SINGAPORE JUNIOR PHYSICS OLYMPIAD 2014
SPECIAL ROUND

30 August, 2014

9:00 – 12:30 pm

Time Allowed: $3\frac{1}{2}$ HOURS

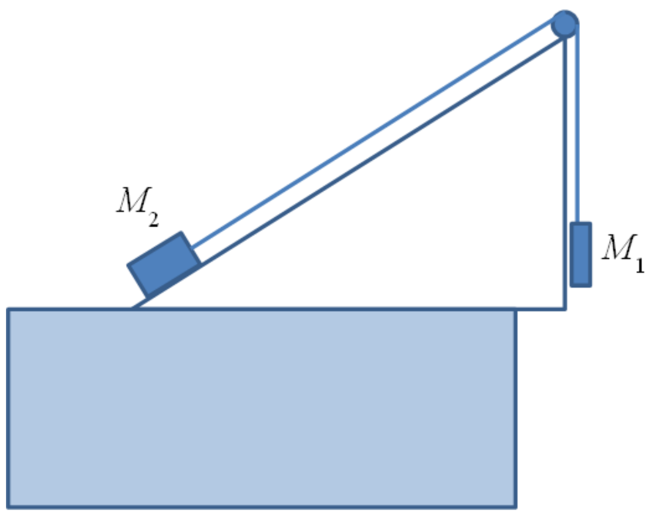
INSTRUCTIONS

1. This paper contains **11** structural questions and **10** printed pages.
2. The mark for each question is indicated at the end of the question.
3. Answer **ALL** the questions in the booklet provided.
4. Scientific calculators are allowed in this test.
5. A table of information is given in page 2. Not all information will be used in this paper.

TABLE OF INFORMATION

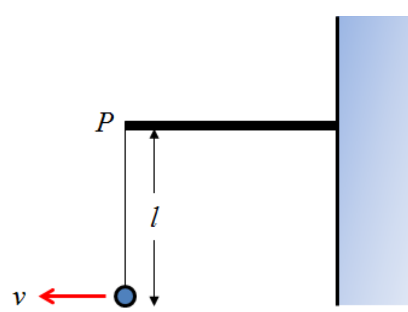
Acceleration due to gravity at Earth surface, g	$=$	9.80 m/s^2
Universal gas constant, R	$=$	$8.31 \text{ J/(mol} \cdot \text{K)}$
Newton's gravitational constant, G	$=$	$6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
Vacuum permittivity, ϵ_0	$=$	$8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$
Vacuum permeability, μ_0	$=$	$4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$
Speed of light in vacuum, c	$=$	$3.00 \times 10^8 \text{ m/s}$
Speed of sound in air, v	$=$	331 m/s
Charge of electron, e	$=$	$1.60 \times 10^{-19} \text{ C}$
Planck's constant, h	$=$	$6.63 \times 10^{-34} \text{ J} \cdot \text{s}$
Mass of electron, m_e	$=$	$9.11 \times 10^{-31} \text{ kg}$
Mass of proton, m_p	$=$	$1.67 \times 10^{-27} \text{ kg}$
Boltzmann constant, k	$=$	$1.38 \times 10^{-23} \text{ J/K}$
Avogadro's number, N_A	$=$	$6.02 \times 10^{23} \text{ mol}^{-1}$
Latent heat of vaporisation of water, L_V	$=$	$2.26 \times 10^6 \text{ J kg}^{-1}$
Radius of Earth, R_E	$=$	$6.37 \times 10^3 \text{ km}$
Standard atmospheric pressure	$=$	$1.01 \times 10^5 \text{ Pa}$
Lorentz Transformation: x'	$=$	$\dfrac{x - ut}{\sqrt{1 - u^2/c^2}}$
y'	$=$	y
z'	$=$	z
t'	$=$	$\dfrac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}$

1. In the setup below, masses M_1 and M_2 form a simple pulley system. Assume all the surfaces and pulleys to be frictionless and that the slope is fixed to the ground. The slope makes an angle of $30°$ with respect to the ground.



Your schoolmate Syam now performs 2 experiments. In the first, M_1 is suspended while M_2 rests on the slope. The time taken for M_2 to traverse from the base of the slope to the top of the slope is t. In the second experiment, Syam now interchanges M_2 with M_1. The time taken now for M_1 to traverse from the base to the top of the slope is $t/3$. What can we figure out about the ratio of M_1 to M_2? [8]

2. A small ball is hung from the tip of a rod P by a string of length l as shown below.



(a) Describe qualitatively the possible motions of the ball for different horizontal speeds v when the ball is directly below P. Include drawings that illustrate the paths of the ball for all the different cases you have described. [5]

(b) For what value of v (in terms of l and g) will the ball hit the point P? [6]

3. In a demonstration known as the ballistics cart, a ball is projected vertically upward from a cart moving with constant velocity along the horizontal direction. The ball lands in the catching cup of the cart because both the cart and the ball have the same horizontal component of velocity.

   Consider a ballistics cart on an incline making an angle θ with the horizontal as shown in Figure 2. The cart (including wheels) has a mass M and the moment of inertia of each of the two wheels is $mR^2/2$. Assuming no friction between cart and axles and pure rolling motion (no slipping), and the initial speed of the ball imparted by the spring in the cart is v_0.

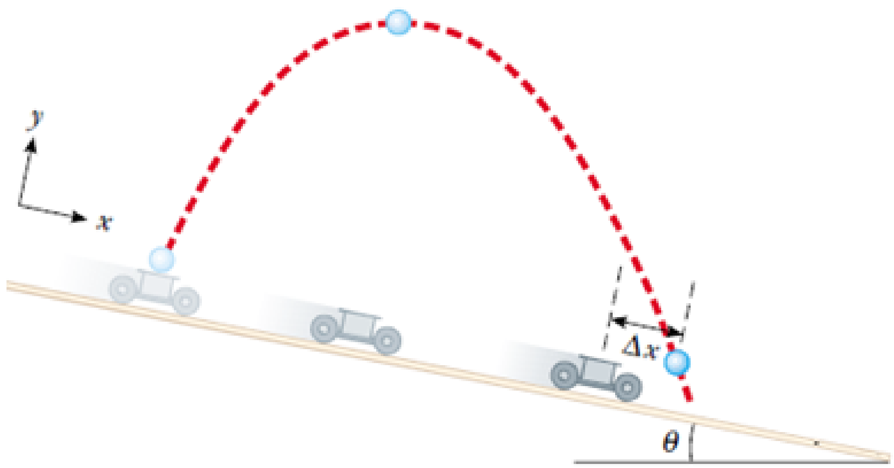


Figure 2

   (a) Show that the acceleration of the cart along the incline is [3]

$$a_x = \left(\frac{M}{M+m}\right) g \sin\theta$$

   (b) Find Δx, where Δx is the distance which the ball overshoots the cart when it landed back along the slope again. [7]

4. Suppose there is a string of length L shown in Figure 1(a), you pluck the taut string in the middle M and move this point a distance A ($A \ll L$) up. In doing so, the tension T in the string does not change.

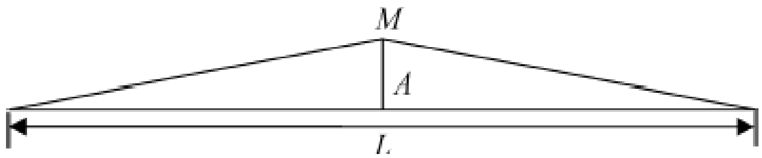


Figure 1(a)

The energy stored in such a configuration is given by

$$E = \frac{(2TA^2)}{L}$$

Now divide the same string into n equal sections, each of length L/n, as shown in Figure 1(b). The midpoints of each of the n sections are moved with a distance A_n.

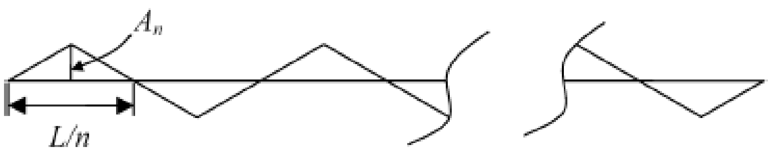


Figure 1(b)

(a) Show that the work you have to do to set up the configuration in Figure 1(b) is

$$E = \frac{(2Tn^2A_n^2)}{L}.$$ [2]

(b) Calculate the energy stored in one triangular wavelength (λ) of a vibrating string (standing wave) in terms of T, A_n and λ. [2]

(c) Given that the energy stored in one sinusoidal wave of a vibrating string is:

$$E_{sine} = \frac{(TA^2\pi^2)}{\lambda}$$

What is the ratio of $E_{sine} : E_{triangular}$? [2]

5. The distribution of the intercepted solar energy into various channels over the globe is given in Table 1 below.

Channel	**Electromagnetic Energy**
Solar radiation on the Earth (Q)	1.74×10^{17} W
Direct reflection (Q_{rmR})	5.2×10^{16} W
Direct conversion to heat (Q_{rmH})	8.2×10^{16} W
Evaporation (Q_{rmEV})	4.0×10^{16} W
Winds, waves, convection and currents (Q_{rmW})	3.7×10^{14} W
Photosynthesis (Q_{rmPH})	4.0×10^{13} W

Table 1

(a) Based on table 1 and other relevant quantities found in the table of constants, what is the volume of water evaporated annually? [3]

(b) If the water evaporated precipitates and is collected over the surface of the Earth in one year as a uniform shell over the surface of the Earth, what is the thickness of this shell? [2]

In addition, the global annual evaporation and precipitation data reported by UNESCO in its publication World Water Balance and Water Resources of the Earth is provided in table 2 below.

Globe	**Ocean**	**Land**	**Total**
Area	3.61×10^{14} m^2	1.50×10^{14} m^2	5.11×10^{14} m^2
Evaporation	5.05×10^{14} m^3	0.72×10^{14} m^3	5.77×10^{14} m^3
Precipitation	4.58×10^{14} m^3	1.19×10^{14} m^3	5.77×10^{14} m^3

Table 2

(c) How does the data in (a) compare with the data in table 2? [1]

(d) For a world population of 7 billions and with a freshwater requirement of 6800 litres per day per person (inclusive of irrigation and industrial needs), what percentage of the precipitation over land is required for the world human population's requirements? [2]

6. Bromine gas at atmospheric pressure and room temperature has a density of 6.4 kgm^{-3}.

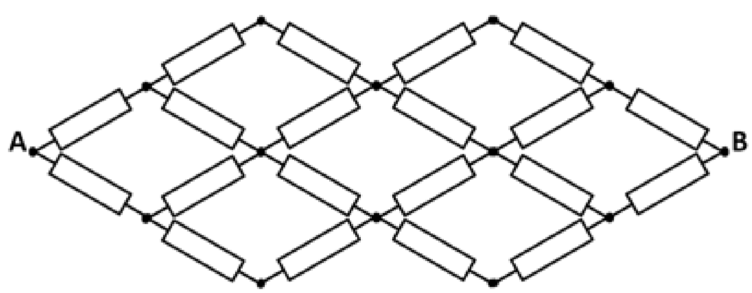
   (a) What is the root mean square speed of bromine molecules? [2]

   (b) We notice that bromine gas diffuse by 10 cm over 500 s in air. This is due to the collisions between the bromine and the air molecules. Between successive collisions, the molecule would have travelled an average distance of λ between successive collisions. For a bromine molecule to have travelled between 2 points on a random path, the distance x between the two points is given by $x^2 = n\lambda^2$ where n is the number of collisions. Estimate the number of collisions suffered by the bromine molecule when it diffuses 0.1 m. What is the mean free path of the molecules? [6]

   (c) The effective volume of the bromine molecule as it passes through air molecules is $\pi d^2 \lambda$ where d is the diameter of the bromine molecule. It is observed that 5 cm^3 of bromine liquid becomes a cloud of bromine gas of diameter 17 cm, estimate the diameter of a bromine molecule. [3]

7. A glass rod placed in air has a refractive index of 1.5 and a length of 45 cm. One end of the rod is shaped as a convex surface with radius of curvature $R_1 = 10$ cm. If we are to use this rod as a telescope, how should the other end be shaped? Be as precise as possible in your answer. (A telescope will allow a faraway object along the principle axis to have its image within the telescope system.) [7]

8. Identical resistors with the resistance r is connected as shown in the figure below. What is the equivalent resistance between A and B?

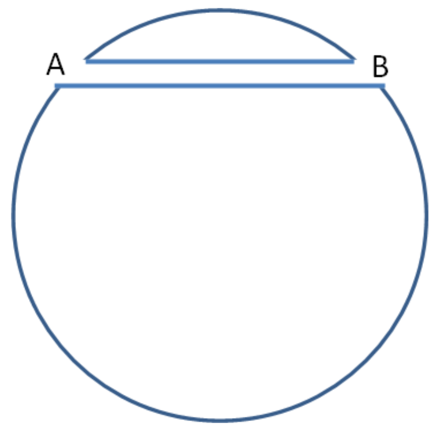


[5]

9. The electric field strength at a point P inside a uniformly charged solid sphere is the same as would be produced at P if all the charges closer to the centre of the sphere than P were concentrated at the centre of the sphere.

(a) Use this result to deduce the electric field strength $\vec{E}$ at a point P located a distance r from the centre of a charged sphere of uniform charge density ρ. Express your answer in terms of r, ρ and any other relevant fundamental constants. [3]

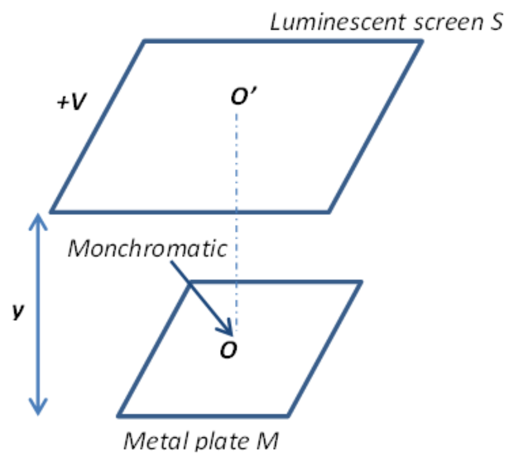
(b) A smooth, narrow, straight tunnel from A to B is bored through a large charged sphere but not passing through the centre as shown below.



i. Show that an object with a test charge (of opposite sign with respect to that on the sphere) and placed at one end A of the tunnel can perform simple harmonic motion, that is, its acceleration a can be expressed in the form $a = -kx$ where x is taken to be the distance from the center of the tunnel and k is a positive constant. Find the expression for k. You may assume that the charge does not touch the sides of the tunnel and the mass of the sphere can be ignored. [4]

ii. Find the time taken for the test charge to reach B, assuming that the sphere has a radius of 6400 km and that the electric field strength is 150 N/C on its surface. [4]

10. The figure below shows a metal plate M of work function ϕ. A luminescent screen S (similar to the screen in a cathode ray tube) is mounted parallel to M and a distance y from it. The arrangement is contained in an evacuated tube. A very small circular area of the metal plate (at O) is illuminated with monochromatic light of wavelength. The screen is at a positive potential V with respect to M, so that photoelectrons emitted from the metal are accelerated towards the screen. When the electrons strike the screen, their kinetic energy is converted to light energy.



(a) Describe and explain what would be seen on the screen M. As a reference, point O' on the screen is vertically above O on the metal plate. [2]

(b) Draw a schematic, showing the dimensions of the pattern as expressed in the above-mentioned quantities and other relevant quantities in the data sheet. Describe, using a graph or otherwise, how the pattern will change as the potential V changes. [6]

11. Lim and Ong have identical spaceships 120 m long. The diagram shows Ong's observations of Lim's ship, which passes at a speed of $c/\sqrt{2}$. Clocks at the back of both ships read 0 just as they pass. Ong is at the centre of his ship and at $t = 0$ on his clock, he peers at a second clock on Lim's ship, as shown in figure 10(a).

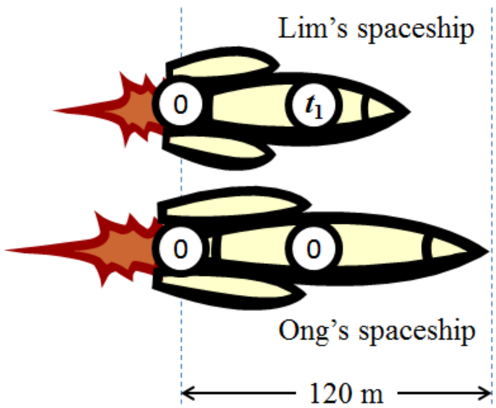


Figure 10(a)

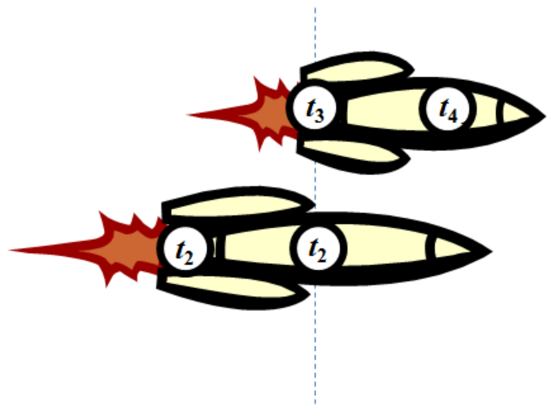


Figure 10(b)

(a) What does this clock read, i.e., t_1? [2]

(b) Later, the back of Lim's ship passes Ong as shown in figure 10(b). At what time, i.e., t_2, does this occur according to Ong? [2]

(c) What will observers at Ong's frame see on Lim's two clocks (t_3 and t_4) at this time? [3]

(d) Identify two events (indicating its position x and time t in both frames) that show time dilation and two events that show length contraction **according to Lim**. Explain your answers. [4]

(e) Sketch three diagrams, similar to figure 10(a) and 10(b), **showing Lim's view** of the passing of the spaceships when the clocks in Lim's frame read -200 ns, 0 ns and 200 ns. Include the times on clocks in Ong's ship that have already been calculated above. [4]

Note: The clocks in each spaceship (denoted by the circles) are properly synchronised in its own frame according to Special Relativity

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