SINGAPORE JUNIOR PHYSICS OLYMPIAD 2018

SPECIAL ROUND

Friday 29 June 2018

1430 – 1730

Time Allowed: Three hours

INSTRUCTIONS

1. This paper contains **10** multiple choice questions, **6** structured questions and **13** printed pages.
2. For the structured questions (60 points):
   - You may use your own approximations and assumptions.
   - Answers which are more **complete** with clear and/or detailed **working** may be awarded bonus points.
   - Open-ended parts of the structured questions may be used as tie breakers, please answer in detail if you have time.
3. For the multiple choice questions (10 points):
   - Each of the questions or incomplete statements is followed by five suggested answers or completions. Select the one that is **best** in each case and state clearly on the answer sheet.
   - **W**orking for the MCQ questions **may** be marked for bonus points.
4. Answer **all** questions. Points will **NOT** be deducted for wrong answers.
5. **Scientific calculators** are **allowed** in this test. Graphic calculators are **not** allowed.
6. All sheets of papers with answers and working should be carefully **labeled** with the **Question number** and placed in the envelope at the end of the competition.
7. Please fill in your **name**, **IC number** and **school** on the answer sheets **before the competition starts.**
8. A general information sheet is given in page 2. You may detach it **when the competition starts** for easy reference.

GENERAL INFORMATION SHEET

Acceleration due to gravity at Earth surface,	$g = 9.80 \text{ m s}^{-2} = \lvert\vec{g}\rvert$
Gravitational constant	$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Radius of the Earth	$R_E = 6.371 \times 10^6 \text{m}$
Universal gas constant,	$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$
Atomic mass unit,	$u = 1.66 \times 10^{-27} \text{ kg}$
Speed of light in vacuum,	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
Charge of electron,	$e = 1.60 \times 10^{-19} \text{ C}$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J s}$
Mass of electron,	$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.000549\,u$
Mass of proton,	$m_p = 1.67 \times 10^{-27} \text{ kg} = 1.007\,u$
Rest mass of alpha particle,	$m_\alpha = 4.003\,u$
Boltzmann constant,	$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
Avogadro's number,	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
Standard atmosphere pressure,	$P_0 = 1.01 \times 10^5 \text{ Pa}$
Density of water,	$\rho_w = 1000 \text{ kg m}^{-3}$
Specific heat (capacity) of water,	$c_w = 4.19 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$
Latent heat of fusion for water,	$L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$
Stefan-Boltzmann constant,	$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Sum of N terms in an arithmetic series,	$\sum_{k=0}^{N-1} a_k = \frac{N(a_0 + a_{N-1})}{2}$
Sum of N terms in a geometric series,	$\sum_{k=0}^{N-1} r^k = \frac{1-r^N}{1-r}$
Approximation for square root, for small x	$\sqrt{1+x} \approx 1 + \frac{x}{2}$
Area under the curve of $y = x^n$ for x between 0 and x_0	$\int_0^{x_0} x^n \, dx = \frac{1}{n+1} x_0^{n+1}$

Section A: Multiple Choice Questions (MCQ)

MCQs 1 to 4 are given below, and questions 5 to 10 are embedded within the structured questions, where the structured questions provide the context for the multiple choice questions.

Draw the table below into the first page of your answer booklet and fill in your answers to the MCQs.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
C	E	B	D	C	B	C	A	C	C

(Table to be drawn into answer booklet)

MCQ 1.

A brick is dropped from rest, off the roof of a building. It passes a 1.2 m tall window in a time of 0.38 s. What is the distance between the roof and the top of the window?

A. 0.010 m

B. 0.078 m

C. 0.086 m

D. 0.12 m

E. 0.43 m

MCQ 2.

A rock is suspended by a light string. When the rock is in air, the tension in the string is 44.6 N. When the rock is totally immersed in water, the tension is 28.3 N. When the rock is totally immersed in an unknown liquid, the tension is 18.2 N. What is the density of the liquid in kg m^{-3}?

A. 590

B. 640

C. 1000

D. 1120

E. 1620

MCQ 3.

The International Space Station makes 15.65 revolutions around the Earth each day. Assuming a circular orbit, and the radius of the Earth as 6371 km, how high is this satellite above the surface of the Earth?

A. 258 km

B. 376 km

C. 572 km

D. 6746 km

E. 6942 km

MCQ 4.

A 63.5 cm long guitar string is tuned to produce note B_3 of frequency 245 Hz when vibrating in its fundamental mode. The displacement of such a standing wave may be given by $y(x,t) = A\sin(kx)\sin(\omega t)$, where A is 1.0 cm, k is the wave number and ω is the angular frequency. What is the maximum transverse velocity of the string at the antinode?

A. 0

B. 6.28 cm/s

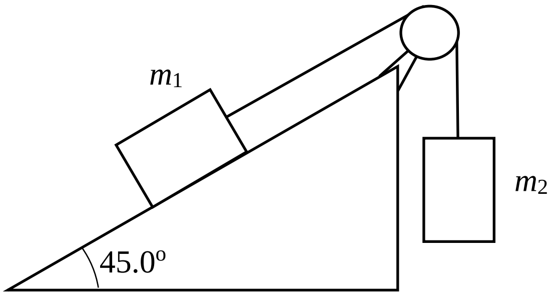
C. 256 cm/s

D. 1540 cm/s

E. 2450 cm/s

Section B: Structured Questions

1. A block of mass m_1 = 9.20 kg is at rest on a ramp inclined at 45.0° to the horizontal. It is connected by a light, inelastic cord over a frictionless pulley to another block of mass m_2 = 5.00 kg. The surface of the ramp is rough, and the blocks are in equilibrium. The mass m_1 is on the verge of sliding along the ramp.



(a) Is the block m_1 on the verge of sliding up or sliding down the ramp? Explain briefly. [2 pts]

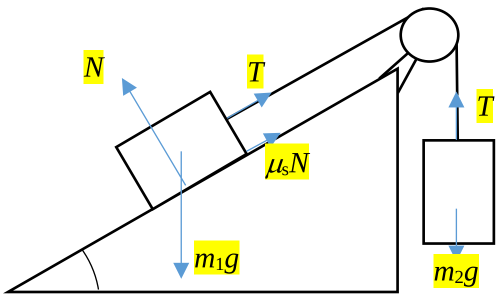
Sliding down. Because the component of weight of m_1 along the ramp is greater than the weight of m_2.

(b) In your answer booklet, sketch two free body diagrams that show separately, the forces acting on the masses. [4 pts]

T: Tension

N: Normal contact force

$\mu_s N$: Static friction



(c) Find the tension in the cord, and the coefficient of static friction. [4 pts]

$$m_1 g \sin(45^0) - \mu_s N - T = 0$$
$$N - m_1 g \cos(45^0) = 0$$
$$m_2 g - T = 0$$

Solving, $T = m_2\, g$ = 49 N and $\mu_s = 0.23$

MCQ 5.

At what angle of inclination of the ramp would static friction be zero?

A. 14^o

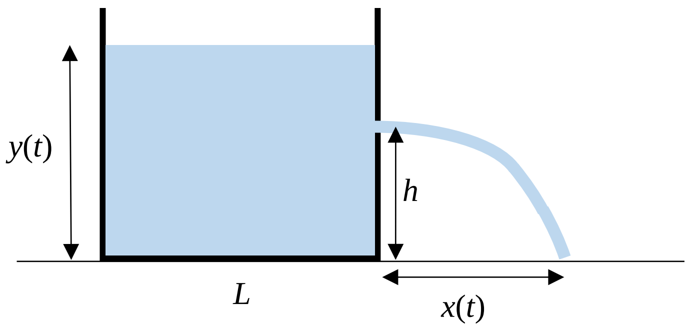
B. 29^o

C. 33^o

D. 57^o

E. Static friction cannot be zero.

2. The figure below shows a rectangular tank of sides L with water filled to an initial height y_0. A tiny circular hole of area A in the side of the tank is opened and water gushes out in a stream to hit the ground at an horizontal distance x away. The hole is at a height h above the base of the tank, measured <u>from the base to the centre of the hole</u>.



(a) The speed of the water jet gushing out of the hole is approximately $v \approx \sqrt{2g[y(t)-h]}$. Derive this, with careful explanation of the principles, laws and/or assumptions used. [2 points]

Assuming wide enough tank, so L >> size of hole such that the speed of the water at the top surface is negligible, and that water is a non-viscous liquid so that energy losses are negligible,

Use conservation of energy for a small mass of water Δm,

$$\frac{1}{2}\Delta m v^2 - \frac{1}{2}\Delta m v_{top}^2 = \Delta m g(y(t)-h)$$
$$\frac{1}{2}v^2 - \frac{1}{2}\cancelto{\sim 0}{v_{top}^2} = g(y(t)-h)$$
$$v \approx \sqrt{2g[y(t)-h]}$$

Or, use Bernoulli's equation.

(b) What should h be, so that the initial horizontal distance of the water jet is maximum? What would this maximum distance be? [3 points]

Using kinematics, we have

$x = vt$ and $h = \frac{1}{2}gt^2$, which gives us $x = 2\sqrt{h(y_0-h)}$

Differentiating x w.r.t. h and setting the derivative to zero, we get $\frac{dx}{dh} = -\frac{\sqrt{h}}{\sqrt{y_0-h}} + \frac{\sqrt{y_0-h}}{\sqrt{h}} = 0$ which gives

$h = y_0/2$.

Maximum distance is then $x_{\max} = y_0$.

(c) Determine the time required for the water level in the tank to reach the <u>top</u> of the hole, where the hole is at the height you found in part (b). [5 points]

Total height of water that needs to be drained is $y_f = y_0 - h + r$, where r is the radius of the hole, $r = \sqrt{A/\pi}$

Volume flow rate of water at the hole is $Av = A\sqrt{2g[y(t)-h]}$

Using continuity equation, $Av = -L^2 \dfrac{dy}{dt}$

These give us

$$-L^2 \frac{dy}{dt} = A\sqrt{2g[y(t)-h]}$$

$$-\int_{y_0}^{y_f} \frac{dy}{\sqrt{y-h}} = \int_0^t \frac{A\sqrt{2g}}{L^2} dt$$

$$-2\sqrt{y_f - h} + 2\sqrt{y_0 - h} = \frac{A\sqrt{2g}}{L^2} t$$

Where $y_f = y_0 - h + r$ and $h = y_0 / 2$

This give us the time as $t = \dfrac{L^2}{A\sqrt{g}} \left(\sqrt{y_0} - \sqrt{2}\left(\dfrac{A}{\pi}\right)^{1/4} \right)$.

3. The magnitude of the air resistance acting on a cyclist is $k(v_{wc})^2$, where v_{wc} is the magnitude of the velocity of the wind relative to the cyclist and k is a constant. The force acts in the direction of the relative velocity.

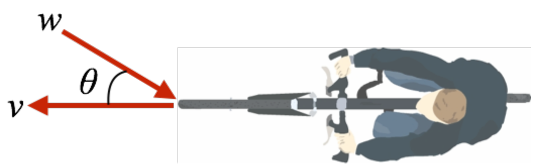


Figure: Top view of cyclist travelling horizontally.

Both $\vec{v}$ and $\vec{w}$ are relative velocity vectors oriented horizontally (i.e. parallel to the ground).

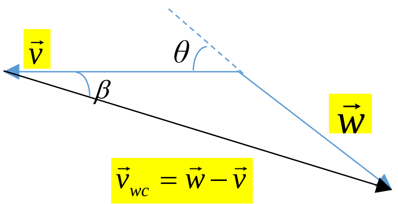
The figure shows a cyclist travelling horizontally with a road velocity $\vec{v}$ (i.e. velocity relative to the road). A wind blows horizontally. The velocity of the wind relative to the road is w, at an angle θ as shown in the figure. The relationship between power developed by the cyclist P, its propulsive force $\vec{F}$ and velocity of the cyclist $\vec{v}$, is given by

$$P = \vec{F} \cdot \vec{v} \quad \text{-- Equation (1).}$$

(a) State the relationship between the propulsive force $\vec{F}$ and the air resistance $\vec{F}_{air}$ when the cyclist is travelling at constant velocity. [1 point]

They are equal and opposite (assuming no other resistive forces).

(b) Sketch a vector diagram showing the relationship between the following vector quantities: the velocity of the cyclist $\vec{v}$ relative to the road, the velocity of the wind relative to the road $\vec{w}$ and the velocity of the wind relative to the cyclist $\vec{v}_{wc}$. [1 point]



(c) Using $\left|\vec{v}_{wc}\right|^2 = w^2 + v^2 + 2wv\cos\theta$, determine the power developed by the cyclist in overcoming a head wind ($\theta = 0^{o}$) when cycling at constant velocity? [4 points]

Since $\vec{F} = -\vec{F}_{air}$, we have $P = -\vec{F}_{air} \cdot \vec{v} = F_{air} v\cos\beta = kv_{wc}^2 v\cos\beta$, where β is the angle between $-\vec{F}_{air}$ and $\vec{v}$. This angle is also the angle between $-\vec{v}_{wc}$ and $\vec{v}$.

This gives us $P = k(w^2 + v^2 + 2wv\cos\theta)v\cos\beta$.

For head wind, (θ=β = 0°), $P_{head} = k(w^2 + v^2 + 2vw)v = k(w+v)^2 v$.

(d) Is the power P_{cross} required to cycle at horizontal speed v (relative to road) in a cross wind (θ = 90°) of speed v (relative to road) larger or smaller than the power P_{still} required to cycle at the same speed v in still air? What is the fraction $P_{\text{cross}}/P_{\text{still}}$? [4 points]

Obviously, it is harder to cycle in cross wind than in still air, so $P_{cross} > P_{still}$.

For cross wind, (θ= 90°), $\cos\beta = \dfrac{v}{\sqrt{v^2 + w^2}}$, where in this question $w = v$.

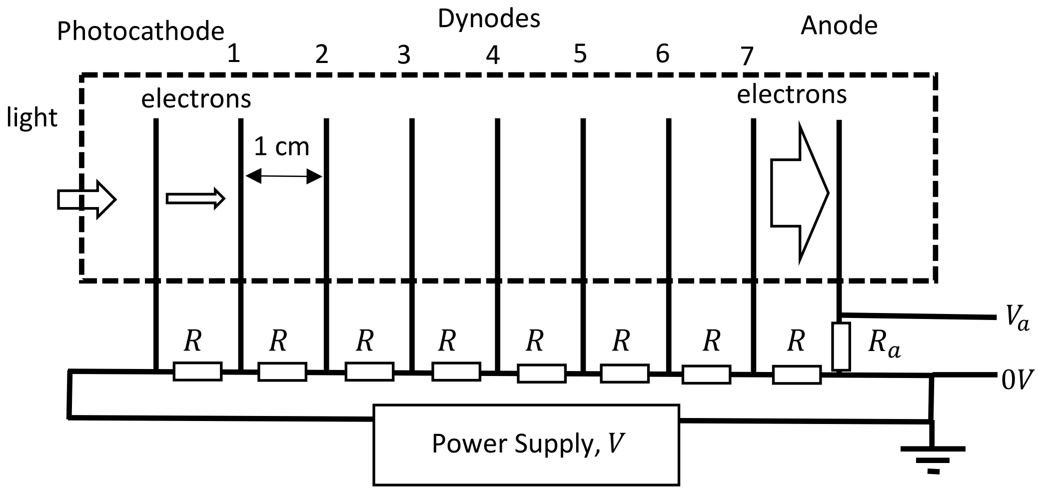
$$P_{cross} = k(w^2 + v^2)v\frac{v}{\sqrt{v^2 + w^2}} = kv^2\sqrt{v^2 + w^2} = kv^3\sqrt{2}$$

$$P_{still} = k(v^2)v = kv^3$$

Hence, $\dfrac{P_{cross}}{P_{still}} = \sqrt{2}$.

Questions 4 and 5 are based on the same background information as below:

A photomultiplier tube converts light to electrical signals and works well at low light levels. The figure shows a diagram of a simplified model of the photomultiplier. The photomultiplier consists of several electrodes; a photo cathode, several identical dynodes and an anode all in a vacuum tube. A potential difference is applied across the whole set of electrodes. When a photon strikes the photo cathode, an electron may be emitted from the photo cathode. This electron accelerates towards the first dynode. When the electron strikes the dynode, it may knock out several secondary electrons. These electrons are accelerated towards the next dynode through a voltage $V_d$where they knock out even more electrons. The same process happens until the last dynode. The average number of secondary electrons per incoming primary electron is called the secondary emission ratio δ. In this simplified model let us assume that it takes on average some energy E_o to knock out a secondary electron and that this average value is a constant. Assume that secondary electrons have negligible energy. A potential difference V is applied across photocathode and anode. An external circuit consisting of resistors $R = 1.0\ \text{k}\Omega$ are connected across each pair of electrodes to try to maintain the same potential difference across each pair of electrodes and the signal voltage across resistor $R_a = 50\Omega$ is measured. This photomultiplier has $n_d = 7$ dynodes. The distance between electrodes is $d = 1$ cm. Assume that on average $\alpha = 3.5$ photons produces 1 useful electron at the photocathode. When $V = -100$ V, the photomultiplier has a gain i.e number of electrons reaching the anode/ number of photons arriving at the photocathode of $G = 1/\alpha$.



4. Consider the photomultiplier operating at very low light levels such that the presence of the electrons do not disturb the applied fields.

(a) Derive an expression in terms of the variables mentioned above to estimate the average speed of the electron.

(a) Acceleration due to electrostatic force, $a = \frac{eE}{m_e} = \frac{eV}{m_e d(n_d+1)}$

Conservation of energy, initial KE =0 $v_{max}^2 = 2ad = \frac{2eV}{m_e(n_d+1)}$

Constant acceleration so average velocity is half, $v_{ave} = \frac{v_{max}}{2} = \sqrt{\frac{eV}{2m_e(n_d+1)}}$

[1 point]

MCQ 6.

Estimate the average speed of the electrons in the photomultiplier at $V = -100$ V?

A. 0 ms^{-1}

B. $1 \times 10^6 \text{ms}^{-1}$

C. $3 \times 10^6 \text{ms}^{-1}$

D. $9 \times 10^6 \text{ms}^{-1}$

E. $2 \times 10^7 \text{ms}^{-1}$

(b) Derive an expression to estimate the gain and calculate the gain when $V = -1000$ V

Gain is combination of effect at photocathode and then subsequently at each dynode.

At each dynode the gain is $\frac{Energy\ of\ electron\ arriving\ at\ dynode}{E_0} = \frac{eV}{(n_d+1)E_0}$

Assume voltage across R_a is negligible and all electrons are captured at anode.

$$G = \frac{1}{\alpha}\left(\frac{eV}{(n_d+1)E_0}\right)^{n_d}$$

When $V = 100$, $G = \frac{1}{\alpha}$, So when $V = 1000$

$$G = \frac{1}{\alpha}\left(\frac{1000}{100}\right)^{n_d} = 2.86 \times 10^6$$

Optionally can find E_0,

$$\left(\frac{eV}{(n_d+1)E_0}\right)^{n_d} = 1$$

$$\frac{eV}{(n_d+1)E_0} = 1$$

$$E_0 = \frac{eVd}{(n_d+1)} = 12.5eV = 2\times 10^{-18}J$$

[1 point]

(c) Derive an expression to estimate the transit time t_t i.e. the time between the photon reaching the photocathode and the time the electrons reach the anode and calculate the transit time when $V = -1000$ V.

(c) From 4(a), $v_{ave} = \sqrt{\frac{eV}{2m_e(n_d+1)}}$

$$t_t = \frac{(n_d+1)d}{v_{ave}} = d\sqrt{\frac{2m_e(n_d+1)^3}{eV}} = 24ns$$

[1 point]

(d) Two identical photomultipliers PMT-A and PMT-B are placed in a 1mT magnetic field in the z-direction. PMT-A is oriented in the z-direction such that the magnetic field is parallel to the direction of the electric field and PMT-B is oriented in the x-direction such that the direction of the magnetic is perpendicular to the direction of the electric field.

(i) Sketch a graph of electron kinetic energy versus distance (x) for the space between the photo cathode and the first dynode of PMT-B.

Total K.E increases linearly with distance.

(ii) Estimate the distance through which the electron is deflected in the y-direction as it travels from from the 1st dynode to the 2nd dynode of PMT-B. Assume that the force due to the magnetic field is small as compared to that due to the electric field but not necessarily always negligible such that, to a first approximation, acceleration a_{0x} in the x-direction is constant and the acceleration in the y direction increases directly proportional to time for electrons in PMT-B i.e. $a_y = j\,t$. Where j is a constant.

(ii) Consider one pair of electrodes first:

Almost constant acceleration due to electrostatic force in the x direction , $a_{0x} \approx \frac{eE}{m_e} = \frac{eV}{m_e d(n_d+1)}$

So $v_x = a_{0x}\mathrm{t} \approx \frac{eE}{m_e} t = \frac{eV}{m_e d(n_d+1)} t$

Acceleration due to electrostatic force in the y direction ,

$a_y \approx \frac{ev_x B}{m_e} = \frac{eB}{m_e} a_{0x}\mathrm{t} = \frac{e^2 VB}{m_e^2 d(n_d+1)} t = jt$

Distance d traveled in the x direction in time $t \approx d\sqrt{\frac{2m_e(n_d+1)}{eV}}$

Velocity in the y direction is (area under acceleration curve)

$$v_y \approx \frac{1}{2} jt^2$$

Distance travelled in the y direction

$$d_y \approx \frac{1}{6} jt^3 = Bd^2\sqrt{\frac{2e(n_d+1)}{9m_e V}} = 1.8mm$$

(iii) In fact the acceleration in the x-direction cannot be exactly constant for PMT-B. Considering conservation of energy, derive a slightly more accurate expression for the velocity of the electron in the x direction as a function of time. Hence or otherwise derive an expression to estimate the difference in transit time of the two cases.

(iii) The transit time for PMT-A must be the same as the one without magnetic field.

The total KE must be due only to the work done by the electric force, the magnetic force transfers some KE due to the velocity in the x direction to the velocity in the y direction So the actual velocity in the x-direction must be smaller than the constant velocity estimated in the previous section.

$$v^2 = v_x^2 + v_y^2$$

$v_x = \sqrt{v^2 - v_y^2} \approx \sqrt{(a_0\mathrm{t})^2 - \left(\frac{1}{2} jt^2\right)^2} \approx a_0\mathrm{t}\left(1 - \frac{1}{8}\frac{j^2}{a_0} t^2\right) = a_0\mathrm{t} - \frac{1}{8a_0} j^2 t^3$

Area under the curve of v_x is d so

$$d = \frac{1}{2}a_0 t_{tB}^2 - \frac{1}{32a_0}j^2 t_{tB}^4$$

$$j^2 t_{tB}^4 - 16a_0^2 t_{tB}^2 + 32 d a_0 = 0$$

Although it is possible to solve this quadratic equation, to get the time difference, it may be easier to rewrite

$$t_{tB}^2 = 2d/a_0 + \frac{1}{16a_0^2}j^2 t_{tB}^4$$

$$t_{tB}^2 = t_{tA}^2 + \frac{1}{16a_0^2}j^2 t_{tB}^4$$

$$t_{tB}^2 \approx t_{tA}^2 + \frac{1}{16a_0^2}j^2 t_{tA}^4$$

$$t_{tB} \approx t_{tA}\left(1 + \frac{1}{32a_0^2}j^2 t_{tA}^2\right)$$

And the difference is $\Delta t_t = \frac{1}{32a_0^2}j^2 t_{tA}^3$

[5 points]

<u>MCQ 7.</u>

For the situation in (e), if $V = -1000$ V , difference in transit time is _____.

(A) 0.00 ns

(B) 0.05 ns

(C) 0.37 ns

(D) 2.6ns

(E) 3.0ns

[MCQ]

(e) Discuss the impact of a fluctuating applied voltage of 1% on the gain and transit time of the photomultiplier at $V = -1000$ V.

There is a big variation in gain,~7% but a small variation in transit time ~0.5%

[2 points]

5. For this question, consider the same photomultiplier in the previous question but this time there may be appreciable amount of current flowing. Do not consider the effects of magnetic field.

MCQ 8.

A steady stream of photons is directed towards the photocathode. If the electrical current flowing **through** the anode from R_a is 1000 μA , and the gain is 286, the current flowing **through** the photo-cathode terminal is _____________ and the current flowing through the whole surface surrounding the photomultiplier vacuum tube including the current through all the terminals is _____________.

A. −1 μA, 0 μA

B. −1 μA, 999 μA

C. 1 μA, 1001 μA

D. 1000μA, 0 μA

E. −1000 μA , 0 μA

(a) Sketch graphs of (i) electron speed, (ii) current versus distance for the space between the last dynode and the anode. Label the lines with an equation or sentence describing its shape.

(i) proportional to $\sqrt{d}$.

(ii) constant.

[2 points]

(b) The photomultiplier is operated at the same voltage as in part 5(a) and not at the voltages mentioned in question 4. Describe in detail the current and voltage related to the resistors in the circuit for the case (i) when no light falls on the photo cathode and the case (ii) when light falls on the photo-cathode such that the electrical current flowing into the anode from R_a is 2.86 mA. Include estimates of numerical values where possible and state clearly the assumptions used in arriving at the estimates.

(i)

Same voltage across all the resistors. Estimate using $V_d \approx 12.5e^{\ln\frac{G}{\alpha}} = 34V$, $V = 268\ V$Assume that current flowing through the PMT is negligible to get numerical answer even for 1mA.

same current through all the resistors $I = \frac{V_d}{R} = 34mA$

(ii)

Current through each resistor decreases going from cathode to anode since some current flows through the PMT terminals

Voltage across each resistor decreases going from cathode to anode since current decreases. But sum of voltage must equal V

[6 points]

(c) Discuss the impact, if any, of light level on the gain of the photomultiplier.

(c) The gain must decrease with light level and the transit time must increase with light level.

Consider the case of 1 dynode, then the voltage b/w PC and dynode increases whereas the voltage b/w dynode and anode must decrease. Since gain only happens b/w PC and dynode, the gain must increase. However for large number of dynodes, voltage across some dynodes will increase, while voltage across other dynodes will decrease. So there may not be a clear increase. If we consider further that an increase in voltage by dV of one stage is matched by a –dV change in voltage in another stage, then the overall gain considering these two stages together is proportional to $(V + dV)(V - dV)$ which will give rise to an overall decrease in the gain.

[2 points]

6. In the past a popular use for roof space is for solar water heating. More recently, it has become increasingly popular to make use of roof space for photovoltaic solar panels. This question investigates some of the physics of a proposed hybrid solar-panel water heating system.

(a) A solar panel with dimensions $L \times W \times D$. It is placed in the sun light with intensity I and absorbs a fraction α of the light. The surrounding temperature is T_r. Derive an expression for the equilibrium temperature T_e. Assume that the average emissivity of the solar panel is ε and the solar panel is not connected electrically to anything. Consider only heat transfer by radiation.

This is the situation when the solar panel is not connected to anything and just left in the sun

By COE,

$$P_{out} = P_{in}$$

$$P_{in} = \alpha ILW + \varepsilon\sigma 2(LW + (L+W)D)T_r^4$$

The 1st term is for radiation from the sun. The second term is radiation from the surroundings. Note that the area used in the two terms are different.

$$P_{out} = \varepsilon\sigma 2(LW + (L+W)D)T_e^4$$

There is only one term here. Based on the question to consider only radiation.

$$T_e^4 = \frac{\alpha ILW}{2\varepsilon\sigma(LW + (L+W)D)} + T_r^4$$

$$T_e^4 = \frac{\alpha I}{2\varepsilon\sigma\left(1 + (\frac{1}{L} + \frac{1}{W}\right) D)} + T_r^4$$

[2 points]

MCQ 9.

Suppose the solar panel is now connected up to a load resistor such that $P_e = 250W$ of electrical power is transferred to the load. Assume that the load is far away from the solar panel and not thermally connected. How would the equilibrium temperature change?

(A) Same as before because the load is not thermally connected.

(B) Higher because the solar panel needs to generate the 250W of power.

(C) Lower because the solar panel transfers 250W of "electrical" power to the load.

(D) Higher because the solar panel absorbs more heat to generate the 250W of power.

(E) Lower because the solar panel transfer 250W of "heat" power to the load.

(b) Suppose that the back of the solar panel $L = 1.64$ m , $W = 0.992$ m $D = 0.04$ m, $\alpha = 0.8$, sun light intensity $I = 1$ kWm^{-2}, $T_r = 25$ °C, $\varepsilon = 0.8$, is thermally connected to a single peltier cooler powered by the 250W from solar panel itself. Assume that the efficiency of the electronics driving the peltier heat pump is $\beta = 0.9$, The peltier heat pump has dimensions $L_p = L$, $W_p = W$ $D_p = 0.0037$ m. Assume that the heat pump has the ideal coefficients

$$COP_{cooling} = \frac{T_{cold}}{T_{hot}-T_{cold}} = \frac{Power\ pumped\ from\ cold\ side}{electrical\ power},$$

$$COP_{heating} = \frac{T_{hot}}{T_{hot}-T_{cold}} = \frac{Power\ pumped\ to\ hotside}{electrical\ power},$$

and that it has thermal conductivity $k = 2.0\ \mathrm{Wm^{-1}K^{-1}}$ and that the internal resistance is negligible. Assume that there is perfect thermal contact between the cold side of the heat pump and the back surface of the solar panel, i.e. $T_{e1} = T_{cold}$ and the whole solar panel is at one temperature. The hot side of the heat pump is connected to a water flow which maintains $T_{hot} = 60°\mathrm{C}$. Derive an equation in symbols involving the new equilibrium temperature T_{e1}. (Hint it may not be necessary to simplify the expression)

$$P_{in} = \alpha ILW + \varepsilon\sigma(LW + 2(L+W)D)T_r^4 + kLW(T_{hot} - T_{e1})$$

$$P_{out} = \varepsilon\sigma(LW + 2(L+W)D)T_{e1}^4 + P_e COP_{cooling}$$

$$P_{out} = \varepsilon\sigma(LW + 2(L+W)D)T_{e1}^4 + P_e \frac{T_{e1}}{T_{hot} - T_{e1}}$$

$$\alpha ILW + \varepsilon\sigma(LW + 2(L+W)D)T_r^4 + kLW(T_{hot} - T_{e1}) = \varepsilon\sigma(LW + 2(L+W)D)T_{e1}^4 + P_e \frac{T_{e1}}{T_{hot} - T_{e1}}$$

[4 points]

MCQ 10.

The equilibrium temperature of the solar panel in the situation described in part (b) is ______.

(A) 15°C

(B) 25°C

(C) 35°C

(D) 45°C

(E) 55°C

(c) Calculate the volume flow rate of water if the water temperature changes by 10K between the entry and exit of the water flow system.

$$\text{Heat flow} = P_e \frac{T_{hot}}{T_{hot} - T_{e1}} \approx 3500W$$

$$\text{Water flow} = \frac{P_e \frac{T_{hot}}{T_{hot} - T_{e1}}}{c_w \Delta T} = 80gs^{-1}$$

[2 points]

(d) In reality it is difficult to get perfect thermal contact between the cold side of the heat pump and the solar panel and also between the hot side of the heat pump with the water flow system. For simplicity, this may be modelled as a thick thermal conductor between the solar panel and the hot side as well as a thick thermal conductor between the hot side of the heat pump and the water system. Discuss the impact and physics of this on trying to maximize temperature of the hot side of the heat pump and maximum amount of heat which can be pumped.

The larger the amount of heat being pumped implies a greater temperature difference across the conductor. This implies that the temperature difference needs to be more extreme. Therefore the COP decreases. In the extreme case if the heating COP is very close to one. It would be simpler to make use of the generated electrical power as ohmic heating.

[2 points]

----End of Paper---