Tutorial 3B (Dynamics) Worked Solutions

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The following only contains the worked solutions to discussion questions in part 2 of Tutorial 3 (Conservation of Linear Momentum/Collisions). I have included some appendices to supplement the textbook.

For Self-Review questions, please refer to the other uploaded document.

[D12] **Solution:** A

Since the trolleys stick together on impact, $\vec{v_x} = \vec{v_y}$. The momentum of the entire system $\vec{p} = 0$ since it was stationary before the trolleys were released. Hence the final speed of Trolley Y $v_y = p/(M+2M) = 0$.

Note that the force of the elastic band on the trolleys is an internal force and hence cannot change the momentum of the entire system.

[D13] **Solution:** C

Total momentum before the collision $= 6 \times 5.0 + 10 \times (-3.0) = 0$

Each trolleys is decelerated to rest. Average force on each trolley = $\dfrac{\Delta P}{\Delta t} = \dfrac{30}{0.20} = 150.$

[D14] (a) **Solution:** $721.4\,\mathrm{m\,s^{-1}}$

All the momentum of the $1.8\,\mathrm{kg}$ block and the embedded bullet comes from the momentum of the bullet after it passes through the first block:

$$\begin{aligned} v_{bullet} &= \frac{p}{m_{bullet}} \\ &= \frac{\left(1.8 + 3.5 \times 10^{-3}\right) \times 1.4}{3.5 \times 10^{-3}} \\ &= 721.4\,\mathrm{m\,s^{-1}} \end{aligned}$$

(b) **Solution:** $937.4\,\mathrm{m\,s^{-1}}$ All the momentum of the 1.8 kg block, the embedded bullet and the 1.2 kg block comes from the original momentum of the bullet:

$$\begin{aligned} u_{bullet} &= \frac{p_{total}}{m_{bullet}} \\ &= \frac{\left(1.8 + 3.5 \times 10^{-3}\right)(1.4) + 1.2(0.63)}{3.5 \times 10^{-3}} \\ &= 937.4\,\mathrm{m\,s^{-1}} \end{aligned}$$

[D15] (a) i. The linear momentum of a body is the product of its mass and its velocity. Mathematically, linear momentum, $\vec{p} = m\vec{v}$.

ii. The total linear momentum of a system is conserved if no net external force acts on the system. Mathematically:

$$\sum_i m_i u_i = \sum_i m_i v_i \tag{1}$$

(b) i. Their relative speed of approach is equal to the relative speed of separation. Mathematically:

$$u_1 - u_2 = v_2 - v_1 \tag{2}$$

ii. The direction of motion before and after the collision are along the same line of motion.

(c) We define right as positive.

i. **Solution:** $2285\,\mathrm{m\,s^{-1}}$ Since the collision is elastic, the relative speed of approach is equal to the relative speed of separation:

$$\begin{aligned} v_{sep} &= u_H - u_O \\ &= 1.88 \times 10^3 - (-405) = 2285\,\mathrm{m\,s^{-1}} \end{aligned}$$

ii. $m_H u_H + m_O u_O = m_H v_H + m_O v_O$

iii.

$$\begin{aligned} v_O - v_H &= 2285 \\ v_O &= 2285 + v_H \end{aligned} \tag{3}$$

Substitute (3) into the equation in (c)ii:

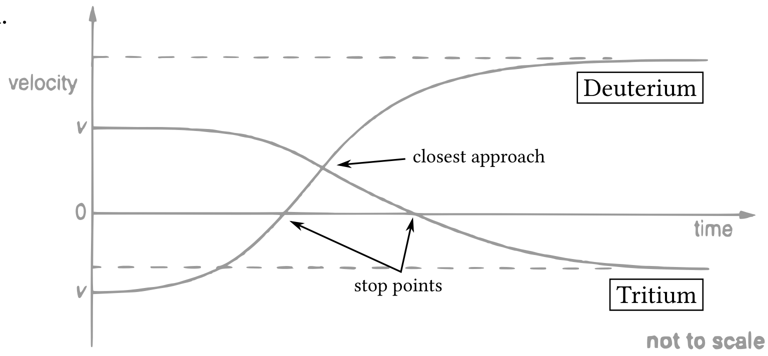
$$\begin{aligned} m_H v_H + m_O(2285 + v_H) &= (2.00)\left(1.88 \times 10^3\right) + (32.0)(-405) \\ v_H(m_H + m_O) &= -9200 - (2285)(32.0) \\ v_H &= \frac{-82320}{2.00 + 32.0} = \underline{-2.42 \times 10^3\,\mathrm{m\,s^{-1}}} \\ v_O &= 2285 + v_H = \underline{-136\,\mathrm{m\,s^{-1}}} \end{aligned}$$

[D16] (a) The total momentum of the 2 nuclei is non-zero $\left(\sum p = 3mv - 2mv = mv\right)$

(b)

$$p = 5mv_s$$
$$v_s = \frac{\cancel{m}v}{5\cancel{m}} = {}^1\!/_5 v$$

(c) i.



ii. We define right as positive.

$$p = (3\cancel{m})v_t + (2\cancel{m})v_d = \cancel{m}v$$
$$2v_d + 3v_t = v \tag{4}$$
$$(u_t - u_d) = \left[v - (-v)\right] = 2v = v_d - v_t$$
$$v_d - v_t = 2v \tag{5}$$

(4) − 2 × (5) :

$$v_t = -{}^3\!/_5 v = -0.6v$$
$$v_d = \left[2 + \left(-{}^3\!/_5\right)\right] v = 1.4v$$

[D17] (a) By the law of conservation of linear momentum, the total momentum of the system must be conserved. Since, by exhausting the gas, the gas from the rocket-gas system gains momentum, the toy car must gain momentum in the opposite direction in order to conserve total momentum.

Since $F = \frac{\mathrm{d}p}{\mathrm{d}t}$, and there is a change in momentum, the rocket produces a forward force on the car.

(b) i. [1] $a = \text{grad} = \dfrac{16.0 - 2.8}{4.90 - 0.00} = 2.7\,\mathrm{m\,s^{-2}}$

[2]

$$\sum F = ma = F_{rocket} - F_r$$
$$F_r = F_{rocket} - ma$$
$$= 4.6 - \left(440 \times 10^{-3}\right)(2.7)$$
$$= 3.4\,\mathrm{N}$$

(c) The acceleration of the toy car at $t = 0$ is approximately $a = 8.4\,\mathrm{m\,s^{-2}}$, which is less than $g = 9.81\,\mathrm{m\,s^{-2}}$. Thus, it can be deduced that the rocket engine will not be able to produce a force large enough for the toy car to overcome the forces of gravity and travel vertically upwards. Thus, this suggestion is not very feasible.

Appendices

Appendix A Solving Collision Problems

Step 0 Illustrate if no diagram is provided.

Step 1 Set up the PCOM equation, choosing an appropriate positive direction. **IF** collision is **perfectly inelastic**, use one variable to represent the final velocities of both bodies (i.e. let $v_1 = v_2 = v$)

Step 2 **IF** collision is elastic, set up the Relative Speed of Approach/Separation equation. If you aren't very sure, apply Conservation of Kinetic Energy.

Step 3 Solve for the unknowns, e.g. mass, initial or final velocities.

You need as many independent equations as there are unknowns to solve them uniquely[1].

Appendix B Coefficient of Restitution

The elasticity of a collision is quantified by a number called coefficient of restitution, ε, defined as the ratio of the speed of separation to the speed of approach:

$$\varepsilon = \frac{|\vec{v_2} - \vec{v_1}|}{|\vec{u_2} - \vec{u_1}|} \tag{6}$$

A perfectly elastic collision has a coefficient of restitution $\varepsilon = 1$ (e.g. two diamonds bouncing off each other).

A perfectly plastic, or inelastic, collision has $\varepsilon = 0$ (e.g. two lumps of clay that don't bounce at all, but stick together.).

So the coefficient of restitution will always be between zero and one.

ε is determined by several factors, like the material of the colliding bodies, and *assumed* to be independent of speed.

[1]If you are curious, for systems of linear equations, google linear independence, general solution to linear systems.

Appendix C Newton's Law in Car Safety

Using Newton's laws, discuss how each of the following features in vehicles increases the safety of its passengers:

1. Seat Belts

   Seat belt exerts a retardation force on the passenger, without which he would have continued in his state of uniform in a straight line into the windscreen (N1L).

   The vehicle is able to decelerate rapidly because of the braking system leads to a frictional force between the road surface and the tyres. Unfortunately, this frictional force acts on the vehicle, not the passenger. Without the seat belt, the only retardation force would be the frictional force between the seat and the passenger's bum. But this force is limited (recall: $f \leq \mu_s N$). If the vehicle decelerates too fast, the frictional force is not large enough to decelerate the passenger at the same rate as the vehicle.

   Be very clear that when passengers crash into the windscreen, it was due to their inertia. It was not due to some mysterious "forward force".

2. Head Restraints

   If the car is hit from the rear, the restraints provides a forward force on the passenger's head so that the head is accelerated at the same rate as the rest of his body, so that he does not suffer from whiplash (neck injury).

3. Air Bags

   Air bags exerts a relatively small force over a relatively long duration of time, safely bringing the passenger to rest by reducing the passenger's momentum at a relatively slower rate (N2L).