Dynamics Problem Set

involving Forces, Momentum and WEP

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1. A small car collides head-on with a large SUV. Which of the following statements concering this collision are correct? (There may be more than one correct choice)

   A. Both vehicles are acted upon by the same magnitude of average force during the collision.

   B. The small car is acted upon by a greater magnitude of average force than the SUV.

   C. The small car undergoes a greater change in momentum than the SUV

   D. Both vehicles undergo the same change in magnitude of momentum

Solution: A and D

2. A ball of mass $0.18\,\mathrm{kg}$ moving with speed $11.3\,\mathrm{m\,s^{-1}}$ collides head-on with an identical stationary ball. (Notice how we do not know the type of collision.) Which of the following quantities can be calculated from this information alone?

   A. The force each ball exerts on the other.

   B. The velocity of each ball after the collision.

   C. Total kinetic energy of both balls after the collision.

   D. Total momentum of both balls after the collision.

Solution: D

3. You drop an egg from rest with no air resistance. As the egg falls,

   A. only its momentum is conserved.

   B. only its kinetic energy is conserved.

   C. both its momentum and its mechanical energy are conserved.

   D. its mechanical energy is conserved, but its momentum is not conserved.

Solution: D. Do note that, however, if we consider the entire system of the egg and the Earth (and the air), momentum is still conserved.

4. You, at $70.0\,\mathrm{kg}$, are standing stationary on a sheet of ice that covers the football stadium parking lot in Buffalo; there is negligible friction between your feet and the ice. A $50.0\,\mathrm{kg}$ friend throws you a $0.400\,\mathrm{kg}$ ball.

   (a) If your friend moves back at a speed of $0.0800\,\mathrm{m\,s^{-1}}$, what is the speed of the ball?

   Solution:

   $$v_b = \frac{50.0 \times 0.0800}{0.400} = \frac{4}{0.4} = 10.0\,\mathrm{m\,s^{-1}}$$

   (b) If you catch the ball, with what speed do you and the ball move afterwards?

   Solution:

   $$v_{combined} = \frac{4}{0.400 + 70.0} = 5.68 \times 10^{-2}\,\mathrm{m\,s^{-1}}$$

   (c) If the ball hits you and bounces off your chest, so that afterwards it is moving horizontally at $8.00\,\mathrm{m\,s^{-1}}$ in the opposite direction, what is your speed after the collision?

   Solution:

   $$\begin{aligned} m_{ball}u_{ball} &= m_{ball}v_{ball} + m_{you}v_{you} \\ v_{you} &= \frac{m_{ball}\left(u_{ball} - v_{ball}\right)}{m_{you}} \\ &= \frac{0.400\left[10.0 - (-8.00)\right]}{70.0} \\ &= 1.03 \times 10^{-1}\,\mathrm{m\,s^{-1}} \end{aligned}$$

5. **[Poorly Maintained Car]** John was driving his 1.50×10^3 kg car up a hill when the engine suddenly dies and the gear disengages, causing his vehicle to free-wheel[1]. Having missed his car servicing for many years, his brakes were also faulty. Hoping to stay alive, he jumps out of the car as his car slows to a momentary stop along the hill at $h = 100\,\mathrm{m}$ above sea level. As he turns around, he sees his car roll down the hill and crash into a 5.00×10^3 kg truck parked at the bottom of the hill.

   You may disregard any resistive forces in the wheels.

   (a) If the car and truck fuse together, at what speed would the car/truck combination be moving?

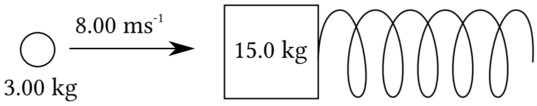
   > **Solution:**
   > $$\begin{aligned} \mathrm{GPE} = \cancel{m}gh &= \frac{1}{2}\cancel{m}v_i^2 = \mathrm{KE} \\ v_i &= \sqrt{2gh} \\ P_i = mv_i = m\sqrt{2gh} &= (m+M)v_f = P_f \\ v_f &= \frac{m\sqrt{2gh}}{m+M} = 10.2\,\mathrm{m\,s^{-1}} \end{aligned}$$

   (b) The car/truck combination then slides without slowing into an abandoned building at the end of the road and crashes into a rigid wall. What is the average force of the rigid wall on the car/truck combination, if the entire collision happens in $0.8\,\mathrm{s}$?

   > **Solution:**
   > $$\begin{aligned} F = \frac{\Delta P}{\Delta t} &= \frac{m\sqrt{2gh}}{\Delta t} \\ &= 8.31 \times 10^4\,\mathrm{N} \end{aligned}$$

[1] The wheels are no longer driven by the engine

6. A $15.0\,\mathrm{kg}$ block is attached to a very light horizontal spring of force constant $500\,\mathrm{N\,m^{-1}}$ and is resting on a frictionless horizontal table. Suddenly it is struck by a $3.00\,\mathrm{kg}$ stone travelling horizontally at $8.00\,\mathrm{m\,s^{-1}}$ to the right, whereupon the stone rebounds at $2.00\,\mathrm{m\,s^{-1}}$ horizontally to the left. Find the maximum distance that the block will compress the spring after the collision. (*Hint:* Break this problem into two parts — the collision and the behaviour after the collision — and apply the appropriate conservation law to each part.)



Solution: We define right as positive. At the moment of collision,

$$\begin{aligned} P_i &= P_f \\ m_{stone}u_{stone} &= m_{stone}v_{stone} + m_{block}v_{block} \\ v_{block} &= \frac{m_{stone}\left(u_{stone} - v_{stone}\right)}{m_{block}} = \frac{3.00(8+2)}{15} \\ &= 2.00\,\mathrm{m\,s^{-1}} \end{aligned}$$

After the collision, the KE imparted on the $15.0\,\mathrm{kg}$ block is all converted to EPE.

$$\begin{aligned} \mathrm{KE} &= \mathrm{EPE} \\ \cancel{\frac{1}{2}}m_{block}v_{block}^2 &= \cancel{\frac{1}{2}}kx_{max}^2 \\ x_{max} &= \sqrt{\frac{m_{block}v_{block}^2}{k}} \\ &= \sqrt{\frac{15.0 \times 2.00^2}{500}} \\ &= \frac{\sqrt{3}}{5}\mathrm{m} \end{aligned}$$

7. **[A Cart of Sand]** Consider a cart full of sand with total mass of M moving at a constant velocity v on a frictionless track.

   (a) A crack forms at the bottom of the cart and sand is leaking out at a rate of $\mathrm{d}m/\,\mathrm{d}t = \sigma$. What will the velocity of the cart be?

   Solution: v

   (b) If you apply a force F to the leaking cart, what is its acceleration?

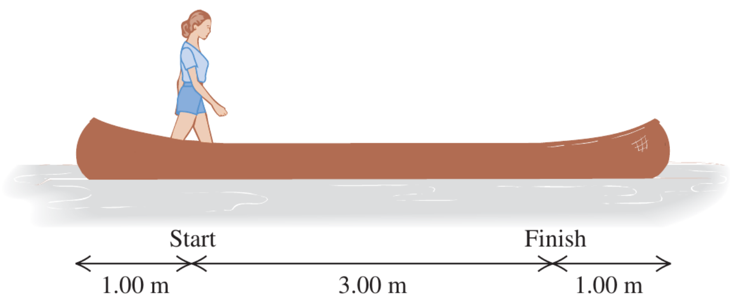
   Solution: $a = \dfrac{F}{M - \sigma t}$

   (c) The sand then falls (vertically) into a passing cart travelling at $2v$ below the tracks. With what force must you push on the cart to keep it moving (horizontally) at a constant speed $2v$?

   Solution: $F = \dfrac{\mathrm{d}p}{\mathrm{d}t} = ma + \dfrac{\mathrm{d}m}{\mathrm{d}t}v = 0 + \sigma v$

Challenging Questions

1. **[Walking in a boat]** A $45.0\,\mathrm{kg}$ woman stands up in a $60.0\,\mathrm{kg}$ canoe of length $5.00\,\mathrm{m}$. She walks from a point $1.00\,\mathrm{meter}$ from one end to a point $1.00\,\mathrm{meter}$ from the other end. If the resistance of the water is negligible, how far does the canoe move during the process?



Solution: Suppose the woman moves at some point with some velocity v_w to the right relative to the stationary river bank. We define right as positive. By conservation of momentum, we know that:

$$\begin{aligned} m_w v_w + m_c v_c &= 0 \\ v_c &= -\frac{m_w}{m_c} v_w \end{aligned}$$

Integrating both sides,

$$\begin{aligned} \int_{t=0}^{t=t} \frac{\mathrm{d}S_c}{\mathrm{d}t}\ \mathrm{d}t &= -\frac{m_w}{m_c}\int_{t=0}^{t=t} \frac{\mathrm{d}S_w}{\mathrm{d}t}\ \mathrm{d}t \\ S_c - S_{c_0} &= -\frac{m_w}{m_c}\left(S_w - S_{w_0}\right) \\ S_c &= -\frac{m_w}{m_c} S_w \end{aligned}$$

Since the woman moves $3\,\mathrm{m}$ relative to the canoe, we obtain $S_w = 3.00 + S_c$:

$$\begin{aligned} S_c &= -\frac{45.0}{60.0}\left(3.00 + S_c\right) \\ S_c\left(1 + \frac{3}{4}\right) &= -\frac{3}{4}(3) \\ S_c &= -\frac{9/4}{7/4} \\ &= -1.29\,\mathrm{m} \end{aligned}$$

Alternatively, using the fact that the centre of mass of the system stays in the same place relative to the river bank:

$$\text{CoM of system} = \frac{(45.0)(1.00) + (60.0)(2.50)}{45.0 + 60.0} = \frac{13}{7}\text{m from the end of the canoe}$$

After walking, taking reference from the same point as previously:

$$\begin{aligned} \text{CoM of system} = \frac{(45.0)(4.00 + \Delta x) + (60.0)(2.50 + \Delta x)}{45.0 + 60.0} &= \frac{13}{7} \\ \Delta x &= \frac{\left[13/7\right](105) - 330}{45.0 + 60.0} \\ &= -9/7 \\ &= -1.29\,\mathrm{m} \end{aligned}$$

The canoe has moved to the left by $1.29\,\mathrm{m}$.

2. **[Stranded on a lake]** You're stuck in a boat in the middle of a lake. Luckily, you brought your physics textbook. You decide to use your textbook to propel you back to the shore. You shoot your $3\,\mathrm{kg}$ textbook horizontally overboard with a speed of $10\,\mathrm{m\,s^{-1}}$ with the slingshot on board. If you and the boat have the combined mass M of $100\,\mathrm{kg}$,

   (a) How long would it take you to reach the shore $60\,\mathrm{m}$ away after shooting your book? (Ignore friction between the water and the boat.)

   Solution: $200\,\mathrm{s}$

   (b) (*CALC) Unfortunately, it starts raining at $t = 100\,\mathrm{s}$ as you float towards the shore. The rain, that is falling straight down, collects in your boat at a rate of $\mathrm{d}m/\,\mathrm{d}t = \sigma$. How long would it take you to reach the shore in total, assuming your boat does not sink?

   Hint: if $y = e^x$, then $\ln y = x$. If you have yet to learn calculus, there are some hints in the appendix.

   Solution:

   Let x be the Δt after $t = 100$

$$\begin{aligned}
M(x) &= 100 + \sigma x \\
v(x) &= \frac{P}{M(x)} = \frac{3 \times 10}{100 + \sigma x} \\
\therefore \frac{\mathrm{d}S}{\mathrm{d}x} &= \frac{30}{100 + \sigma x} \\
\int_{S(x=0)}^{S(x=x)} \mathrm{d}S &= 30 \int_0^x \frac{1}{100 + \sigma x}\,\mathrm{d}x \\
S &= 30 \left[\frac{1}{\sigma} \ln(100 + \sigma x)\right]_0^x \\
&= \frac{30}{\sigma} \left[\ln(100 - \sigma x) - \ln(100)\right]
\end{aligned}$$

   We know from part (a) that $S = 30$:

$$\begin{aligned}
30 &= \frac{30}{\sigma} \left[\ln(100 - \sigma x) - \ln(100)\right] \\
\sigma &= \ln(100 - \sigma x) - \ln(100) \\
&= \ln\left(\frac{100 - \sigma x}{100}\right) \\
100e^{\sigma} &= 100 - \sigma x \\
\implies x = \Delta t &= \frac{100\left(e^{\sigma} - 1\right)}{\sigma} \\
t &= 100 + \frac{100\left(e^{\sigma} - 1\right)}{\sigma}
\end{aligned}$$

   And indeed, we can check the limits:

$$\begin{aligned}
\lim_{\sigma \to 0} \Delta t &= \lim_{\sigma \to 0} \frac{100\left(e^{\sigma} - 1\right)}{\sigma} \\
&= \lim_{\sigma \to 0} \frac{100e^{\sigma}}{1} \quad \text{By L'Hôpital's Rule} \\
&= 100 \\
\implies \lim_{\sigma \to 0} t &= 200\,\mathrm{s}
\end{aligned}$$

   which is the answer from part (a)

Appendices

Appendix A Calculus

Should any of the questions in this set require calculus, I have provided the necessary formulae for you to use and apply. You may make use of them before you formally learn them in your curriculum.

Question 2(b)

$$\begin{aligned}\int_a^b f(x)\ \mathrm{d}x &= \left[F(x)\right]_a^b \\ &= F(b) - F(a) \quad \text{where } F(x) \text{ is the integral of } f(x) \qquad (1)\end{aligned}$$

$$\int \frac{1}{Ax+B}\ \mathrm{d}x = \frac{1}{A}\ln(Ax+B) + \textit{Constant} \qquad (2)$$

Appendix B Other useful properties

Question 2(b)

$$e^{\ln x} = x$$

$$\ln x - \ln y = \ln\left(\frac{x}{y}\right)$$