SINGAPORE JUNIOR PHYSICS OLYMPIAD 2015
SPECIAL ROUND

29 August, 2015

0900 – 1230

Time Allowed: $3\frac{1}{2}$ HOURS

INSTRUCTIONS

1. This paper contains **13** structural questions and **8** printed pages.
2. The mark for each question is indicated at the end of the question.
3. Answer **ALL** the questions in the booklet provided.
4. Scientific calculators are allowed in this test.
5. A table of information is given in page 2. Not all information will be used in this paper.

TABLE OF INFORMATION

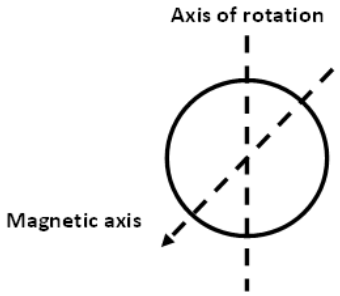
Acceleration due to gravity at Earth surface, g	=	$9.80\ \text{m/s}^2$
Universal gas constant, R	=	$8.31\ \text{J/(mol} \cdot \text{K)}$
Newton's gravitational constant, G	=	$6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$
Vacuum permittivity, ϵ_0	=	$8.85 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)$
Vacuum permeability, μ_0	=	$4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}$
Speed of light in vacuum, c	=	$3.00 \times 10^8\ \text{m/s}$
Speed of sound in air, v	=	$331\ \text{m/s}$
Charge of electron, e	=	$1.60 \times 10^{-19}\ \text{C}$
Planck's constant, h	=	$6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$
Mass of electron, m_e	=	$9.11 \times 10^{-31}\ \text{kg}$
Mass of proton, m_p	=	$1.67 \times 10^{-27}\ \text{kg}$
Boltzmann constant, k	=	$1.38 \times 10^{-23}\ \text{J/K}$
Avogadro's number, N_A	=	$6.02 \times 10^{23}\ \text{mol}^{-1}$

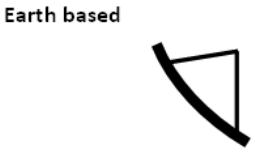
1. (a) Newton's Law of motion relates the rate of change of momentum of a particle to the force applied to it. The angular momentum L and the torque τ are related in a similar manner.

When nuclear reactions in the Sun cease for lack of fuel, the Sun will collapse inwards and the angular velocity of its spin will increase. If the Sun collapses till a certain critical radius, it will be spinning so fast that material will be lost from the Sun's surface at the equator. By considering the conservation of angular momentum, what is this critical radius and what is the corresponding period of rotation and density of solar matter when this happens?

The present radius and the period of rotation of the Sun are 7.0×10^8 m and 2.3×10^6 s respectively. The Sun's mass is 2.0×10^{30} kg.

(b) A neutron star is of mass $M = 3.0 \times 10^{30}$ kg and radius $R = 10$ km.





Pulsars are spinning neutron stars where the magnetic pole axis is misaligned with the axis of rotation by an angle as shown in the figure below. The strongest electromagnetic field emanates from the magnetic pole. A receiver on earth would then receive pulses of electromagnetic radiation (hence the name pulsars). In the above case, what is the lower bound on the value of the period of the pulses? [10]

2. Suppose that in rain or fog, we have V m^3 of water in a volume of 1 m^3 of air. Each water droplet in the rain or fog is approximated to be a sphere of L m in diameter.

   (a) What is the total area blocked off by the water droplets? Express your answer in the variables given and other relevant constants.

   (b) The depth of seeing may be defined at the distance traversed by light such that 90% of the viewing area is blocked off (10% visibility). If $V = 10^{-6}$ m^3, estimate the depth of seeing into

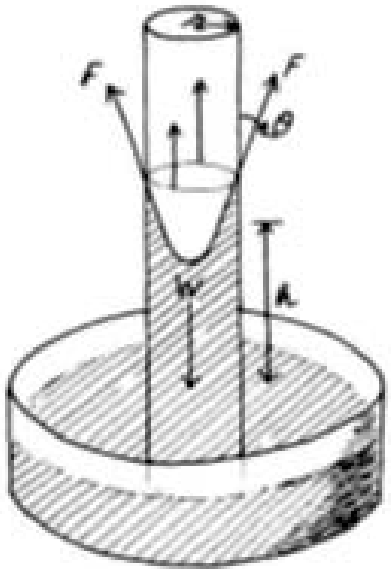
      i. the rain ($L = 10^{-3}$ m)

      ii. the fog ($L = 10^{-5}$ m)

      State any assumption(s) you have made in your estimation. *(Hint: Imagine looking through a linear stack of s one-meter cubes with total volume s m^3.)* [8]

3. A regular polygon has n sides of length d each. The vertices of the polygon are numbered consecutively: $1, 2, 3, \ldots, n$. An ant is placed at each vertex. At a moment, all ants begin to chase each other with equal and constant speed v in the following manner: The ant starting at vertex 1 will run towards the ant at vertex 2 who, in turn, chases the one starting at vertex 3 and so on. Finally, the ant starting at vertex n chases the one starting out at vertex 1. As the ants change positions, they continue to aim at each other as in the beginning of the chase. Eventually, all ants end up at the center of the polygon. How long does the chase last? [8]

4. Consider the diagram below where the meniscus is concave. The angle of contact between the liquid and the surface is θ and the surface tension is γ. The liquid with density ρ is held in a cylindrical tube with radius r and the height of the liquid held up by the capillary force is h. F is the force due to surface tension.



(a) By equating the vertical component of the force F with the weight of the column of liquid, derive an expression for the height of liquid column h in terms of the $\gamma, r, \theta, g, \rho$ and other numerical constants.

(b) In the case of water, $\theta = 25°$. The density of water is 1000 kg/m^3 and $\gamma_{\text{water}} = 0.0728$ N/m. If it is being held in a cylindrical capillary tube with a radius of 1 mm, what is the height of the water column due to capillary force?

(c) If the liquid is changed to mercury, the meniscus is convex and the contact angle between the surface of the mercury and the wall is 140°. The surface tension and the density of mercury are 0.47 N/m and 13.4×10^3 kg/m^3 respectively. If it is being held in the same capillary tube, what and how is the height changed due to capillary force? [8]

5. A cylinder with adiabatic walls is closed at both ends and is divided into two volumes by a frictionless piston that is also thermally insulating. The volume, pressure and temperature of the ideal gas in each side of the cylinder are initially equal at V_0, P_0 and T_0 respectively. The gas on the right is then slowly heated by a heating coil until the pressure reaches $64P_0/27$. If the heat capacity C_v of the gas is independent of temperature, and $C_p/C_v = 1.5$, find the following in terms of V_0, P_0 and T_0:

   (a) the entropy change of the gas on the left;

   (b) the final volume of the gas on the left;

   (c) the final temperature of the gas on the left;

   (d) the final temperature of the gas on the right;

   (e) the work done on the gas on the left. [12]

6. Tall chimneys that fell would usually rupture into two or more pieces in mid-air. The mechanism of a falling chimney can be modelled as a falling stick.

   (a) A rigid stick of mass m, length L is released from an angle θ measured from horizontal, and there is a small mass on the top of the stick initially. Find the maximum value of the angle θ such that the mass will lose contact with the stick at the moment of release.

   (b) Now the same stick is placed vertically on the table. The stationary stick started falling towards the table; at certain instant it forms an angle θ with the horizontal. Find the force on the stick at its base. You may assume that there is no slipping between the base of the stick and the table.

   (c) Explain why a falling chimney will rupture in midair. [10]

   (The moment of inertia of a stick of mass M and length L about one end is $\frac{1}{3}ML^2$.)

7. A small ball is bouncing down a staircase with step width and height both being L. The ball bounces once on each step at the same relative position, and after each bounce the ball reaches the same height H with respect to that step. The coefficient of restitution is e. Find the expressions for

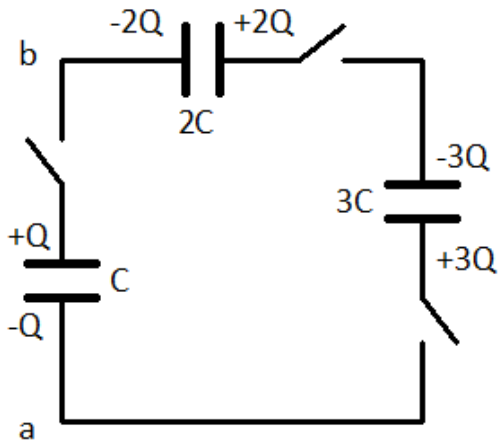
   (a) the height H and

   (b) the horizontal velocity of the ball v_x

   in terms of L, e and other constants. (Hint: The coefficient of restitution may be defined as ratio of the separation velocity over the approach velocity and is 1 for elastic collision, and 0 for completely inelastic collision.) [8]

8. A small sphere of mass m is attached to the bottom of a light spring with spring constant k. The sphere is placed on the table with the spring vertically upwards. The top of the spring was pulled upwards with a constant velocity v, until the spring reached its maximum extension. Find the work done by the pulling force during the process. [8]

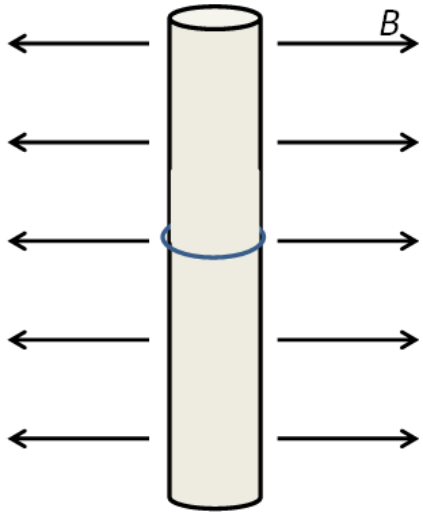
9. Three capacitors are charged initially as shown in the figure below. If $V = Q/C$, what is the potential difference across point a and b, $V_{ab} = V_a - V_b$ in terms of V after all the switches are closed?



[7]

10. Potassium naturally occurs in 3 isotopes: ^{39}K (93.258%), ^{40}K (0.012%), ^{41}K (6.730%). Radioactive isotope ^{40}K is used in radiometric dating called the potassium-argon dating. During ^{40}K decay, 89.1% of the nuclei are converted into ^{40}Ca (calcium-40) while the remaining 10.9% are converted into ^{40}Ar (argon-40). By measuring the ^{39}K and ^{40}Ar content in the surrounding rocks, the age of the largest known dinosaur, Argentinosaurus fossil is estimated to be around 96 million years old. In common practice, the ^{40}K content is rarely measured directly from the rock sample. Instead, it is inferred from the ^{39}K content by assuming the above isotopes ratio. Suppose 25 g of ^{39}K is measured and 19 μg of trapped ^{40}Ar are recovered from the rock sample, what is the half-life of ^{40}K (potassium-40)? [8]

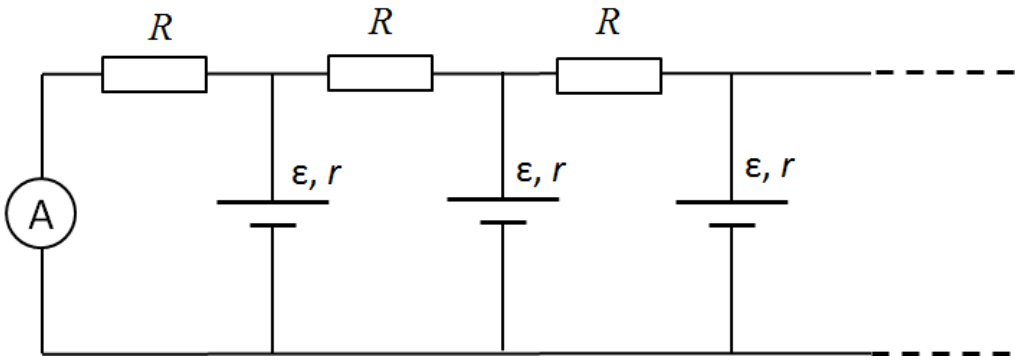
11. A very long rod magnet has a circular cross section with radius R from which a radially symmetrical magnetic field B emanates from the rod. A circular aluminum ring with radius approximated to R and originally at rest is allowed to drop freely, as shown in the diagram. The ring is made of aluminum wire that has a cross-section radius r_0. The density of the aluminum is D and the resistivity is ρ.



(a) What is the terminal velocity of the falling ring? Express your answer in terms of the above stated quantities and any other constants.

(b) Express the electrical power expended by the aluminum ring as the ring is dropping with the terminal velocity. [8]

12. The circuit shown in the figure below extends to the right into infinity. Each battery has the emf of $\epsilon = 1.5$ V and the internal reistance $r = 0.50\ \Omega$. Earch resistance in the circuit has a resistance $R = 2.0\ \Omega$. What would the ammeter connected in this circuit as shown below read?



[7]

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