SINGAPORE JUNIOR PHYSICS OLYMPIAD 2009
SPECIAL ROUND

19 August, 2009

2:30 pm – 5:30 pm

Time Allowed: THREE HOURS

INSTRUCTIONS

1. This paper contains **12** structural questions and **8** printed pages.
2. The mark for each question is indicated at the end of the question.
3. Answer **ALL** the questions in the foolscape sheets provided.
4. Scientific calculators are allowed in this test.
5. A table of information is given in page 2.

TABLE OF INFORMATION

$$
\begin{aligned}
\text{Acceleration due to gravity at Earth surface, } g &= 9.80\ \text{m/s}^2 \\
\text{Universal gravitational constant } G &= 6.67 \times 10^{-11}\ \text{m}^3/(\text{kg}\cdot\text{s}^2) \\
\text{Universal gas constant, } R &= 8.31\ \text{J}/(\text{mol}\cdot\text{K}) \\
\text{Vacuum permittivity, } \epsilon_0 &= 8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2) \\
\text{Vacuum permeability, } \mu_0 &= 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A} \\
\text{Atomic mass unit, } u &= 1.66 \times 10^{-27}\ \text{kg} \\
\text{Speed of light in vacuum, } c &= 3.00 \times 10^{8}\ \text{m/s} \\
\text{Speed of sound in air, } v &= 331\ \text{m/s} \\
\text{Charge of electron, } e &= 1.60 \times 10^{-19}\ \text{C} \\
\text{Planck's constant, } h &= 6.63 \times 10^{-34}\ \text{J}\cdot\text{s} \\
\text{Mass of electron, } m_e &= 9.11 \times 10^{-31}\ \text{kg} \\
\text{Mass of proton, } m_p &= 1.67 \times 10^{-27}\ \text{kg} \\
\text{Boltzmann constant, } k &= 1.38 \times 10^{-23}\ \text{J/K} \\
\text{Avogadro's number, } N_A &= 6.02 \times 10^{23}\ \text{mol}^{-1} \\
\text{Standard atmosphere pressure} &= 1.01 \times 10^{5}\ \text{Pa} \\
\text{Density of water} &= 1000\ \text{kg/m}^3 \\
\text{Lorentz Transformation: } x' &= \frac{x - ut}{\sqrt{1 - u^2/c^2}} \\
y' &= y \\
z' &= z \\
t' &= \frac{t - ux/c^2}{\sqrt{1 - u^2/c^2}}
\end{aligned}
$$

1. A cube of ice whose edges measure 20.0 mm is floating in a glass of ice-cold water with one of its faces parallel to the water's surface.

   (a) How far below the water surface is the bottom face of the block?

   (b) Ice-cold ethyl alcohol is gently poured onto the water surface to form a layer 5.0 mm thick above the water. The alcohol does not mix with the water. When the ice again attains hydrostatic equilibrium, what will be the distance from the top of the water to the bottom face of the block?

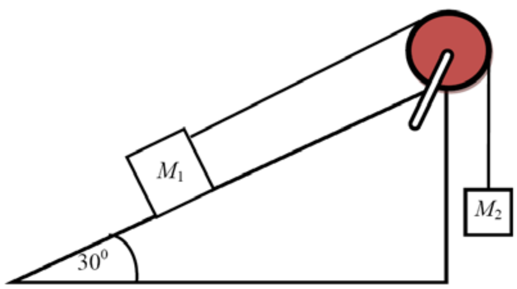
   (c) Additional cold ethyl alcohol is poured onto the water's surface until the top surface of the ice cube (in hydrodynamic equilibrium). How thick is the required layer of ethyl alcohol?

   (At 0°C, density of water = 1000 kg/m^3, density of ice = 917 kg/m^3 and density of ethyl alcohol = 806 kg/m^3.) [8]

2. A wire of mass 9.8×10^{-3} kg per meter passes over a frictionless pulley fixed on the top of an inclined frictionless plane which makes an angle of 30° with the horizontal. Masses M_1 and M_2 are tied at the two ends of the wire. The mass M_1 rests on the plane and the mass M_2 hangs vertically. The whole system is in equilibrium. Now, a transverse wave propagates along the wire with a velocity of 100 m/s. Find the magnitude of masses M_1 and M_2.

   You have probably assumed that the mass of the wire to be small compared to the M_1 and M_2 in your calculation. If this assumption is not made, explain how the speed would vary along the wire. Would it take a longer or shorter time for the wave to travel from one mass to the other?

   Hint: The speed of a transverse wave in the wire is given by $v = \sqrt{\dfrac{F}{\mu}}$ where F is the tension in the wire and μ is the mass per unit length of the wire. [8]



3. Two trains approach the train station from opposite sides each moving at an initial velocity of 120 km/h with respect to the station and slowing down to a halt in 20 seconds. If both trains are whistling at 5000 Hz, find the distance between the zero and first order maxima as a function of time. [6]

4. (a) A Singapore Polytechnic student sets out to take flight: she wants to calculate the size of a hot air balloon to support her (45 kg) her friend (50 kg) and a basket (30 kg). She is planning on using the sun alone to warm up the air inside a new super light and strong black balloon. Assuming that the balloon can stretch effortlessly (the internal pressure remains about 1 atm) and its weight negligible and given that the initial air temperature is 30°C and the final is 60°C (density of air $\rho_{AIR} = 1.165$ kg/m^3); help her calculate the initial radius of this spherical balloon.

   (b) An ice cube of mass 0.10 kg at 0°C is placed in an isolated container which is at 227°C. The specific heat S of the container varies with temperature T according to the empirical relation, $S = A + BT$, where $A = 420$ J/kg $\cdot$ K and $B = 8.4 \times 10^{-2}$ J/kg $\cdot$ K^2. If the final temperature of the container is 27°C, determine the mass of the container. (Specific latent heat of fusion of water=3.3×10^5 J/kg, specific heat of water=4.2×10^3 J/kg $\cdot$ K. You may use calculus or a graphical method to solve this problem.) [10]

5. In the spectacular sport of bungee jumping,, a light elastic cord (a bungee cord) is tied tightly around the ankles of someone who jumps from a bridge of height H to which the other end of the cord is attached. The length of the cord is calculated so that the jumper, of mass m, will not quite reach the surface of the water below the bridge before he or she springs back up. Suppose the cord behaves like a spring of spring constant $10mg/H$, where g is the acceleration due to gravity.

   (a) How long must the cord be so that a jumper just touches the water before being pulled back up? Neglect the height of the jumper and any effects due to friction.

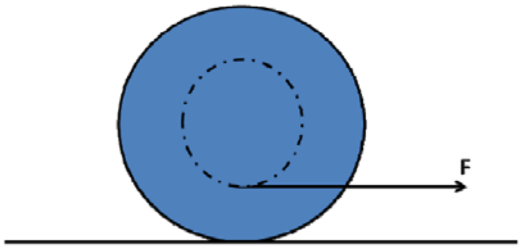
   (b) Friction damps the up-and-down motion of the jumper that results after the initial jump. How far above the water would the jumper be when the oscillations have ceased?

   Express your answers in terms of H. [6]

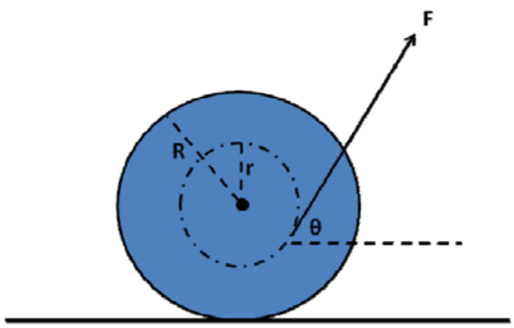
6. The rockets of the Goths and the Huns are each 1000 m long in their respective rest frame. The rockets pass each other, virtually touching, at relative speed of $0.8c$. The Huns have a laser cannon at the rear of their rocket that shoots a deadly laser beam at right angles to the motion. The captain of the Hun rocket wants to send a threatening message to the Goths by "firing a shot across their bow." He tells his first mate, "The Goths rocket is length contracted to 600 m. Fire the laser cannon at the instant the nose of our rocket passes the tail of their rocket. The laser beam will cross 400 m in front of them." But things are different from the Goths' perspective. The Goth captain muses, "The Huns' rocket is length contracted to 600 m, 400 m shorter than our rocket. If they fire the laser cannon as their nose passes the tail of our rocket, the lethal laser blast will go right through our side."

   The first mate on the Hun rocket fires as ordered. Does the laser beam blast the Goths or not? Resolve this paradox. Show that, when properly analyzed, the Goths and the Huns agree on the outcome. Your analysis should contain both quantitative calculations and written explanation. [8]

7. (a) A circular yo-yo as shown rests on a level surface. A gentle horizontal pull is exerted on the cord so that the yo-yo rolls without slipping. Which way does it roll and why?



   (b) The yo-yo consists of an axle of radius r and an outside circle of radius R which rolls on the ground. A thread is wrapped around the axle and is pulled with tensional force F, at an angle θ with the horizontal.



   i. Given R and r, what should θ be so that the spool does not move? Assume that the friction between the spool and the ground is large enough so that the spool does not slip.

ii. Given R, r and the coefficient of friction μ between the spool and the ground, what is the largest value of F for which the spool remains at rest? (*Note:* μ is the ratio of the maximum possible frictional force to the force normal to the surface of contact with which the bodies press against each other. [8]

8. The speed of sound v in an ideal gas is given by the expression

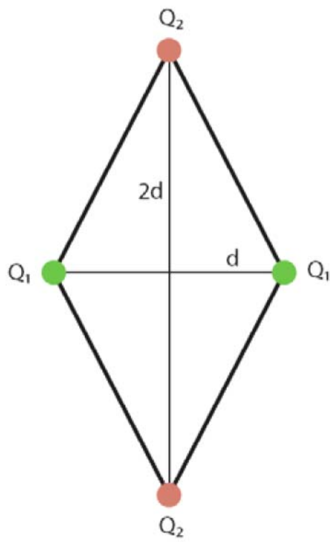
$$v = k\sqrt{T}$$

where T is the absolute temperature and k is a constant.

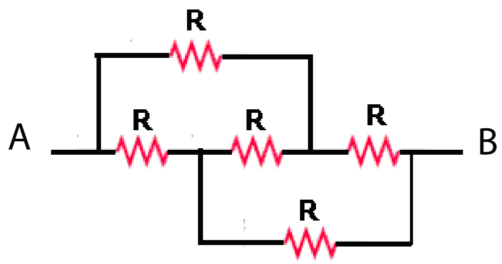
(a) A parallel beam of sound passing through an ideal gas at 17° C makes an angle of incidence of 50° on a plane thin membrane separating the gas from another sample of the same ideal gas at 100°C. What will be the angle of refraction of the beam? (You may assume that the membrane itself produces no deviation.)

(b) If the incident beam is to be totally reflected at the boundary, calculate the minimum temperature to which the gas on the other side of the membrane must be raised. [6]

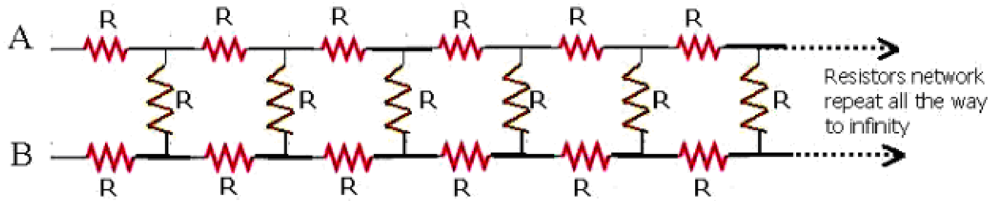
9. 4 positive charges are placed on the corners of rhomboid. The sides are hinged on the corners and free to rotate. The charges along the vertical axis are identical and the charges along the horizontal axis are also identical. If the axes length is d and $2d$, find the value of the ratio Q_1/Q_2 such that the system is perfectly in equilibrium and maintains its original shape. [8]



10. (a) Find the equivalent resistance across points A and B for each of the following circuit.

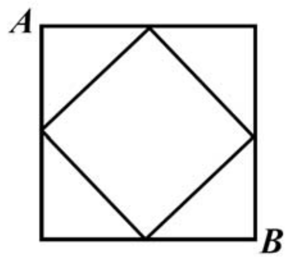


(i)



(ii)

(b) A wire has linear resistance ρ (in Ω/m). Find the resistance between points A and B if the length of each side of the big square is d metres.



(c) A battery has an emf of $\epsilon = 6$ V. The battery is connected in series with an ammeter and a voltmeter. If a certain resistor is connected in parallel with the voltmeter, the voltmeter reading decreases by a factor 3, and the ammeter reading increases by the same factor. What is the initial reading V of the voltmeter? All elements of the circuit have unknown internal resistances.

[14]

11. An experiment is performed on a sample of atoms known to have a ground state of $-5.0\,\text{eV}$. The gas is illuminated with "white light" (400–700 nm). A spectrometer capable of only analyzing radiation in this range is used to measure the radiation. The sample is observed to absorb light at only 400 nm. After the "white light" is turned off, the sample is observed to emit visible radiation of 400 nm and 600 nm.

   (a) Determine the values of the energy levels and sketch an energy-level diagram showing the energy values in eVs and the relative positions of:

      i. the ground state

      ii. the energy level to which the system was first excited

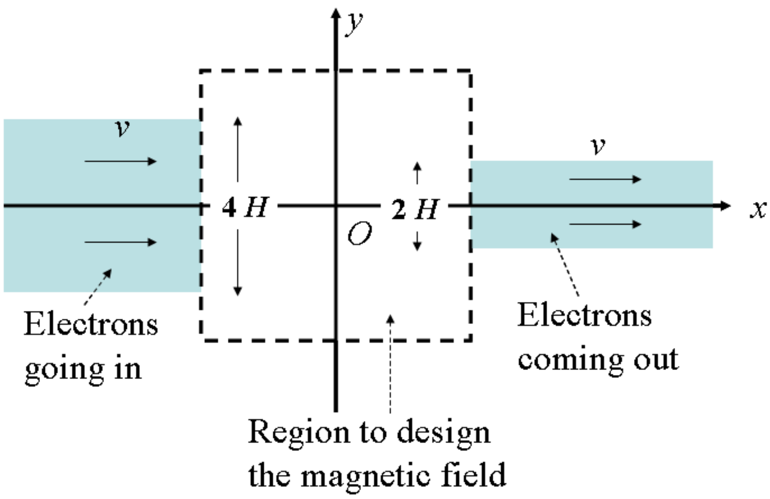
      iii. one other energy level that the experiment suggests may exist

   (b) What is the wavelength of any other radiation, if any, that might have been emitted in this experiment? Why was it not observed? [8]

12. In the diagram below, electrons are travelling horizontally towards the right at uniform velocity v and are confined within $-2H < y < 2H$. Assume the electrons are travelling very far from the left towards the y-axis. Design a magnetic field (within the dotted square) such that

   (a) all the electrons will pass through the origin O **AND**

   (b) all the electrons will then be emitted with velocity v travelling towards the right but now confined within a region $-H < y < H$. [10]



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