Time-series classification by matrix-based methods: Application to blackhole state identification

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***Abstract*—Classification of timeseries as stochastic (noise-like) or non-stochastic (which has a well-defined structure), helps understand the underlying phenomenon, in domains as diverse as medicine, weather, finance etc. Traditional integration-based methods (such as Correlation Integral (CI)-approach, Entropy, Mutual Information-based), are computationally expensive (atleast $O(N^2)$). In this work, we propose a computationally simple decomposition-based algorithm O(Linear in N) that utilizes two classical matrix-based methods to achieve stochastic vs non-stochastic classification. The two legs of the proposed algorithm carry out complementary analysis. Temporal dynamics is studied using SVD-decomposition followed by topological analysis (Betti numbers), which produces the SVD-label. Parallely, temporal-ordering agnostic PCA analysis is carried out, and PCA-based features are computed. These features are fed to linear SVM to produce the PCA-label. The proposed methods have been applied to synthetic data, which are 48 distinct realisations of white noise, pink noise (stochastic timeseries), Logistic map and Lorentz system (non-stochastic timeseries), as proof of concept. The utility of the proposed algorithm is illustrated on astronomy data which are 12 categories of timeseries pertaining to blackhole *GRS 1915 + 105*, obtained from RXTE satellite. For a given timeseries, if SVD-label and PCA-label both concur, then the corresponding spectral features are combined in order to determine the state of the blackhole. Else, it is deemed "inconclusive" requiring further investigation. Comparison of obtained results with those in literature are also presented. It is found that out of 12 categories of timeseries considered, the state of the blackhole can be inferred in 11 categories. However, it is more important to find the "inconclusive" categories since investigations into them are expected to have long standing implications in astrophysics and otherwise.**

***Index Terms*—Timeseries classification, stochastic, non-stochastic, SVD analysis, PCA analysis**

I. Introduction

Several real-world phenomena are studied by collecting associated measurements over time, called as timeseries. Timeseries classification as stochastic (noise-like) or non-stochastic (which has a well-defined structure), is the first step in understanding the underlying physical phenomenon. A stochastic timeseries is defined as a sequence of realizations of a random variable which are independent. Standard stochastic signals such as white noise, pink noise, etc. exhibit characteristics such as nearly zero auto-correlation coefficients for all possible values of lags and a power spectral density that decays with frequency. The rate of decay determines the kind of noise. Standard non-stochastic signals such as Logistic map (at growth rate = 4), Lorenz system result in timeseries that exhibit a well-defined structure, such as having a certain number of fixed points. The trajectory of the points taken pair-wise, also exhibits a distinct pattern, called "attractor behavior" [1].

The most popular method to classify timeseries as stochastic or non-stochastic is Correlation Integral (CI)-based [1]. This approach determines correlation dimension, which explains the underlying dynamics of the phenomenon. It is computation-intensive, since the correlation dimension, for a fixed value of embedding dimension, is defined as the rate of change of CI with respect to the neighborhood radius, for values of radii that tend to zero. This computation is needed for different choices of embedding dimension. Besides, it is to be computed for different ranges of neighbourhood radii. It is well-known that the value of correlation dimension does not saturate for a stochastic time series. Hence to establish if the timeseries is stochastic, this computation needs to be repeated for multiple values of embedding dimension, leading to higher computational complexity. Another popular set of approaches to determine if the timeseries is stochastic or non-stochastic, are based on Entropy. Entropy-based approaches [2], [3], [4] utilize concepts of phase-space reconstruction, approximate entropy and recurrence plots. These approaches are also known to be computationally intensive since they involve computing parameters such as Lyapunov exponent.

The problem of stochastic vs non-stochastic classification is important for one of the challenging problems in astrophysics, which could lead to the understanding of black holes. As a black hole cannot be seen directly, to identify it, one has to look for its environment forming a disc-like structure by the infalling matter called accretion disc. In this work, we focus on the black hole source *GRS 1915+105*, which presents several intriguing facets. It has been divided into 12 different temporal categories: α, β, γ, δ, λ, κ, μ, ν, ρ, ϕ, χ and θ [5], with their respective distinct timeseries. One fundamental aspect of the understanding is to determine if the black hole source is stochastic or non-stochastic (implying turbulent nature of the system). There are studies reported that utilize CI-based approach to determine the characterization of this specific black hole data [6], [7]. However, in this work, we propose to utilize classical matrix-based methods, Singular Value Decomposition (SVD) and Principal Component Analysis (PCA), to understand the same data. It is useful to compare the inferences obtained using these distinct approaches; the implications of the (dis)similarities in inferences, if any, could lead to better understanding of the system.

It is widely known that the true nature of the source is understood by studying both temporal and spectral features. Spectral features are impacted by dependence on temperature. If the source radiation is temperature dependent, it is called

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multicolour blackbody or "diskbb" [8]. On the other hand, if it is independent of temperature, it exhibits power-law tail ("PL") [9], [10]. The difference lies in the fact that the former leads the underlying accretion disc around the black hole to be geometrically thin, while the latter leads to a geometrically thick disc. Studies in literature combine these observations into four possible blackhole states [11]:

1) Non-stochastic and diskbb: Keplerian disc [8].
2) Non-stochastic and PL: Advection Dominated Accretion Flow (ADAF) [10].
3) Stochastic and diskbb: Slim disc [12].
4) Stochastic and PL: General Advective Accretion Flow (GAAF) [9], [13].

Blackhole state identification is critically impacted by the nature of the timeseries. Hence it is imperative to analyze the data from diverse perspectives, to cross-check obtained inferences. This would help identify discrepancies, if any, that would need further investigation.

The contributions of this paper :

- Classification of a timeseries as stochastic or non-stochastic using classical matrix-based decompositions, SVD (with topological descriptors) and PCA (Derived features followed by SVM classification)
- Blackhole state identification by combining temporal and spectral properties

Block diagram of the proposed approach is shown in Fig. 1.

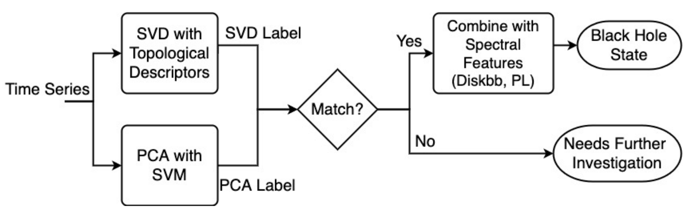


Fig. 1. Block diagram of the proposed algorithm

II. Related Work

Several groups have worked on distinguishing between stochastic and non-stochastic time series, in studies of general non-linear systems, some of which are also applied to astronomy data. In CI-based approach, a $M \times K$ data matrix is constructed, where each column is an observation vector with M components (embedding dimension), such that each of the components are independent. Each row contains K consecutive samples taken from the time series. Correlation sum is then computed for various values of radius and embedding dimension, M, as explained in [1]. This method is known to be computationally expensive, $O(N^2M)$, where N is the length of the timeseries, and M is the embedding dimension. Among entropy-based approaches a recent study explored the idea of utilizing Permutation Entropy (PE) to determine the complexity measure of a timeseries [14]. This idea was utilized in [4], where PE was used to parameterize a given time series followed by classification using Neural Network. PE was used to determine the measure of resemblance with known stochastic signals such as pink noise. The claim was that for non-stochastic signals the deviation of the parameter is relatively large as compared to that of the parameter of a stochastic signal. The authors report results on synthetic signals such as various types of noise, chaotic systems as well as emperical datasets such as human gait data and Heart rate variability data. This approach, however, assumes prior knowledge on the length of ordinal sequences.

Apart from CI-based and entropy-based approaches, there are also reported works that utilize graph-based methods and dictionary-based methods. In [15], the authors have utilized the horizontal visibility algorithm in order to distinguish between stochastic and non-stochastic processes. A recent work, reported in [16], mapped timeseries into graphs and computed various topological properties, which they called *NetF*, capturing measures such as centrality, distance, connectivity etc. PCA was applied on the *NetF* feature matrix and clustering was performed on the principal components. The authors illustrated the method on diverse datasets such as insect wing beat, timeseries obtained from electric devices etc. In [17], the authors combined the idea of sparsity and machine learning with non-linear dynamical systems, in order to determine the governing dynamics. Sparse regression was used to determine the fewest terms in the equations that govern the dynamics of the phenomenon. The authors utilize the method on diverse scenarios such as oscillators, the fluid vortex shedding behind an obstacle, etc. The user-defined dictionary of basis functions consists of well-known functions such as polynomials, trigonometric functions and exponentials. However, the optimal choice of dictionary for a specific choice of problem remains a challenge.

In this work, we propose to utilize computationally simple classical matrix based methods which do not require any assumptions about the underlying phenomenon.

III. Proposed Method

In this work, we propose an algorithm with two parallel legs as shown in Fig. 1, each of which perform complementary analysis, in order to determine if the given timeseries is stochastic or non-stochastic. They are 1) SVD decomposition followed by topological analysis (using Betti number descriptors) and 2) PCA derived features followed by SVM classification. If the labels obtained using the two approaches concur, the corresponding label is retained; Else the result is declared "Inconclusive". Proof of concept on synthetic signals is presented below. In the following sections, $z_1, z_2 \dots z_N$ denotes the time series of length N

A. SVD based approach

In this approach, we form uncorrelated observation vectors from the raw time series data by choosing an appropriate value of embedding dimension, M, as in [7] using autocorrelation plot. Data matrix, D given in Equation 1, is formed with each row as the time shifted version of the original time series.

$$D = \begin{bmatrix} z_1 & \dots & z_k \\ z_{1+\tau} & \dots & z_{K+\tau} \\ z_{1+2\tau} & \dots & z_{K+2\tau} \\ \dots & & \\ z_{1+(M-1)\tau} & \dots & z_{K+(M-1)\tau)} \end{bmatrix} \quad (1)$$

Here $z(1), z(2), \ldots, z(K)$ are K consecutive samples of the time series. The time shift, τ, is chosen to be large enough so that each column can be viewed as a different observation vector of the same time-evolving phenomenon. Temporal dynamics is understood by utilizing the right singular vectors of the SVD decomposition of D as given in equation (2) below. We consider the top two right singular vectors, E1 and E2, for our analysis.

$$D = U \Sigma V^T. \tag{2}$$

We observe the topology of the plot E1 vs E2. For non-stochastic time series this plot is expected to show a specific pattern (attractor behavior, where the plot follows a structured trajectory leaving well-defined voids). On the other hand, E1 Vs E2 plot for a stochastic time series, appears as a single blob without any voids. The toplogy of the E1 vs E2 plot is captured using Betti numbers [18]. Betti number descriptor for a d-dimensional manifold is a vector of d integers which is represented as $\beta = (\beta_0, \beta_1 \ldots \beta_{d-1})$. Here β_0 is the number of blobs (connected components) and β_k represents number of k-dimensional holes for $k > 0$. The E1 vs E2 plots are 2-D manifolds, which are described by $\beta = (\beta_0, \beta_1)$. Here, we utilize $L1$-norm of the β descriptor vector, $\|\beta\|_1$, for classification. For a stochastic time series the values of β_0 is expected to be 1 (single connected component) and that of β_1 is expected to be 0 (no holes). Hence for a stochastic time series, $\|\beta\|_1 = 1$. However, for a non-stochastic time series, we observe that the value of β_0 can be greater than 1 (can have one or more connected components) and the value of β_1 is always greater than 0 (presence of holes) due to the attractor behavior. Hence for a non-stochastic timeseries $\|\beta\|_1 > 1$.

B. PCA Based approach

PCA decomposition is carried out to infer if the given time series possesses a dominant orientation or not. This is computed by hierarchally splitting the timeseries into two halves, and computing the covariance matrix of these split observations. The eigenvalues of this 2×2 covariance matrix will show one of the signatures: If the data indeed show any dominant direction (as in non-stochastic timeseries), then the larger eigenvalue will be significantly greater than the other. This will lead to a large eigen value ratio. On the other hand, if the data do not show any dominant direction (as in stochastic time series), then the two eigenvalues of the covariance matrix will be comparable. This will lead to small eigen value ratio. This observation is utilized in devising features for stochastic Vs non-stochastic classification. The steps are outlined as below.

For a time series consisting of N values $z_1, z_2 \ldots z_N$.

- Split the series into two halves $(z_1, z_2 \ldots z_{\lfloor \frac{N}{2} \rfloor})$ and $(z_{\lfloor \frac{N}{2} \rfloor + 1}, \ldots z_N)$.
- Compute covariance matrix, C, by treating the samples in two halves as $\lfloor \frac{N}{2} \rfloor$ observations of 2-D vectors.
- Find eigenvalues of C, λ_1 and λ_2; the eigenvalue ratio is computed as λ_1 / λ_2 where $\lambda_1 > \lambda_2$ (eigenvalues of a covariance matrix are real).

If the eigenvalue ratio for an interval is greater than a value of threshold, Th (computing optimal value of Th is described in subsection III-B1 below), the interval is further split into two sub-intervals of equal size. Subsequently, the eigenvalue ratio for each sub-interval is computed. The process is repeated as long as the length of the sub-interval is greater than a predefined number of samples (here taken as 100). For a fixed value of Th, the following features are derived

- **V**ariance of **E**igenvalue **R**atio (**VER**): This is the variance of the eigenvalue ratios of covariance matrices across sub-intervals in the entire time series: $VER = var(e_1, e_2, \ldots e_N)$ where e_i denotes the eigenvalue ratio for sample i . This measure captures the spread in values of the eigenvalue ratio, reflecting the presence or absence of patterns in a timeseries.
- **A**rea **U**nder the **E**igenvalue **R**atio curve (**AUER**): This measure captures the area under the curve of the eigenvalue ratio for the entire time series computed using equation $AUER = \sum_{i=1}^{N} e_i t_i$ where t_i is the length of interval i. High values of this measure indicate that relatively higher eigenvalue ratios prevail for long time intervals, capturing structure in a timeseries.

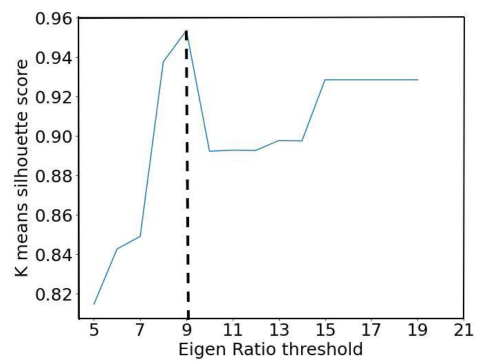
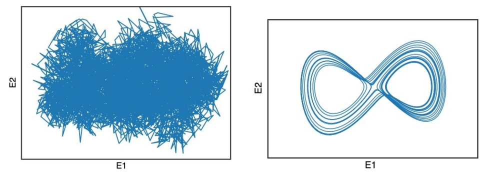


Fig. 2. Plot of Silhoutte score vs eigenratio threshold. Maximum Silhoutte score value, which indicates the best clustering, is obtained at a threshold of 9.

1) Computing optimal value of threshold Th: In order to arrive at the optimal value of Th, we observe the plot of the Silhoutte score of K-Means clustering, with $K = 2$ (stochastic and non-stochastic), performed using the above feature set, as a function of the threshold value. The value of the threshold that results in the best Silhoutte clustering score is taken as Th. This process is illustrated in the Silhoutte score plot shown in Fig. 2. For the time series considered here for illustration, it is evident from the plot that the best clustering is obtained at threshold value 9, resulting in the maximum value of Silhoutte score. Hence we use the corresponding $Th = 9$ to arrive at the optimal hierarchical splitting and subsequent computing of the devised PCA-based features, VER and AUER.

C. Proof of Concept on Synthetic Data

The proposed approaches have been applied to standard synthetic signals. For stochastic class of signals, white noise and pink noise are considered; for non-stochastic class of signals, Lorenz system and Logistic map (for growth rate = 4) are considered.



(a) (b)

Fig. 3. Comparison of E1 Vs E2 plots for: (a) white noise (stochastic) (b) Lorentz system (non-stochastic)

SVD-Decomposition based technique : The SVD decomposition of the data is computed, followed by the plot of the top two right singular vectors. This plot is utilized to determine the Betti descriptors. The plot in Fig. 3(a) corresponds to the E1 vs E2 plot for a realization of white noise, which is known to be stochastic. The plot shows one single blob implying Betti descriptor of (1 0), $\|\beta\|_1 = 1$. As discussed before, this timeseries is labelled as stochastic. On the other hand, the plot in Fig. 3(b) corresponds to the E1 vs E2 plot of a realization of Lorentz system, which is known to be non-stochastic. The plot shows two distinct voids, implying Betti descriptor to be (1 2), $\|\beta\|_1 = 3$, due to which the timeseries is non-stochastic. This inference mechanism has been utilized on real data described in section IV. The order of computations needed is mainly determined by the complexity of SVD Decomposition (For M x N matrix = $O(M^2N)$, which is linear in N).

PCA-Decomposition based technique: PCA-based features (i) VER and (ii) AUER, are computed for the considered synthetic signals which are 24 different realizations of white noise and Logistic map (growth rate = 4). The scatter plot of these features is shown Fig 4 below. Since the dynamic range of values for the features VER and AUER is large, we use log scale for the scatter plot. We observe the following:

- VER: For a stochastic signal since the variation in the eigenvalue ratios is typically small, the computed variance across the values is also small. On the other hand, for a non-stochastic signal, since the eigenvalue ratios occupy diverse values, VER is typically high.
- AUER: For a stochastic time series, since the eigenvalue ratios are small across the entire span, the value of AUER is also small. However, for a non-stochastic signal, the eigenvalue ratios remain high for longer time intervals. Hence the value of AUER is significantly higher.

The plot makes it evident that in this feature space the two classes of timeseries, stochastic and non-stochastic, become linearly separable. Hence a linear SVM classifier is utilized. For training the SVM, computed features from white noise and Logistic map are utilized. For validating the trained SVM, 12 realizations of pink noise and Lorentz system are used. The classification on all 12 realizations of pink noise, yields the label stochastic, and the classification label for Lorentz system is obtained as non-stochastic, leading to perfect validation accuracy. This trained SVM is used to classify real data as described in section IV

The order of computations needed can be split into 1) computations for covariance matrix (which involves computing X^TX where X is of dimensions $\lfloor \frac{N}{2} \rfloor \times 2$: $O(N)$), 2) computing eigen values of 2×2 matrix which takes constant time and 3) number of iterations (For N-length timeseries = assuming maximum number of splits, (log N)). Since final SVM 2-class classification step also takes constant time, the overall time complexity of PCA based approach is $O(NlogN)$

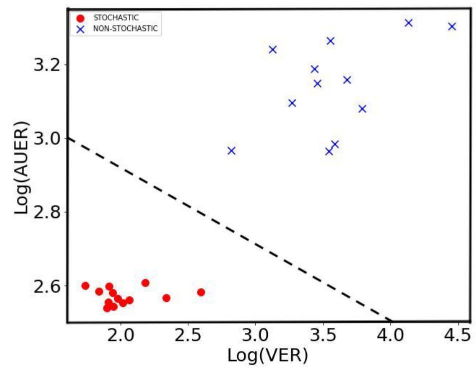


Fig. 4. Scatter plot of PCA-based features for synthetic data

IV. Results and Discussion

In this section, we present the real data used, results obtained using proposed approaches and comparison with results in literature. All codes and results are available at the link https://github.com/sunilvengalil/ts_analysis_pca_eig

A. Real Data

The proposed approaches are illustrated on the publicly available data of *GRS 1915 + 105* taken from website [19]. 12 categories of time series are utilized from the available data. All these time series are re-sampled with a sampling interval of 0.1 second. The length of the time series considered across the 12 categories vary from a minimum of TODO to a maximum of TODO. These datasets were also used in the work reported in [11], where the authors use CI based approaches, leading us to be able to compare our obtained results with theirs.

B. Results of SVD based analysis

The data matrix D is constructed with $K = 5000$ columnns. From SVD decomposition of the data matrix, we plot the top 2 right singular vectors (E1 vs E2) to understand the temporal dynamics for each time series. For classification, we use the $L1$-norm of the Betti descriptor, β, $\|\beta\|_1$. If $\|\beta\|_1 = 1$, the time series is classified as stochastic, else the time series is non-stochastic. The obtained values of $\|\beta\|_1$, along with the classification labels have been tabulated in Table I.

C. Results of PCA based analysis

PCA-derived features, VER and AUER, are computed for each of the timeseries. These features are input to the SVM classifier to obtain the class labels. The obtained feature values along with the classification labels have been tabulated in Table I. Comparison of results using proposed approaches with CI based approach [11] is shown in Table I.

TABLE I
TIMESERIES: COMPARISON BETWEEN CI BASED LABEL AND INFERENCE USING PROPOSED APPROACHES. THE MISMATCHED TIME SERIES CLASS, δ, IS SHOWN IN BOLD. (HERE NS STANDS FOR NON-STOCHASTIC AND S STANDS FOR STOCHASTIC)

Class	diskbb	PL	CI Label	Betti Norm	SVD Label	VER	AUER	PCA Label	State by CI	State by SVD	State by PCA	Match
β	46	52	NS	4	NS	483	43	NS	ADAF	ADAF	ADAF	Yes
θ	11	88	NS	5	NS	778	58	NS	ADAF	ADAF	ADAF	Yes
λ	54	46	NS	4	NS	6782	314	NS	Keplerian	Keplerian	Keplerian	Yes
κ	59	51	NS	4	NS	5199	144	NS	Keplerian	Keplerian	Keplerian	Yes
μ	56	41	NS	2	NS	51	12	NS	Keplerian	Keplerian	Keplerian	Yes
ν	28	72	NS	7	NS	32	16	NS	ADAF	ADAF	ADAF	Yes
α	23v	77	NS	6	NS	1.9	27.7	NS	ADAF	ADAF	ADAF	Yes
ρ	28	72	NS	2	NS	147	35	NS	ADAF	ADAF	ADAF	Yes
δ	48	50	S	1	S	9.7	26.2	NS	ADAF	ADAF	ADAF	No
ϕ	50	34	S	1	S	0.5	15	S	Slimdisc	Slimdisc	Slimdisc	Yes
γ	60	31	S	1	S	1	16	S	Slimdisc	Slimdisc	Slimdisc	Yes
χ	09	89	S	1	S	0.25	6.05	S	GAAF	GAAF	GAAF	Yes

D. SVD Vs PCA approach

SVD decomposition is utilized to study the temporal dynamics in the timeseries considered. This is in contrast to the PCA approach which does not consider the temporal ordering of data. Hence each of the approaches follows a perspective that is complementary to the other. Utilizing them both together to obtain inferences that concur makes the study reliable. In case of non-concurrence, it is clear that the problem would need further investigations.

E. Comparison of Blackhole state Inference

Comparison of Blackhole state inference using proposed approaches as against CI based approach [11] is shown in Table I. We observe that results obtained using SVD based analysis are consistent with CI based results for all the 12 categories of time series. However, with the PCA based approach the inference for δ time series is not consistent with the other two approaches. According to the CI based analysis δ turns out to be in between states slim disc and GAAF [11]. However, the present analysis shows that δ falls in between ADAF and Keplerian disc.

V. CONCLUSION

Exploring different techniques in order to have a conclusive inference for black hole systems turns out to be indispensable. We propose an algorithm that utilizes two separate legs (SVD and PCA decomposition), each of which determines an independent label for a timeseries, using complementary approaches. SVD decomposition studies the temporal dynamics, while PCA decomposition is agnostic to temporal ordering. If the two independent labels concur, then the blackhole state is identified by combining with available spectral features. This is illustrated on *GRS 1915+105* black hole using the time series obtained from *RXTE* satellite data. We compare inferences of the CI based approach with those obtained using the proposed algorithm. The proposed algorithm was applied on 12 categories of timeseries, where concurrence was obtained on 11 of them. Based on our analysis, we are able to identify two extreme temporal dynamical classes of accretion around black holes.

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