SINGAPORE JUNIOR PHYSICS OLYMPIAD 2010
SPECIAL ROUND

4 September, 2010

9:30 am – 12:30 pm

Time Allowed: THREE HOURS

INSTRUCTIONS

1. This paper contains **12** structural questions and **7** printed pages.
2. The mark for each question is indicated at the end of the question.
3. Answer **ALL** the questions in the booklets provided. Answers for Questions 1 – 6 are to be written in the **green** booklets provided while answers to Questions 7 – 12 are to be written in the **orange** booklets.
4. Scientific calculators are allowed in this test.
5. A table of information is given in page 2.

TABLE OF INFORMATION

Acceleration due to gravity at Earth surface, g	$=$	$9.80\ \mathrm{m/s^2}$
Universal gas constant, R	$=$	$8.31\ \mathrm{J/(mol \cdot K)}$
Vacuum permittivity, ϵ_0	$=$	$8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$
Vacuum permeability, μ_0	$=$	$4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$
Speed of light in vacuum, c	$=$	$3.00 \times 10^{8}\ \mathrm{m/s}$
Speed of sound in air, v	$=$	$331\ \mathrm{m/s}$
Charge of electron, e	$=$	$1.60 \times 10^{-19}\ \mathrm{C}$
Planck's constant, h	$=$	$6.63 \times 10^{-34}\ \mathrm{J \cdot s}$
Mass of electron, m_e	$=$	$9.11 \times 10^{-31}\ \mathrm{kg}$
Mass of proton, m_p	$=$	$1.67 \times 10^{-27}\ \mathrm{kg}$
Boltzmann constant, k	$=$	$1.38 \times 10^{-23}\ \mathrm{J/K}$
Avogadro's number, N_A	$=$	$6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Rydberg constant, R_∞	$=$	$1.097 \times 10^{7}\ \mathrm{m^{-1}}$
Density of air, ρ_{air}	$=$	$1.20\ \mathrm{kg/m^3}$
Density of gaseous helium, ρ_{He}	$=$	$0.178\ \mathrm{kg/m^3}$
Standard atmosphere pressure	$=$	$1.01 \times 10^{5}\ \mathrm{Pa}$

Lorentz Transformation:

$$
\begin{aligned}
x' &= \frac{x - ut}{\sqrt{1 - u^2/c^2}} \\
y' &= y \\
z' &= z \\
t' &= \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}
\end{aligned}
$$

1. A party balloon (of mass 0.0025 kg when empty) is filled with helium to a volume of 0.0045 m^3. It is tied to a small stone of mass 0.015 kg by a light string of length 1.5 m to prevent it from flying away. A child holds the balloon at ground level and then releases it.

   (a) How long does it take for the balloon to rise 1.5 m, that is, for the string to become taut? [3]

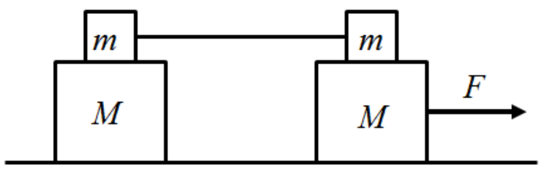
   (b) What is the velocity of the stone when it is lifted off the ground? You may assume that the time needed to bring the stone to this velocity is very short (impulse approximation) once the string is taut, and the string remains taut throughout. [3]

   (c) How long after lifting off will the stone touch the ground again? (If you think that the stone will never touch the ground again, explain why it is so.) [3]

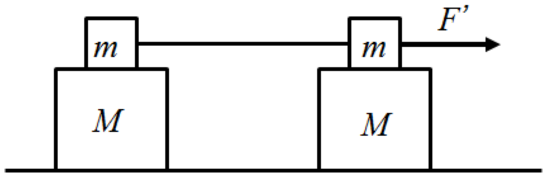
   You may neglect air resistance and treat the stone as a particle.

2. Four blocks are arranged on a smooth horizontal surface as shown. The masses of the blocks are given (see the diagram). A light rope connects the two top blocks. The maximum static friction that can exist between each top and the bottom blocks is f_s.

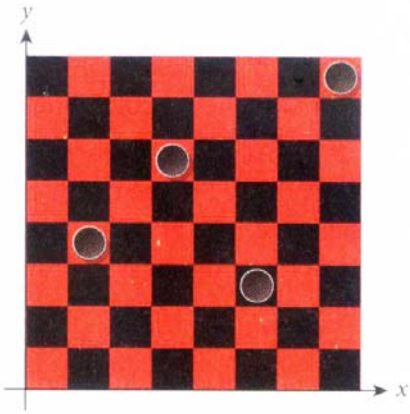
   (a) What is the maximum value of the horizontal force F, applied to one of the bottom blocks as shown below, that makes all four blocks move with the same acceleration? [5]



   (b) If the horizontal force F' is applied to the top block instead, as shown below, what is its maximum value to make all four blocks move with the same acceleration? [3]

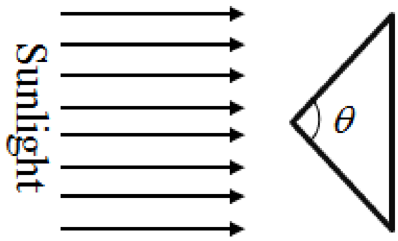


3. (a) A 32-cm-by-32-cm checkerboard has a mass of 100 g. There are four 20-g checkers located on the checkerboard as shown in the figure. Take the bottom left corner of the checkerboard to be the origin of our co-ordinate system (0, 0). What are the co-ordinates of the centre of mass of the checkerboard-checkers system? [4]



(b) Supposed all the checkers are removed from board, and a circular hole of radius $r = 6$ cm with centre at $(x, y) = (8, 8)$ is cut in the board. What are the co-ordinates of the centre of mass of checkerboard with the hole? [5]

4. A cone-shaped spaceship of the future uses the solar radiation pressure to propel itself away from the Sun. The axis of the cone points directly at the Sun as shown. The conical surface of the ship is painted black. The astronauts then try to increase their acceleration by covering the conical surface with a highly reflective material. To their dismay, the acceleration actually drops 30%. Find the angle θ. [6]



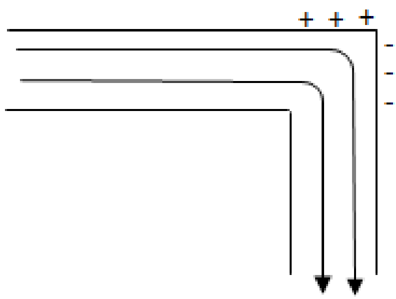
5. "Vibrato" in a violin is produced by sliding the finger back and forth along the vibrating string. The G-string on a particular violin measures 30 cm between the bridge and its far end and is clamped rigidly at both points. Its fundamental frequency is 197 Hz.

(a) How far from the end should the violinist place a finger so that the G-string plays the note A (440 Hz)? [5]

(b) If the violinist executes vibrato by moving the finger 0.50 cm to either side of the position in part (a), what is the resulting range of frequencies? [3]

6. A vertical cylindrical container contains some helium gas that is in thermodynamic equilibrium with the surroundings. The gas is confined by a movable heavy piston. The piston is slowly elevated a distance H from its equilibrium position and then kept in the elevated position long enough for the thermodynamic equilibrium to be reestablished. After that, the container is insulated and then the piston is released. After the piston comes to rest, what is the new equilibrium position of the piston? [8]

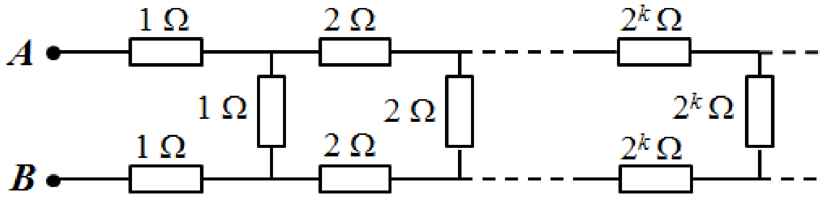
7. In order to lead an electric current around a right angle in a conducting wire, surplus charge is required on the outer walls of the wire as shown in the picture below. The field lines coming from the left will seem to terminate at the negative charges at the corner and field lines going down will seem to originate from the positive charges.



(a) Assuming that a 1.0 A current exists in a copper wire of 1.0 mm^2 uniform cross-section, calculate the electric field that exists in the wire. (Resistivity of copper is $1.7 \times 10^{-8}\ \Omega \cdot \mathrm{m}$.) [4]

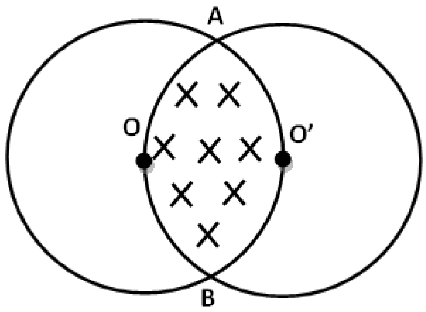
(b) Hence, make a good estimate of the amount of charge on each surface at the right angle turn as indicated. (Hint: the electric field E is related to the density of the charges on a surface by $E = \sigma/\epsilon_0$) [3]

8. Find the resistance between points A and B of an infinite circuit shown. The resistance of the resistors in each loop is twice those of the previous loop on its left.



[5]

9. The following figure shows two circular loops of insulated conducting wires, each of radius r, is placed in a configuration such that the wires intersect at points A and B. O and O' denote the respective centers of the wires. In the area of intersection, a uniform magnetic field pointing into the paper is applied. The magnetic field is dependent linearly on the time, i.e., $B = kt$ where k is a large positive constant.



(a) What is the direction and magnitude of the current in wire loop 1? [4]

(b) When $B = B_0$, what is the force on the loop 1? [3]

10. (a) A spaceship at rest in a certain reference frame S is given a speed increment of $0.50c$. Relative to its new frame, it is then given a further $0.50c$ increment. This process is continued until its speed with respect to its original frame S exceeds $0.999c$. How many increments does this process require? [5]

(b) The job of the Stanford Linear Accelerator Center (SLAC) in the United States is to accelerate electrons to very high energies, to crash these electrons into things and then to see what happens. The acceleration is accomplished with a 3.2 km long linear accelerator (linac) which is capable of accelerating electrons such that each electron will have energy of 50 GeV at the exit point of the accelerator.

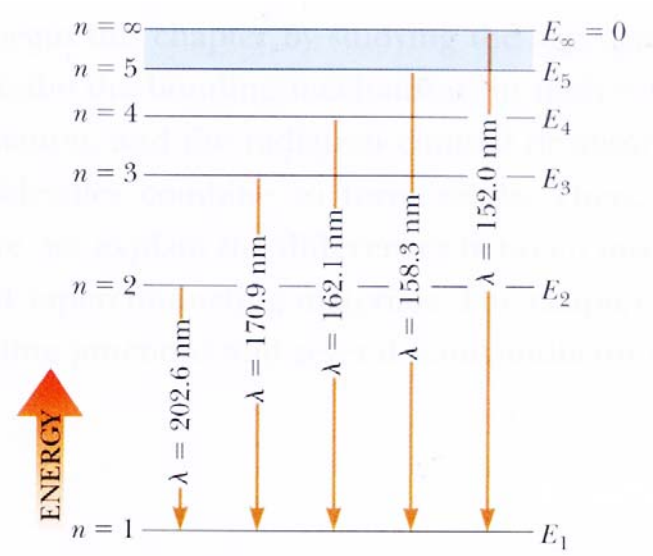
i. Assuming that the electrons accelerate constantly down the linac, what is the speed of the electron after going 1 m down the accelerator? [5]

ii. When the electrons exit the linear accelerator, magnets are used to curve the electron beams. Construction plans indicate that the radius of the curvature is 280 m. By considering the momentum of these now relativistic electrons, what is the minimum magnetic field required to curve the electron beams? [5]

11. (a) Liquid helium (atomic weight 4.003) has a density $\rho = 0.13$ g/cm^3. Estimate the radius of a helium atom, assuming that the atoms are packed in the densest possible configuration, which fills 74% of the space. [3]

(b) Canal rays, i.e., positive ion rays are generated in a gas discharge tube. How often does an ion ($r = 0.05$ nm) collide with an atom of the ideal filler gas ($r = 0.1$ nm) if it travels 1 m in a straight path through the discharge tube and if the pressure in the tube is 1 mbar? 1 mbar corresponds to 10^2 Pa. All the particles are assumed to have the same velocity. [4]

12. Astronomers observed a series of spectral lines in the light from a distant galaxy. On the hypothesis that the lines form the Lyman series for a (new?!) one-electron atom, they start to construct the energy-level diagram shown in the figure below, which gives the wavelengths of the first four lines and the short-wavelength limit of this series.



(a) Calculate the energies of the ground state and the first four excited states for this one-electron atom. [3]

(b) Calculate the wavelengths of the first three lines and the short-wavelength limit in the Balmer series for this atom. [2]

(c) Show that the wavelengths of the first four lines and the short-wavelength limit of the Lyman series for the hydrogen atom are all 60.0% of the wavelengths for the Lyman series in the one-electron atom described in part (b). [3]

(d) Based on this observation, explain clearly (with calculation if necessary) why this atom could be hydrogen. [3]

End of Paper