SPhO 2018

Paper 1 (2 hour) Answer All Questions

1. A particles X is projected with speed V in a direction which makes an angle of 30° from the horizontal. When this particle reaches the highest point of its trajectory, another particle Y is dropped from the roof of a tall building. The two particles collide at the base of the building. The particle Y takes a time of 0.17 s to fall a 5.0 m tall window in the building. The base of the window is 50.0 m above the ground. Ignore the effect of air resistance, find

a. the height of the building [4]

b. the value of V [4]

c. the distance of the point of projection of X from the foot of the building [2]

2. A horizontal platform vibrates with simple harmonic motion in the horizontal direction with a period of 2.0 s. A small object placed on the platform starts to slide when the amplitude of vibration reaches 0.4 m.

a. Calculate the coefficient of static friction between the object and the platform. [5]

b. The platform now excutes vertical simple harmonic motion with a period of 1.5 s. What is the maximum amplitdue of the motion if the object were to be in contact with the plate throughout the motion?[5]

3. Consider a gas with molecular mass m in a constant gravitational field $\vec{g}$.

a. Write down an equation relating a small change in pressure ΔP over a small change in height Δz. [1]

b. Show that, if the temperature T is constant, the pressure of a gas $P(z)$, in a uniform gravitational field decreases with height, z according to the expression $P(z) = P(0)e^{-\frac{mgz}{KT}}$. [9]

4. A copper wire with mass m is stretched between two fixed points, distance l apart and a tension, F_T is applied to it. When the copper wire is vibrating in the fundamental mode together with a 256-Hz tuning fork, a beat of frequency 5 Hz is observed. The copper wire is removed and a brass (which is an alloy made of copper and zinc) wire, with the same length and diameter, is stretched between the same two fixed points. The same tension is again applied to the brass wire. It is found that in this case, the brass wire, vibrating in the fundamental mode, resonates with the 256-Hz tuning fork when the two are vibrated together.

[Densities: copper: 8940 kg m^{-3}, zinc: 7140 kg m^{-3}]

a. State an equation for the speed of the wave on the string in terms of m, l and F_T only. [1]

b. Determine the percentage by mass of zinc in the brass wire. State any assumptions you make in your calculation. [9]

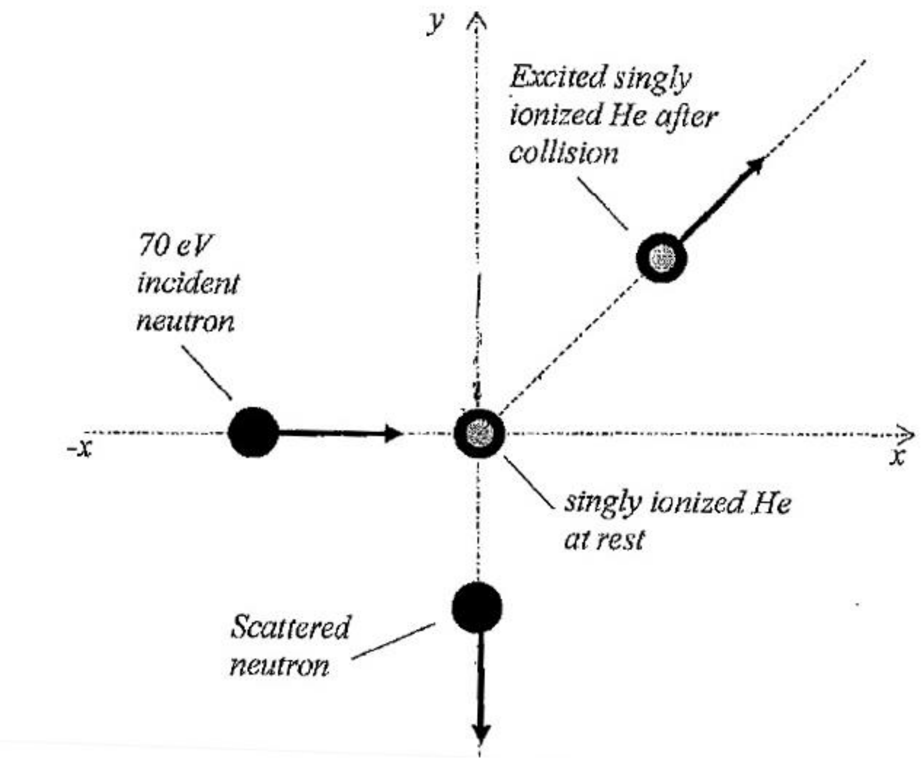
5. A 5.00 ml solution was injected into the bloodstream of a patient. The solution contains radioactive iodine ^{131}I wih a half-life of 8.025 days at a concentration of $1.00 \times 10^{-10}\ kg\,m^{-3}$. The activity of a 5.00 ml blood sample taken 24 hours later is found to be 3171 counts in 30 minutes.

a. Calculate the decay constant for ^{131}I. [1]

b. Calculate the total volume of blood in the patient's body given by these results. State any assumptions you make in your calculation. [9]

Paper 2 (2 hour) Answer All Questions

1. A singly ionized helium, with electron in its ground state i.e. n = 1, is at rest at position O. A neutron of kinetic energy of 68.2 eV is moving from the left towards O and collides inelastically with the singly ionized helium. The neutron is scattered at an angle of 90° with respect to its original direction of motion i.e. towards y-axis. The singly ionized helium after collision is excited and moves at shown. It may be noted that the mass of singly ionized helium is four times the mass of the neutron and the energy levels in the ionized helium atom are given by $E_n = -\frac{54.4}{n^2}$ eV.



a. Find the allowed values of the kinetic energy of the scattered neutron and the kinetic energy of the singly ionized helium after collision. [8]

b. If the singly ionized helium get de-excited subsequently by emitting radiation, find the shortest and longest wavelengths of the emitted radiation. For this part assume that the momentum of the emitted photon is negligible. [4]

c. When an atom emits a photon in a transition from the quantum state with energy E_i to the quantum state with energy E_f, the energy of the photon is not exactly equal to $E_i - E_f$. This is because the atom must recoil and thus some of the transition energy becomes the kinetic energy of recoil of the atom. Estimate the percentage of this transition energy, $E_i - E_f$, that becomes the recoil kinetic energy of the recoiling atom for the shortest wavelength case of the previous part. [6]

d. Estimate also the change in speed of the helium ion due to this transition. [2]

2. A particle of mass 100 g is dropped from a great height and falls vertically downwards. The force due to air resistance is $-kv$ where v is the speed of the particle and k is a constant having the value $1.09 \times 10^{-2}\ kg\, s^{-1}$.

a. What is the terminal velocity of the particle during the fall? [1]

b. How much time does it take for the particle to achieve 99% of the terminal velocity? [4]

c. How far has the particle fallen when its velocity achieves 99% of the terminal velocity? [4]

3. If a charged particle with charge q, mass m, having velocity $\vec{v} = (v_x, v_y, 0)$ travels in a magnetic field $\vec{B} = (0, 0, B_z)$ then a Lorentz force $\vec{F} = (F_x, F_y, 0)$ acts on the particle. As a result, the particle moves in a circular orbit. Assume a non-relativistic situation and that radiation is negligible.

a. Derive an expression for T, the time taken for the charged particle to make one complete revolution in the orbit in terms of m, q, B_z. [2]

b. Write down expressions for the acceleration of the particle in the x, y and z direction, i.e. $a_x(t)$, $a_y(t)$, $a_z(t)$ in terms of $v_x(t)$, $v_y(t)$, $v_z(t)$ and T. [3]

c. To compute the trajectory of the particle, one can estimate and compute the velocity of the particle after a short time step Δt. For method 1, to a first approximation, assume that the acceleration in the x and y directions are constant during this short period of time such that $v_x(t+\Delta t) = v_x(t) + a_x(t)\Delta t$. Write down equations which will allow $x(t+\Delta t)$ and $y(t+\Delta t)$ to be calculated from $x(t)$ and $y(t)$ in terms of T, B_z, q, m and Δt. [3]

d. Derive an equation for the change in kinetic energy ΔE_k after time Δt in terms of T, B_z, q, m and Δt. [3]

e. Calculate the ratio $\frac{E_k'}{E_{ko}}$ where E_k' is the kinetic energy at the end of an orbit and E_{ko} is the initial kinetic energy if the time step Δt is $0.01\,T$. [2]

f. Let us consider another way to compute the trajectory, method 2. We still assume that the acceleration in the x and y directions are constant during this short period of time Δt. However for $t > 2\Delta t$, in our computation, we will approximate by making use of the average velocity from time $t - \Delta t$ to time $t + \Delta t$ when calculating the Lorentz force and work out the acceleration at time t as the change in average velocity $t - \Delta t$ to t and t to $t + \Delta t$ i.e.

$$v_x(t) = \frac{x(t+\Delta t) - x(t-\Delta t)}{2\Delta t}$$

$$a_x(t) = \frac{\frac{x(t+\Delta t) - x(t)}{\Delta t} - \frac{x(t) - x(t-\Delta t)}{\Delta t}}{\Delta t}$$

Obtain equations to calculate $x(t+\Delta t)$ and $y(t+\Delta t)$ from $x(t)$, $x(t-\Delta t)$ and $y(t)$, $y(t-\Delta t)$ in terms of B_z, q, m and Δt. [6]

g. Derive an equation for the change in kinetic energy ΔE_k after time Δt in terms of B_z, q, m and Δt introduced by computing using method 2. [4]

h. Sketch a diagram to illustrate the difference in the trajectory of the particle as computed using method 1 and method 2. Also illustrate the difference from the real trajectory. [2]

4. A rocket of proper length 600 m is moving directly away from the earth with uniform velocity. A radar pulse is sent out from the earth and is reflected from the reflectors at the back end and the front end of the rocket. The first reflected radar pulse is received back at the base 5.00 minutes after emission and the second reflected pulse is received 12.0 μs later.

a. Calculate the distance of the rocket from the earth at the instant the outgoing radar pulse hits the back end reflector. [2]

b. Calculate the velocity of the rocket relative to the earth. [5]

c. Calculate the time interval between the reflections at the back end and front end of the rocket measured in the inertial frame of the rocket. [3]

d. Explain why the time interval between the reflections in the two frames (i.e. the earth frame and the rocket frame) are not related by the time dilation formula. [3]