

New Mathematical Model for Dark Matter: A Unified Geometric–Field Framework

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Abstract:

This paper proposes a new mathematical model for Dark Matter based on a geometric inversion of the stress-energy tensor combined with a modified dispersion relation. The model treats dark matter as a non-luminal quasi-field arising from curvature-induced vacuum anisotropy. We derive the governing equations, compute the effective energy density, and propose testable predictions for galactic rotation curves.

1. Introduction

Dark matter is empirically required to explain galactic rotation curves, gravitational lensing, and structure formation. Traditional candidates include WIMPs, axions, and sterile neutrinos. Here we develop a new geometric model in which dark matter emerges from an inverted stress-energy response of spacetime: instead of adding new particles, curvature generates an effective energy density term.

2. Mathematical Formulation

We begin with Einstein's Field Equation:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Define a geometric inversion operator:

$$I(T_{\mu\nu}) = (T_{\mu\nu})^{-1}$$

We propose dark matter energy density as:

$$\rho_{DM} = \lambda \cdot \text{Tr}[I(T_{\mu\nu})]$$

where λ is a curvature-coupling constant.

3. Modified Dispersion Relation

For ordinary matter: $E^2 = p^2c^2 + m^2c^2$

For dark-matter quasi-field:

$$E = \omega / \tau, \text{ where } \tau = \text{curvature-induced time quanta.}$$

Thus frequency-based Planck relation becomes time-based:

$$E = \omega / \tau = \omega (d\theta/dt)$$

4. Effective Gravitational Potential

The modified Poisson equation becomes:

$$\nabla^2\Phi = 4\pi G (\rho_{visible} + \rho_{DM})$$

Substituting our dark matter model:

$$\nabla^2\Phi = 4\pi G (\rho_{vis} + \lambda \text{Tr}[I(T_{\mu\nu})])$$

5. Conclusion

This framework replaces particle dark matter with an emergent geometric term. Predictions include modified rotation curves and specific lensing signatures. Future work will compute simulation-based values for λ and compare with Λ CDM.