Time-series classification using matrix-based methods: Application to blackhole state identification of RXTE satellite data

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***Abstract*—Across diverse domains such as medicine, weather, finance, agriculture, astronomy, etc., it is required to deal with timeseries of measurements. Classification of timeseries as stochastic (noise-like) or non-stochastic (which has a well-defined structure), helps understand the underlying phenomenon. The methods used to accomplish this classification are either : (i) Correlation Integral (CI)-based or (ii) Entropy-based approaches, both of which are computationally expensive. In this work, we propose two matrix-based methods to achieve stochastic vs non-stochastic classification, without requiring the computation-intensive phase space. The proposed matrix-based methods are: (a) SVD-decomposition followed by topological analysis (b) PCA-based technique. The proposed methods have been applied to synthetic data, as proof of concept. The utility of the methods is illustrated on astronomy data which are 12 categories of timeseries pertaining to blackhole *GRS 1915 + 105*, obtained from RXTE satellite. Comparisons of obtained results with those in literature are also presented. The order of computational complexity using the proposed approaches is XXXXXXX of N, where N is the length of the timeseries. In contrast, CI based approaches require XXXX, while Entropy-based approaches need XXXXX. It is found that among the proposed matrix based methods, SVD analysis concurs with CI based analysis on all 12 categories of time series utilized. However, the inference using PCA based approach illustrates that one class among the 12 turns out to be inconsistent with the other approaches. Investigation into these (in)consistencies is expected to have long standing implications in astrophysics and otherwise.**

***Index Terms*—Timeseries classification, stochastic, non-stochastic, SVD analysis, PCA analysis**

I. Introduction

Several real-world phenomena are studied by collecting associated measurements over time, popularly called as time-series. Timeseries classification as stochastic (noise-like) or non-stochastic (which has a well-defined structure), is the first step in understanding the underlying physical phenomenon. Standard stochastic signals such as white noise, pink noise, etc. exhibit characteistics such as nearly zero auto-correlation coefficients for all possible values of lags and a power spectral density that decays with frequency. The rate of decay determines the kind of noise. On the other hand, standard non-stochastic signals such as Logistic map (at growth rate = 4), Lorenz system result in timeseries that

Stochastic timeseries may be seen as noise, while non-stochastic timeseries could reveal more about the associated physcial phenomenon. In literature, methods that accomplish this classification can be broadly categorised as : (i) Correlation Integral based (ii) Entropy-based. the method followed in literature [New Refs 1-5 ukraine paper] computes the Embedding Dimension, using concepts of phase-space. It is a computation-intensive process, since the Correlation Integral (CI) needs to be computed for different choices of the Embedding dimension check is it correlation dimension , given by the equation below. The rate of change of log(CI(r)) with respect to log (r), as r tends to 0 is taken as the ...... It is well-known that this value of correlation dimension does not saturate for a stochastic time series. Hence to establish if the considered timeseries is stochastic, this computation needs to be repeated for a large range of values of Embedding Dimension, making the order of computations needed greater by that factor. The knowledge of the Embedding Dimension would indeed further the understanding of the physical process. However, if an initial screening test to determine if the timeseries is indeed stochastic or not, it would be advantageous to utilize a technique that does not require the computation of Correlation Dimension.

standard algorithm to compute th Most methods of chaotic dynamics used for time series analysis, based on the reconstruction space of single realization using the procedure Packard-Takens [1-4]. The reconstruction of the pseudo-phase space allows us to compute the embedding dimension, which is the main means of distinguishing chaotic and random processes [3, 5]. This approach allows us to well distinguish between chaotic dynamics and uncorrelated random noise, however, because this method is based on the estimation of the fractal dimension and detection autocorrelation relations, it has no effect for the fractal random processes having long dependence [6, 7].

One of the challenging problems in astrophysics is the understanding of black holes. As a black hole cannot be seen directly, to identify it, one has to look for its environment forming a disc-like structure by the infalling matter called accretion disc. In this work, we focus on the black hole source *GRS 1915+105*, which presents several intriguing facets. It has been classified into 12 different temporal classes: α, β, γ, δ,

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λ, κ, μ, ν, ρ, ϕ, χ and θ [?], with their respective distinct time series. One of the fundamental aspects of the understanding is to determine if the black hole source is a stochastic system or a non-stochastic one. The latter one is related to the well-known turbulent nature of the system. There are several studies that utilize the Correlation Integral (CI) approach to determine the characterization of the black hole data [?], [?]. However, there can also be other approaches to understanding the same data by applying, for e.g., matrix-based methods such as Principal Component Analysis (PCA) and Singular Value Decomposition (SVD). It is useful to compare the inferences obtained using these two distinct approaches; the implications of the (dis)similarities in inferences, if any, could lead to questions about understanding the temporal dynamics of the system.

Interestingly, to quantify the properties of a black hole source, along with temporal features one has to look for spectral features as well, they together lead to the true nature of the source. If the source radiation is temperature dependent, it produces more like a blackbody radiation, namely multicolour blackbody or "diskbb" [?]. On the other hand, the temperature independent radiation consists of a power-law tail, named as "PL" [?], [?]. While the former leads the underlying accretion disc around the black hole to be geometrically thin, the latter leads to a geometrically thick disc.

In the present study, black hole states are determined by classifying the given time series, which is photon count rate as a function of time, as being either stochastic or non-stochastic. This classification is performed using classical matrix based methods, SVD and PCA. However, the novelty of the study lies in (i) quantifying temporal complexity obtained by SVD decomposition, using topological techniques and (ii) utilizing features derived from PCA for classification. Based on our analysis there are four possible black hole states [?]:

1) Non-stochastic and diskbb: Keplerian disc [?].
2) Non-stochastic and PL: Advection Dominated Accretion Flow (ADAF) [?].
3) Stochastic and diskbb: Slim disc [?].
4) Stochastic and PL: General Advective Accretion Flow (GAAF) [?], [?].

Utility of the proposed approach is illustrated by comparing the results with previously established methods. Following are the major contributions of this paper:

- SVD decomposition of the data matrix is used for identifying the temporal dynamics of the time series as in [?]. A plot involving the top two right singular vectors of the data matrix shows a clear distinction between stochastic and non-stochastic time series. This distinction is captured using the topological descriptor called Betti numbers [?]. This descriptor for a stochastic time series is topologically simpler than that for a non-stochastic one.
- PCA, which is a widely used approach for decorrelating features and dimensionality reduction, is utilized for characterizing a time series as stochastic vs non-stochastic. We propose a novel approach by iteratively computing eigenvalue ratios of covariance matrix for different subintervals of the time series. We derive multiple features from the eigenvalue ratios and use them to characterize the time series.

II. Related Work

Several groups have worked on distinguishing between stochastic and non-stochastic time series. The idea of utilizing Permutation Entropy (PE) to determine the complexity measure of a time series was explored in [?]. In the work reported in [?], PE was used to parameterize a given time series followed by classification using Neural Network. The paper explored the idea of utilizing PE of a time series to determine if it is strongly correlated with known stochastic signals (noise). The claim was that for non-stochastic signals the deviation of the parameter is relatively large as compared to that of the parameter of a stochastic signal. Another set of reported studies are based on graph theory. In the work reported in [?], the authors have utilized the horizontal visibility algorithm in order to distinguish between stochastic and non-stochastic processes. A recent work, reported in [?], mapped time series into graphs and computed various topological properties, which they called *NetF*, capturing measures such as centrality, distance, connectivity etc. PCA was applied on the *NetF* feature matrix and clustering was performed on the principal components.

In the approach outlined in [?], the authors combined the idea of sparsity and machine learning with non-linear dynamical systems, in order to determine the governing dynamics. Sparse regression was used to determine the fewest terms in the equations that govern the dynamics of the phenomenon. The user-defined dictionary of basis functions consists of well-known functions such as polynomials, trigonometric functions and exponentials. However, the optimal choice of dictionary for a specific choice of problem remains a challenge.

In this work, we propose to utilize classical matrix based methods which do not require any assumptions about the underlying phenomenon.

III. Proposed Method

In this work, we propose two different matrix based approaches to characterize time series as stochastic vs non-stochastic. They are 1) SVD decomposition followed by Betti number descriptors and 2) PCA derived features followed by SVM classification. Proof of Concept on synthetic signals is also presented.

A. SVD based approach

In this approach, we form uncorrelated observation vectors from the raw time series data by utilizing the optimal value of embedding dimension [?]. A data matrix, D, is formed with each row as the time shifted version of the original time series. The time shift is chosen to be large enough so that each column can be viewed as a different observation vector of the same time evolving phenomenon. Temporal dynamics is understood by utilizing the right singular vectors of the SVD decomposition of D as given in equation (1) below. Columns of U and V form the left and right singular vectors respectively and Σ is a block diagonal matrix with diagonal elements as the singular values, given by

$$D = U\Sigma V^T. \quad (1)$$

We observe the plot of the top two right singular vectors (E1 vs E2). For non-stochastic time series this plot is expected to show a specific pattern (attractor behavior, where the plot follows a trajectory leaving a well-defined gap). On the other hand, for stochastic time series, this behavior is absent.

The characteristics of the E1 vs E2 plot are captured using Betti numbers [?]. Betti number descriptor for a d-dimensional manifold is a vector of d integers which is represented as $\beta = (\beta_0, \beta_1 \dots \beta_{d-1})$. Here the E1 vs E2 plots are 2-d manifolds, which are described by $\beta = (\beta_0, \beta_1)$. β_0 is the number of blobs (connected components) while β_1 is the number of 1-d holes. For a stochastic time series the values of β_0 and β_1 are expected to be 1 and 0 respectively, as the E1 vs E2 plot consists of one single blob. However, for a non-stochastic time series, we observed that the value of β_0 can be greater than 1 and the value of β_1 is always greater than 0 due to the attractor behavior.

B. PCA Based approach

We utilize PCA to understand if the available data possess a dominant orientation. This can be computed by splitting the time series into two halves, and computing the covariance matrix of these observations. The eigenvalues of this 2×2 covariance matrix will show one of the signatures: If the data indeed show any dominant direction (as in non-stochastic time series), then the larger eigenvalue will be significantly greater than the other. This will lead to a large ratio of the eigenvalues. On the other hand, if the data do not show any dominant direction (as in stochastic time series), then the two eigenvalues of the covariance matrix will be comparable. This will lead to small values of eigenvalue ratio.

Consider a time series consisting of n values $z_1, z_2 \dots z_n$. We begin by computing the eigenvalue ratio for the entire series using the following steps:

- Split the series into two halves $(z_1, z_2 \dots z_{\lfloor \frac{n}{2} \rfloor})$ and $(z_{\lfloor \frac{n}{2} \rfloor + 1}, \dots z_n)$.
- Compute covariance matrix, C, by treating the samples in two halves as $\lfloor \frac{n}{2} \rfloor$ observations of two dimensional vectors.
- Find eigenvalues of C, λ_1 and λ_2; the eigenvalue ratio is computed as λ_1/λ_2 where $\lambda_1 > \lambda_2$ (eigenvalues of a covariance matrix are real).

If the eigenvalue ratio for an interval is less than a value of threshold, τ, the interval is further split into two sub-intervals of equal size. Subsequently, the eigenvalue ratio for each sub-interval is computed. The process is repeated as long as the length of the sub-interval is greater than a predefined number of samples (here taken as 100). For a chosen value of τ, the following features are derived

- **V**ariance of **E**igenvalue **R**atio (**VER**): This is the variance of the eigenvalue ratios of covariance matrices across sub-intervals in the entire time series.
- **A**rea **U**nder the **E**igenvalue **R**atio curve (**AUER**): This measure captures the area under the curve of the eigenvalue ratio for the entire time series.

In order to arrive at the optimal value of τ, we observe the plot of the Silhoutte score of K-Means clustering performed on the feature set, as a function of the threshold value. The value of the threshold that results in the best Silhoutte clustering score is taken as τ. The figure below shows the variation in the Silhoutte score of K-means clustering across various values of Threshold.

C. Proof of Concept on Synthetic Data

The proposed approaches have been applied to standard synthetic signals. For stochastic class of signals, white noise and pink noise are considered; for non-stochastic class of signals, Lorenz system and Logistic map (for growth rate = 4) are considered.

SVD-Decomposition based technique : The SVD decomposition of the data is computed, followed by the plot of the top two right singular vectors. The plot in Fig. **??** is used for determining the Betti descriptors. PCA-based features,

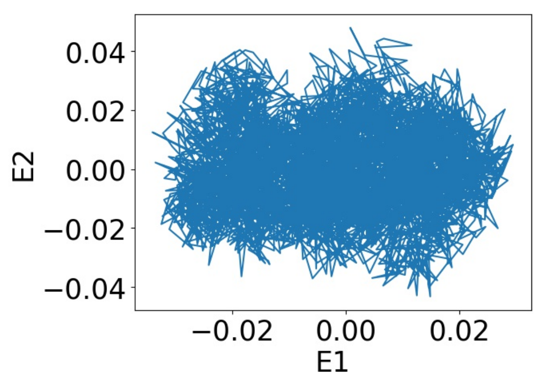

Fig. 1. Plot of Top-2 Right singular vectors for a stochastic timeseries

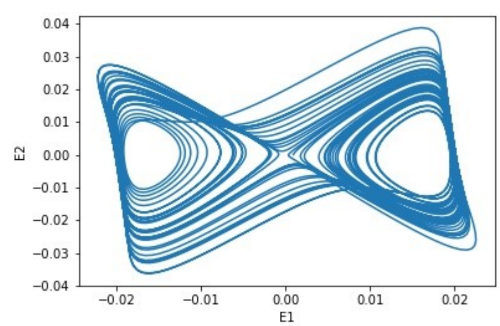

Fig. 2. Plot of Top-2 Right singular vectors for a non-stochastic timeseries

(i) VER (Variance of Eigen ratios) and (ii) AUER (Area under the curve of the eigen ratios) are computed. The scatter plot of these features for multiple realizations (total = 24) of the considered synthetic signals is shown below. The plot makes it evident that a decision boundary separating the two classes, stochastic and non-stochastic can be easily found in this feature space. Hence a linear SVM classifier is utilized. For training the SVM, features from white noise and Logistic map are utilized. For testing the trained SVM, pink noise and Lorentz system are used.

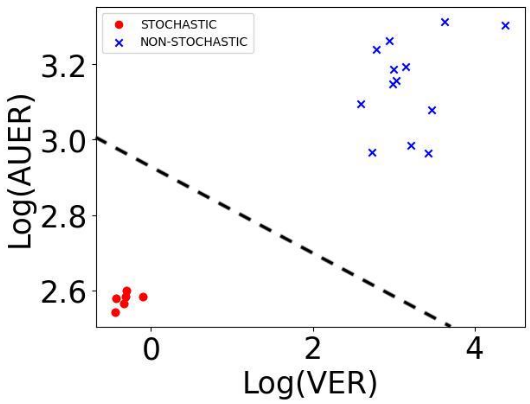

Fig. 3. Scatter plot of PCA-based features for synthetic data

IV. Results and Discussion

In thin section, we present the real data used, results obtained using proposed approaches and comparison with results in literature.

A. Real Data

The proposed approaches are illustrated on the publicly available data of *GRS 1915 + 105* taken from website [?]. 12 distinct classes of time series are utilized from the available data. All these time series are re-sampled with a sampling interval of 0.1 second. These datasets were also used in the work reported in [?], where the authors use CI based approaches, leading us to be able to compare our obtained results with theirs. Figures **??** and **??** show a representative time series of stochastic and non-stochastic nature respectively.

B. Results of SVD based analysis

From SVD decomposition of the data matrix, we plot the top 2 right singular vectors (E1 vs E2) to understand the temporal dynamics for each time series. Figure **??** shows representative E1 vs E2 plots for time series that are classified as non-stochastic and Figure **??** shows the corresponding plots for time series that are classified as stochastic. The Betti number descriptors for each of the E1 vs E2 plots are tabulated in Table I under the column *Betti descriptor*. In order to infer the label of the time series from the Betti descriptors, we use the L1-norm of β, $\|\beta\|_1$. If $\|\beta\|_1 > 1$, the time series is classified as non-stochastic, else the time series is stochastic.

TODO :Table of Betti number - Time series, Betti Descriptor, l1 noram , SVD label

C. Results of PCA based analysis

TODO : table of derived features and SVM inference - Timer series, VER, AUER, SVM label

Figures **??** and **??** show the eigenvalue ratio plots for stochastic time series shown in Figure **??** and non-stochastic time series shown in Figure **??** respectively. We compute the three PCA based features which are MER, VAR and Area and utilize them for classification using the following observations:

- VER: For a stochastic signal since the range of eigenvalue ratios is typically small, the VAR is also small. On the other hand, for a non-stochastic signal, since the eigenvalue ratios occupy a large range of values, VAR is typically high.
- AUER: For a stochastic time series, since the eigenvalue ratios are small the Area is also small. However, for a non-stochastic signal the eigenvalue ratios remain high for longer time intervals. Hence the Area is significantly higher.

D. Consolidated Results

1) Comparison of Results: Table I tabulates the computed features and the respective inferences using the proposed approaches. Comparison of our results with CI based approach [?] is also presented. The columns of the table are described below.

1) Column 1 (*Class*) gives the class of the time series [?].
2) Column 2 (*diskbb*) and column 3 (*PL*) give quantities diskbb and PL, respectively, which indicate the spectral states of the black hole [?].
3) Column 4 (CI Inference) gives the inference about the state of the time series using CI approach [?].
4) Column 5 SVD based inference.
5) Our inference using these PCA features is given in column XX.
6) Finally the last column gives if there is a match between all three inferences.

E. Identification of black hole states

We observe that SVD based analysis results in classification are consistent with CI based results for all the 12 classes of time series. However, with the PCA based approach the inference for δ time series is not consistent with the other two approaches. We observe that the PCA based features, VAR and Area, result in visible clustering as shown in Figure **??**. This could be attributed to the fact that these features take into account the entire span of time series and hence form robust feature space. According to the CI based analysis δ turns out to be in between states slim disc and GAAF [?]. However, the present analysis shows that δ falls in between ADAF and Keplerian disc.

V. Conclusion

Exploring different techniques in order to have a conclusive inference for black hole systems turns out to be indispensable. We explore two different classical matrix based techniques to identify states of *GRS 1915+105* black hole using the time series obtained from *RXTE* satellite data. Based on our analysis, we are able to identify two extreme temporal dynamical classes of accretion around black holes. In the

TABLE I
TIMESERIES: COMPARISON BETWEEN CI BASED LABEL AND INFERENCE USING PROPOSED APPROACHES. THE MISMATCHED TIME SERIES CLASS, δ, IS SHOWN IN BOLD. (LC STANDS FOR LIMIT CYCLE [?] WHICH IS NON-STOCHASTIC, F STANDS FOR FRACTAL WHICH IS ALSO NON-STOCHASTIC AND S STANDS FOR STOCHASTIC)

Class	diskbb	PL	CI Inference	Betti descriptor	SVD Inference	MER	Variance	Area	PCA Inference	Match
β	46	52	F	(1,3)	Non-stochastic	214	483	43	Non-stochastic	Yes
θ	11	88	F	(3, 2)	Non-stochastic	577	778	58	Non-stochastic	Yes
λ	54	46	F	(1,3)	Non-stochastic	600	6782	314	Non-stochastic	Yes
κ	59	51	F	(1,3)	Non-stochastic	700	5199	144	Non-stochastic	Yes
μ	56	41	F	(1,1)	Non-stochastic	50	51	12	Non-stochastic	Yes
ν	28	72	F	(1,6)	Non-stochastic	30	32	16	Non-stochastic	Yes
α	23	77	F	(6,0)	Non-stochastic	30	1.9	27.7	Non-stochastic	Yes
ρ	28	72	LC	(1,1)	Non-stochastic	60	147	35	Non-stochastic	Yes
δ	**48**	**50**	**S**	**(1,0)**	**Stochastic**	**42**	**9.74**	**26.2**	**Non-stochastic**	**No**
ϕ	50	34	S	(1,0)	Stochastic	7	0.5	15	Stochastic	Yes
γ	60	31	S	(1,0)	Stochastic	12	1	16	stochastic	Yes
χ	09	89	S	(1,0)	Stochastic	5.6	0.25	6.05	Stochastic	Yes

first approach we extend SVD decomposition to understand temporal dynamics, by adding topological descriptors, to classify time series as stochastic vs non-stochastic. In yet another approach, a novel application of PCA to characterize the time series is proposed. We compare inferences of the CI based approach with those obtained using the proposed matrix based methods. Of the 12 classes of time series analysed, a mismatch is observed in the PCA based inference of only one class, while all other classes concur.