Chapter 6

Circular Motion

The Singapore Flyer is currently the tallest Ferris wheel in the world. Described by its operators as an observation wheel, it reaches 42 stories high, with a total height of 165 m (541 ft), and is 5 m (16 ft) taller than the Star of Nanchang and 30 m (98 ft) taller than the London Eye.

Chapter 6: MOTION IN A CIRCLE

H2 Physics Syllabus 9646

Content

- Kinematics of uniform circular motion
- Centripetal acceleration
- Centripetal force

Learning Outcomes

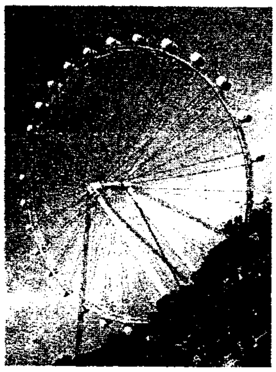
Candidates should be able to:

(a) Express angular displacement in radians.
(b) Understand and use the concept of angular velocity to solve problems.
(c) Recall and use $v = r\omega$ to solve problems.
(d) Describe qualitatively motion in a curved path due to a perpendicular force, and understand the centripetal acceleration in the case of uniform motion in a circle.
(e) Recall and use centripetal acceleration $a = r\omega^2$, $a = v^2/r$ to solve problems.
(f) Recall and use centripetal force $F = mr\omega^2$, $F = mv^2/r$ to solve problems.

Further Readings / References

1) University Physics with Modern Physics, 11th Ed., *Young & Freedman, Chapter 5, Pg 181.*
2) Advanced Level Physics, 6th Ed., *Nelkon & Parker, Chapter 2, Pg 48.*
3) College Physics, 8th Ed., *Young & Geller, Chapter 6, Pg 161.*
4) Physics for Scientists and Engineers with Modern Physics, 6th Ed., *Serway Jewett, Chapter 6, Pg 150.*

6.1 Introduction



Circular motion is a common occurrence in our daily lives, e.g. the bicycle rounding a bend, the capsules of the Ferris wheel, and children on a merry-go-round.

In astronomy, we know that the moon circles the Earth, which circles the Sun, which circles the centre of the Milky Way. (Note: Strictly speaking, these are elliptical paths, but let's keep things simple for the time being.)

Have you ever wondered:
- Why must a cyclist lean into a turn?
- Why must an airplane tilt when executing a turn?
- Why don't people in the roller coaster fall out at the top of the loop? (No, it is not because of the belts)

The exploration of the fascinating world of circular motion begins here.

A Short Revision on basic ideas of Newton's Laws of Motion

Newton's 1st law:
Describes the motion of any object not subjected to a net external force, specifically that
- an object at rest remains at rest, or
- an object continues with constant speed in a straight line.

In the presence of a net external force, a constant mass experiences an acceleration governed by *Newton's 2nd Law*:

$$\bar{a} = \frac{\bar{F}_{net}}{m}$$

Note:
- Forces which are parallel to the motion result in its motion speeding up or slowing down.
- Forces which are perpendicular to its motion do not affect its speed, but instead changes its direction of motion.

6.2 Kinematics of Circular Motion

6.2.1 Circular Measure

The S.I. unit for angles is not the degree but the radian. Angle measured in radian is actually the ratio of two lengths: arc length and radius. The *arc length s* is the distance travelled along the circular path, and the angle θ is said to *subtend* the arc length at the circle centre (Fig. 1).

Hence, note that θ is actually a dimensionless quantity since it is a ratio of 2 lengths.

$$\theta \text{ (in rad)} = \frac{\text{arc length}}{\text{radius}} = \frac{s}{r}$$

$$s = r\theta$$

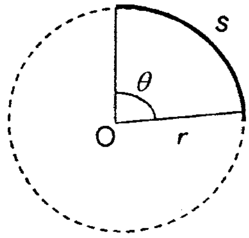


Figure 1

One radian is the angle subtended at the centre of the circle by an arc equal in length to the radius of the circle.

If circumference of a circle $s = 2\pi r$, $\theta = s/r = 2\pi$ rad .
Since for a complete circle, the angle subtended is 360°,

$$360° \equiv 2\pi \text{ rad}$$

We can deduce that

$$1 \text{ radian} = \left(\frac{360}{2\pi}\right)° = 57.3°$$

Conversion between degrees and radians:

$$180° \equiv \pi \text{ rad}$$

$$Z \text{ (degrees)} \equiv Z \times \frac{\pi}{180} \text{ (radians)}$$

6.2.2 Angular Displacement ($\Delta\theta$) vector

Angular displacement $\Delta\theta$ is the angle an object turns about a fixed point. In circular motion, the fixed point is taken to be the centre of the circle (Fig. 2).

$$\Delta\theta = \theta - \theta_0$$

If $\theta_0 = 0$, then $\Delta\theta = \theta$.

S.I. unit of $\Delta\theta$: radian (rad)

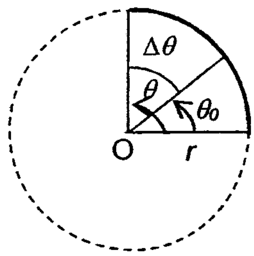


Figure 2

6.2.3 Angular Velocity (ω)

The description of circular motion in angular form is analogous to the description of linear motion. Angular velocity ω (pronounced as *omega*) is defined as the rate of change of angular displacement.

$$\text{Average angular velocity, } \bar{\omega} = \frac{\text{Angular displacement}}{\text{Elapsed time}}$$

$$\bar{\omega} = \frac{\theta - \theta_0}{t - t_0} = \frac{\Delta\theta}{\Delta t}$$

The *instantaneous angular velocity* ω is the angular velocity that exists at any given instant. Analogous to instantaneous linear velocity,

$$\text{Instantaneous angular velocity, } \quad \omega = \frac{d\theta}{dt}$$

If an object has a **constant** angular velocity, the instantaneous value and the average value are **the same** (i.e. $\omega = \bar{\omega}$).

Note:
- Magnitude of the instantaneous angular velocity is called the *instantaneous angular speed.*
- S.I. unit of ω : rad s^{-1}

6.2.4 Relationship between Tangential Speed *v* and Angular Speed ω

A particle moving in a circle has an instantaneous velocity tangential to its circular path. For a **constant** angular speed, the particle's **orbital or *tangential speed*** *v* is also constant. (*v* is also known as ***linear speed***.)

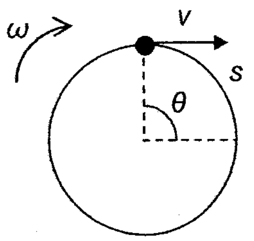


Figure 3

Since $s = r\theta = r(\omega t)$. Thus $\quad s = r\omega t$
The arc length, or distance, *s* is also given by $s = vt$

Thus,

$$\text{tangential speed, } v = r\omega$$

Alternatively, $\quad s = r\theta$

Taking derivative with respect to time, $\quad \dfrac{ds}{dt} = r\dfrac{d\theta}{dt} \quad \rightarrow \quad v = r\omega$

Unit of linear/orbital speed: m s^{-1}

6.2.5 Period (T) and Frequency (f)

Period (T) is the time it takes for an object in circular motion to make one complete revolution, or cycle.

Frequency (f) is the number of revolutions, or cycles, made per unit time. The unit of frequency is s^{-1}, which is called the *hertz* (Hz) in the SI.

$$\text{frequency, } f = \frac{1}{T}$$

Since an angular displacement of 2π rad is travelled in 1 period,

$$\textit{angular velocity, } \omega = \frac{2\pi}{T} = 2\pi f$$

Example 1

A Compact Disc player is spinning a standard CD (120 mm diameter) at 300 revolutions per minute (RPM). Determine

i) the frequency of the rotation (in revolutions per second)
ii) the period of the rotation
iii) the angular speed

Example 2

Dishes are placed on lazy susan at reunion dinner during Chinese New Year. Which dishes will have higher speed, those in the outer circle or those in the inner circle? Why?

6.3 Uniform Circular Motion

Uniform circular motion is the motion of an object travelling at a **constant** (uniform) speed in a circular path.

An object moving in a uniform circular motion (Fig. 4) has a *constant angular velocity* (ω). The ***magnitude*** of the velocity vector (or the linear speed, v) is ***constant*** but the ***direction*** of the velocity vector is ***changing***. Thus, the *linear velocity vector is continuously changing* ($\boldsymbol{v_1}$, $\boldsymbol{v_2}$ & $\boldsymbol{v_3}$) as the object moves around the circle. Since there is a change in the velocity vector with respect to time, the object should be under acceleration.

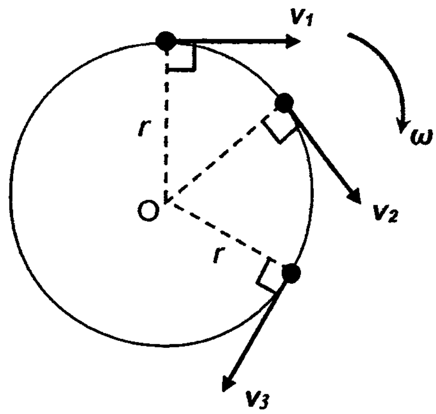


Figure 4

Derivation of the equation of centripetal acceleration:

Consider a particle moving in a circle of radius r with uniform angular speed (Fig. 5a). At one instant the particle is at A, and its instantaneous velocity is v in the direction AA'.

A very short time later δt later, the particle has moved to B, a distance $r\delta\theta$ along the arc, where $\delta\theta$ is a small angle and its velocity is v in the direction BB' at this time.

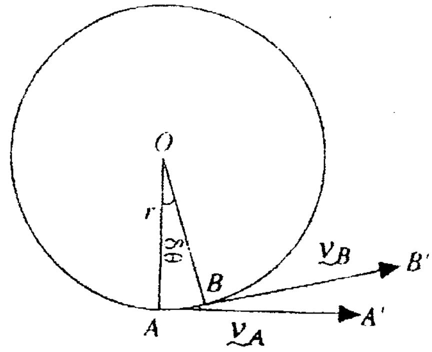


Figure 5a

With reference to the vector triangle in Fig. 5b, the arc length can be approximated to a chord.
For small angle $\delta\theta$,

$$\delta v \approx v\delta\theta$$
$$\delta v/\delta t \approx v\delta\theta/\delta t$$
$$\frac{dv}{dt} = v\frac{d\theta}{dt}$$
$$a_c = v\omega$$

Similarly,

$$a_c = (r\omega)\omega = r\omega^2$$
$$a_c = v(\frac{v}{r}) = \frac{v^2}{r}$$

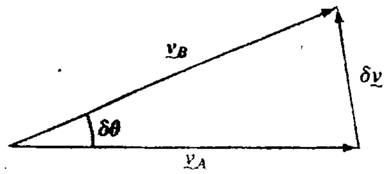


Figure 5b

As δt (or $\delta\theta$) (Fig. 5b) becomes smaller, the angle between δv and either v_A or v_B will tend to 90°. Since either v_A or v_B are the tangents to the circular path, this means δv will point towards the centre of the circular path. Similarly, its acceleration will point towards the centre of the circular path because δv and its acceleration are in the same direction. This type of acceleration in uniform circular motion is called ***centripetal acceleration***, and the net force which is causing the circular motion is called ***centripetal force***.

The centripetal acceleration is directed **radially inward** towards the centre of the circular path. The direction of the centripetal acceleration is continuously changing.

For an object in ***uniform*** *circular motion*, there is **no** acceleration component in the tangential direction (i.e. angular acceleration), or else the magnitude of the velocity vector (tangential or linear speed) would change.

Hence,

$$\text{centripetal acceleration, } a_c = \frac{v^2}{r} = r\omega^2 = v\omega$$

Conceptual Questions:

(a) Is an object travelling at non-uniform speed accelerating?
Yes, acceleration involves any change in velocity (magnitude + direction), and the speed is changing in this case.

(b) Can an object travelling at constant speed be accelerating?
Yes, although its speed may be constant, its direction of motion, thus its velocity, may change, hence there must be an acceleration.

Centripetal Force (F_c)

To provide acceleration, there must be a net force. Thus to produce a centripetal (inward) acceleration, there must be a resultant force towards the centre of the circular motion. As the resultant force is centre-seeking, we call it the centripetal force.

From Newton's second law ($\sum F = ma$),

$$F_c = ma_c = \frac{mv^2}{r} = mr\omega^2 = mv\omega$$, and it is **always directed toward the centre** of the circle.

Note: In general, when a force is continuously applied **at an angle of 90° to the direction of motion** (as is centripetal force), ***only the direction*** of the velocity changes.

<u>Conceptual Question:</u> Is there work done by a centripetal force?

A centripetal force does no work because there is no displacement in the direction of the centripetal force. The centripetal force is always perpendicular to the direction of motion (displacement).

Note: **For uniform circular motion,**

- Centripetal force **does not** denote a new and separate force created by nature.
- Centripetal force is provided by the **net force** pointing toward the centre of the circular path, and this net force is the **vector sum of all the force components that point along the radial direction**.

6.4 Problem solving for Circular Motion

6.4.1 General strategy for solving circular motion problems

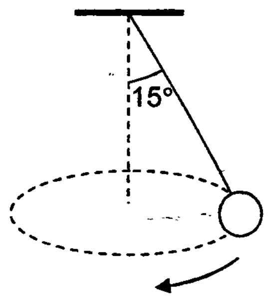
1. Identify the object undergoing circular motion (and the relevant known quantities such as m, v, r, ω...).
2. Draw a **free body diagram** of the object, carefully identifying the individual forces *experienced by* the object.
3. **Resolve forces** into relevant perpendicular "directions".
4. Apply Newton's 2nd Law and solve.

6.4.2 Horizontal Circular Motion

Example 3 - Conical pendulum

In a conical pendulum system, a small pendulum bob of mass 0.50 kg is rotating in a fixed horizontal plane. The string is 30 cm long and makes an angle of 15° to the vertical.

Calculate the (a) tension in the string,
(b) linear speed of the bob, and
(c) period of oscillation of the bob.



Example 4

Explain with the aid of a diagram, why the mass at the end of a light inelastic string cannot be whirled in uniform circular motion in such a way that the string is horizontal.

Example 5 - Car going around a bend

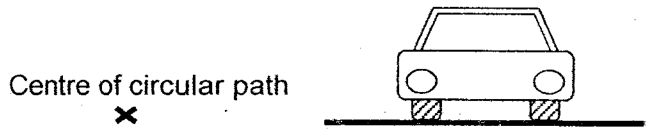
(a) A bend in the road has a 50 m radius of curvature. A car of mass 600 kg takes the bend at 45 km h^{-1}.

i) What is the centripetal acceleration of the car?

ii) What is the centripetal force experienced by the car and what provides it?

iii) If the coefficient of friction, μ, between the tires and the road is 0.50, what is the maximum speed at which the car can turn this bend safely? (Recall: frictional force = μN)

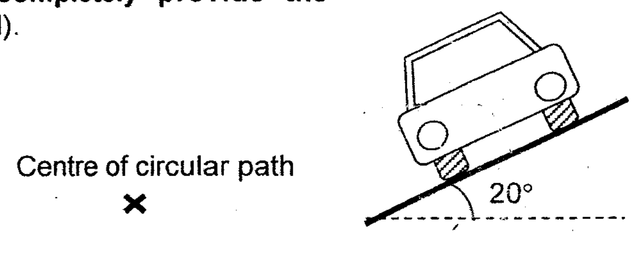
iv) What will happen to the car if the driver decided to take the bend at 60 km h^{-1} instead?



(b) When the car negotiates a corner on horizontal ground, the frictional force between the tyres and the ground is the only force providing the centripetal force. As there is a limit to this frictional force for a particular road surface, there is a maximum speed which the car can make the turn safely, above which skidding will occur.

Hence, some corners (especially at race-tracks) have raised embankments to increase the maximum speed at which a vehicle can take the corner than if on a level road. It does so by making the **normal contact force** contribute a component to the centripetal force.

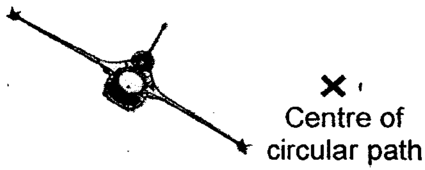
For an embankment inclined at 20° to the horizontal, find the speed at which the normal contact force is able to **completely provide the centripetal force** (i.e. no frictional force required).



Example 6 - Aircraft banking turn

When an airplane flies, it's the flow of air around the wings that generates a force (also known as lift), which is assumed to be perpendicular to the wings. If the plane flies level at constant speed, then the forces acting on it are:

When an airplane executes a turn, it tilts its wings, so that the lift makes an angle θ with the vertical (known as the banking angle), and now has a component in the horizontal direction, that helps the plane to turn. This maneuver is known as ***banking***.



Express θ in terms of the linear speed of the aircraft v and the radius of the turn r.

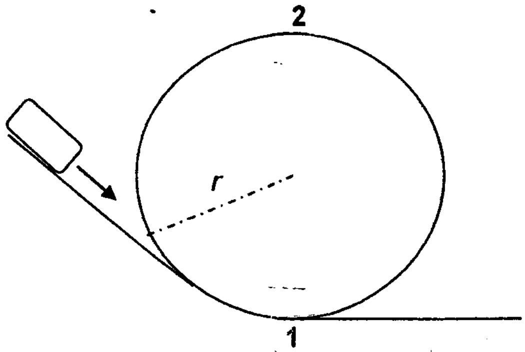
N.B. Lift on the plane is always **perpendicular** to the wings.

6.4.2 Vertical Circular Motion

When objects travel along a vertical circular path, the speed of the objects is often not constant due to the influence of gravity, resulting in non-uniform circular motion. If, at each point of the motion, we resolve the net force into its tangential and centripetal components, then the former changes only the speed of the motion, the latter changes only the direction of the motion. Calculations-wise, the equation for centripetal acceleration is still $\frac{v^2}{r}$, just that centripetal acceleration must now vary as the speed changes. The **principle of conservation of energy** provides the most convenient method to determine the speed at any position.

Example 7 - Roller coaster

Some roller coasters have several loops along the track. The picture on the right illustrates such a coaster executing a loop-the-loop.



(a) If a passenger car for a roller coaster enters a loop of radius 19 m at position 1 with a speed of 33 m s^{-1}, determine the normal contact force the track exerts on it at
   i) the bottom (position 1) and
   ii) the top (position 2) of the loop.

(b) Find the minimum speed at which the passenger car must travel at while it is at the top of the loop in order to clear the loop safely.

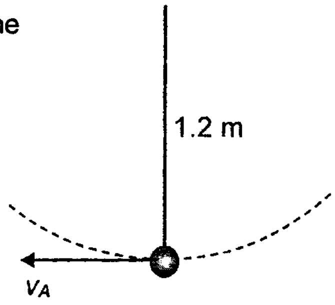
Assume the total mass of the vehicle and its passengers to be 170 kg and no friction.

Example 8 – String and bob

A stone of mass 800 g is tied to one end of a string and is whirled in a vertical circle. The string is inextensible and of length 1.2 m. The stone is projected with a certain speed v_A as shown below.

(i) Determine the minimum speed the stone must have at the top of the circular motion if the string is to be taut at the top.

(ii) Hence, show that the stone can complete a vertical circular motion if $v_A = 9.0 \text{ m s}^{-1}$.



In some other instances of vertical circular motion, the speeds of the objects are made constant (e.g. Ferris wheel). In these cases, the objects are undergoing uniform circular motion. You will encounter some of these applications in the tutorial.

Tutorial 6 Circular Motion

Self Review Questions

S1 (a) An object cannot move in a circle unless there is a resultant force acting ________ the centre of the circle. This is called a ___________ force. If this force is removed, the object will continue moving in a ___________ line, because of Newton's __________ law.

(b) The centripetal force causes a centripetal __________. The larger the linear __________ (in m s^{-1}) and the smaller the __________ of the circle, the larger the acceleration.

(c) The number of revolutions in one second is known as the ___________. This is measured in ___________. The time taken for one complete revolution is called the _________.

S2 A disc is rotating about an axis through its centre and perpendicular to its plane. A point P on the disc is twice as far from the axis as a point Q.

At a given instant what is the value of $\frac{\textit{the linear velocity of } P}{\textit{the linear velocity of } Q}$? *(N96/I/9)*

A 4 **B** 2 **C** ½ **D** ¼

S3 A particle travels in uniform circular motion. Which of the following correctly describes the linear velocity, angular velocity and centripetal acceleration of the particle? *(edited N91/I/9)*

	Linear velocity	*Angular velocity*	*Centripetal acceleration*
A	constant	constant	varying
B	constant	constant	zero
C	constant	varying	constant
D	varying	constant	varying
E	varying	varying	constant

S4 Which of the following statements is correct for a particle moving in a horizontal circle with constant angular velocity? *(N93/I/6)*

A The linear momentum is constant but the kinetic energy varies.

B The kinetic energy is constant but the linear momentum varies.

C Both the kinetic energy and linear momentum are constant.

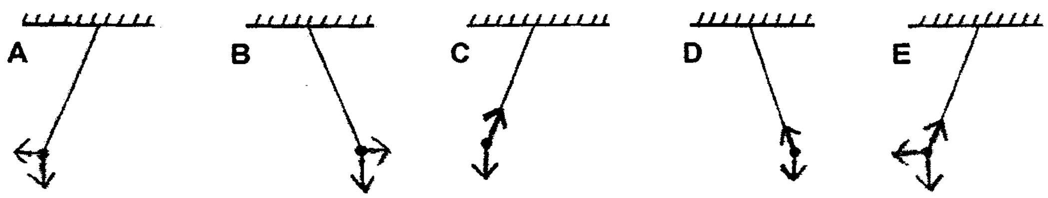
D Both speed and linear velocity are constant.

E Neither the linear momentum nor the kinetic energy is constant.

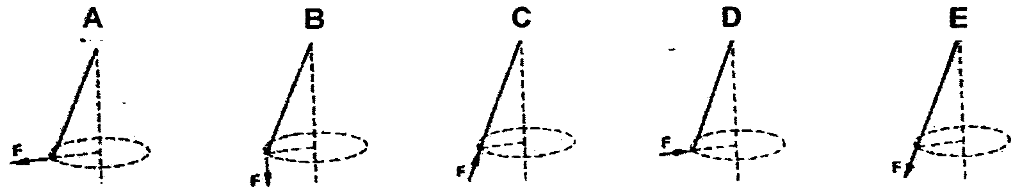
S5 A mass of 2 kg rotates at constant speed in a horizontal circle of radius 5 m and the time for one complete revolution is 3 s. The force, in N, acting on the mass is *(J79/II/1)*

A $\frac{2\pi^2}{9}$ **B** $\frac{4\pi^2}{9}$ **C** $\frac{40\pi^2}{9}$ **D** $\frac{100\pi^2}{9}$ **E** $\frac{400\pi^2}{9}$

S6 A passenger is sitting in a railway carriage facing in the direction in which the train is travelling. A pendulum hangs down in front of him from the carriage roof. The train travels along a circular arc bending to the right. Which one of the following diagrams shows the position of the pendulum as seen by the passenger and the directions of the forces acting on it? *(J81/II/7)*



S7 A mass on the end of a string is set in motion so that it describes a circle in a horizontal plane. Which diagram shows the direction of the resultant force acting on the mass at an instant in its motion? *(J91/I/8)*



S8 A car of mass m moving at a constant speed v passes over a humpback bridge of radius of curvature r. Given that the car remains in contact with the road, what is the net force R exerted by the car on the road when it is at the top of the bridge? *(J82/II/6; N85/I/4)*

A $R = mg + \dfrac{mv^2}{r}$ **B** $R = \dfrac{mv^2}{r}$

C $R = mg - \dfrac{mv^2}{r}$ **D** $R = mg$

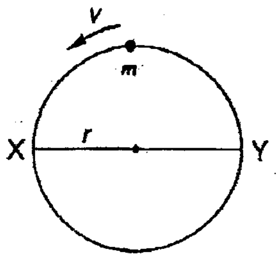
S9 An artificial satellite travels in a circular orbit about the Earth. Its rocket engine is then fired and produces a force on the satellite exactly equal and opposite to that exerted by the Earth's gravitational field. The satellite would then start to move *(J88/I/7)*

A along a spiral path towards the Earth's surface.
B along a tangent to the orbit.
C in a circular orbit with a longer period.
D in a circular orbit with a shorter period.

Discussion Questions

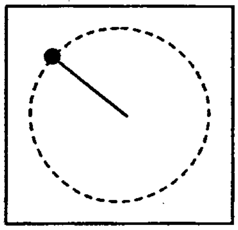
D1 A body of mass *m* moves in a horizontal circle of radius *r* at constant speed *v*. Which pair of values correctly gives (i) the work done by the centripetal force, (ii) the change in the linear momentum of the body, when it moves from X to Y? (XY is the diameter) *(N79/II/7)*

	work done	change in linear momentum
A	$2mv^2$	$2mv$
B	πmv^2	$2mv$
C	0	0
D	0	$2mv$
E	πmv^2	0



D2 A mass of 0.050 kg is attached to one end of a piece of elastic of unstretched length 0.50 m. The force constant of the elastic is 40 N m^{-1}. The mass is rotated steadily on a smooth table in a horizontal circle of radius 0.70 m as shown. What is the approximate speed (in m s^{-1}) of the mass? *(J87/I/9)*

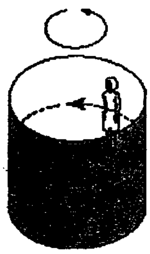
A 11
B 15
C 20
D 24
E 28



D3 The maximum safe speed of a car rounding a unbanked corner is 20 m s^{-1} when the road is dry. The maximum frictional force between the road surface and the wheels of the car is halved when the road is wet, what is the maximum safe speed (in m s^{-1}) for the car to round the corner? *(J97/I/8)*

A $\frac{20}{4}$ **B** $\frac{20}{2\sqrt{2}}$ **C** $\frac{20}{2}$ **D** $\frac{20}{\sqrt{2}}$

D4 In a popular amusement park ride known as the Rotor, people stand against the wall of a cylinder that is rotated. When rotating fast enough, the fall drops away, leaving the riders "pressed" against the wall in a vertical position as shown in the picture below.



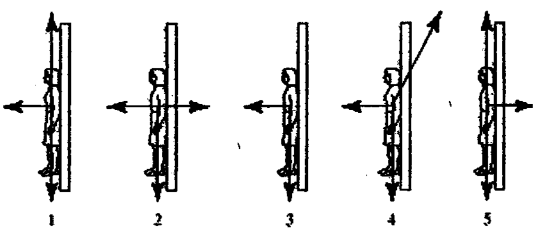
For a cylinder of radius 3.00 m rotating at 5.00 rad s^{-1},

(a) identify which of the diagrams in the figure on the right correctly identifies the forces acting on the person, and suggest what keeps the person from sliding down.

Calculate the
(b) centripetal acceleration and

(c) centripetal force, experienced by a 60 kg person.

What physical force provides for the centripetal acceleration?

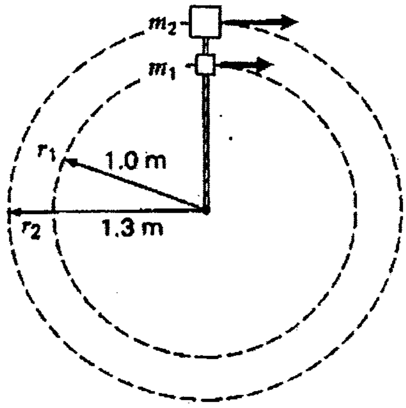


D5 Suppose that two masses, m_1 = 2.5 kg and m_2 = 3.5 kg, respectively, are connected by light strings and are in uniform circular motion on a horizontal frictionless surface as shown below where r_1 = 1.0 m and r_2 = 1.3 m. The forces acting on the masses are T_2 = 2.9 N and T_1 = 4.5 N.

Find the

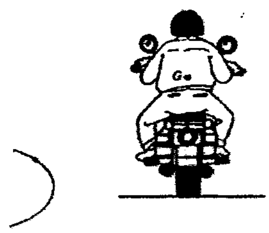
(a) centripetal accelerations,

(b) magnitude of the tangential velocities of the masses.

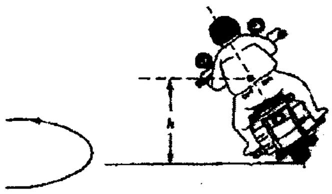


D6 (a) When making a bend, motorcycles tend to lean inwards. Why?

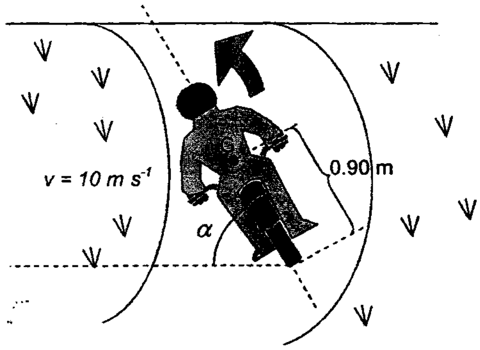
- Draw a free-body diagram of the motorcyclist and his bike going round the bend without leaning inwards. Describe what would happen to the motorcyclist.



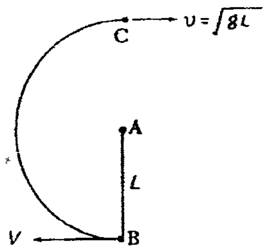
- Now, draw a free-body diagram of the motorcyclist and his bike going round the bend leaning inwards.



(b) In order to make a turn around a corner of radius 15 m, a motorist traveling at 10 m s^{-1} on a horizontal gravel road has to tilt his motorcycle at an angle, as shown in the diagram below. The combined mass of the motorist and his motorcycle is 300 kg, and the combined centre of mass is at G, at a distance of 0.90 m from the point of contact with the ground, as shown in the diagram. Calculate the angle α that he makes to the horizontal while he is making the turn.



D7 A particle is suspended from a point A by an inextensible string of length L. It is projected from B with a velocity v, perpendicular to AB, which is just sufficient for it to reach the point C. (*N81/I/1*)



(a) Show that, if the string is just to be taut when the particle reaches C, its speed there is $\sqrt{gL}$.

(b) Find the speed v with which the particle should be projected from B.

D8 Serway and Faugh. (7E) Pg 223. P7.59.

A frictionless roller coaster is given an initial velocity of v_o at height h, as in the figure below. The radius of curvature of the rack at point A is R.

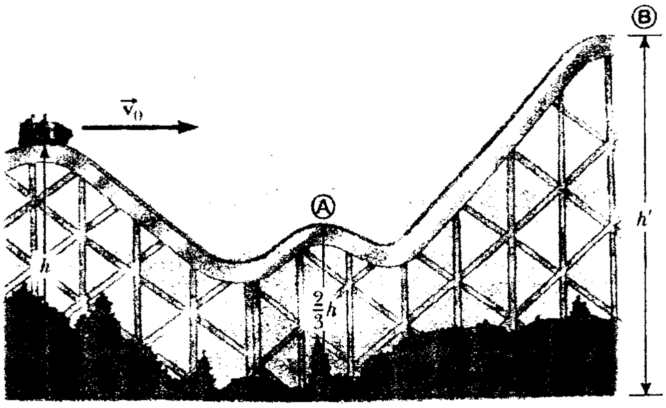


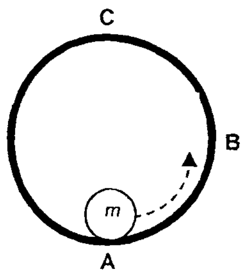
Fig. P7.59

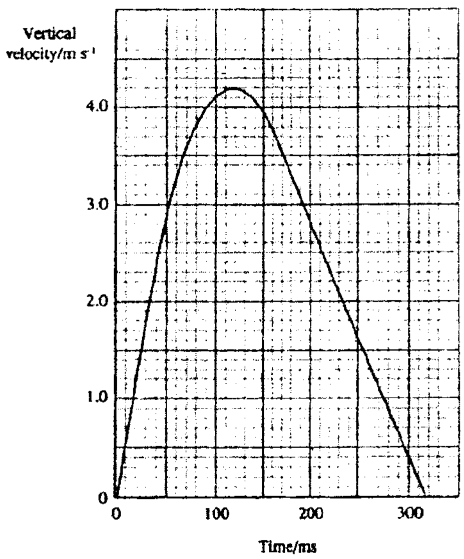
(a) Find the maximum value of v_o so that the roller coaster stays on the track at **A** solely because of gravity.

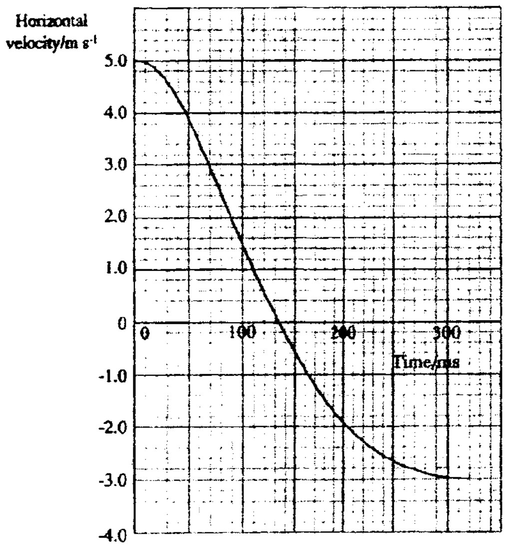
(b) Using the value of v_o calculated in (a), determine the value of h' that is necessary if the roller coaster just makes it to point B.

(c) Consider solution in (b), why do we not use the equations of motion that we learnt in the earlier chapter in kinematics to solve this question?

D9 A particle of mass m performs vertical circular motion as show in the diagram. The following two graphs show the vertical and horizontal components of the velocity of the particle along path **ABC**.







Calculate the centripetal acceleration at point **C**. *(MJC Prelims 10/I/12)*

A zero **B** 4.91 m s^{-2} **C** 9.81 m s^{-2} **D** 22.1 m s^{-2}

D10 Sasha's favourite ride at the fair is the Ferris wheel that has a radius of 7.0 m.

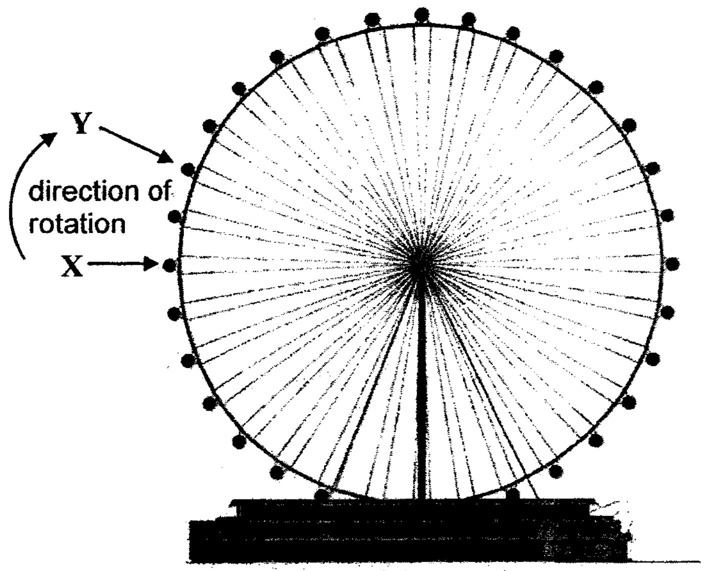
(a) If the ride takes 20.0 s to make one full revolution, what is the linear speed of the wheel?

(b) What centripetal force will the ride exert on Sasha's 50.0 kg body?

(c) In order for Sasha to feel weightless at the top of the ride, at what linear speed must the Ferris wheel turn?

(d) At this speed, how much will she appear to weigh at the bottom of the Ferris wheel?

D11 The Singapore Flyer (as shown in figure below) is a giant observation wheel that is set to be Asia's most visible iconic visitor attraction, providing breathtaking, panoramic views of Singapore and beyond. Completed in Mar 2008, it is one of the world's largest man-made moving land objects. It has a height of 178 m, a diameter of 150 m and sits on a 28 m high three-storey terminal building.



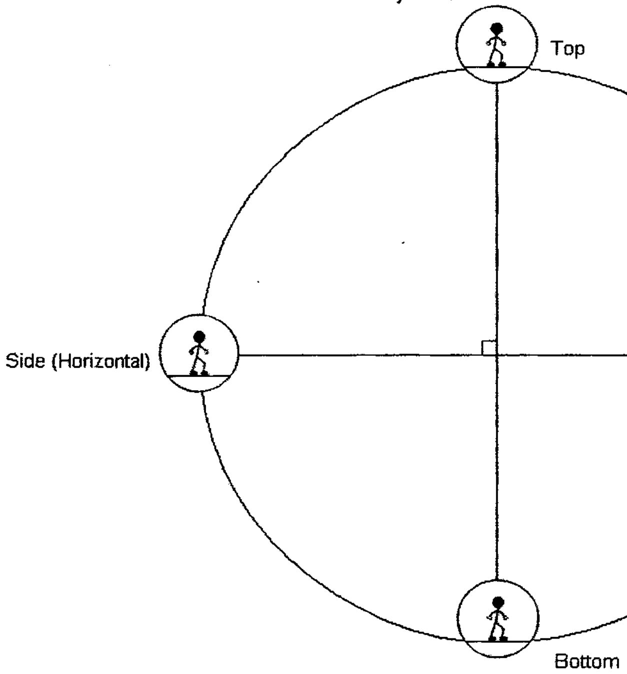
During the testing phase, the Singapore Flyer goes through a series of continuous revolutions without stopping. Assume that it revolves at constant angular speed ω and each complete revolution takes 37 minutes.

(a) Calculate the angular speed of the Singapore Flyer. [2]

(b) Hence or otherwise, determine the speed of a passenger capsule which is located at the circumference. [2]

(c) Given that there are 32 equally spaced capsules on the Flyer, find the time taken for a capsule to go from position X to position Y. [2]

(d) The Singapore Flyer now revolves at a much faster rate such that the centripetal force on the passenger becomes about half his weight. Draw and label the forces acting on a passenger standing inside a capsule at the three positions in the figure on the next page. [4]



(e) Since the Singapore Flyer revolves at a constant angular speed, no energy is required to keep it revolving once it is moving. Discuss briefly the validity of the statement. [2]

Suggested Answers

D1	**D**	
D2	**A**	
D3	**D**	
D4	**(b)**	75.0 m s^{-2}
	(c)	4500 N
D5	**(a)**	a_1 = 0.64 m s^{-2}
		a_2 = 0.83 m s^{-2}
	(b)	v_1 = 0.80 m s^{-1}
		v_2 = 1.0 m s^{-1}
D6	**(b)**	55.9°
D7	**(b)**	$\sqrt{5gL}$
D8	**(a)**	$v_o = \sqrt{g(R - \frac{2h}{3})}$
	(b)	$h' = \frac{R}{2} + \frac{2h}{3}$
D9	**D**	
D10	**(a)**	2.20 m s^{-1}
	(b)	34.6 N
	(c)	8.29 m s^{-1}
	(d)	981 N
D11	**(a)**	2.83×10^{-3} rad s^{-1}
	(b)	0.212 m s^{-1}
	(c)	139 s