Joint Optimization for Power-Constrained MIMO Systems: Is Low-Resolution DAC Still Optimal?

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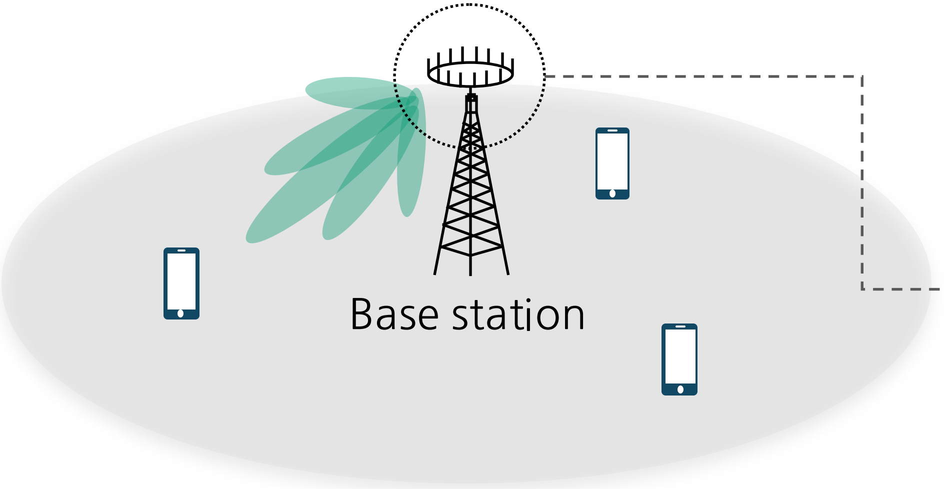
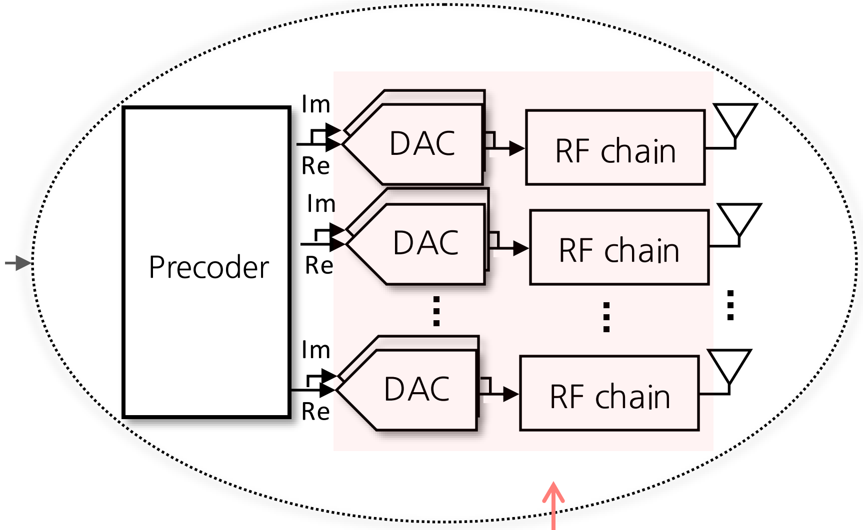
01 Power Consumption Challenge

Multi-user multiple-input multiple-output (MU-MIMO) systems

- Adding more antennas at a base station can boost the spectral efficiency of users.





  – **Circuit power consumption** can become **very large**

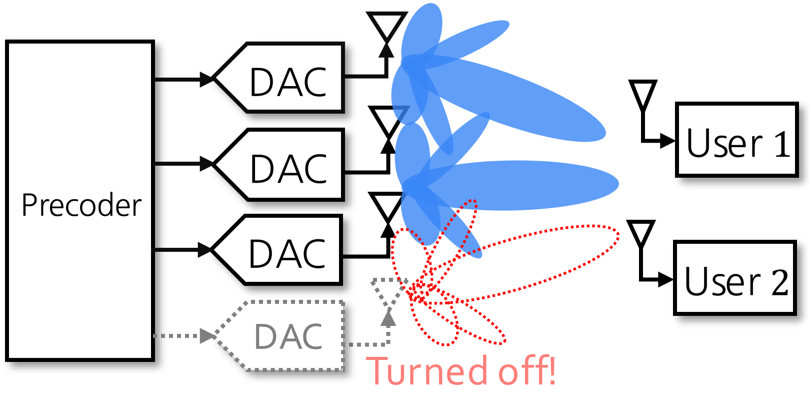
Excessive power consumption!

$$P_{\mathrm{cir}} = P_{\mathrm{LO}} + \sum_{i=1}^{N} (2P_{\mathrm{DAC}} + P_{\mathrm{RF}})$$

01 Conventional Approaches

Two approaches to tackle the power consumption problem

- Antenna selection
  - Norm-based antenna selection [1]



- Low-resolution DACs

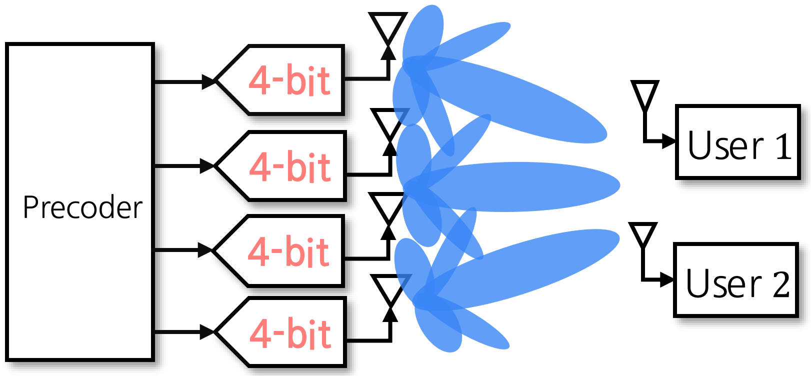
  \# of quantization bits

  - Lower power $\because$ $P_{\mathrm{DAC}} \propto f_s \times 2^{2b_{\mathrm{DAC}}}$
    - Significant quantization error

[2]

b_{DAC}	1	2	3	4	5
β_{DAC}	0.3634	0.1175	0.03454	0.009497	0.002499

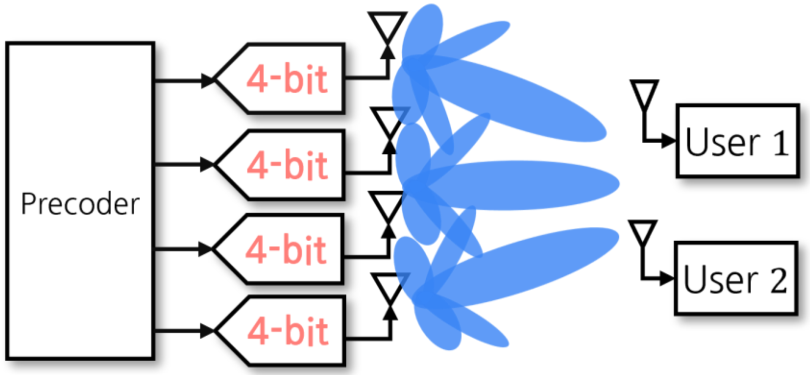
$$\beta_{\mathrm{DAC}} = \frac{\mathbb{E}[|x - Q(x)|^2]}{\mathbb{E}[|x|^2]}$$



01 Limitation of Conventional Approaches

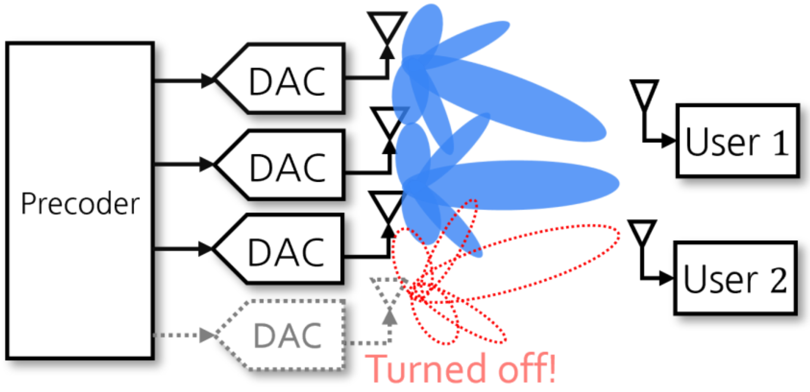
Prior Works

Precoding using low-resolution DACs



- 3~5 bits are the most power-efficient [3]
- Circuit power consumption was ignored [4], [5]

Precoding using antenna selection

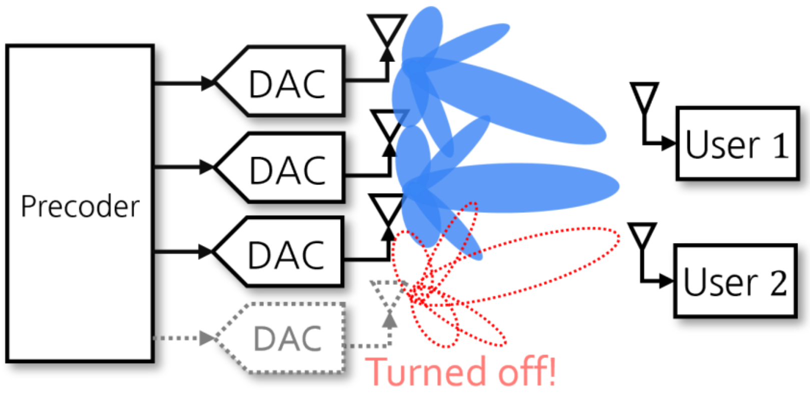


- Circuit power consumption was considered
- Energy efficiency was mostly optimized [6], [7]
  → Did not explicitly optimize spectral efficiency

A more comprehensive optimization with a fixed power budget at the base station?

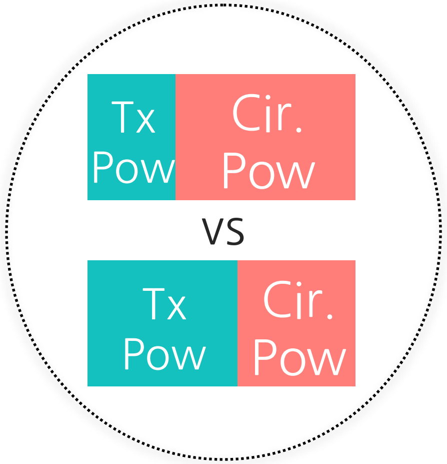
01 Motivation

A comprehensive optimization for maximizing the sum spectral efficiency



$$\kappa^{-1} P_{\text{tx}} + P_{\text{cir}} \leq P_{\text{tot}}$$

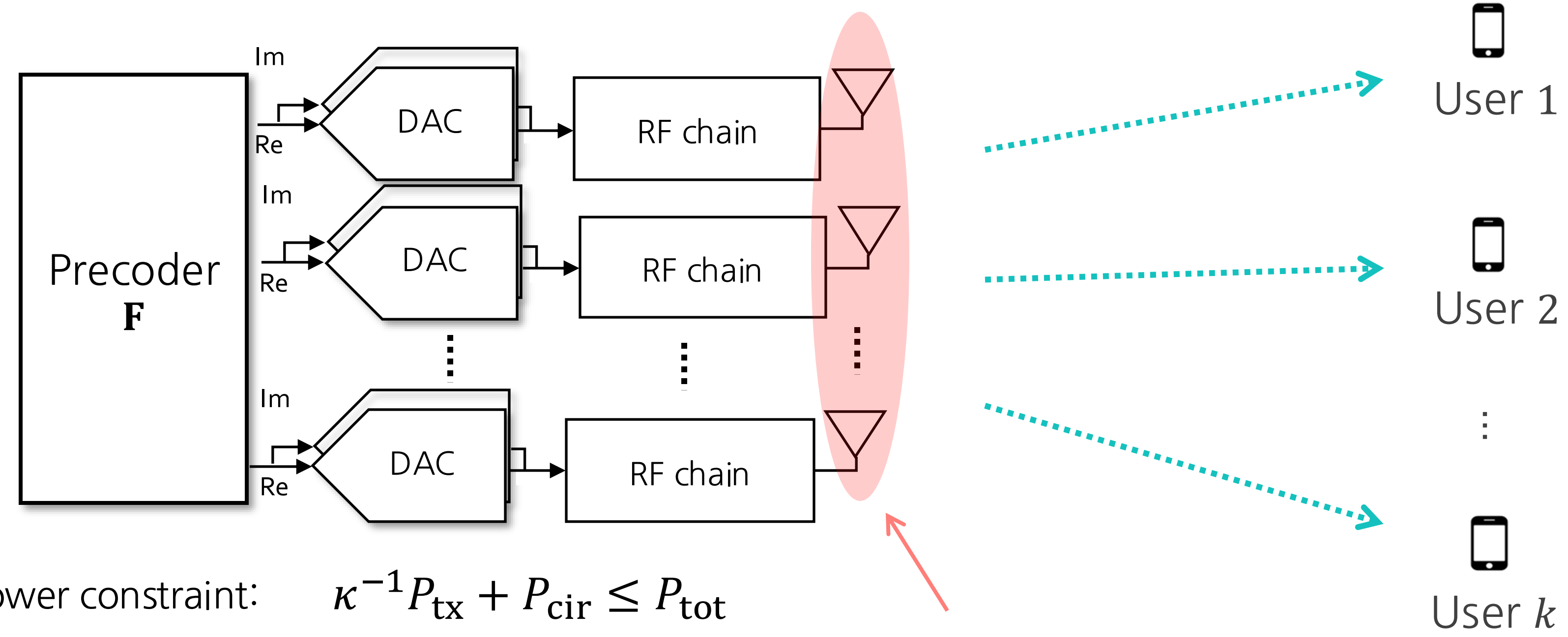
Transmit power $P_{\text{tx}} = \tau P$



- Joint optimization of **antenna selection**, **precoding**, and **transmit power control**
- Strategic balancing of the transmit power and antenna deployment

More room for the use of medium/high-resolution DACs?

Downlink MU-MIMO communication system



Total power constraint: $\kappa^{-1} P_{\mathrm{tx}} + P_{\mathrm{cir}} \leq P_{\mathrm{tot}}$

Tx power constraint: $P_{\mathrm{tx}} \leq P$

Goal: maximize the sum spectral efficiency of users

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Additive quantization noise model (AQNM)

Tx Signal

$$\mathbf{x} = \sqrt{P}\sum_{k=1}^{K} \mathbf{f}_k s_k$$

symbol of user k

precoder of user k

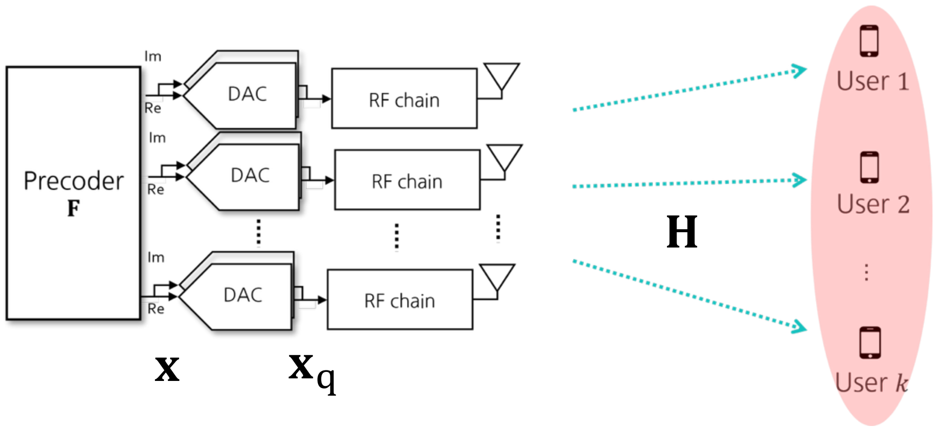
AQNM

Quantized Signal

$$Q(\mathbf{x}) \approx \mathbf{x}_\mathrm{q} = \sqrt{P}\boldsymbol{\Phi}_{\alpha_\mathrm{DAC}}\sum_{k=1}^{K} \mathbf{f}_k s_k + \mathbf{q}_\mathrm{DAC} \qquad [8]$$

quantization loss matrix

$$\begin{bmatrix} 1-\beta_\mathrm{DAC} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1-\beta_\mathrm{DAC} \end{bmatrix}$$



Rx Signal

$$\mathbf{y} = \mathbf{H}^\mathrm{H}\mathbf{x}_\mathrm{q} + \mathbf{n}$$

Linear approximation of the quantization process

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Spectral efficiency of user k

Without antenna selection

$$R_k(\mathbf{F}) = \log_2\left(1 + \frac{P\left|\mathbf{h}_k^{\mathrm{H}}\mathbf{\Phi}_{\alpha_{\mathrm{DAC}}}\mathbf{f}_k\right|^2}{\mathrm{IUI}_k(\mathbf{F}) + \mathrm{QE}_k(\mathbf{F}) + \sigma^2}\right)$$

$$\mathrm{IUI}_k(\mathbf{F}) = P\sum_{i=1,i\neq k}^{K}\left|\mathbf{h}_k^{\mathrm{H}}\mathbf{\Phi}_{\alpha_{\mathrm{DAC}}}\mathbf{f}_i\right|^2$$

$$\mathrm{QE}_k(\mathbf{F}) = \mathbf{h}_k^{\mathrm{H}}\mathbf{R}_{\mathbf{q}_{\mathrm{DAC}}}\mathbf{h}_k$$

With antenna selection

$$R_k(\mathbf{F}^{\mathrm{sel}}) = \log_2\left(1 + \frac{P\left|\left(\mathbf{h}_k^{\mathrm{sel}}\right)^{\mathrm{H}}\mathbf{\Phi}_{\alpha_{\mathrm{DAC}}}^{\mathrm{sel}}\mathbf{f}_k^{\mathrm{sel}}\right|^2}{\mathrm{IUI}_k(\mathbf{F}^{\mathrm{sel}}) + \mathrm{QE}_k(\mathbf{F}^{\mathrm{sel}}) + \sigma^2}\right)$$

$$\mathrm{IUI}_k(\mathbf{F}^{\mathrm{sel}}) = P\sum_{i=1,i\neq k}^{K}\left|\left(\mathbf{h}_k^{\mathrm{sel}}\right)^{\mathrm{H}}\mathbf{\Phi}_{\alpha_{\mathrm{DAC}}}^{\mathrm{sel}}\mathbf{f}_i^{\mathrm{sel}}\right|^2$$

$$\mathrm{QE}_k(\mathbf{F}^{\mathrm{sel}}) = \left(\mathbf{h}_k^{\mathrm{sel}}\right)^{\mathrm{H}}\mathbf{R}_{\mathbf{q}_{\mathrm{DAC}}^{\mathrm{sel}}}\mathbf{h}_k^{\mathrm{sel}}$$

However, antenna selection may not bring rate losses!

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Sum rate maximization problem & challenges

$$\max_{\mathcal{A},\mathbf{f}_1,\ldots,\mathbf{f}_K} \sum_{k=1}^{K} R_k\big(\mathbf{F}(\mathcal{A})\big)$$

→ 1. Non-convex

$$\text{subject to } \kappa^{-1}P_{\text{tx}}\big(\mathbf{F}(\mathcal{A})\big) + P_{\text{cir}}\big(\mathbf{F}(\mathcal{A})\big) \leq P_{\text{tot}},$$
$$P_{\text{tx}}\big(\mathbf{F}(\mathcal{A})\big) \leq P$$

2. Transmit power control

3. Circuit power consumption

$$P_{\text{cir}} = P_{\text{LO}} + \sum_{i=1}^{N} \boxed{\mathbb{1}_{\{i\in\mathcal{A}\}}}\{2P_{\text{DAC}}(b_{\text{DAC}}, f_s) + P_{\text{RF}}\}$$

Ant. power consumption

non-smooth function

Biggest challenge: need to deal with the indicator function!

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Problem reformulation: sum spectral efficiency

Power control:

$$P_{\text{tx}} = \tau P$$

$$0 < \tau \leq 1$$

Weighted precoding vector:

$$\mathbf{w}_k = \frac{1}{\sqrt{\tau}} \mathbf{\Phi}_{\alpha_{\text{DAC}}}^{1/2} \mathbf{f}_k$$

$$\text{tr}\left(\mathbf{W}\mathbf{W}^{\text{H}}\right) = 1$$

$$\leftrightarrow \quad P_{\text{tx}} = \text{tr}\left(\mathbb{E}\left[\mathbf{x}_{\text{q}}\mathbf{x}_{\text{q}}^{\text{H}}\right]\right) = \tau P$$

Spectral efficiency in Rayleigh quotient form:

$$R_k = \log_2\left(\frac{\left(\overline{\mathbf{w}}(\mathcal{A})\right)^{\text{H}} \mathbf{A}_k(\mathcal{A}) \overline{\mathbf{w}}(\mathcal{A})}{\overline{\mathbf{w}}(\mathcal{A})^{\text{H}} \mathbf{B}_k(\mathcal{A}) \overline{\mathbf{w}}(\mathcal{A})}\right)$$

Reformulated problem (given τ):

$$\underset{\mathcal{A},\overline{\mathbf{w}}}{\text{maximize}} \quad \sum_{k=1}^{K} \log_2\left(\frac{\left(\overline{\mathbf{w}}(\mathcal{A})\right)^{\text{H}} \mathbf{A}_k(\mathcal{A}) \overline{\mathbf{w}}(\mathcal{A})}{\left(\overline{\mathbf{w}}(\mathcal{A})\right)^{\text{H}} \mathbf{B}_k(\mathcal{A}) \overline{\mathbf{w}}(\mathcal{A})}\right)$$

$$\text{subject to} \quad \|\overline{\mathbf{w}}(\mathcal{A})\| = 1 \ \ \text{and} \ \ \frac{\tau}{\kappa} P + P_{\text{cir}} \leq P_{\text{tot}}$$

The objective function is now GPI-friendly!

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Problem reformulation: indicator function approximation

$$\mathbb{1}_{\{|x|^2>0\}} \approx \frac{\log_2\left(1+\frac{|x|^2}{\rho}\right)}{\log_2\left(1+\frac{1}{\rho}\right)}$$

[9]

$$\mathbb{1}_{\{i\in\mathcal{A}\}} = \mathbb{1}_{\{\|\mathbf{f}_i\|^2>0\}} \approx \log_2\left(1+\rho^{-1}\left\|\frac{\widetilde{\mathbf{w}}_i}{\sqrt{\alpha_{\mathrm{DAC}}}}\right\|^2\right)^{\frac{1}{\log_2(1+\rho^{-1})}}$$

Approximation becomes tight as $\rho \to 0$

$$P_{\mathrm{cir}} \approx \tilde{P}_{\mathrm{cir}}(\overline{\mathbf{w}}) = P_{\mathrm{LO}} + \sum_{i=1}^{N} \log_2\left(1+\rho^{-1}\left\|\frac{\widetilde{\mathbf{w}}_i}{\sqrt{\alpha_{\mathrm{DAC}}}}\right\|^2\right)^{\frac{P_{\mathrm{ant}}}{\log_2(1+\rho^{-1})}}$$

$$= P_{\mathrm{LO}} + \sum_{i=1}^{N} \log_2\left(\overline{\mathbf{w}}^{\mathrm{H}}\mathbf{E}_i\overline{\mathbf{w}}\right)^{\frac{P_{\mathrm{ant}}}{\log_2(1+\rho^{-1})}}$$

$$\underset{\overline{\mathbf{w}}}{\text{maximize}} \quad \sum_{k=1}^{K} \log_2\left(\frac{\overline{\mathbf{w}}^{\mathrm{H}}\mathbf{A}_k\overline{\mathbf{w}}}{\overline{\mathbf{w}}^{\mathrm{H}}\mathbf{B}_k\overline{\mathbf{w}}}\right)$$

$$\text{subject to} \quad \|\overline{\mathbf{w}}\| = 1 \quad \text{and} \quad \frac{\tau}{\kappa}P + \tilde{P}_{\mathrm{cir}}(\overline{\mathbf{w}}) \le P_{\mathrm{tot}}$$

Approximated problem

P_{cir} is now tractable!

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Generalized power iteration (GPI) algorithm

Lagrangian function:

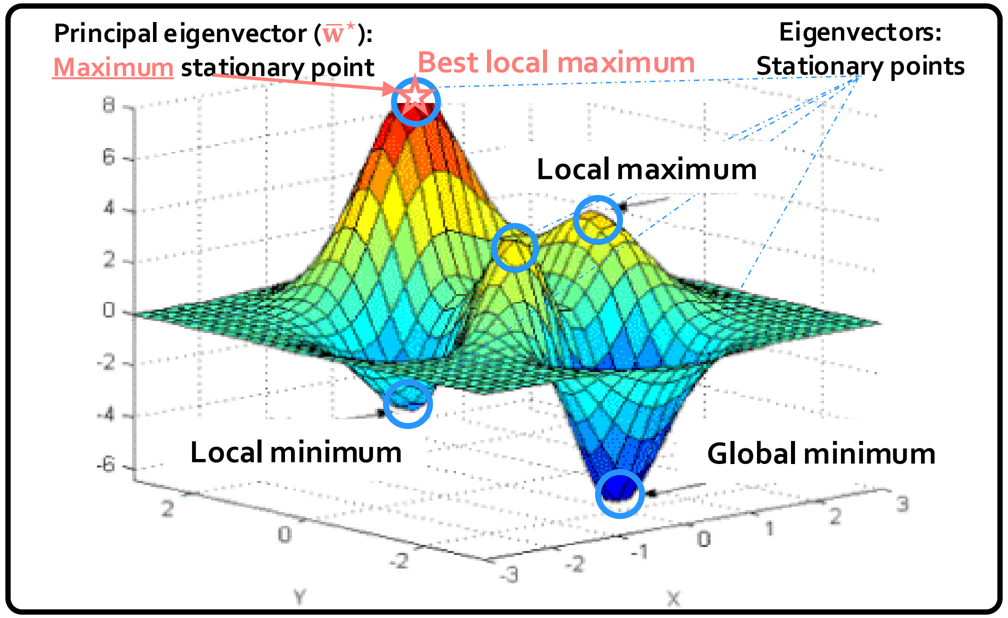
$$L(\bar{\mathbf{w}},\tau,\mu) = \sum_{k=1}^{K} \log_2\left(\frac{\bar{\mathbf{w}}^{\mathrm{H}}\mathbf{A}_k\bar{\mathbf{w}}}{\bar{\mathbf{w}}^{\mathrm{H}}\mathbf{B}_k\bar{\mathbf{w}}}\right) - \mu\left(\frac{\tau}{\kappa}P + \tilde{P}_{\mathrm{cir}}(\bar{\mathbf{w}}) - P_{\mathrm{tot}}\right)$$
$$= \ln\big(\lambda(\bar{\mathbf{w}},\tau,\mu)\big)$$

First-order optimality condition:

$$L(\bar{\mathbf{w}},\tau,\mu) = \frac{1}{\ln 2}\sum_{k=1}^{K}\left(\frac{\mathbf{A}_k\bar{\mathbf{w}}}{\bar{\mathbf{w}}^{\mathrm{H}}\mathbf{A}_k\bar{\mathbf{w}}} - \frac{\mathbf{B}_k\bar{\mathbf{w}}}{\bar{\mathbf{w}}^{\mathrm{H}}\mathbf{B}_k\bar{\mathbf{w}}}\right) - \frac{\mu}{(\ln 2)\log_2(1+\rho^{-1})}\sum_{i=1}^{N} P_{\mathrm{ant},i}\frac{\mathbf{E}_i\bar{\mathbf{w}}}{\bar{\mathbf{w}}^{\mathrm{H}}\mathbf{E}_i\bar{\mathbf{w}}} = 0$$

$$\mathbf{B}_{\mathrm{KKT}}(\bar{\mathbf{w}},\tau,\mu)^{-1}\mathbf{A}_{\mathrm{KKT}}(\bar{\mathbf{w}},\tau,\mu)\bar{\mathbf{w}} = \lambda(\bar{\mathbf{w}},\tau,\mu)\bar{\mathbf{w}}$$

Finding principal eigenvector   ∝ objective function



The first-order optimality condition is interpreted as a generalized eigenvalue problem

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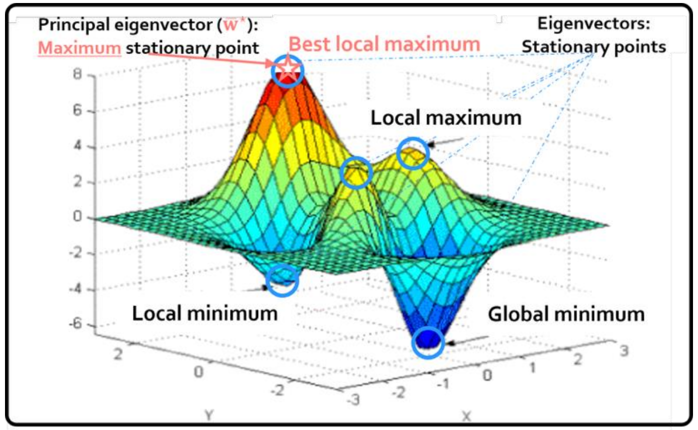
Algorithm 1

Algorithm 1: Optimization of $\mathbf{W}$ (Joint precoding & AS)

initialize: $\bar{\mathbf{w}}^{(0)}$ → Initialize RZF
Set the iteration count $t = 0$
while $\left\|\bar{\mathbf{w}}^{(t+1)} - \bar{\mathbf{w}}^{(t)}\right\| > \epsilon$ & $t < t_{\max}$ **do**
  Build matrices $\mathbf{A}_{\mathsf{KKT}}$ and $\mathbf{B}_{\mathsf{KKT}}$ in (25) and (26)
  Compute $\bar{\mathbf{w}}^{(t+1)}$ using (29)
  $t \leftarrow t + 1$
return $\bar{\mathbf{w}}^{(t)}$



Finds the best local optimal solution

Algorithm 2

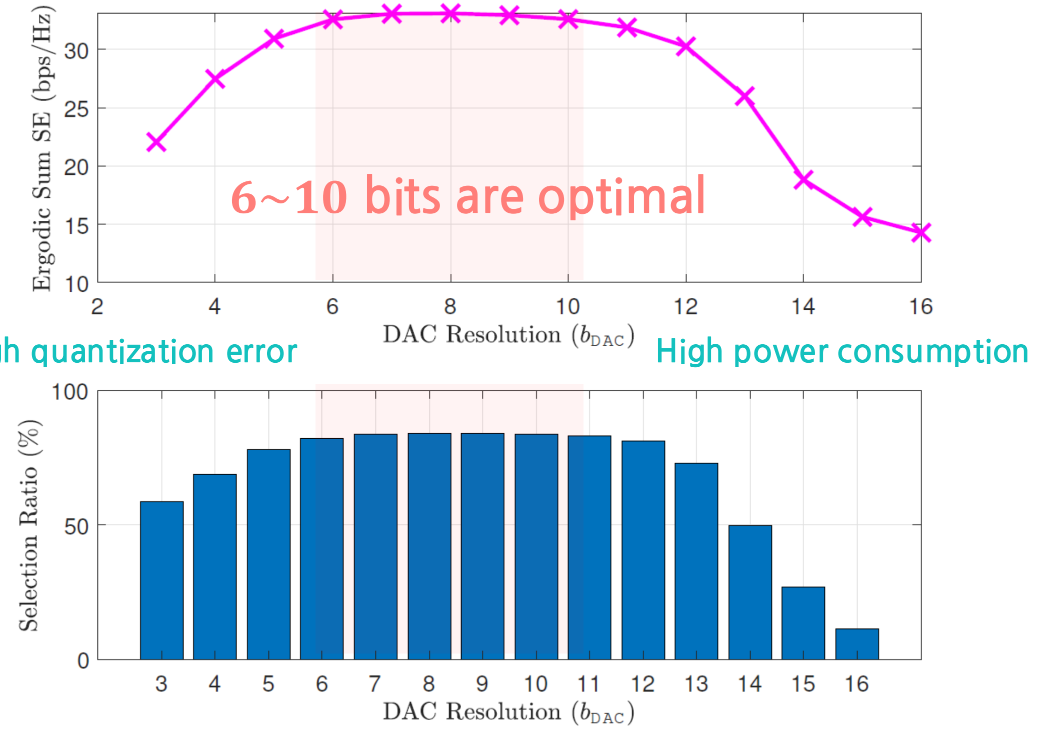
Algorithm 2: Power-Constrained Antenna Selection

initialize: $\bar{\mathbf{w}}^{(0)}, \tau^{(0)}$, and $\mu^{(0)}$
Set the iteration count $t_\mu = 0$.
while $t_\mu < t_{\mu,\max}$ **do**
  Set the iteration count $t_{\mathbf{F}} = 0$
  while $\frac{\left\|\mathbf{F}^{(t_{\mathbf{F}}+1)} - \mathbf{F}^{(t_{\mathbf{F}})}\right\|_F}{\left\|\mathbf{F}^{(t_{\mathbf{F}})}\right\|_F} > \epsilon_{\mathbf{F}}$ & $t_{\mathbf{F}} < t_{\mathbf{F},\max}$ **do**
    Set the iteration count $t_\tau = 0$
    while $\frac{\left|\tau^{(t_\tau+1)} - \tau^{(t_\tau)}\right|}{\left|\tau^{(t_\tau)}\right|} > \epsilon_\tau$ & $t_\tau < t_{\tau,\max}$ **do**
      Update $\tau^{(t_\tau)}$ according to (30)
      $\tau^{(t_\tau)} = \max\left\{0, \min\left\{\tau^{(t_\tau)}, 1\right\}\right\}$
      $t_\tau \leftarrow t_\tau + 1$
    (Power control)
    $\bar{\mathbf{w}} = \text{Alg. 1}\left(\bar{\mathbf{w}}, \tau^{(t_\tau)}, \mu^{(t_\mu)}\right)$
    Compute $\mathbf{F}^{(t_{\mathbf{F}})} = \sqrt{\tau^{(t_\tau)}}\,\mathbf{\Phi}_{\alpha\mathsf{DAC}}^{-1/2}\left[\mathbf{w}_1, \cdots, \mathbf{w}_K\right]$
    $t_{\mathbf{F}} \leftarrow t_{\mathbf{F}} + 1$
  Compute $\hat{\mathbf{W}} = \tilde{\mathbf{W}} \,/\, \max_j \left\|\tilde{\mathbf{w}}_j / \sqrt{\alpha_{\mathsf{DAC}}}\right\|$
  Set $\tilde{\mathbf{f}}_i^{(t_{\mathbf{F}})} = \mathbf{0}_{1\times K}$ if $\left\|\hat{\mathbf{w}}_i / \sqrt{\alpha_{\mathsf{DAC}}}\right\|^2 < \epsilon_{\mathsf{as}}$: Thresholding
  Update $\mu^{(t_\mu)}$ using the bisection method
  $t_\mu \leftarrow t_\mu + 1$
$\tau = \max\left\{0, \min\left\{\frac{\kappa}{P}\left(P_{\mathsf{tot}} - P_{\mathsf{cir}}\right), 1\right\}\right\}$
$\mu = 0$
$\bar{\mathbf{w}} = \text{Alg. 1}\left(\bar{\mathbf{w}}, \tau, \mu\right)$
Compute $\mathbf{F} = \sqrt{\tau}\,\mathbf{\Phi}_{\alpha\mathsf{DAC}}^{-1/2}\left[\mathbf{w}_1, \cdots, \mathbf{w}_K\right]$
return $\mathbf{F}$

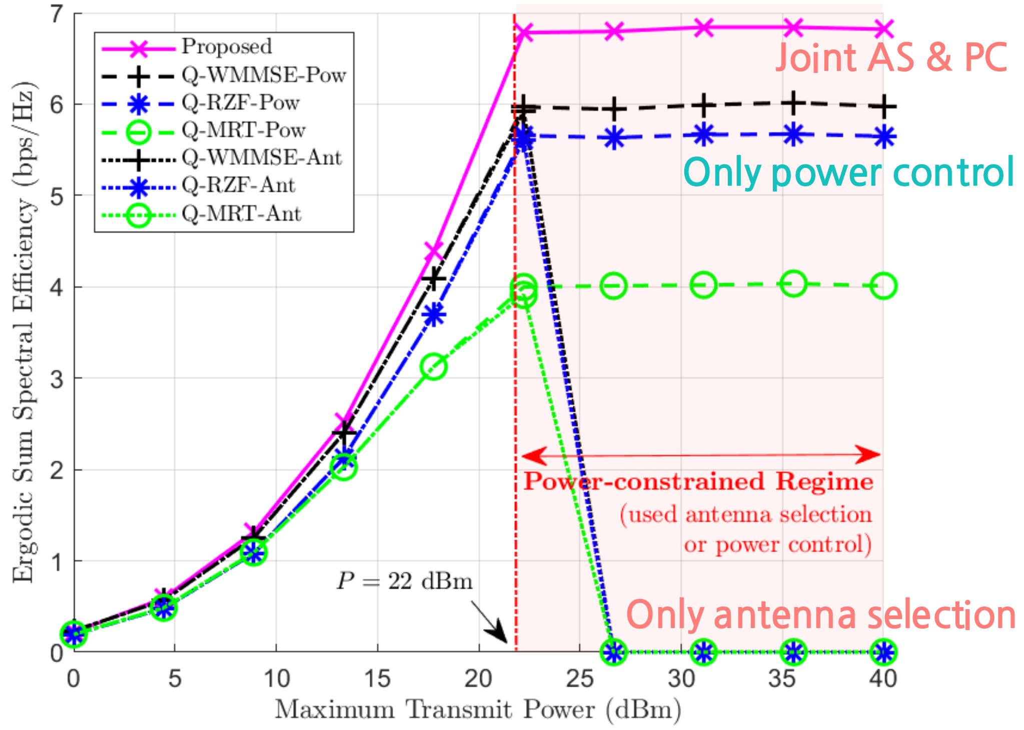
04 Numerical Results

Joint Optimization of the Precoder, Antenna Selection, and Power Control

Sum SE w.r.t. DAC resolution



Sum SE w.r.t. the maximum transmit power



Medium-resolution DACs can actually be more power efficient!

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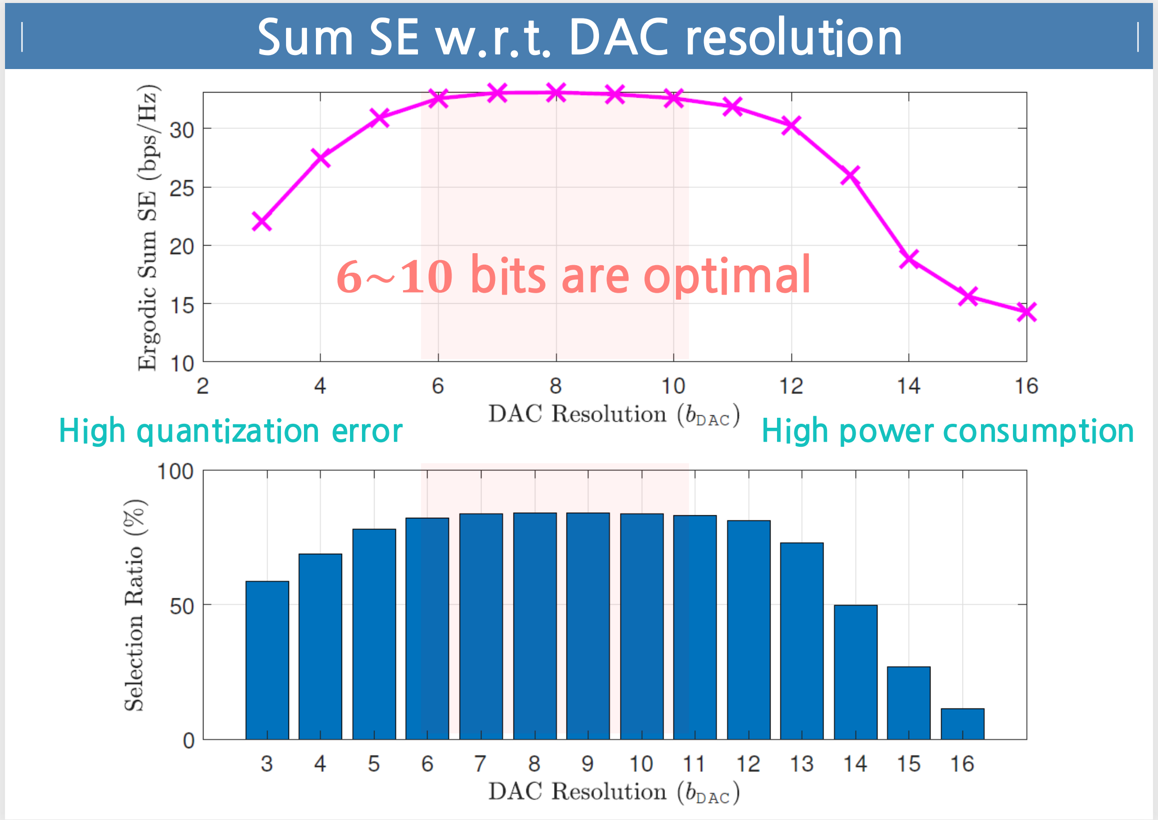

Conclusion

- Challenge: circuit power consumption

$$P_{\mathrm{cir}} = P_{\mathrm{LO}} + \sum_{i=1}^{N} (2P_{\mathrm{DAC}} + P_{\mathrm{RF}})$$

- Solution: joint optimization of the **precoder**, **AS**, and **PC**
- Takeaway 1: **medium-resolution DACs** may be more **power efficient**!
- Takeaway 2: Such comprehensive optimization is needed to use the **full potential** of the power budget at the BS!

05 References

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Thank you for your attention

Any questions are welcome

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