SJPO Training 2018
– Summary of Topics –

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The following summary is based on the SJPO 2018 Syllabus. Not all bullet points in the syllabus have been elaborated upon. The complete syllabus has been uploaded onto Google Drive.

Good luck
Don't Panic

Contents

1   Mechanics of Particles

1.1   **Kinematics** – position, displacement, velocity, acceleration, vectors

Position   Position is described by displacement vector $\vec{x}$ from a fixed point, origin

Velocity $\vec{v} = \dfrac{\mathrm{d}\vec{x}}{\mathrm{d}t}$ Acceleration $\vec{a} = \dfrac{\mathrm{d}\vec{v}}{\mathrm{d}t}$

1.1.1   Motion of point mass

When considering translational motion, we can consider all forces acting on an object to be acting through its centre of mass.

1.1.2   Motion in 1-dimension

These equations are derived by integrating a constant acceleration, a, using the limits where $t_i = 0$, $t_f = t$, $v_i = u$, $v_f = v$. Clearly define the coordinate system ie which direction is positive before use.

$$s = ut + \frac{1}{2}at^2 \qquad\qquad v^2 = u^2 + 2as \qquad\qquad v = u + at$$

1.1.3   Motion in 2-dimension

Motion in more than one dimension can be uniquely separated into its dimensions as long as they are orthogonal (perpendicular) to one another. Each dimension has its own set of independent kinematic equations.

1.1.4   Vector description of motion of a point mass

The motion of a point mass can be uniquely defined when its position and momentum (velocity) is known.

1.1.5   Relative velocity

In an inertial frame travelling at $\vec{v}_1$ relative to a rest frame, the velocity of an object travelling at $\vec{v}_2$ in the rest frame appears to be

$$\vec{v}_{\text{apparent}} = \vec{v}_2 - \vec{v}_1$$

1.1.6 Motion with constant acceleration *(e.g. free fall, projectile motion)* or with variable acceleration *(e.g. car on straight road)*

Motion with constant acceleration is parabolic/quadratic.

Consider graphical methods when the velocity of an object changes multiple times in a question.

A general rule-of-thumb of integration is that any **polynomial** function, when integrated, will have its power increased by 1. $\left(\text{e.g. } x^2 \to x^3\right)$.[1] Similarly, any **polynomial** function, when differentiated, will have its power decreased by 1. $\left(\text{e.g. } x^3 \to x^2\right)$.

1.1.7 Motion in a circle *(involving centripetal acceleration)*

For an object to undergo circular motion, the net force acting on the object must be

$$\vec{\boldsymbol{F}}_{\text{net}} = m\vec{\boldsymbol{a}}_{\text{centripetal}} = m\frac{v^2}{r} = mr\omega^2$$

in the radial direction.

Note: Centripetal force isn't a physical force, rather it is a condition for the net force acting on an object that is undergoing circular motion.

1.2 Dynamics – inertia, momentum, impulse, forces, energy

Inertia	A property of matter by which it continues in its existing state of rest or uniform motion in a straight line, unless that state is changed by an external force. Related to the mass of an object
Impulse	$\vec{\boldsymbol{J}} = \Delta\vec{\boldsymbol{P}} = \vec{\boldsymbol{F}}_{\text{avg}}\Delta t = m\Delta\vec{\boldsymbol{v}}$
Forces	$\vec{\boldsymbol{F}} = \dfrac{\mathrm{d}\vec{\boldsymbol{p}}}{\mathrm{d}t} = m\,\dfrac{\mathrm{d}\vec{\boldsymbol{v}}}{\mathrm{d}t}$ (for constant mass) aka Newton's second law
	$\vec{\boldsymbol{F}}_{\text{avg}} = m\vec{\boldsymbol{a}}_{\text{avg}} = m\dfrac{\Delta v}{\Delta t}$

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[1]Note that this does not apply to $\int \frac{1}{x} = \ln x + C$

1.2.1   Newton's laws of motion/Application of Newton's laws

Newton's first law states that every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force. This is normally taken as the definition of inertia. The key point here is that if there is no net force acting on an object (if all the external forces cancel each other out) then the object will maintain a constant velocity. If that velocity is zero, then the object remains at rest. If an external force is applied, the velocity will change because of the force.

The second law explains how the velocity of an object changes when it is subjected to an external force. The law defines a force to be equal to change in momentum (mass $\times$ velocity) per change in time. Newton also developed the calculus of mathematics, and the "changes" expressed in the second law are most accurately defined in differential forms. (Calculus can also be used to determine the velocity and location variations experienced by an object subjected to an external force.) For an object with a constant mass m, the second law states that the force $\vec{F}$ is the product of an object's mass and its acceleration $\vec{a}$:

$$\vec{F} = m\vec{a}$$

For an external applied force, the change in velocity depends on the mass of the object. A force will cause a change in velocity; and likewise, a change in velocity will generate a force. The equation works both ways.

The third law states that for every action (force) in nature there is an equal and opposite reaction. In other words, if object A exerts a force on object B, then object B also exerts an equal force on object A. Notice that the forces are exerted on different objects. The third law can be used to explain the generation of lift by a wing and the production of thrust by a **jet engine**.

https://www.grc.nasa.gov/www/k-12/airplane/newton.html

1.2.2   Motion affected by dissipative forces *(e.g. friction, fluid resistance)*

Kinetic friction is constant and acts in the opposite direction of motion $F_{\text{kinetic}} = \mu_k N$.

Other kinds of resistance are proportional to the v or v^2 of the object and act in the opposing direction. The resultant motion is exponential decay for the former while the latter is similar in shape but has a sharper decrease.

1.2.3   Elastic, friction, normal, gravity including Newton's law of gravitation, electric and magnetic

Elastic Force	$F_{\text{elastic}} = -kx$
Friction	$f_{\text{static}} \leq \mu_s N$
	$f_{\text{kinetic}} = \mu_k N$
Normal Contact Force	Normal force occurs between two surfaces and is perpendicular to the surface
Gravity	$F_g = mg$ near surface of Earth
	$F = \dfrac{GMm}{r^2}$ towards each other
Coloumb's law	$\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 q_2}{r^2}$
Force by an electric field	$\vec{F_E} = q\vec{E}$
Magnetic Force	$\vec{F_B} = q\vec{v} \times \vec{B}$

1.3   Conservation of energy, conversion of energy

Energy is always conserved in a closed system in an inertial frame. Energy can be converted to other forms by cannot be destroyed.

1.3.1 Work Done, Power, Potential Energy (PE), Kinetic Energy (KE)

Work Done	$W = \vec{F} \cdot \vec{d}$ Work done by an object through a force is equal to the product of the magnitude of the force acting parallel to the direction of motion.
Power	$P = \dfrac{\mathrm{d}W}{\mathrm{d}t}$
Elastic PE	$\mathrm{EPE} = \frac{1}{2}kx^2$
Work done against Friction	$W_f = \vec{f} \cdot \vec{d}$
Gravitational PE	$\mathrm{GPE} = mgh$ (for small values of h at the surface of earth) $\mathrm{GPE} = -\dfrac{GMm}{r}$ (in general)
Electric PE	$\mathrm{EPE} = q\Delta V = q \displaystyle\int_{r_0}^{r_1} e\,\mathrm{d}r$ $\mathrm{EPE} = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 q_2}{r}$ for two particles in free space
Kinetic Energy	$\mathrm{KE} = \frac{1}{2}mv^2$

1.4 Impulse and Momentum / Conservation of Linear Momentum

Momentum $\vec{p} = m\vec{v}$ is conserved in any closed system due to newton's third law.

1.4.1 Collisions/explosions

There are two types of collisions: Inelastic and elastic. For elastic collisions, both energy and momentum is conserved. This leads to the condition that the velocity of approach must be equal and in opposite direction to the receding velocity:

$$u_1 - u_2 = -(v_1 - v_2)$$

1.4.2 Coefficient of Restitution

e is the ratio of the final to initial relative velocity between two objects after they collide, usually a positive, real number between 0 and 1:

$e = 0$	This is a perfectly inelastic collision. The objects do not move apart after the collision, but instead they coalesce. Kinetic energy is converted to heat or work done in deforming the objects.
$0 < e < 1$	This is a real-world inelastic collision, in which some kinetic energy is dissipated.
$e = 1$	This is a perfectly elastic collision, in which no kinetic energy is dissipated, and the objects rebound from one another with the same relative speed with which they approached.

1.4.3 External and internal forces for a system of particles, centre of mass

Internal forces cannot be detected from outside the system. They do not cause a change in total momentum or energy of a system. The resultant force of an internal force acts on another object in the system.

External forces causes a change in the total momentum and energy of a system and its resultant force acts on an object outside the system.

COM is the weighted average of the displacement of the masses

$$x_{\text{COM}} = \frac{1}{M} \sum_i m_i r_i \text{ where } M = \sum_i m_i$$

For continuous distribution:

$$x_{\text{COM}} = \frac{1}{M} \int r \, \mathrm{d}m$$

2   Mechanics of Rigid Bodies

$\boxed{2.1}$ **Statics** – equilibrium, stability

Static equilibrium	is attained when the net forces acting on an object is zero.
Rotational equilibrium	is attained when the net torque acting on an object is zero.
Stable equilibrium	is attained when a small disturbance from its position is met with a restoring force.
Dynamic equilibrium	is attained when a small disturbance from its position causes a net force acting in the same direction.
Neutral equilibrium	is attained when the system experiences not net force after a small disturbance.

$\boxed{2.2}$ **Kinematics** – angular position, angular displacement, angular velocity, angular acceleration, vector products

θ	Angular displacement relative to an angle
Angular Velocity	$\vec{\omega} = \dfrac{\mathrm{d}\vec{\theta}}{\mathrm{d}t}$
Angular acceleration	$\vec{\alpha} = \dfrac{\mathrm{d}\vec{\omega}}{\mathrm{d}t}$

The direction of angular displacement/velocity/acceleration is given by the right-hand-rule.

2.2.1   Rotation with constant angular acceleration

Akin to kinematic equations:

Moment of Inertia	$I = \displaystyle\int r^2 \, \mathrm{d}m$
Displacement	$\theta = \omega t + \dfrac{1}{2}\alpha t^2$
Velocity	$\omega_f^2 = \omega_i^2 + 2\alpha\theta$
$\vdots$	$\vdots$

2.2.2   Relationship between linear and angular quantities

$$\vec{v} = \vec{r} \times \vec{\omega}$$

$$\vec{a} = \vec{r} \times \vec{\alpha}$$

2.3   Dynamics – moment of inertia, torque

$$\vec{\tau} = I\vec{\alpha} = \vec{r} \times \vec{F}$$

2.3.1   Angular momentum

$$\vec{L} = \vec{r} \times \vec{p} = mvr \sin \theta \, \hat{\mathbf{n}}$$

Angular motion is conserved in a closed system. It remains constant if no torque is acting upon the body.

For circular motion, $L = mvr$.

2.3.2   Rotational KE

$$\text{Rotational KE} = \frac{1}{2}I\omega^2$$

3   Fluid Mechanics

3.1 **Fluid statics** – Density, pressure, buoyancy, surface tension

Pressure	$P = \dfrac{F}{A} = \rho g h$ in a fluid where $\rho = \dfrac{M}{V}$
Buoyancy Force	Buoyancy force is equal to the weight of fluid displaced caused due to difference of pressure between the top and body of an object.
Surface tension	$\sigma = \gamma l$ where γ is a constant based on the property of the fluid and l is the length of surface exposed to the liquid. For a film, the total surface tension is double since there are two surfaces.

3.1.1   Pascal's law, Archimedes' principle

Pascal's Law	Pascal's law (also Pascal's principle or the principle of transmission of fluid-pressure) is a principle in fluid mechanics that states that a pressure change occurring anywhere in a confined incompressible fluid is transmitted throughout the fluid such that the same change occurs everywhere. (i.e the pressure of a fluid at a height is always equal.)
Archimedes' principle	Archimedes' principle states that the upward buoyant force that is exerted on a body immersed in a fluid, whether fully or partially submerged, is equal to the weight of the fluid that the body displaces and acts in the upward direction at the center of mass of the displaced fluid.

$$F_u = \rho V_{\text{displaced}} g = \rho V g$$

3.2 Fluid Dynamics

3.2.1 Bernoulli's principle

Bernoulli's principle states that an increase in the speed of a fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy.

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{constant}$$

where:

$v =$ Fluid flow speed at a point on a streamline

$g =$ Acceleration due to gravity

$z =$ Elevation of the point above a reference plane, with the positive z-direction pointing upward – so in the direction opposite to the gravitational acceleration

$p =$ Pressure at the chosen point

$\rho =$ Density of the fluid at all points in the fluid

The following assumptions must be met for this Bernoulli equation to apply:

- the flow must be steady, i.e. the flow parameters (velocity, density, etc...) at any point cannot change with time

- the flow must be incompressible – even though pressure varies, the density must remain constant along a streamline

- friction by viscous forces must be negligible

4   Oscillations and Waves

4.1   Simple Harmonic Motion

The general form of a SHM is as follows:

$$A = A_0 \sin(\omega t + \phi)$$

Solution of the SHM equation:

$$\vec{a} = -k\vec{x} \qquad \text{where } k = \omega^2 = \left(\frac{2\pi}{T}\right)^2$$

4.2   Mechanical, Electrical

4.2.1   Frequency, Period, Phase Difference

Frequency f	Number of oscillations that occur per second
Period T	Time taken for an oscillation to occur, the inverse of frequency
Phase Difference $\Delta\phi$	The difference in the phase angle between two oscillating systems ie how much one system is leading another
	If they are in phase, the phase difference is equal to 2π and the two systems reach their maxima and minima at the same time.
	If they are out of phase, the phase difference is π and the maxima of one system occurs at the minima of another system.

4.2.2   Qualitative understanding of damping and resonance

Damping occurs when a force opposes a system's change, it is modelled as proportional to the system's rate of change i.e drag force. There are three types of damping:
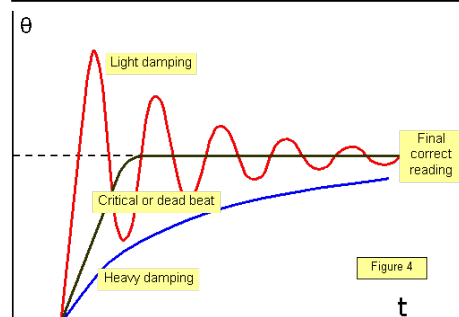
Lightly Damped
Still sinusoidal but the amplitude is contained in an exponential decay envelope.

Overdamped
Never passes the origin, tends towards equilibrium

Critically Damped
May pass through the origin at most once. Reaches the origin in the fastest possible time.

4.3 Waves: Mechanical *(Sound, String, Fluid)*, Electromagnetic

For a wave traveling in the positive x, $k > 0$, it has a general equation:

$$y = A \sin(\omega t - kx + \phi)$$

Attenuation	The loss of energy of a wave over a distance.
Wavelength λ	The distance of space over which a wave repeats
Wave Speed v	The speed at which a wave propagates. $v = f\lambda$
Amplitude A, Intensity I	$I \propto A^2$
Transverse Waves	Transverse waves oscillate in the plane **perpendicular** to the direction of propagation.
Longitudinal Waves	Longitudinal waves oscillate **along** the direction of propagation.

4.3.1 Polarization, Malus' law

Polarized light occurs when the vibrations only occur along a single plane.

Malus' law states that the intensity of a beam of plane-polarized light after passing through a rotatable polarizer varies as the square of the cosine of the angle through which the polarizer is rotated from the position that gives maximum intensity:

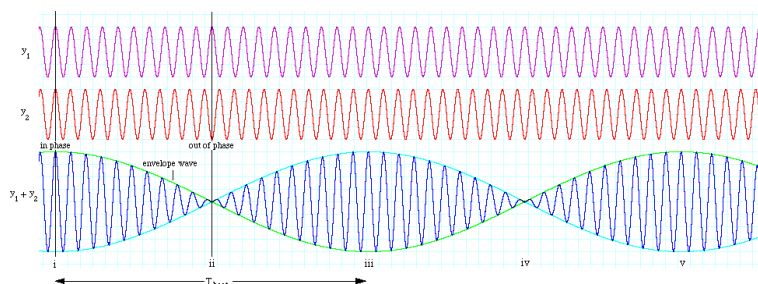$$I = I_0 \cos^2 \theta \iff A = A_0 \cos \theta$$
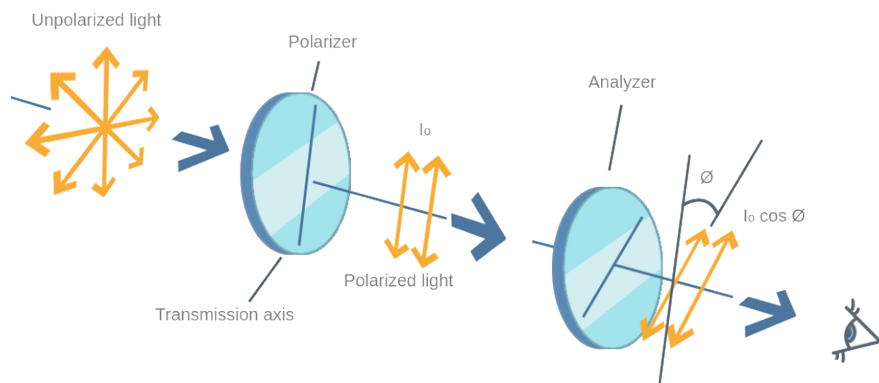
4.3.2 Principle of superposition

The net displacement of the sum of waves at a point is equal to the sum of the displacements caused by each wave separately. (i.e. waves can be added on each other)

4.3.3   Standing waves, interference, diffraction, beats

Standing waves	Standing waves occur when two waves travelling in opposite directions interfere and form a wave where the nodes and antinodes have fixed positions.
Interference	Interference occurs when two waves in the same medium superpose.
	Constructive interference occurs when the waves are in phase.
	Destructive interference occurs when the waves are exactly out of phase.
Diffraction	Diffraction occurs when waves bend around corners of an object or aperture into the geometrical shadow of the obstacle.
Beats	Beats occur when two waves interfere. It results in a sinusoidal pattern where the amplitude is controlled by another sinusoidal pattern.

More at http://www.physicsclassroom.com/class/sound/Lesson-3/Interference-and-Beats.

4.4   **Geometric Optics**

Reflection	$\theta_{\text{incident}} = \theta_{\text{reflected}}$
Refraction	Optical Density of medium $\eta = \dfrac{c}{v_{\text{light in medium}}}$
	$$\frac{\sin \theta_i}{\sin \theta_r} = \frac{\eta_i}{\eta_r}$$
	Refer to hyperphysics.phy-astr.gsu.edu/hbase/Tables/indrf.html for common refractive indices.
Total Internal Reflection	From medium with greater optical density to a lower one:
	$$\theta_i > \theta_c \ \text{ where } \ \frac{1}{\sin \theta_c} = \frac{\eta_i}{\eta_r}$$

5   Electric Charge and Electric Field

Coulomb's Law	$\dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q_1 q_2}{r^2}$
Electric Field	$E = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{\sum_i q_i}{r^2} = \dfrac{1}{4\pi\varepsilon_0} \sum \dfrac{\mathrm{d}q}{r^2}$
Electric Flux	$\Phi = \displaystyle\int \vec{E} \cdot \mathrm{d}\vec{A} = \vec{E} \cdot \vec{A}$
$\Rightarrow$ Analogous to the number of field lines cutting through a surface per area	
Electric Potential	$V = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{\sum_i q_i}{r} = \dfrac{1}{4\pi\varepsilon_0} \sum \dfrac{\mathrm{d}q}{r}$
Electric Potential Energy	$U = qV$
Electric Force on a Charged Particle in an E Field	$F = qE$
Work done to assemble a system of charges	$\dfrac{1}{4\pi\varepsilon_0} \displaystyle\sum_{i \neq j} \dfrac{q_i q_j}{r^2}$

Work done to bring a positive charge in the direction of electric field lines is negative.

Work done to bring a positive charge against the direction of the electric field lines is positive.

5.1   Motion of charged particles in an electric field

The motion of charged particles in an electric field is affected by the electric force, which is given by:
$$\vec{F_E} = q\vec{E}$$

5.2   Capacitance

Capacitance is the ratio of charge stored between conductors to potential difference. Capacitance is only dependent on the geometrical factors of the capacitor. Doubling the magnitude of charge on each conductor will double the charge density, electric field and also the potential difference.

Capacitance	$C = \dfrac{Q}{V}$
Equivalence Capacitance of a system of capacitors	$C_{\text{series}} = \left(\displaystyle\sum_i \dfrac{1}{C_i} \right)^{-1}$
	$C_{\text{parallel}} = \displaystyle\sum_i C_i$

Electric Potential Energy stored in a capacitor is defined as the work done to charge a capacitor, which is to separate opposite charges and place them on different conductors:

$$\mathrm{d}W = V\,\mathrm{d}q = \frac{Q}{C}\,\mathrm{d}q$$

$$W = \frac{1}{C}\int q\,\mathrm{d}q = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Electric Energy Density $\qquad\qquad\qquad\qquad U = \dfrac{E}{\mathrm{Vol}} = \dfrac{0.5CV^2}{Ad} = \dfrac{1}{2}\varepsilon_0 E^2$

Gauss Law (for symmetrical continuous charge distributions) $\qquad \oiint \vec{E}\cdot\mathrm{d}\vec{A} = \dfrac{Q}{\varepsilon_0}$

5.3   Conservation and quantization of charge

Charge is a conserved quantity. The minimum unit of charge is $e = 1.60 \times 10^{-19}$ C, the charge of an electron or proton.

6 Current and Magnetic Field

6.1 Current, impedance, and potential difference in DC and AC circuits

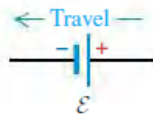|	
Ohm's Law	$V = IR$
Resistance	$R = \dfrac{V}{I} = \rho \dfrac{l}{A}$
Current	$I = \dfrac{\mathrm{d}Q}{\mathrm{d}t} = \dfrac{ne}{t} = \dfrac{nev}{L}$

(a) Sign conventions for emfs

$+\mathcal{E}$: Travel direction from $-$ to $+$:
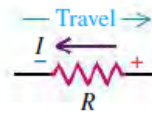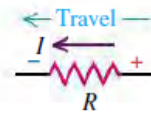
$-\mathcal{E}$: Travel direction from $+$ to $-$:

(b) Sign conventions for resistors

$+IR$: Travel *opposite to* current direction:

$-IR$: Travel *in* current direction:

Power	$P = VI = I^2 R = \dfrac{V^2}{R}$
Impedance	Impedance extends the concept of resistance to AC circuits, and possesses both magnitude and phase, unlike resistance, which has only magnitude.
Kirchoff's Loop Rule	The sum of voltage drops across all circuital elements (in a loop) is zero.
Kirchoff's Junction Rule	At any point where there is a node formed by the junction of various current carrying branches, by current conservation, the sum of the currents going into the node must equal to the sum of the currents going out of the node.

6.1.1 Circuits containing non-ohmic devices with known V-I characteristics

Semiconductors	The resistance at a point is equal to the potential difference across the semiconductor divided by the current, <u>**NOT necessarily the slope at the point**</u>

6.2   Magnetic field and Magnetic Forces

Magnetic Force (f. wire)	$\vec{F} = I\vec{L} \times \vec{B}$
Lorentz Force (f. moving point charge)	$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$
Biot-Savart Law (f. wire)	$\mathrm{d}\vec{B} = \dfrac{\mu_0 I}{4\pi}\dfrac{\mathrm{d}\vec{l} \times \vec{r}}{|r|^3}$
Biot-Savart Law (f. moving point charge)	$\mathrm{d}\vec{B} = \dfrac{\mu_0 q}{4\pi}\dfrac{\vec{v} \times \vec{r}}{|r|^3}$
Magnetic Dipole Moment	$\mu = N\vec{I} \times \vec{A}$
Ampere's Law	$\oint \vec{B} \cdot \mathrm{d}\vec{l} = \mu_0 I$

6.3   Electromagnetic induction and inductance

Magnetic Flux	$\phi = \vec{B} \cdot \vec{A}$
Faraday's Law	$\varepsilon = -N\dfrac{\mathrm{d}\phi}{\mathrm{d}t}$
Lenz's Law	$\uparrow$ accounts for negative sign
Inductance	$V = L\dfrac{\mathrm{d}I}{\mathrm{d}t}$

6.3.1   Self-Inductance

Consider a circuit consisting of a switch, power source and wires.

Initially, current flows in the wire. When the switch is turned off, the current in the wire does not instantaneously drop to zero but slowly falls to zero. According to Faraday's Law, this effect results in the induction of a magnetic field to oppose this change. This induced magnetic field is in the opposite direction of the magnetic field of the original current. This property of the loop in which its own magnetic field opposes any change in current is called self-inductance.

$$\varepsilon_{\text{back}} = -L\frac{\mathrm{d}I}{\mathrm{d}t} \qquad \text{where } L = \frac{N\phi}{I}$$

6.3.2   Mutual Inductance

Consider two coils placed near to each other.

Suddenly, a current flows in the first coil (with N turns and current I_A) which gives rise to a magnetic field. Some of the magnetic field lines through coil A will also pass through coil B. This results in an increase in magnetic flux linkage through coil B. By Faraday's Law, an induced emf occurs in coil B.

$$N_B \frac{\mathrm{d}\phi_B}{\mathrm{d}t} = M \frac{\mathrm{d}I_A}{\mathrm{d}t} \qquad \text{where } M = \frac{N_B \phi}{I_A}$$

7   Thermodynamics

7.1   Laws of Thermodynamics and Absolute Temperature

0th If two systems are in thermal equilibrium with a third system, they are in thermal equilibrium with each other.

1st Increase in internal energy of system is sum of heat absorbed by system and work done on system.
$$\Delta U = Q + W_{on}$$

Absolute temperature (unit: Kelvin) is defined with reference to absolute zero. At absolute zero, particles have zero vibrational motion. For reference,

$$\Delta 1\,^{\circ}\text{C} = \Delta 1\,\text{K} \ \text{ and } \ 0\,^{\circ}\text{C} = 273.15\,\text{K}$$

7.2   Kinetic theory of an ideal gas

Ideal gas equation:

$$PV = nRT = NkT$$

For a monatomic gas particle:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M_{\text{molar}}}}$$

$$E_k = \frac{3}{2}NkT = \frac{3}{2}nRT$$

where:

P	Pressure	T	Absolute Temperature
V	Volume	k	Boltzmann Constant
n	No. of moles of gases	N	Number of particles
R	Ideal gas constant	M_{molar}	Molar mass of the gas (in kg mol^{-1})

Note: $1\,\text{mol} = 6.023 \times 10^{23}$ particles. This number is called the Avogadro's number N_A.

Note: We usually measure M_{molar} in g mol^{-1}. In this unit, it is numerically equal to the atomic mass of a gas particle/molecule.

| 7.3 | **Thermal properties of Materials** |

Heat Capacity c $Q = mc\Delta\theta$

Latent Heat Q $Q = mL$ where L is the process's *specific latent heat*

Latent heat is thermal energy released or absorbed, by a body or a thermodynamic system, during a constant-temperature process (e.g. boiling).

Note: Heat capacity is different for different materials at different states. You will mostly be using the specific heat capacity c of an object, and not $C = mc$, the heat capacity of the object.

Note: Q is heat energy in joules.

7.3.1 Thermal Conductivity

In an isotropic medium the thermal conductivity is the parameter k in the Fourier expression for the heat flux $\vec{q} = -k\vec{\nabla}T$, where $\vec{q}$ is heat flux (amount of heat flowing per second and per unit area), and $\vec{\nabla}T$ the temperature gradient.

In a one-dimensional case, it reduces to:

$$H = -kA\frac{\mathrm{d}T}{\mathrm{d}x}$$

where $q = \dfrac{H}{A} \implies H = qA = \dfrac{\mathrm{d}Q}{\mathrm{d}t}$.

H is thus **the amount of heat flowing per second through a surface** with area A.

If the temperature difference is small, k can be taken as constant. The equation thus reduces to:

$$H = kA\frac{T_{\mathrm{H}} - T_{\mathrm{L}}}{L}$$

7.3.2 Thermal Expansion

Linear Expansion	Area Expansion	Volume Expansion
$\dfrac{\Delta L}{L_0} = \alpha\Delta T$	$\dfrac{\Delta A}{A_0} = 2\alpha\Delta T$	$\dfrac{\Delta V}{V_0} = 3\alpha\Delta T$

7.4   Thermodynamic Processes

The 1$^{\text{st}}$ Law of Thermodynamics is relevant to many Thermodynamics Processes.

For any process, the work done by an expanding gas $W_{\text{by}} = -W_{\text{on}}$ is given by:

$$W_{\text{by}} = P\Delta V \impliedby \mathrm{d}W_{\text{by}} = \int P\, \mathrm{d}V$$

We assume, for the following P-V graphs, that the arrow points towards the positive-V direction.

Isobaric	Constant Pressure	$W_{on} < 0\,,\ \Delta U > 0$
Isochoric	Constant Volume	$W_{on} = 0\,,\ \Delta U > 0$
Isothermal	Constant Temperature	$W_{on} < 0\,,\ \Delta U = 0$
Adiabatic	Thermally Insulated	$W_{on} < 0\,,\ Q = 0$
Cyclic	Start and end at the same state	$\Delta U = 0$

Work is done **by the gas** when **expanding**;
Work is done **on the gas** when **contracting**.

The internal energy U is the microscopic kinetic energy E_k of the particles. For a monatomic particle

$$U = \frac{3}{2}nRT$$

7.4.1   Thermodynamic efficiency

The thermal efficiency η_{th} is given by:

$$\eta_{\text{th}} \equiv \frac{W_{out}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$