

Genetic Algorithms

Aim:

To apply genetic algorithm for a given problem of Flow Shop Scheduling

Problem Statement: Examining various genetic operators to design a genetic algorithm for the flowshop scheduling problem with an objective to minimize the makespan of the jobs.

Tool/Language: Python 3

Report Summary:

1. Problem that the paper solves:

Flowshop scheduling for minimizing the makespan is one of the most well-known problems in the area of scheduling. Therefore, this paper aims at examining various genetic operators to design an efficient genetic algorithm for solving this flowshop scheduling problem by minimising the makespan of the jobs. This paper indicates that two-point crossover and shift change mutation schemes prove to be an effective solution to solving this problem. The efficiency of this process is compared with many search algorithms as well to figure out the best possible algorithm.

2. Components involved in Genetic Algorithm

a. Type of Encoding Scheme used:

In this paper, each solution of the optimization problem is encoded as a string. A sequence of jobs is represented as a string in this paper for the flowshop scheduling problem. For example, a string 'ABCDEF' represents a sequence of jobs where job A is processed first, followed by job B and so on. If a combination of string is 'ABCDE' then it cannot be counted as a valid solution to the problem as it contains 2 occurrences of job A and no occurrence of job F. We denote the sequence of n jobs by an n-dimensional vector $x = (x_1 \dots x_j \dots x_n)$ where x_j denotes the j-th processing job. That is, the n-dimensional vector x is handled as a string in genetic algorithms.

b. Type of Selection, Mutation and Crossover Schemes used:

Selection:

Selection is an operation to select two parent strings for generating a new string (i.e., child). Let N_{pop} be the number of solutions in each population in genetic algorithms, i.e., N_{pop} is the population size.

Conventionality calls for the random selection of parents, the beneficial effects of genetic search being gained by the replacement of "poor" chromosomes. But it is also possible to allow at least one parent to be selected according to its fitness, with lower-valued chromosomes being the favoured ones. The selection of parents was then made in accordance with the probability distribution $p([k]) = 2k/(M*(M+1))$, where $[k]$ is the kth chromosome in ascending order of fitness (i.e. descending order of makespan). This implies that the median value has a chance of $1/M$ of being selected, while the Mth (the fittest) has a chance of $2/(M + 1)$, roughly twice that of the median.

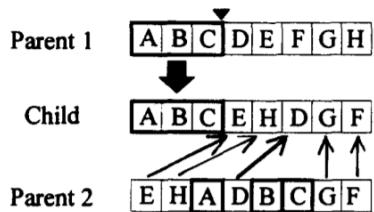
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Crossover:

Crossover is an operation to generate a new string (i.e., child) from two parent strings. Since our flowshop scheduling problem is a sequencing problem of n jobs, various crossover operators proposed for traveling salesman problems and scheduling problems are applicable.

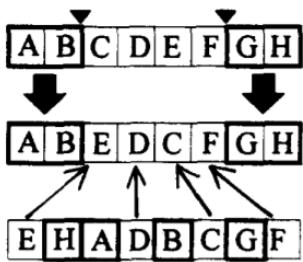
There were 10 crossover operators analysed in this paper and the following 6 turned out to be the most effective.

i. *One-point crossover*: One point is randomly selected for dividing one parent. The set of jobs on one side (each side is chosen with the same probability) is inherited from one parent to the child, and the other jobs are placed in the order of their appearance in the other parent.

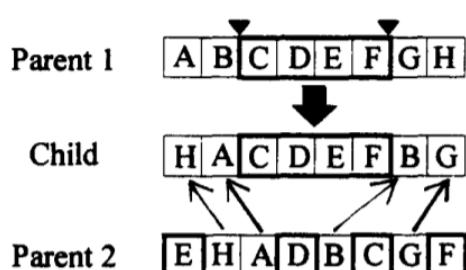


(a) One-point crossover

ii. *Two-point crossover*: There are three versions of this crossover. At first two points are randomly selected for dividing one parent. In the first version, jobs outside the parent are inherited from parent 1 and other jobs are arranged similar to one-point crossover. In version 2, the set of jobs between two randomly selected points is always inherited from one parent to the child, and the other jobs are placed in the same manner as the one-point crossover. Version 3 is a mixture of the version 1 and 2.



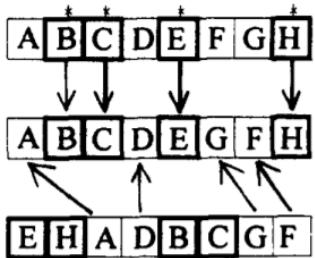
(b) Two-point crossover Ver. I



(c) Two-point crossover Ver. II

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iii. *Position based crossover:* There are two versions in this type of crossover. The jobs at randomly selected positions marked by "*" are inherited from one parent to the child, and the other jobs are placed in the order of their appearance in the other parent. In version 1, the number of those positions is first determined as a random integer in $[1, n]$, then positions are randomly selected. In version 2, each position is independently marked with the probability of 0.5, while the number of marked positions is the same as that defined in version 1.

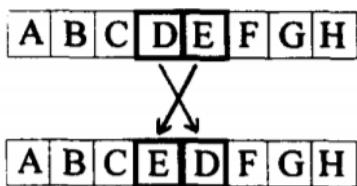


(d) Position based crossover

Mutation:

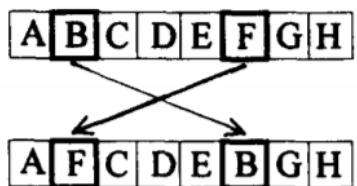
Mutation is an operation to change the order of n jobs in each string generated by a crossover operator. Such a mutation operation can be viewed as a transition from a current solution to its neighborhood solution in local search algorithms. There are four types of mutations reviewed in this paper which are as follows:

i. *Adjacent two-job change:* In this type of mutation, the adjacent two jobs to be changed are randomly selected.



(a) Adjacent two-job change

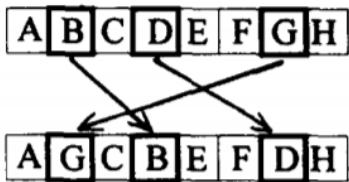
ii. *Arbitrary two-job change:* In this type of mutation, the two jobs to be changed are arbitrary and randomly selected. This mutation includes the adjacent two-job change as a special case.



(b) Arbitrary two-job change

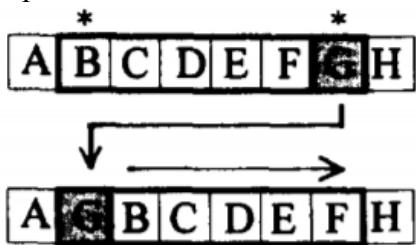
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iii. *Arbitrary three-job change*: In this type of mutation, the three jobs to be changed are arbitrary and randomly selected, and the order of the selected jobs after the mutation is randomly specified. This mutation includes the above two mutation operators as a special case.



(c) **Arbitrary three-job change**

iv. *Shift change*: In this mutation, a job at one position is removed and put at another position. The two positions are randomly selected. This mutation includes the adjacent two-job change as a special case and has an intersection with the arbitrary three-job change.



(d) **Shift**

c. Fitness Function Used:

In minimization problems, the low valued chromosomes are the “good” chromosomes. One approach is to define fitness as ($V_{max} - V$), but of course we are unlikely to know what V_{max} is. We could use the largest value found so far as a surrogate for V_{max} , but in practice this method was not very good at distinguishing between chromosomes. In essence, if the population consists of many chromosomes whose fitness values are relatively close, the resolution between "good" and "bad" ones is not of a high order. This approach was therefore abandoned, in favour of a simple ranking mechanism. Sorting the chromosomes is a simple task, and updating the list is also straightforward.

The selection of parents was then made in accordance with the probability distribution $p([k]) = 2k/(M*(M+1))$, where $[k]$ is the k^{th} chromosome in ascending order of fitness (i.e. descending order of makespan). This implies that the median value has a chance of $1/M$ of being selected, while the M^{th} (the fittest) has a chance of $2/(M + 1)$, roughly twice that of the median.

3. Results observed by the authors:

The authors have applied the genetic algorithm, local search algorithm, the taboo search algorithm and the simulated annealing algorithm to 100 test problems with 20 jobs and 10 machines. For comparison, we also applied a random sampling technique with the same computation load as the other algorithms. It was observed that the average makespans obtained by each algorithm are normalized using the results by the simulated annealing algorithm with

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200,000 evaluations. We can also see that the genetic algorithm, which is much superior to the random sampling technique, is a bit inferior to the other search algorithms.

4. Observation and Conclusion:

In this paper, we examined the performance of genetic algorithms in order to specify some genetic operators (e.g., crossover and mutation) and parameters (e.g., population size, crossover probability and mutation probability) for the flowshop scheduling problem. By computer simulations, we showed that the genetic algorithm was much superior to a random sampling technique but it was a bit inferior to other search algorithms such as local search, taboo search and simulated annealing. In order to improve the performance of the genetic algorithm, we examined two hybrid algorithms: genetic local search and genetic simulated annealing. We also examined some variants of their hybrid algorithms. By computer simulations, we showed that the hybrid algorithms outperformed the other algorithms (i.e., the genetic algorithm, the local search algorithm, the taboo search algorithm and the simulated annealing algorithm).

Algorithm:

Step 1: Initialization - Randomly generate an initial population W_t of N_{pop} strings (i.e., N_{pop} solutions).

Step 2: Selection - Select N_{pop} pairs of strings from a current population according to the selection probability.

Step 3: Crossover - Apply one of the above crossover operators to each of the selected pairs in Step 2 to generate N_{pop} solutions with the crossover probability P_c . If the crossover operator is not applied to the selected pair, one of the selected solutions remains as a new string.

Step 4: Mutation - Apply one of the above mutation operators to each of the generated N_{pop} strings with the mutation probability P_m (we assign the mutation probability not to each bit but to each string).

Step 5: Elitist - Randomly remove one string from the current population and add the best string in the previous population to the current one.

Step 6: Termination - If a prespecified stopping condition is satisfied, stop this algorithm. Otherwise, return to Step 2

Code:

```
import numpy as np
import math
import time
import random
import itertools
import queue
import pandas as pd
from IPython.display import display, Markdown
# Dataset number. 1, 2 or 3
dataset = "2"
```

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```
if dataset == "1":  
    optimalObjective = 4534  
elif dataset == "2":  
    optimalObjective = 920  
else:  
    optimalObjective = 1302  
  
filename = "data" + dataset + ".txt"  
f = open(filename, 'r')  
l = f.readline().split()  
  
# number of jobs  
n = int(l[0])  
  
# number of machines  
m = int(l[1])  
  
# ith job's processing time at jth machine  
cost = []  
  
for i in range(n):  
    temp = []  
    for j in range(m):  
        temp.append(0)  
    cost.append(temp)  
  
for i in range(n):  
    line = f.readline().split()  
    for j in range(int(len(line)/2)):  
        cost[i][j] = int(line[2*j+1])  
  
f.close()  
def initialization(Npop):  
    pop = []  
    for i in range(Npop):  
        p = list(np.random.permutation(n))
```

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```
while p in pop:  
    p = list(np.random.permutation(n))  
    pop.append(p)  
  
return pop  
  
def calculateObj(sol):  
    qTime = queue.PriorityQueue()  
  
    qMachines = []  
    for i in range(m):  
        qMachines.append(queue.Queue())  
  
    for i in range(n):  
        qMachines[0].put(sol[i])  
  
    busyMachines = []  
    for i in range(m):  
        busyMachines.append(False)  
  
    time = 0  
  
    job = qMachines[0].get()  
    qTime.put((time+cost[job][0], 0, job))  
    busyMachines[0] = True  
  
    while True:  
        time, mach, job = qTime.get()  
        if job == sol[n-1] and mach == m-1:  
            break  
        busyMachines[mach] = False  
        if not qMachines[mach].empty():  
            j = qMachines[mach].get()  
            qTime.put((time+cost[j][mach], mach, j))  
            busyMachines[mach] = True  
        if mach < m-1:  
            if busyMachines[mach+1] == False:
```

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```
qTime.put((time+cost[job][mach+1], mach+1, job))
busyMachines[mach+1] = True

else:
    qMachines[mach+1].put(job)

return time

def selection(pop):
    popObj = []
    for i in range(len(pop)):
        popObj.append([calculateObj(pop[i]), i])

    popObj.sort()

    distr = []
    distrInd = []

    for i in range(len(pop)):
        distrInd.append(popObj[i][1])
        prob = (2*(i+1)) / (len(pop) * (len(pop)+1))
        distr.append(prob)

    parents = []
    for i in range(len(pop)):
        parents.append(list(np.random.choice(distrInd, 2, p=distr)))

    return parents

def crossover(parents):
    pos = list(np.random.permutation(np.arange(n-1)+1)[:2])

    if pos[0] > pos[1]:
        t = pos[0]
        pos[0] = pos[1]
        pos[1] = t
```

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```
child = list(parents[0])

for i in range(pos[0], pos[1]):
    child[i] = -1

p = -1
for i in range(pos[0], pos[1]):
    while True:
        p = p + 1
        if parents[1][p] not in child:
            child[i] = parents[1][p]
            break

return child


def mutation(sol):
    pos = list(np.random.permutation(np.arange(n))[:2])

    if pos[0] > pos[1]:
        t = pos[0]
        pos[0] = pos[1]
        pos[1] = t

    remJob = sol[pos[1]]

    for i in range(pos[1], pos[0], -1):
        sol[i] = sol[i-1]

    sol[pos[0]] = remJob

    return sol


def elitistUpdate(oldPop, newPop):
    bestSolInd = 0
    bestSol = calculateObj(oldPop[0])
```

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```
for i in range(1, len(oldPop)):
    tempObj = calculateObj(oldPop[i])
    if tempObj < bestSol:
        bestSol = tempObj
        bestSolInd = i

rndInd = random.randint(0, len(newPop)-1)

newPop[rndInd] = oldPop[bestSolInd]

return newPop

# Returns best solution's index number, best solution's objective value
and average objective value of the given population.

def findBestSolution(pop):
    bestObj = calculateObj(pop[0])
    avgObj = bestObj
    bestInd = 0
    for i in range(1, len(pop)):
        tObj = calculateObj(pop[i])
        avgObj = avgObj + tObj
        if tObj < bestObj:
            bestObj = tObj
            bestInd = i

    return bestInd, bestObj, avgObj/len(pop)

popSize1 = [3,3,3,3,3,5,5,5,5,10,10,10,10,10]
objVal1 = [4534,4534,4534,4534,4534,4534,4534,4534,4534,4534,4534,4534,4534,4534,4534,4534,4534]
avgObj1 = [4929.33,4840.33,5002.00,5154.33,4836.00,5066.80,5356.20,5663.20,4850.20,5050.60,5530.40,5839.50,5671.60,5725.70,5459.10]
gap1 = [0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00]
cpuTime1 =
```

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```
[24.80,25.36,26.22,26.35,25.44,43.37,43.84,43.72,44.11,43.64,86.21,86.90,8  
2.90,80.21,68.50]

popSize2 = [3,3,3,3,3,5,5,5,5,5,10,10,10,10,10]
objVal2 = [920,929,920,920,937,931,937,937,926,920,937,931,929,938,937]
avgObj2 = [1001.00,1002.33,1055.00,964.67,1028.67,1052.40,1185.20,1074.00,1076.00,10  
63.60,1177.00,1166.00,1139.80,1182.50,1181.40]
gap2 = [0.00,0.98,0.00,0.00,1.85,1.20,1.85,1.85,0.65,0.00,1.85,1.20,0.98,1.96,1.8  
5]
cpuTime2 = [41.74,42.15,  
45.23,40.75,39.75,64.36,64.04,64.76,67.21,66.68,123.27,122.97,122.32,  
122.29,122.42]

popSize3 = [3,3,3,3,3,5,5,5,5,5,10,10,10,10,10]
objVal3 = [1302,1302,1302,1302,1302,1302,1302,1302,1302,1323,1302,1302,1302,1302,132  
3]
avgObj3 = [1363.00,1390.00,1341.00,1302.00,1346.33,  
1521.20,1403.40,1398.40,1497.00,1453.00,1610.50,  
1619.90,1654.90,1600.30,1678.80]
gap3 = [0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,0.00,1.61,0.00,0.00,0.00,0.00,1.6  
1]
cpuTime3 = [57.17,57.09,56.90,57.42,56.91,94.06,93.96,  
93.88,93.97,93.81,182.90,189.32,199.76,200.02,193.59]

#dSet = dSet1 + dSet2 + dSet3
popSize = popSize1 + popSize2 + popSize3
objVal = objVal1 + objVal2 + objVal3
avgObj = avgObj1 + avgObj2 + avgObj3
gap = gap1 + gap2 + gap3
cpuTime = cpuTime1 + cpuTime2 + cpuTime3

dfDict1 = {'Pop Size':popSize1, 'Obj Val':objVal1, 'Pop Avg Obj':avgObj1,
```

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```
'%Gap':gap1, 'CPU Time (s)':cpuTime1 }
ds1 = pd.DataFrame(dfDict1)
ds1 = ds1[['Pop Size', 'Obj Val', 'Pop Avg Obj', '%Gap', 'CPU Time (s)']]

dfDict2 = {'Pop Size':popSize2, 'Obj Val':objVal2, 'Pop Avg Obj':avgObj2,
'%Gap':gap2, 'CPU Time (s)':cpuTime2 }
ds2 = pd.DataFrame(dfDict2)
ds2 = ds2[['Pop Size', 'Obj Val', 'Pop Avg Obj', '%Gap', 'CPU Time (s)']]

dfDict3 = {'Pop Size':popSize3, 'Obj Val':objVal3, 'Pop Avg Obj':avgObj3,
'%Gap':gap3, 'CPU Time (s)':cpuTime3 }
ds3 = pd.DataFrame(dfDict3)
ds3 = ds3[['Pop Size', 'Obj Val', 'Pop Avg Obj', '%Gap', 'CPU Time (s)']]

dfDict = {'Pop Size':popSize, 'Obj Val':objVal, 'Pop Avg Obj':avgObj,
'%Gap':gap, 'CPU Time (s)':cpuTime }
ds = pd.DataFrame(dfDict)
ds = ds[['Pop Size', 'Obj Val', 'Pop Avg Obj', '%Gap', 'CPU Time (s)']]

display(Markdown("_*Dataset1:_"))
display(ds1)
print()

display(Markdown("_*Dataset2:_"))
display(ds2)
print()

display(Markdown("_*Dataset3:_"))
display(ds3)

# Number of population
Npop = 3
# Probability of crossover
Pc = 1.0
# Probability of mutation
Pm = 1.0
# Stopping number for generation
stopGeneration = 500
```

Experiment 2: Genetic Algorithms

```
# Start Timer
t1 = time.clock()

# Creating the initial population
population = initialization(Npop)
s = []
# Run the algorithm for 'stopGeneration' times generation
for i in range(stopGeneration):
    # Selecting parents
    parents = selection(population)

    child� = []

    # Apply crossover
    for p in parents:
        r = random.random()
        if r < Pc:
            child�.append(crossover([population[p[0]], population[p[1]]]))
        else:
            if r < 0.5:
                child�.append(population[p[0]])
            else:
                child�.append(population[p[1]])

    # Apply mutation
    for c in child�:
        r = random.random()
        if r < Pm:
            c = mutation(c)

    # Update the population
    population = elitistUpdate(population, child�)

    print("Generation "+str(i)+": "+ str(population))
    # print(population)
        print("Time          of          the          Best          Solution:
```

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```
"+str(findBestSolution(population) [1]))  
    s.append(findBestSolution(population) [1])  
  
# Stop Timer  
t2 = time.clock()  
  
# Results Time  
  
bestSol, bestObj, avgObj = findBestSolution(population)  
  
print("Population:")  
print(population)  
print()  
  
print("Solution:")  
print(population[bestSol])  
print()  
  
print("Time (In secs):")  
print(bestObj)  
print()  
  
print("Average Objective Value of Population:")  
print("%.2f" %avgObj)  
print()  
  
print("%Gap:")  
G = 100 * (bestObj-optimalObjective) / optimalObjective  
print("%.2f" %G)  
print()  
  
print("CPU Time (s)")  
timePassed = (t2-t1)  
print("%.2f" %timePassed)  
  
import matplotlib.pyplot as plt
```

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```
def get_plot(s): #input all variables

    plt.plot(s, color='green', linewidth = 3)
    plt.xlabel('Number of Generations')
    plt.ylabel('Time (in seconds)')
    plt.title('Flow Shop Scheduling')
    plt.show()

get_plot(s)
```

Results:

	Pop	Size	Obj	Val	Pop	Avg Obj	%Gap	CPU	Time (s)
0	3	4534	4929.33	0.0	24.80				
1	3	4534	4840.33	0.0	25.36				
2	3	4534	5002.00	0.0	26.22				
3	3	4534	5154.33	0.0	26.35				
4	3	4534	4836.00	0.0	25.44				
5	5	4534	5066.80	0.0	43.37				
6	5	4534	5356.20	0.0	43.84				
7	5	4534	5663.20	0.0	43.72				
8	5	4534	4850.20	0.0	44.11				
9	5	4534	5050.60	0.0	43.64				
10	10	4534	5530.40	0.0	86.21				
11	10	4534	5839.50	0.0	86.90				
12	10	4534	5671.60	0.0	82.90				
13	10	4534	5725.70	0.0	80.21				
14	10	4534	5459.10	0.0	68.50				

Experiment 2: Genetic Algorithms

Dataset2:

	Pop	Size	Obj	Val	Pop	Avg	Obj	%Gap	CPU	Time (s)
0	3	920		1001.00	0.00		41.74			
1	3	929		1002.33	0.98		42.15			
2	3	920		1055.00	0.00		45.23			
3	3	920		964.67	0.00		40.75			
4	3	937		1028.67	1.85		39.75			
5	5	931		1052.40	1.20		64.36			
6	5	937		1185.20	1.85		64.04			
7	5	937		1074.00	1.85		64.76			
8	5	926		1076.00	0.65		67.21			
9	5	920		1063.60	0.00		66.68			
10	10	937		1177.00	1.85		123.27			
11	10	931		1166.00	1.20		122.97			
12	10	929		1139.80	0.98		122.32			
13	10	938		1182.50	1.96		122.29			
14	10	937		1181.40	1.85		122.42			

Dataset3:

	Pop	Size	Obj	Val	Pop	Avg	Obj	%Gap	CPU	Time (s)
0	3	1302		1363.00	0.00		57.17			
1	3	1302		1390.00	0.00		57.09			
2	3	1302		1341.00	0.00		56.90			
3	3	1302		1302.00	0.00		57.42			
4	3	1302		1346.33	0.00		56.91			
5	5	1302		1521.20	0.00		94.06			
6	5	1302		1403.40	0.00		93.96			
7	5	1302		1398.40	0.00		93.88			
8	5	1302		1497.00	0.00		93.97			
9	5	1323		1453.00	1.61		93.81			
10	10	1302		1610.50	0.00		182.90			
11	10	1302		1619.90	0.00		189.32			
12	10	1302		1654.90	0.00		199.76			
13	10	1302		1600.30	0.00		200.02			
14	10	1323		1678.80	1.61		193.59			

[24] # Number of population
Npop = 3

Experiment 2: Genetic Algorithms

```
Generation 0: [[3, 0, 8, 6, 2, 10, 5, 11, 1, 4, 7, 12, 9], [2, 3, 5, 9, 11, 10, 0, 1, 4, 8, 6, 7, 12], [3, 0, 8, 12, 2, 5, 10, 6, 1, 11, 7, 4, 9]]  
Time of the Best Solution: 1075  
Generation 1: [[2, 3, 5, 9, 11, 10, 0, 1, 4, 8, 6, 7, 12], [2, 3, 5, 9, 1, 11, 10, 0, 4, 8, 6, 7, 12], [3, 0, 8, 2, 10, 12, 6, 5, 11, 1, 4, 7, 9]]  
Time of the Best Solution: 1075  
Generation 2: [[6, 3, 0, 8, 2, 10, 12, 5, 1, 11, 4, 7, 9], [2, 3, 12, 5, 9, 1, 11, 10, 0, 8, 6, 4, 7], [2, 3, 5, 9, 11, 10, 0, 1, 4, 8, 6, 7, 12]]  
Time of the Best Solution: 1075  
Generation 3: [[2, 3, 5, 9, 11, 10, 0, 1, 4, 8, 6, 7, 12], [6, 3, 0, 10, 8, 2, 12, 5, 1, 11, 4, 7, 9], [8, 2, 3, 5, 9, 11, 10, 0, 1, 4, 6, 7, 12]]  
Time of the Best Solution: 1075  
Generation 4: [[6, 3, 0, 10, 8, 2, 12, 7, 5, 1, 11, 4, 9], [2, 3, 5, 9, 11, 10, 0, 1, 4, 8, 6, 7, 12], [11, 2, 3, 5, 9, 10, 6, 0, 8, 1, 4, 7, 12]]  
Time of the Best Solution: 1075  
Generation 5: [[6, 7, 3, 0, 10, 8, 2, 12, 5, 1, 11, 4, 9], [6, 3, 0, 10, 2, 8, 11, 12, 7, 5, 1, 4, 9], [2, 3, 5, 9, 11, 10, 0, 1, 4, 8, 6, 7, 12]]  
Time of the Best Solution: 1075  
Generation 6: [[2, 3, 5, 9, 11, 10, 0, 1, 4, 8, 6, 7, 12], [7, 6, 3, 0, 10, 8, 2, 12, 5, 1, 11, 4, 9], [4, 2, 3, 5, 9, 11, 10, 0, 8, 1, 6, 7, 12]]  
Time of the Best Solution: 1003  
Generation 7: [[4, 2, 3, 5, 9, 11, 10, 0, 8, 1, 6, 7, 12], [7, 6, 3, 0, 10, 8, 2, 12, 5, 1, 4, 11, 9], [2, 3, 5, 0, 10, 1, 11, 4, 9, 8, 7, 6, 12]]  
Time of the Best Solution: 1003  
Generation 8: [[2, 3, 5, 9, 0, 10, 1, 11, 4, 8, 7, 6, 12], [4, 2, 3, 5, 9, 11, 10, 0, 8, 1, 6, 7, 12], [4, 2, 3, 0, 10, 11, 5, 9, 8, 1, 6, 7, 12]]  
Time of the Best Solution: 1003  
Generation 9: [[4, 2, 5, 3, 9, 11, 10, 0, 8, 1, 6, 7, 12], [4, 2, 3, 5, 9, 11, 10, 0, 8, 1, 6, 7, 12], [4, 2, 3, 10, 0, 11, 6, 5, 9, 8, 1, 7, 12]]  
Time of the Best Solution: 1003  
Generation 10: [[4, 2, 3, 10, 0, 11, 9, 6, 5, 8, 1, 7, 12], [4, 2, 3, 10, 1, 0, 11, 6, 5, 9, 8, 7, 12], [4, 2, 5, 3, 9, 11, 10, 0, 8, 1, 6, 7, 12]]  
Time of the Best Solution: 1003  
Generation 11: [[4, 2, 5, 2, 3, 10, 0, 11, 9, 6, 8, 1, 7, 12], [4, 2, 5, 3, 9, 11, 10, 0, 8, 1, 6, 7, 12], [4, 2, 3, 10, 1, 0, 11, 6, 5, 9, 7, 8, 12]]  
Time of the Best Solution: 1003  
Generation 12: [[4, 5, 2, 3, 10, 7, 0, 11, 9, 6, 8, 1, 12], [4, 2, 5, 3, 9, 11, 10, 0, 8, 1, 6, 7, 12], [4, 2, 3, 5, 10, 1, 0, 11, 6, 9, 7, 8, 12]]  
Time of the Best Solution: 1003  
Generation 13: [[4, 2, 3, 5, 10, 1, 9, 0, 11, 6, 7, 8, 12], [4, 2, 5, 3, 9, 11, 10, 0, 8, 1, 6, 7, 12], [4, 2, 3, 5, 1, 10, 0, 11, 6, 9, 7, 8, 12]]  
Time of the Best Solution: 1003  
Generation 14: [[4, 2, 3, 9, 5, 1, 10, 0, 11, 6, 7, 8, 12], [4, 2, 9, 3, 5, 10, 11, 0, 8, 1, 6, 7, 12], [4, 2, 5, 3, 9, 11, 10, 0, 8, 1, 6, 7, 12]]  
Time of the Best Solution: 1003  
Generation 15: [[4, 2, 5, 3, 9, 0, 10, 11, 8, 1, 6, 7, 12], [4, 2, 0, 5, 3, 9, 11, 10, 8, 1, 6, 7, 12], [4, 2, 5, 3, 9, 11, 10, 0, 8, 1, 6, 7, 12]]  
Time of the Best Solution: 995  
Generation 16: [[4, 2, 0, 5, 3, 9, 11, 10, 8, 1, 6, 7, 12], [4, 2, 5, 3, 10, 9, 0, 11, 8, 1, 6, 7, 12], [0, 4, 2, 5, 3, 9, 11, 10, 8, 1, 6, 7, 12]]
```

```
Generation 49: [[7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10], [11, 7, 10, 8, 4, 12, 0, 2, 9, 1, 3, 5, 6], [7, 0, 11, 4, 12, 2, 9, 1, 8, 3, 6, 5, 10]]  
Time of the Best Solution: 940  
Generation 492: [[7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10], [11, 0, 7, 10, 8, 4, 12, 2, 9, 1, 3, 5, 6], [7, 0, 1, 11, 4, 12, 2, 9, 8, 3, 6, 5, 10]]  
Time of the Best Solution: 940  
Generation 493: [[7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10], [7, 0, 11, 8, 4, 12, 2, 10, 9, 1, 3, 6, 5], [7, 0, 1, 11, 4, 5, 12, 2, 9, 8, 3, 6, 10]]  
Time of the Best Solution: 940  
Generation 494: [[7, 11, 4, 12, 2, 1, 9, 0, 8, 3, 6, 5, 10], [7, 0, 1, 11, 4, 12, 2, 9, 5, 6, 8, 3, 10], [7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10]]  
Time of the Best Solution: 940  
Generation 495: [[7, 12, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10], [7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10], [7, 0, 1, 11, 4, 12, 2, 9, 5, 8, 3, 6, 10]]  
Time of the Best Solution: 940  
Generation 496: [[7, 4, 0, 1, 11, 12, 2, 9, 5, 8, 3, 6, 10], [7, 1, 11, 4, 12, 2, 9, 0, 8, 3, 6, 5, 10], [7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10]]  
Time of the Best Solution: 940  
Generation 497: [[12, 7, 4, 0, 1, 11, 2, 9, 8, 3, 5, 6, 10], [7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10], [7, 4, 0, 1, 11, 12, 8, 2, 9, 5, 3, 6, 10]]  
Time of the Best Solution: 940  
Generation 498: [[7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10], [12, 8, 7, 4, 0, 1, 11, 2, 9, 3, 5, 6, 10], [7, 4, 0, 3, 1, 11, 12, 8, 2, 9, 5, 6, 10]]  
Time of the Best Solution: 940  
Generation 499: [[1, 7, 11, 4, 12, 2, 9, 0, 8, 3, 6, 5, 10], [7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10], [7, 11, 0, 4, 12, 2, 9, 1, 8, 3, 5, 6, 10]]  
Population: [[1, 7, 11, 4, 12, 2, 9, 0, 8, 3, 6, 5, 10], [7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10], [7, 11, 0, 4, 12, 2, 9, 1, 8, 3, 5, 6, 10]]  
Solution: [7, 11, 4, 12, 2, 9, 0, 1, 8, 3, 6, 5, 10]  
Time(In secs): 940  
Average Objective Value of Population: 971.00  
%Gap: 2.17  
CPU Time (s) 3.98
```

Experiment 2: Genetic Algorithms

The screenshot shows a Google Colab notebook titled "SC_GeneticAlgo.ipynb". The code in cell [25] imports matplotlib.pyplot and defines a function get_plot(s) to create a plot of Time (in seconds) vs Number of Generations. The plot, titled "Flow Shop Scheduling", shows a green step-down graph starting at approximately 1080 seconds and reaching a plateau around 940 seconds by generation 300. The x-axis ranges from 0 to 500, and the y-axis ranges from 940 to 1080.

```
[25]: import matplotlib.pyplot as plt

def get_plot(s): #input all variables
    plt.plot(s, color='green', linewidth = 3)
    plt.xlabel('Number of Generations')
    plt.ylabel('Time (in seconds)')
    plt.title('Flow Shop Scheduling')
    plt.show()
```

[]

Conclusion:

Successfully implemented Flow Shop Scheduling using the genetic algorithm. The type of crossover and mutation used in this code are two point crossover and shift-change mutation which provide the best results in solving this problem.

Experiment 2: Genetic Algorithms
