MA2101 (Linear Algebra 2)

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1 Definitions and Formula

2 General

Reference Book: Axler's Linear Algebra Done Right

2.1 Well-defined functions

I am inspired to discuss this by the section on quotient spaces.
For a vector space V, subspace U of V, define $V/U = \{v + U : v \in V\}$.
Define vector addition on V/U by $(v + U) + (w + U) = ((v + w) + U)$. Define scalar multiplication on V/U by $\lambda(v + U) = ((\lambda v) + U)$.

One of the steps in proving that our newly defined $+$ is a binary operation on V/U, is to show that it is well-defined. That is, for each distinct element in the domain (V/U), there exists a unique image in the codomain.

For sets like integers, the representation of any element in the domain is unique, so we usually don't have to worry about well-definition of functions. However, in the case of quotient spaces, $v - v' \in U$ implies $(v+U) = (v'+U)$, even though we can have $v \neq w$. It is because of these multiple representations that we have to worry about "well-defined-ness". We need to make sure when $(v+U) = (v'+U), (w+U) = (w' + U)$, we have $(v + U) + (w + U) = (v' + U) + (w' + U)$.

2.2 Matrices vs Linear Maps

Other than determinants and the section on eigenvectors, MA1101R (Linear Algebra 1) is sufficiently rigorous. The key difference between MA1101R and Axler's Linear Algebra is the approach taken to building up the theory.