SINGAPORE JUNIOR PHYSICS OLYMPIAD 2011
SPECIAL ROUND

3 September, 2011

2:00 – 5:00 pm

Time Allowed: THREE HOURS

INSTRUCTIONS

1. This paper contains **11** structural questions and **9** printed pages.
2. The mark for each question is indicated at the end of the question.
3. Answer **ALL** the questions in the booklets provided. Answers for **Questions 1 – 5** are to be written in the **green** booklets provided while answers to **Questions 6 – 11** are to be written in the **yellow** booklets.
4. For Question 5, use the loose sheet provided with the relevant figures and attach it to the green booklet.
5. Graph papers are provided for Question 11. You may use up to two graph papers and attach them to the yellow booklet.
6. Scientific calculators are allowed in this test.
7. A table of information is given in page 2. Not all information will be used in this paper.

TABLE OF INFORMATION

Acceleration due to gravity at Earth surface, g	$=$	$9.80\ \mathrm{m/s^2}$
Universal gas constant, R	$=$	$8.31\ \mathrm{J/(mol \cdot K)}$
Newton's gravitational constant, G	$=$	$6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
Vacuum permittivity, ϵ_0	$=$	$8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$
Vacuum permeability, μ_0	$=$	$4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$
Speed of light in vacuum, c	$=$	$3.00 \times 10^{8}\ \mathrm{m/s}$
Speed of sound in air, v	$=$	$331\ \mathrm{m/s}$
Charge of electron, e	$=$	$1.60 \times 10^{-19}\ \mathrm{C}$
Planck's constant, h	$=$	$6.63 \times 10^{-34}\ \mathrm{J \cdot s}$
Mass of electron, m_e	$=$	$9.11 \times 10^{-31}\ \mathrm{kg}$
Mass of proton, m_p	$=$	$1.67 \times 10^{-27}\ \mathrm{kg}$
Boltzmann constant, k	$=$	$1.38 \times 10^{-23}\ \mathrm{J/K}$
Avogadro's number, N_A	$=$	$6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Density of water, ρ_{water}	$=$	$1000\ \mathrm{kg/m^3}$
Standard atmosphere pressure	$=$	$1.01 \times 10^{5}\ \mathrm{Pa}$

1. A body of mass 6.0 kg and density 450 kg/m^3 is dropped from rest at a height 7.5 m into a lake. Calculate

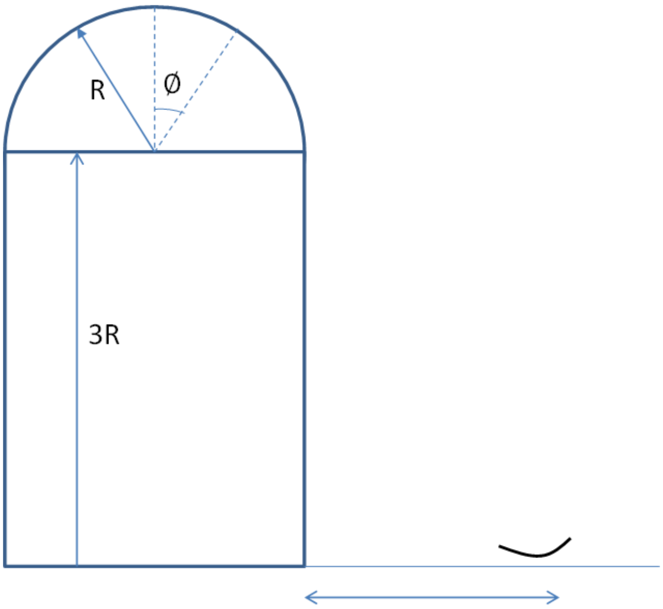
   (a) the speed of the body just before entering the lake,

   (b) the acceleration of the body while it is in the lake, and

   (c) the maximum depth to which the body sinks before returning to float on the surface.

   You may neglect the air resistance and the surface tension and viscous force of the water. [6]

2. A daredevil astronomer stands at the top of his observatory dome wearing roller skates and starts with negligible velocity to coast down over the dome surface.

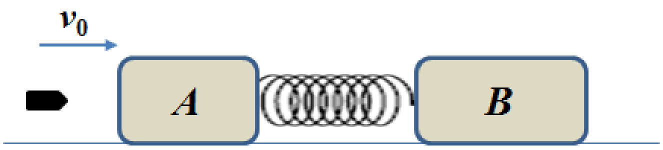


   (a) Neglecting friction, at what angle ϕ does he leave the dome's surface?

   (b) If he were to start with an initial velocity v_0, at what angle ϕ would he leave the dome?

   (c) For the observatory shown above, how far from the base should his assistant position a net to break his fall, as in situation (a)? Evaluate your answer for $R = 8.0$ m. [12]

3. Two wooden blocks A and B, connected by an unstretched spring with a spring constant $k = 950$ N/m, are initially at rest on a frictionless surface. A bullet of mass 50 g moving horizontally with a initial speed of $v_0 = 120$ m/s hits Block A and becomes embedded in it. The embedding takes place within a very short time. The mass of Block A is 1.2 kg and that of Block B is 2.0 kg.



Calculate

(a) the maximum compression (Δx_{max}) of the spring.

(b) the maximum and minimum speeds of Block B in its subsequent motion. [8]

4. 2.00 moles of gas is held in a cylinder with a piston and is initially held at 0.300 atm and has an initial volume of 0.200 m^3. The molar heat capacity of the gas at constant volume is 24.94 J mol^{-1}K^{-1}. The gas is then brought from this initial state (State A) through the following processes:

From state A to B: Gas is allowed to expand isothermally.

From state B to C: The temperature of the gas drops by 100 K while it is being held at constant volume.

From state C to A: The volume of the gas is then compressed in an adiabatic process back to its initial state.

(a) What is the initial temperature of the gas in state A?

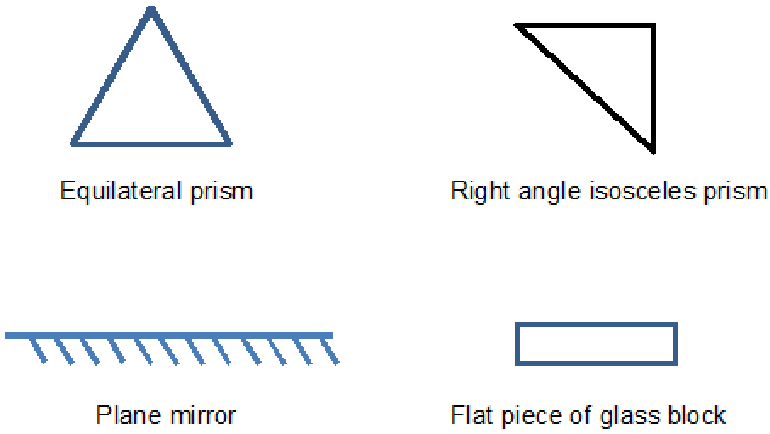
(b) What is the ratio of the molar heat capacity at constant pressure (C_P) to the molar heat capacity at constant volume (C_V) of the gas?

(c) What is the volume of the gas at state C? Hence, sketch a $P - V$ curve depicting the processes, indicating the pressure and volume at each point.

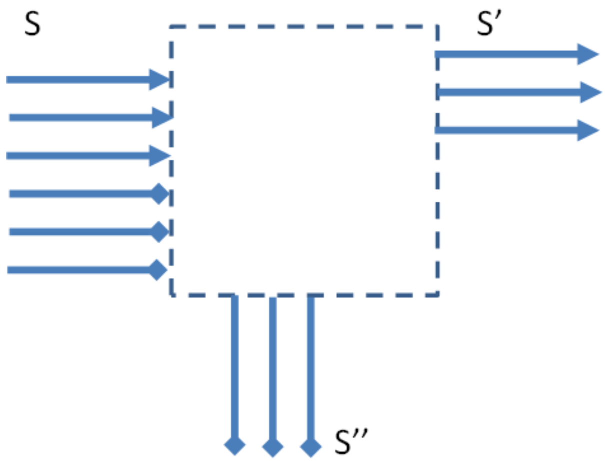
(d) In which of the processes is heat being transferred to the system and in which process is the heat being expelled from the system? Hence, calculate the net work done by the system.

(e) Assume that process B to C is instead stated as "The temperature of the gas rises by 100 K while it is being held at constant volume." Is it possible then to return the gas to its initial state via an adiabatic process? Why or why not? [15]

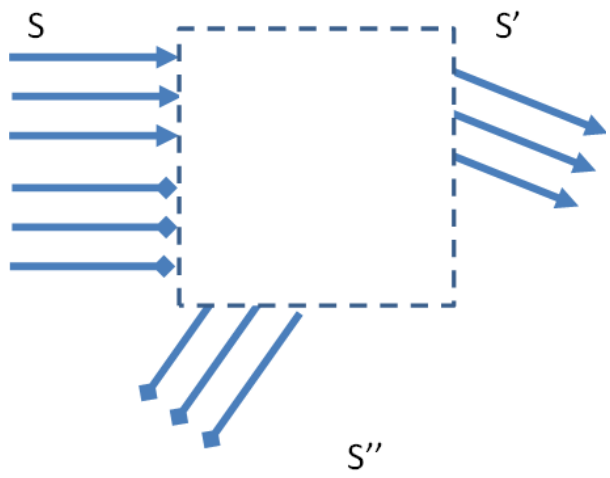
5. You are given four optical elements as indicated below.



Draw the types and positions of the above optical elements that can be placed in the dashed boxes in (a) and (b) so that S indicates the incident rays and S′ and S″ indicate the outgoing rays. Rays with single arrowheads and diamond arrowheads indicate the respective matching incident and outgoing rays. Note: all glass elements are made of glass of refractive index $n = 1.6$ and each optical element is to be used only once. [8]

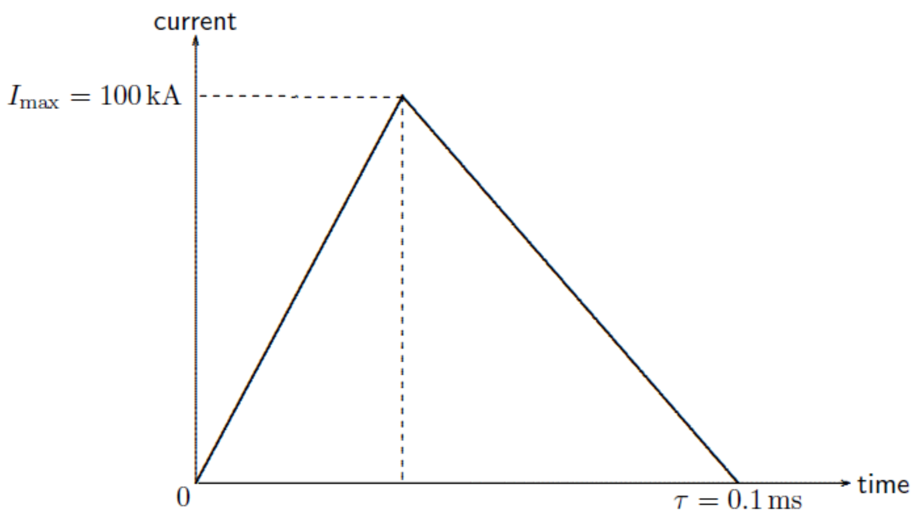


(a)



(b)

6. An oversimplified model of lightning is as follows. Lightning is caused by the build-up of electrostatic charge in clouds. As a consequence, the bottom of the cloud usually gets positively charged and the top gets negatively charged, and the ground below the cloud gets negatively charged. When the corresponding electric field exceeds the breakdown strength value of air, a disruptive discharge occurs: this is lightning.



Distance between the bottom of the cloud and the ground: $h = 1$ km;
Breakdown electric field of humid air: $= 300$ kV m^{-1};
Total number of lightning striking Earth per year: 32×10^6;
Total human population: 6.5×10^9 people.

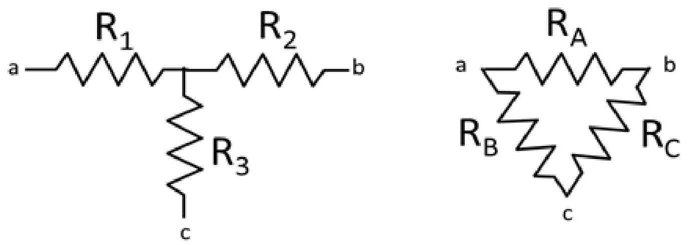
Using the information given in the graph and statements above, answer the following questions.

(a) What is the total charge Q released by the lightning?

(b) What is the average current I flowing between the bottom of the cloud and the ground during lightning?

(c) Imagine that the energy of all the storms of one year is collected and shared equally among all people. For how long could you continuously light up a 100 W light bulb for your share? [6]

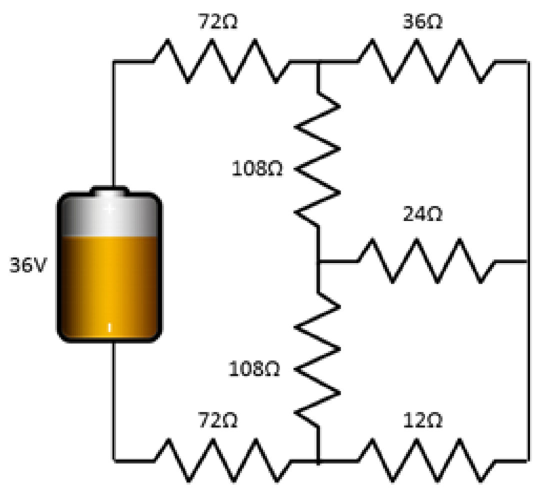
7. It can be shown that as far as any effect on the external circuit is concerned, a T-network is equivalent to what is called a delta network. For example, the T network on the left can be replaced by the delta circuit on the right.



(a) Prove that

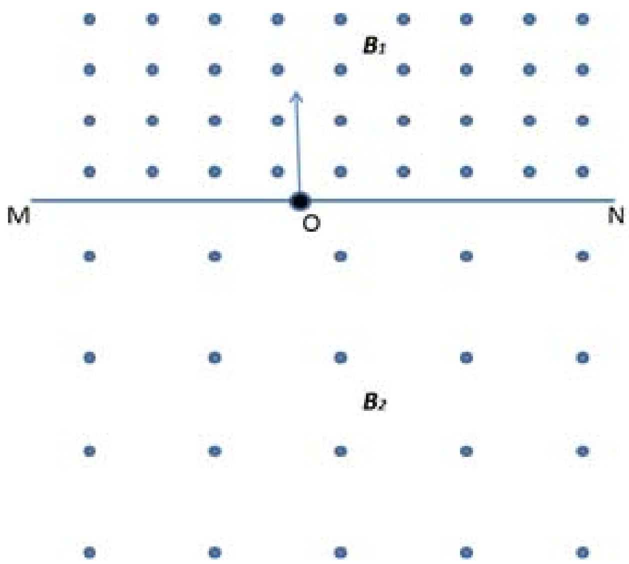
$$\begin{aligned} R_1 &= \frac{R_A R_B}{R_A + R_B + R_C} \\ R_2 &= \frac{R_A R_C}{R_A + R_B + R_C} \\ R_3 &= \frac{R_B R_C}{R_A + R_B + R_C} \end{aligned}$$

(b) Hence or otherwise, compute the current that passing through the 72 Ω resistors in the circuit below



[12]

8. As shown in the diagram below, 2 uniform magnetic fields of strengths B_1 and $B_2(= 0.5B_1)$ are divided by the line MN. Two charged particles A and B, each with the same charge $+q$ and mass m, but with different speeds, v_A and v_B respectively, are ejected into the magnetic field of strength B_1 from point O in a direction perpendicular to MN.



   (a) Sketch the paths taken by the two particles if $v_A < v_B$. Label your diagram appropriately.

   (b) Will the two particles reach O again? If so, how long does it takes for each particle to reach O again for the first time? If not, calculate the time it takes for each particle to get to the point closest to O. Simplify your answers as much as possible. [8]

9. Fundamental quanta of length and time are the Planck length l_P and the Planck time t_P. Using the fact that they depend only on the Newton's gravitational constant G, Planck's constant h, and speed of light in vacuum c and no other constant, derive expressions for them and then calculate their numerical values.

   Hint: Every quantity of an equation related by "=", "+" or "-" must have the same units. For example, in the kinematic equation for constant acceleration $v^2 = u^2 + 2as$, the quantities v^2, u^2 and $2as$ have the common units of $\mathrm{m^2s^{-2}}$. You may use the same idea to find the expressions for l_P and t_P. [6]

10. In 1972, atomic clocks were put on flight and flown around the Earth. They were then compared with the atomic clock which stayed fixed on Earth to verify the effects of relativity. We consider here a simplified version of the experiment. A clock is flown once around the circumference of the Earth (approximately 4.0×10^7 m) and the typical airline plane speed is about 300 m/s.

   (a) What is the flight time T_0 as measured by the flown clock? Would the clock on the ground measure a longer time or shorter time for the flight time as compared to the flown clock according to the theory of special relativity?

   (b) Show that the time difference between the two clocks can be given by $\frac{1}{2}\beta^2 T_0$ where $\beta = v/c$. Show your steps clearly.

   (c) Hence, to what precision (fractional or percentage uncertainty) must the atomic clock be able to measure in order to discern the time difference between the flown clock and the ground clock?

   (d) The time difference in (c) differs from the actual measured difference in the 1972 experiment. Suggest a reason for this discrepancy. [8]

11. Two radioactive elements are found in a specimen. It is known that element A in the specimen has a half-life less than 30 minutes while element B has a half-life greater than 1 hour. The results of activity measurements on this specimen over a period of time are given in the table.

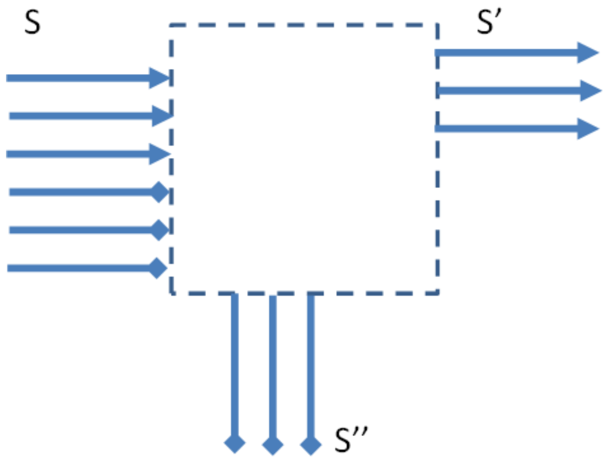
Time (h)	**Decays/s**
0.0	21500
0.5	11760
1.0	7620
1.5	5530
2.0	4260
2.5	3370
3.0	2710
4.0	1770
5.0	1160
6.0	765
7.0	501
8.0	330
9.0	217
10.0	142
12.0	61

Determine the half-lives of element A and element B as accurate as you can from the data in the above table. You may use a graphical method or otherwise. [11]

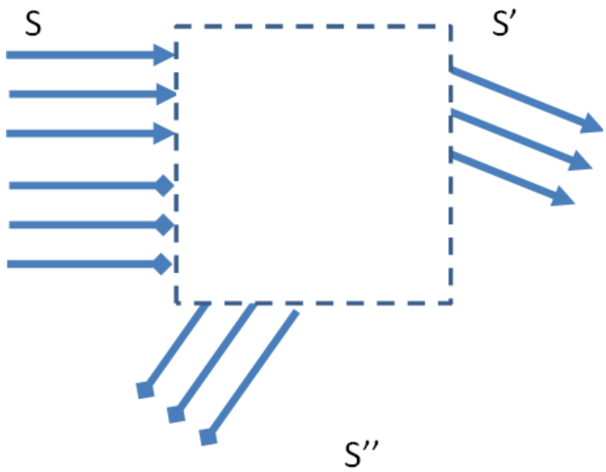
— End of Paper —

Name: ___________________ School: ___________________ Serial No: ______

For Question 5 only



(a)



(b)

Please attach to the Green Booklet