CS2100

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1 Definitions

2 Number representations

Definition $=$ is the identity relation between 2 mathematical objects.

Example $(x)_{b_1} = (x)_{b_2}$, where b_1, b_2 are any 2 representations of x. If we indicate b_1, b_2 by numbers $n, m \in \mathbb{N}$, then such representations could be base-n/m representations respectively.

Definition $\equiv$ is the equivalence relation between 2 object representations.

Example Let S be a set. Let r be a representation of elements of set S. Then $(a)_r \equiv (b)_r$ if the *representation* of a, b under r are identical. Note that it is not necessary for $a = b$.

Note that I'm not too sure how a "representation" is defined rigorously.

2.1 Ones complement

Definition For this section, let representation $r := 1s$. Then

$$\forall x \in [0, (2^{n-1} - 1)] \cap \mathbb{Z}, (x)_r \equiv (x)_2$$

Definition Let $S = [-(2^{n-1} - 1), (2^{n-1} - 1)] \cap \mathbb{Z}$. Suppose the representation is restricted to n bits. Then

$$\forall x \in [-(2^{n-1} - 1), 0) \cap \mathbb{Z}, (-x)_r \equiv (2^n - 1 - x)_r$$

Note: I have yet to prove these. I think this might be the case.

Yet to prove
Proposition For all $a, b \in S$, $(a + b)_r = (a)_r + (b)_r$

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Proposition For all $a, b \in S$, $(a + b)_r \equiv (a)_r + (b)_r$

Proposition For negative x, $(-x)_r \equiv (2^n - 1 - x)_r$

Proposition $(-(-x))_r \equiv (x)_r$

Proof Trivially, $- - x = x$ and hence their representations are the same as well. However, we may wish to verify this purely using the definition of the representation.

Then, we have,

$$
\begin{aligned}
(-(-x))_r &\equiv (2^n - 1 - (-x))_r \\
&\equiv (2^n - 1)_r - (-x)_r \\
&\equiv (2^n - 1)_r - ((2^n - 1)_r - (x)_r) \\
&\equiv ((2^n - 1)_r - (2^n - 1)_r) + (x)_r \\
&\equiv (x)_r
\end{aligned}
$$