Hwa Chong Institution

Singapore Junior Physics Olympiad 2018

Mechanics Diagnostic Test (Secondary 3)

120 marks

Time allocated: **3 hours**

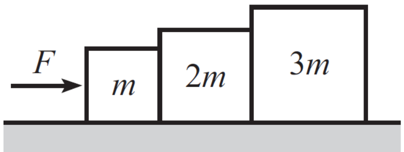
INSTRUCTIONS

1. This paper contains two sections and **12** printed pages.
   (a) **8** Short-Answer Questions (**16** marks)
   (b) **7** Structured Questions (**104** marks)
2. The mark for each question is indicated at the end of the question.
3. Answer all questions on foolscap paper.
4. Scientific calculators are allowed in this test.

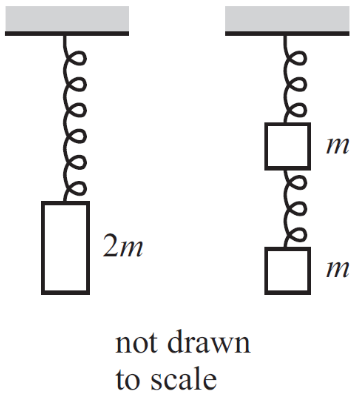
A Short-Answer Questions (16 marks)

1. An object on Earth is thrown vertically upwards with an initial velocity v. When it reaches its maximum height, the mass of Earth is suddenly halved. How long does it take to return to its original position? [2]

2. Three boxes are pushed with a force F across a frictionless table, as shown in the diagram below. Let N_1 be the normal force between the left two boxes, and let N_2 be the normal force between the right two boxes. Find N_1 and N_2 in terms of F. [2]

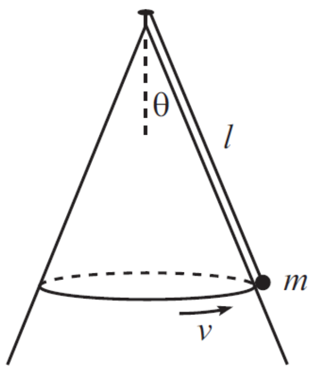


3. A mass $2m$ is suspended from a spring. The mass and the spring are then each cut into two identical pieces and connected as shown in the diagram below. Is the bottom of the lower mass higher than, lower than, or at the same height as the bottom of the original mass? Explain your answer clearly. [2]



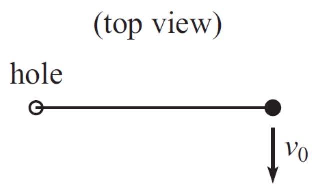
4. A paddle hits a ping-pong ball. The ping-pong ball moves with speed v rightwards towards the paddle. Assuming that the collision is elastic, and that the paddle is massive compared to the ping-pong ball, at what speed and direction should the paddle be moved so that the ping-pong ball becomes stationary after collision? Is it even possible? Explain your answer clearly. [2]

5. A mass is attached by a massless string to the tip of a cone, as shown in the diagram below. Friction exists between the mass and the cone. Suppose that the mass travels in a horizontal circle on the cone. Explain qualitatively how each of these forces enables the mass to travel in circular motion: weight, tension, normal force, and friction. [2]



6. A mass slides on a frictionless horizontal table.

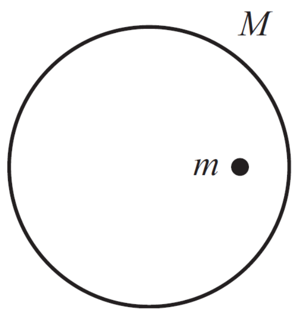
   (a) A string connected to the mass passes through a small hole in the table, and someone below the table holds the other end. The mass circles around the hole. The person below the table then steadily pulls the string downward, causing the mass to gradually spiral inward.



   (b) A string connects the mass to a pole, and it circles around the pole. The radius of the pole is small but nonzero, so as the string wraps around the pole, the mass gradually spirals inward.

Let E be the kinetic energy of the mass, and L be the angular momentum of the mass relative to the center of the hole or pole. During this process, state separately whether E and L are conserved for each scenario in (a) and (b). [2]

7. A mass m is located off-center in the interior of an object with mass M, as shown in the diagram below.

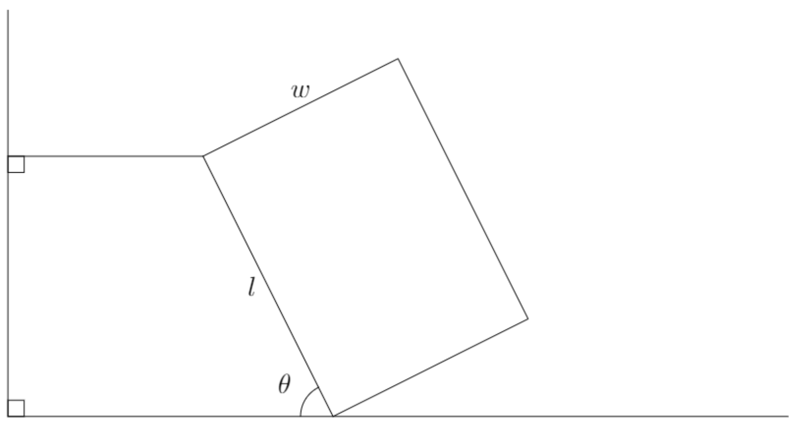


(a) Suppose that this mass M is a uniform ring.

(b) Suppose that this mass M is a uniform solid sphere.

What is the direction of the gravitational force on m due to M in both cases (a) and (b)? Explain your reasoning for both cases clearly. [2]

8. A uniform block of negligible thickness is tilted by an angle above the surface. Friction exists between the block and the surface. An ideal string connects the corner of the block to the wall.



Chris analyses this setup by considering the forces acting on the block. Taking the torque on the block about its center of mass, he finds that the tension, friction, and normal forces all rotate the block in the anti-clockwise direction. He then concludes that the setup can never be in static equilibrium. What's wrong with his reasoning? [2]

B Structured Questions (104 marks)

1. A uniform cube A of length l and density ρ_1 is placed into a large basin containing liquid of density ρ_2. At equilibrium, half of cube A floats above the surface of the liquid. [12]

   (a) State the relation between ρ_1 and ρ_2. [1]

   (b) An identical cube B is placed atop cube A. Predict and explain the height of the two cubes above the surface of the liquid at equilibrium. [1]

   (c) Cube B is then replaced with one of density $\rho_3 = 2\rho_1$ and equal dimensions. The height of the two cubes above the surface of the liquid, h, is recorded. Show that $h = l/2$. [3]

   (d) Cube B is then removed. After cube A has reached equilibrium, it is pushed upwards a distance $l/2$ and released. It begins oscillating vertically. Find:

      (i) the period of oscillations [5]

      (ii) the position of the top of the cube from the surface of the water as a function of time [2]

   State the assumptions you have made in your calculations.

2. An object is placed on flat, horizontal ground. It is launched at an angle θ above the horizontal with an initial velocity v. [12]

   (a) Consider a point on the ground at a horizontal distance x away from the object's starting position.

      (i) Given a fixed angle θ, derive an expression for v such that the object reaches the point. [3]

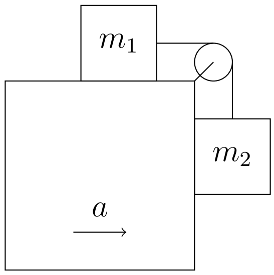
      (ii) For what value of angle θ is the value of v minimised? What is this corresponding value of v? [2]

   (b) Consider a point in the air is at a horizontal distance x and vertical distance y away from the object's starting position.

      (i) Given a fixed angle θ, derive an expression for v such that the object reaches the point. [5]

      (ii) For what value(s) of angle θ is this value of v defined? [2]

3. Two masses, m_1 and m_2, are connected by a pulley system on a massive block. [10]



(a) Suppose that all surfaces are frictionless. Show that, for the two masses to remain stationary relative to the block, the acceleration of the system, a, is equivalent to:

$$a = \frac{m_2}{m_1} g$$

[2]

(b) Suppose that all surfaces have coefficient of static friction μ_s and coefficient of kinetic friction μ_k. For the two masses to remain stationary relative to the block, the acceleration of the system, a, falls within the following range:

$$a_1 \leq a \leq a_2$$

where a_1 and a_2 are the minimum and maximum accelerations respectively that the system can have.

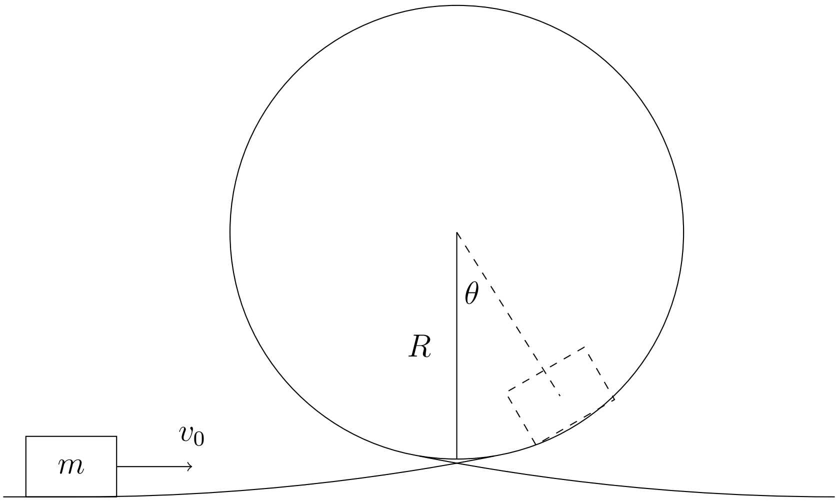
(i) Explain why a specific value of a is required in (a), whereas a range of values of a is acceptable for (b). [1]

(ii) Sketch four separate free-body diagrams for both m_1 and m_2 in the case where $a = a_1$ and $a = a_2$. [2]

(iii) Hence, derive expressions for a_1 and a_2 in terms of m_1, m_2, μ_s, μ_k, g. [4]

(iv) Verify your answer in (iii) by letting $\mu_s = \mu_k = 0$. [1]

4. A car of mass m and initial velocity v_0 goes through a frictionless circular loop of radius R in the vertical plane. [16]



(a) For the car to be able to complete one full loop, the following condition on v_0 is imposed: $v_0 \geq \sqrt{\alpha g R}$. Find the value of α. [3]

(b) Let θ be the angle of the car from the vertical through the center of the loop. Derive an expression for the velocity of the car as a function of θ, $v(\theta)$. [3]

(c) Derive an expression for the normal force acting on the car as a function of θ, $N(\theta)$. [4]

(d) In this part, consider the range $\sqrt{2gR} < v_0 < \sqrt{\alpha g R}$.

  (i) Briefly explain, qualitatively, the physical meaning of this range of values of v_0. [1]

  (ii) Let ϕ be the angle the car makes (with the vertical through the center of the loop) when it loses contact with the loop. Derive an expression for this angle ϕ. [3]

  (iii) Verify that this angle is only defined for $\sqrt{2gR} < v_0 < \sqrt{\alpha g R}$. [1]

  (iv) Hence, show that the velocity of the car when it loses contact with the loop is $\sqrt{\frac{1}{3}(v_0^2 - 2gR)}$. [1]

5. Consider two point particles of equal mass m. Throughout this question, consider only gravitational forces. [20]

   (a) In this part, initial conditions are set up such that the two particles are orbiting each other in the same circular orbit. Each particle has the same velocity v.

      (i) State the relationship between the radius of the orbit and the distance between the two particles. [1]

      (ii) Derive an expression for the radius of the orbit. [3]

      (iii) Hence, suggest a possible configuration for the initial conditions of the two particles in this situation. [1]

   (b) In this part, one of the particles is held stationary by an external force throughout the entire process. This particle will be referred to as the "stationary particle". The other particle will be named the "free particle". The free particle is initially fixed at a distance l away from the stationary particle. It is then released with a velocity v in the direction away from the stationary particle.

      (i) Describe, qualitatively, the possible subsequent motion(s) of the free particle. [1]

      (ii) Derive an expression for the largest distance reached between the free particle and the stationary particle throughout its motion. [3]

      (iii) For what value(s) of v is this largest distance infinite? What is the significance of this value? [2]

   (c) In this part, both particles may freely move. They are initially placed a distance l apart.

      (i) What is the time taken for the two particles to meet? [3] (Hint: make a suitable generalization to an important law involving planetary motion - no calculus required)

      (ii) One of the particles is released with a velocity v in the direction away from the other particle. The largest distance reached between the two particles throughout their motion, x, is recorded.

         A. Explain whether you expect the value of x to be larger than, smaller than, or equal to your answer in (b)(ii). [1]

         B. Confirm your prediction by deriving an expression for x. [5]

6. A solid uniform cylinder of mass M, radius R, and length L has a moment of inertia $I = MR^2/2$ about its central axis. A flat horizontal ground has a coefficient of static friction μ_s and coefficient of kinetic friction μ_k. [18]

   (a) Suppose that the cylinder initially slides across the ground with a linear velocity v_0 and no rotation.

      (i) Describe fully the subsequent motion of the cylinder as time passes, up till the moment when the cylinder is not slipping with respect to the ground. [1]

      (ii) Find the amount of time that passes before the moment when the cylinder is not slipping with respect to the ground. [3]

      (iii) Find the velocity of the center of the cylinder at the moment when the cylinder is not slipping with respect to the ground. [2]

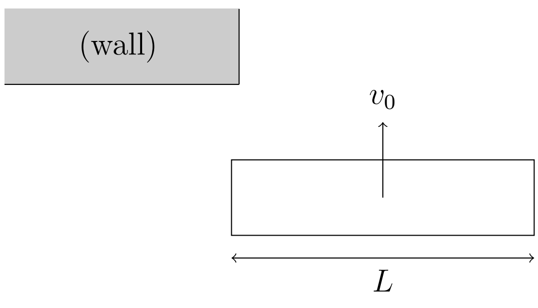
      (iv) Suggest, in theory and in practice, whether the cylinder will stay in motion indefinitely. [1]

   (b) Suppose that the ground is now vertically inclined by an angle θ. For what value(s) of θ can the cylinder achieve a state of motion with no slipping with respect to the ground? [5]

   (c) Consider the scenario where $\mu_s = \mu_k = 0$. The cylinder initially slides across the ground with a linear velocity v_0 and no rotation.

      (i) Explain why the cylinder travels at constant velocity as time passes. [1]

      (ii) Suppose that the cylinder collides elastically with a fixed wall on its end, as shown in the diagram below (top view).



      Derive expressions for the resulting motion of the cylinder. [5]
      Note: Moment of inertia of a cylinder about its central diameter $= MR^2/4 + ML^2/12$

7. This is a question on fluids. [16]

(a) An open water tap has a circular mouth of radius r_0. Water exits the mouth of the tap vertically at a velocity v_0, landing on a sink at a distance h below.

Show that the radius of the water stream at the instant before it lands on the sink is equal to:

$$r_0\sqrt{\frac{v_0}{\sqrt{v_0^2+2gh}}}$$

You may neglect the effects of surface tension, and assume that the flow of water is steady, continuous, and laminar. [3]

(b) A water fountain that shoots vertically upwards can deliver sufficient impulse to balance the weight of a heavy object, enabling it to levitate in mid-air, as shown in the picture below.

Mr Sim wishes to investigate this phenomenon. A circular hose shoots a stream of water of density ρ vertically upwards. Water leaves the hose at speed v_0 and the hose has a radius of r_0. A circular disc of negligible thickness has radius r (where $r > r_0$) and mass per unit area σ. Mr Sim wants to position the disc over the water stream such that it is able to hover steadily in place.

In your calculations, assume unrealistically, that when water crashes into the disc, it bounces off essentially sideways. Also assume that the flow of water is steady, continuous, and laminar.

(i) The disc is placed right above the mouth of the hose. Show that the value of v_0 required for the disc to hover in place is given by:

$$v_0 = \frac{r}{r_0}\sqrt{\frac{g\sigma}{\rho}}$$

[3]

(ii) However, since the radius of the water stream at this distance is very small, the disc is unable to maintain a stable hovering position.
As such, Mr Sim decided to place the disc at a distance h above the mouth of the hose, such that the water stream exactly covers the entire area of the disc. Show that this distance h is given by:

$$h = \frac{v_0^2}{2g}\left(1 - \frac{r_0^4}{r^4}\right)$$

[3]

(iii) Mr Sim uses the value of v_0 in (i), and places the disc at the height h above the mouth of the hose in (ii).

A. Explain why the disc is unable to hover steadily at this height. [1]

B. Instead, the disc travels in an unstable vertical motion. When it finally reaches equilibrium, it stays at a height h_0 above the mouth of the hose. What is this distance h_0? [1]

(iv) The situation in (iii) is not desired. Find a set of values of v_0 and h that Mr Sim should use simultaneously, such that the disc can steadily hover at the same height, with the water stream covering the entire area of the disc. [5]

End of Paper