that $a_n a_{n+1} a_{n+2} a_{n+3} = a_{n+2}^4 - 1$ for $n \geq 1$.
8. Show that $(a_n a_{n+3})^2 + (2a_{n+1}a_{n+2})^2 = a_{2n+3}^2$ for $n \geq 1$.
9. Let $n \in \mathbb{N}$. Show that
   (a) $a_{4n} + 1 = a_{2n-1}(a_{2n} + a_{2n+2})$.
   (b) $a_{4n+1} + 1 = a_{2n+1}(a_{2n-1} + a_{2n+1})$.
   (c) $a_{4n+2} + 1 = a_{2n+2}(a_{2n-1} + a_{2n+1})$.
   (d) $a_{4n+3} + 1 = a_{2n+1}(a_{2n+1} + a_{2n+3})$.
   (e) $a_n + 1$ is composite for $n \geq 4$.
10. $a_{2n-1}a_{2n+5}$ is a sum of two squares.
11. Show that the sum of any consecutive Fibonacci numbers is divisible by a_{10}.
12. Find a positive integer n such that $\mu(n) + \mu(n+1) + \mu(n+2) = 3$.
13. Let $n \in \mathbb{N}$. Show that $\mu(n)\mu(n+1)\mu(n+2)\mu(n+3) = 0$.
14. Let $m \in \mathbb{N}$. Show that there exist infinitely many $n \in \mathbb{N}$ such that $\mu(n+1) = \mu(n+2) = \cdots = \mu(n+m)$.
15. Find all Pythagorean triples (a, b, c) such that a, b, c are in GP.