SINGAPORE JUNIOR PHYSICS OLYMPIAD 2013
SPECIAL ROUND

31 August, 2013

9:15 a.m. – 12:15 p.m

Time Allowed: THREE HOURS

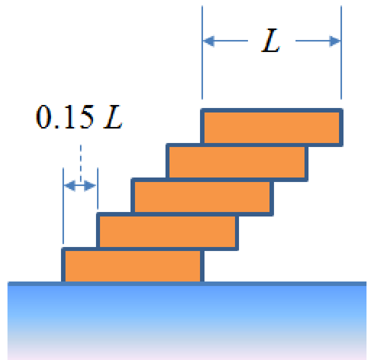
INSTRUCTIONS

1. This paper contains **11** structural questions and **7** printed pages.
2. The mark for each part/question is indicated at the end of the part/question.
3. Answer **ALL** the questions in the booklet provided.
4. Scientific non-graphical calculators are allowed in this test.
5. A table of information is given in page 2. Not all information will be used in this paper.

TABLE OF INFORMATION

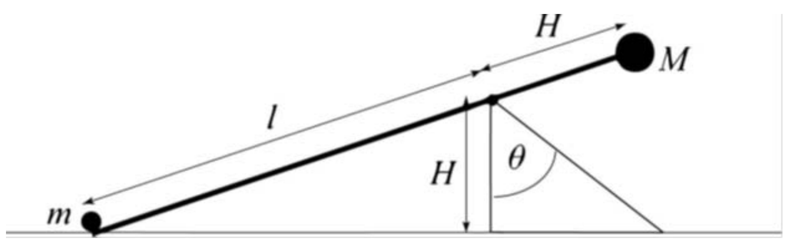
Acceleration due to gravity at Earth surface, g	$=$	$9.80\ \mathrm{m/s^2}$
Universal gas constant, R	$=$	$8.31\ \mathrm{J/(mol \cdot K)}$
Newton's gravitational constant, G	$=$	$6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
Vacuum permittivity, ϵ_0	$=$	$8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$
Vacuum permeability, μ_0	$=$	$4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$
Speed of light in vacuum, c	$=$	$3.00 \times 10^{8}\ \mathrm{m/s}$
Speed of sound in air, v	$=$	$331\ \mathrm{m/s}$
Charge of electron, e	$=$	$1.60 \times 10^{-19}\ \mathrm{C}$
Planck's constant, h	$=$	$6.63 \times 10^{-34}\ \mathrm{J \cdot s}$
Mass of electron, m_e	$=$	$9.11 \times 10^{-31}\ \mathrm{kg}$
Mass of proton, m_p	$=$	$1.67 \times 10^{-27}\ \mathrm{kg}$
Mass of neutron, m_n	$=$	$1.67 \times 10^{-27}\ \mathrm{kg}$
Atomic mass unit, u	$=$	$1.66 \times 10^{-27}\ \mathrm{kg}$
Boltzmann constant, k	$=$	$1.38 \times 10^{-23}\ \mathrm{J/K}$
Avogadro's number, N_A	$=$	$6.02 \times 10^{23}\ \mathrm{mol^{-1}}$
Standard atmospheric pressure	$=$	$1.01 \times 10^{5}\ \mathrm{Pa}$

1. A uniform brick of length L is laid on a smooth horizontal surface. Other equal bricks are now piled on as shown, so that the sides form a continuous plane, but the ends are offset at each block from the previous brick by a distance $0.15\,L$. How many bricks can be stacked in this manner before the pile topples over?



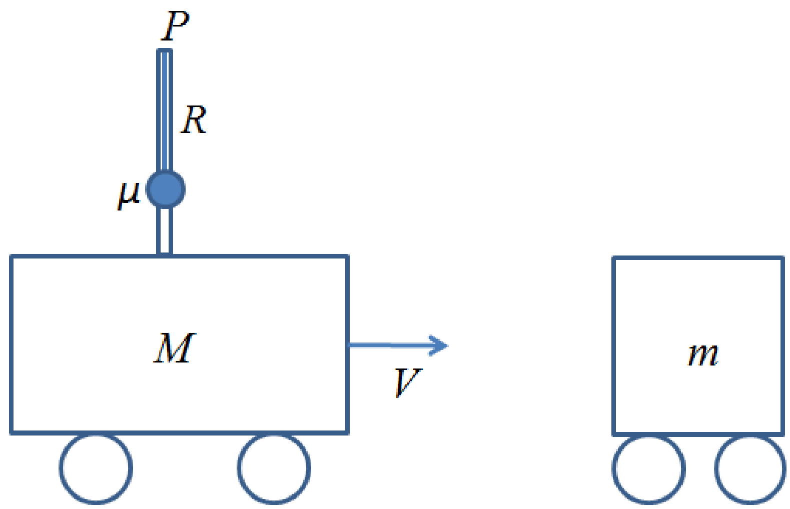
[8]

2. The trebuchet is a siege engine that was employed in the Middle Ages to smash castle walls or to lob projectiles over them. A simplified version of a trebuchet is shown in the following figure. A heavy weight of mass M falls under gravity, and thereby lifts a lighter weight of mass m. The motion of the mass M is blocked as shown in the figure, which launches the lighter mass m; the blockade forms an angle θ with the vertical. The mass of the blockade is much larger than all other masses. The shorter arm of the trebuchet is of length H, whereas the longer arm is of length l; the whole beam (both arms) are of mass μ.



Calculate the angular velocity ω at which the projectile is launched. Express the range R of the projectile in terms of ω and other quantities with respect to the turning point. [9]

3. A cart of mass M has a pole on it from which a ball of mass μ hangs from a thin string attached at point P. The cart and ball have initial velocity V. The cart crashes onto another initially stationary cart of mass m and sticks to it. If the length of the string is R, show that the smallest initial velocity for which the ball can go round in circles around point P is of the form $\left[1+\frac{M}{m}\right](\alpha g R)^{\beta}$ where α and β are constants which you have to determine and g is the gravitational acceleration. Neglect friction and assume $M,\ m \gg \mu$.



[6]

4. A 0.75-m rod has a uniform linear mass density of λ. A small mass m with negligible volume is attached to one end of the rod. The rod with the attached mass is placed in a container of unknown fluid and after oscillating briefly, comes to rest at its equilibrium position. At equilibrium, the rod floats vertically with 2/3 of its length submerged and mass m in the fluid. If the rod were fully submerged it would displace 7.5×10^{-4} kg of fluid.

(a) What is the maximum value that the mass m can have?

(b) What is the minimum value that the mass m can have?

(c) Sketch a graph that shows the values of λ as a function of m. [12]

5. Consider the spherically symmetric expansion of a homogeneous, self-gravitating gas with negligible pressure. The initial conditions of expansion are unspecified; instead, you are given that when the density is ρ_0, a fluid element at radius R_0 from the origin has a velocity of v_0.

By considering the motion of a unit mass at the surface of the gas, find $v(R)$ and describe the ultimate fate of the gas in terms of v_0, R_0 and ρ_0 [6]

6. 1 mole of ideal gas with internal energy $U = \frac{3}{2}RT$, expands from initial volume $V_i = \frac{1}{10}V_0$ following the equation

$$p = -\frac{p_0}{V_0}V + p_0.$$

Find

(a) the highest temperature reached by the gas during the expansion and

(b) the maximum amount of heat taken in by the gas. [8]

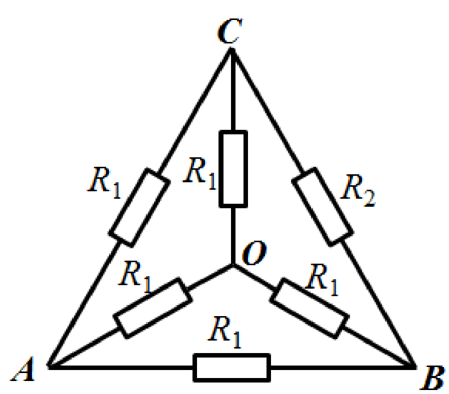
7. (a) A capillary tube immersed in water (refractive index =1.33) is made of glass with index of refraction 1.55. The outer radius of the tube is 2.5 mm. The tube is filled with a liquid with the index of refraction 1.45. What should the minimum internal radius of the tube be so that any ray that hits the tube would enter the liquid in the capillary? [5]

(b) A radio receiver is set up on a mast in the middle of a calm lake to track the radio signal from a satellite orbiting the Earth. As the satellite rises above the horizon, the intensity of the signal varies periodically. The intensity is at a maximum when the satellite is $\theta_1 = 3°$ above the horizon and then again at $\theta_2 = 6°$ above the horizon. What is the wavelength of the satellite signal? The receiver is $h = 4.0$ m above the lake surface. [6]

8. (a) If $R_1 = R_2 = 3\ \Omega$ in the circuit below, determine the resistance of the network

i. between points A and B and

ii. between points A and O. [5]



(b) Redo the calculations above for the case where $R_1 = 3\ \Omega$ and $R_2 = 6\ \Omega$. [7]

9. In a nuclear reactor that uses U-235 as the nuclear fuel, a typical uranium nucleus will emit 2 or 3 fast-moving neutrons during nuclear fission after absorbing a slow neutron having kinetic energy of 0.025 eV. Fast moving neutrons are not useful for a sustained chain fission reactions and the trick to produce slow neutrons is to use graphite (carbon-12) rods as velocity reducing agents. Assume that the carbon nucleus and the neutron are point objects undergoing elastic collisions and that a fast neutron has initial kinetic energy of 1.75 MeV, what is the minimum number of times that the neutron must collide with carbon nuclei before it becomes a slow neutron? [6]

10. As indicated in the diagram below, a sphere of radius R and center O has charge evenly distributed throughout and of electric potential at the surface as 1000 V. At a point O' far away from the sphere lies a proton p which is fired at the sphere with kinetic energy 2000 eV in a direction parallel to OO'. The distance between the two parallel lines denoting OO' and the direction of travel of the proton is denoted as l.

(a) If we are to have the proton graze the surface of the sphere, what should l be? [6]

(b) If we now have an electron in place of a proton and to have it graze the surface of the sphere, what should l be? [4]

11. The surface temperature and radius of the Sun are approximately 5800 K and 6.96×10^8 m respectively. You may assume that the Sun behaves as a blackbody radiator.

(a) Calculate the total energy radiated by the Sun in each second, given that the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$.

(b) What is the intensity of the solar radiation reaching the top of the Earth's atmosphere? (Distance from the Earth to the Sun $= 1.50 \times 10^{11}$ m)

(c) If the Earth has no atmosphere, what would be the Earth's equilibrium temperature be so that at steady state, it radiates as much as it absorbs? (Radius of Earth $= 6.38 \times 10^6$ m)

(d) From measurements of the temperature at different parts of the Earth, the Earth's average temperature is estimated to be about 15°C. Give possible reasons for the difference between this temperature and that you have calculated in part (c).

(e) Estimate the temperature on Mars' surface, given that the distance from Mars to the Sun $= 2.28 \times 10^{11}$ m and radius of Mars $= 3.40 \times 10^{6}$ m. [12]

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