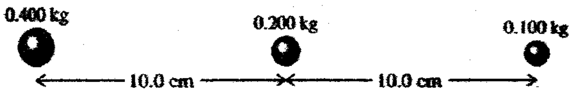
Tutorial 7A: Newton's Law of Gravitation & Gravitational Field

Self Review Questions

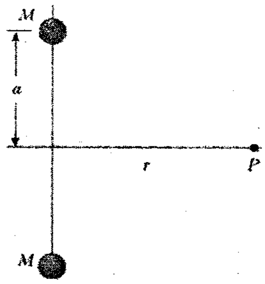
S1 The gravitational constant *G* has the SI base unit

A m s^{-2} **B** m^{-1} kg^{-1} s^{-2} **C** m^3 kg^{-1} s^{-2} **D** m^2 kg^{-2} **E** m^3 s^{-2}

S2 Coletta, Physics Fundamentals, Pg 158. Problem 6.10.
Find the resultant force on (a) the 0.100 kg mass and (b) the 0.200 kg mass in the diagram below (the masses are isolated from the Earth).



S3 Serway and Faugh, 6th Edition, P13.25.
Compute the magnitude and direction of the gravitational field at a point *P* on the perpendicular bisector of the line joining two objects of equal mass separated by a distance 2*a* as shown in the figure to the right.



S4 An experimental satellite is found to have a weight *W* when assembled before launching from a rocket site. It is placed in a circular orbit at a height $h = 6R$ above the surface of the Earth (of radius *R*). What is the gravitational force acting on the satellite whilst in orbit?

A *W*/6 **B** *W*/7 **C** *W*/36 **D** *W*/49

S5 Which statement about a geostationary satellite is true?

A It can remain vertically above any chosen fixed point on the Earth.
B Its linear speed is equal to the speed of a point on the Earth's equator.
C It has the same angular velocity as the Earth's rotation on its axis.
D It is always travelling from east to west.

S6 The moon remains in its orbit around the Earth rather than falls to the Earth because

A it is also attracted by the gravitational forces from the sun and other planets
B the net force on the Moon is zero
C the gravitational force exerted by the Earth on the moon provides a net force that provides the Moon's centripetal acceleration.
D the magnitude of the gravitational force from the Earth is too small to cause any appreciable acceleration of the Moon

S7 Coletta, Physics Fundamentals, Pg 159, Problem 6.23.
Find the speed of a satellite moving in a circular orbit just above the surface of the moon. The moon has radius of 1.74×10^6 m and a mass of 7.36×10^{22} kg. Also find the satellite's orbital period.

S8 A small part of the Earth's gravitational field close to the surface of the Earth is uniform. Which of the following statements is **not** correct?

A The field lines are parallel to each other.
B The gravitational force on an object is proportional to its height above the Earth's surface.
C The units of gravitational field strength are equivalent to m s^{-2}.
D The direction of the field lines is towards the Earth.

S9 The values of the acceleration of free fall, g, on the surfaces of two planets will be the same provided that the planets have the same

A mass **B** radius **C** mass/radius **D** mass/(radius)^2 **E** mass/(radius)^3

S10 Coletta, Physics Fundamentals, Pg 161. Problem 6.39.
Find the radius of a planet made of solid lead, such that gravitational acceleration at the surface of the planet is 1.00 m s^{-2}. The density of lead is 1.13×10^4 kg m^{-3}.

S11 The gravitational field strength outside a uniform sphere of mass M is the same as that due to a point mass M at the centre of the sphere. The Earth may be taken to be a uniform sphere of radius r. The gravitational field strength at its surface is g.

What is the gravitational field strength at a height h above the surface?

A $\dfrac{gr^2}{(r+h)^2}$ **B** $\dfrac{gr}{(r+h)}$ **C** $\dfrac{g(r-h)}{r}$ **D** $\dfrac{g(r-h)^2}{r^2}$

S12 (J83/II/6) Assuming that Earth to be a uniform sphere rotating about an axis through the poles, the weight of a body at the Equator compared with its weight at a pole would be

A greater, because the angular velocity of the Earth is greater at the equator than at the pole.
B greater, because the weight at the Equator is given by the sum of the gravitational attraction of the Earth and the centripetal force due to the circular motion of the body.
C the same, because the weight is the gravitational attraction of the Earth and for a uniform sphere, even when rotating, this is independent of the body's position on the Earth.
D smaller, because the gravitational attraction of the Earth must provide both the weight and the centripetal force due to the circular motion of the body.
E smaller, because the gravitational attraction at the pole is greater than that at the Equator.

Discussion Questions

D1 Coletta, Physics Fundamentals, Pg 158. Problem 6.13.
A particle of mass m is between a 1.00×10^2 kg mass and a 4.00×10^2 kg mass, which are 10.0 m apart. Find the distance of the particle from the 1.00×10^2 kg mass such that the resultant force on the particle is zero.

D2 Coletta, Physics Fundamentals, Pg 161. Problem 6.44. (Similar to N08/1/14)
Europa, one of the moons of Jupiter, was discovered by Galileo in 1610. Europa has a circular orbit of radius 6.708×10^5 km and period 3.551 days. Find the mass of Jupiter.

D3 Lowe & Rounce, 3rd Edition, p85, Ex12.2, Q2.
The acceleration due to gravity at the Earth's surface is 9.81 m/s^2. Calculate the acceleration due to gravity on a planet which has
(a) the same mass and twice the radius,
(b) the same radius and twice the density,
(c) half the radius and twice the density.

D4 The planet Saturn has a mass that is 95 times Earth's mass and a radius that is 9.4 times Earth's radius. What is the acceleration due to gravity on Saturn?

D5 The acceleration of free fall on the surface of the Earth is 6 times its value on the surface of the Moon. The mean density of the Earth is 5/3 times the mean density of the moon.
What is the ratio of the radius of the Earth over the radius of the Moon?

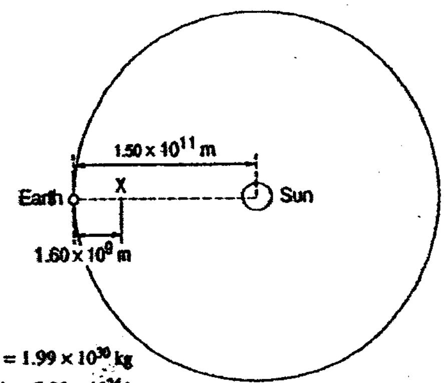
A 1.9 **B** 3.6 **C** 6.0 **D** 10

D6 Coletta, Physics Fundamentals, Pg 161, Problem 6.42.
Suppose the Earth rotates on its axis so quickly that an object at the equator has an apparent weight of zero. What would then be the length of an Earth day, given the radius of Earth is 6400 km?

D7 N2000/III/2
(a)(i) Define angular velocity for an object travelling in a circle.

(ii) Calculate the angular velocity of the Earth in its orbit around the Sun. Assume that the orbit is circular and give your answer in terms of the SI unit for angular velocity.

(b) In order to observe the Sun continuously, a satellite of mass 425 kg is at point X, a distance of 1.60×10^9 m from the centre of the Earth, as shown in Fig. 8 (below).



mass of Sun = 1.99×10^{30} kg
mass of Earth = 5.98×10^{24} kg
Earth-Sun distance = 1.50×10^{11} m

Fig. 8

(i) Calculate, using the data given,
1. the pull of the Earth on the satellite,
2. the pull of the Sun on the satellite.

(ii) Using Fig. 8 as a guide, draw a sketch to show the relative positions of the Earth, the Sun and the satellite. On your sketch, draw arrows to represent the two forces acting on the satellite. Label the arrows with the magnitude of the forces.

(iii) Calculate
1. the magnitude and direction of the resultant force on the satellite,
2. the acceleration of the satellite.

(iv) The satellite is in a circular orbit around the Sun. Calculate the angular velocity of the satellite.

(v) Using your answer to **(a)(ii)** describe the motion of the satellite relative to the Earth. Suggest why this orbit around the Sun is preferable to a satellite orbit around the Earth.

(vi) Suggest two disadvantages of having a satellite in this orbit.

D8 In a binary star system, two stars A and B follow circular orbits, of radius R and r respectively, centred on their common centre of mass O, as shown in Fig. 8.1.
The mass of star A is M, and that of star B is m.

(a) Explain why the period of rotation of star A is equal to the period of rotation of star B.

(b) Show that the period T of rotation of the stars is given by

$$T^2 = \frac{4\pi^2(R+r)^3}{G(M+m)}$$

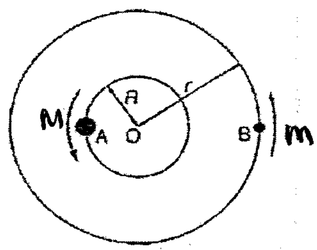


Fig. 8.1

D9 (N09/III/5)

(a)(i) Define gravitational field strength.

(ii) State Newton's law of gravitation and hence, using your definition in **(i)**, show that the gravitational field strength g at a distance R from a point mass M is given by $g = \frac{GM}{R^2}$

(b) A neutron star has mass 5.2×10^{30} kg and radius 1.7×10^4 m.

(i) Calculate the mean density of the star.

(ii) Suggest, with a reason, whether the density is likely to vary with the distance from the centre of the star.

(c) The mass of the star in (b) may be considered to be a point mass at its centre.

(i) Calculate the gravitational field strength at the surface of the star.

(ii) Determine the centripetal acceleration of a particle moving in a circular path of radius 1.7×10^4 m and with a period of rotation of 0.21 s.

(iii) The star rotates about its axis with a period of 0.21 s. Use your answers in (i) and (ii) to suggest whether particles on the surface of the star leave the surface owing to the high speed of rotation of the star.

D10 CIE J96/III/2(part)

(c) The Earth may be considered to be uniform sphere of radius 6370 km, spinning on its axis with a period of 24.0 hours. The gravitational field at the Earth's surface is identical with that of a point mass of 5.98×10^{24} kg at the Earth's centre. For a 1.00 kg mass situated at the Equator,

(i) calculate, using Newton's law of Gravitation, the gravitational force on the mass.

(ii) determine the force required to maintain the circular path of the mass,

(iii) deduce the reading on an accurate newton-meter (spring balance) supporting the mass.

(d) Using your answers to (c), state what would be the acceleration of the mass at the Earth's surface due to

(i) the gravitational force alone,

(ii) the force measured on the newton-meter.

(e) A student situated at the Equator releases a ball from rest in a vacuum and measures its acceleration towards the Earth's surface. He then states that this acceleration is 'the acceleration due to gravity'. Comment on his statement.

D11 CIE J91/III/1 (part)

(b) Modern gravity meters can measure *g*, the acceleration of free fall, to a high degree of accuracy. The principle on which they work is of measuring *t*, the time of fall of an object through a known distance *h* in a vacuum. Assuming that the object starts from rest, deduce the relation between *g*, *t* and *h*. [2]

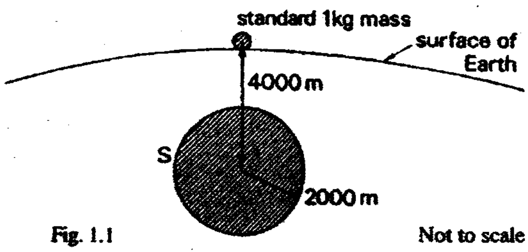


Fig. 1.1

(d) Fig. 1.1 shows a standard kilogram mass at the surfaced of the Earth and a spherical region **S** of radius 2 000 m with its centre 4 000 m from the surface of the Earth. The density of the rock in this region is 2 800 kg m^{-3}. What force does the matter in region **S** exert on the standard mass?

(e) If region **S** consisted of oil of density 900 kg m^{-3} instead of the rock, what difference would there be in the force on the standard mass?

(f) Suggest how gravity meters may be used in oil prospecting. Find the uncertainty within which the acceleration of free fall needs to be measured if the meters are to detect the (rather large) quantity of oil stated in (e).

Numerical solutions

S2) 2.00×10^{-10} N to the left, 4.01×10^{-10} N towards the left
S3) $g = 2MGr(r^2+a^2)^{-3/2}$ towards the centre of mass.
S7) 1.68×10^3 m s^{-1} and 1.81 h
S10) 3.17×10^5 m

D1) 3.33 m
D2) 1.90×10^{27} kg
D3) (a) 2.45 m s^{-2}, (b) 19.6 m s^{-2}, (c) 9.8 m s^{-2}
D4) 10.5 m s^{-2}
D6) 5.06×10^3 s or 1.4 hrs
D7) (a)(ii) 1.99×10^{-7} rad s^{-1}, (b)(i) 0.0662 N, 2.56 N, (b)(iii) 2.49 N towards the sun, 5.87×10^{-3} m s^{-2}, (b)(iv) 1.99×10^{-7} rad s^{-1}
D9) (b)(i) 2.53×10^{17} kg m^{-3} (c)(i) 1.2×10^{12} N kg^{-1} (c)(ii) 1.52×10^7 m s^{-2}
D10) (c)(i) 9.83 N (ii) 0.0337 N (iii) 9.80 N (d)(i) 9.83 m s^{-2} (ii) 9.80 m s^{-2}
D11) (d) 3.91×10^{-4} N (e) 2.65×10^{-4} N; (f) $\pm 3 \times 10^{-4}$ m s^{-2} or to 4 d.p. when measured in m s^{-2}

Tutorial 7B: Gravitational Potential and Potential Energy

Self-Review Questions

S1 CIE J80/II/5 (modified) X and Y are two points at respective distances R and $2R$ from the centre of the Earth, where R is greater than the radius of the Earth. The gravitational potential at X is –800 kJ kg^{-1}. When a 3 kg mass is taken from X to Y, the work done on the mass is

A –400 kJ B –1200 kJ

C +400 kJ D +1200 kJ

S2 N12/1/15 (modified) A satellite of mass 810 kg is to be raised from the Earth to a height of 92.0 km above the surface of the Earth. What is the necessary increase in the potential energy of the satellite?

The mass of the Earth is 5.98 x 10^{24} kg. The radius of the Earth is 6370 km.

S3 Lowe and Rounce, 3rd Edition, Pg. 86, Ex. 12.3, Q2. The moon has mass 7.7 x 10^{22} kg and radius 1.7 x 10^{6} m. Calculate

(a) the gravitational potential at its surface and

(b) the work needed to completely remove a 1.5 x 10^{3} kg space craft from its surface into outer space.

Neglect the effect of earth, other planets, sun etc.

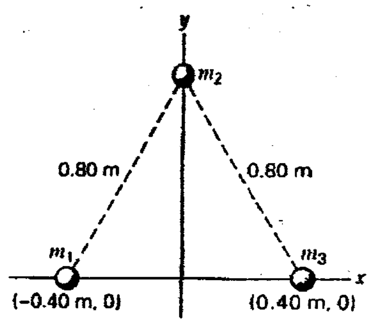
S4 N09/1/17 (modified) A satellite orbits a planet at a distance r from its centre. Its gravitational potential energy is -3.2 MJ. Another identical satellite orbits the planet at a distance $2r$ from its centre. What is the sum of the kinetic energy and the gravitational potential energy of this second satellite?

S5 Lowe and Rounce, 3rd Edition, Pg. 86, Ex. 12.3, Q4. A neutron star has radius 10 km and mass 2.5 x 10^{29} kg. A meteorite is drawn into its gravitational field. Calculate the speed with which it will strike the surface of the star. Neglect the initial speed of the meteorite.

Discussion Questions

D1 (a) What is the total gravitational potential energy of the configuration shown in the figure on the right if all the masses are 1.0 kg?

(b) What is the work done (by an external agent) if we were to add in an additional 1.0 kg mass into the centre of the system of masses? Explain why this answer is negative.



D2 Serway and Vuille, 8th Edition, P7.45. To take advantage of the Earth's rotation, a satellite of mass 200 kg is launched from a site on Earth's equator into an orbit 200 km above the surface of Earth.

(a) Assuming a circular orbit, what is the orbital period of this satellite?

(b) What is the satellite's speed in this orbit?

(c) What is the minimum energy necessary to place the satellite in orbit, assuming no air friction?

You may take the Earth to be a uniform sphere of radius 6400 km for this question.

D3

A space station of mass 450,000 kg is in a low Earth orbit (LEO) at an altitude of 415 km. Calculate the energy required to bring it to a medium Earth orbit (MEO) of altitude 20,200 km. You will have to use that the mass of the Earth is 6.0×10^{24} kg and that its radius is 6,378 km. You may assume that, as the space station is in LEO or MEO, it is in uniform circular motion about the centre of the Earth.

D4 Adapted from UCLES N08/II/3 and N04/II/3

A satellite of mass m orbits a planet of mass M and radius R_p with a speed of v. The radius of the orbit is R. The satellite and the planet may be considered to be point masses with their masses concentrated at their centres. They may be assumed to be isolated in space.

(a) (i) Show that speed v is given by the expression

$$v^2 = \frac{GM}{R}.$$

(ii) The mass of the satellite is m. Determine an expression for the kinetic energy E_k of the satellite in terms of G, M, m and R.

(b) (i) State an expression, in terms of G, M, m and R, for the gravitational potential energy E_p of the satellite.

(ii) Hence, show that the satellite in orbit, the ratio of its *gravitational potential energy* to the *kinetic energy* is equal to -2.

(iii) Also show that the total energy E_t of the satellite is given by $E_t = -\dfrac{GMm}{2R}$.

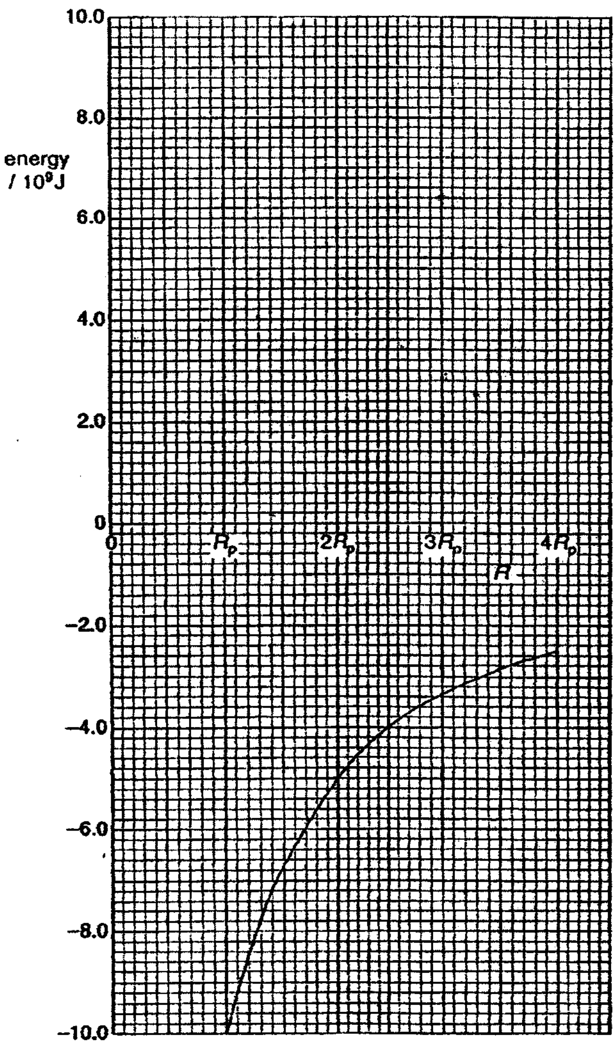
(c) The variation with orbital radius R of the gravitational potential energy of the satellite is shown in the figure on the right.

On the figure, draw the variation with orbital radius of

(i) the kinetic energy of the satellite,

(ii) the total energy of the satellite.

Your line should extend from $R = 1.5\ R_p$ to $R = 4\ R_p$.



(d) As the satellite orbits the planet, it gradually loses energy because of air resistance by the atmosphere of the planet.

(i) State whether the total energy E_t becomes more or less negative.

(ii) Hence, state and explain the effect of this change on

1. the radius of the orbit,
2. the speed of the satellite.

(iii) The mass m of the satellite is 1600 kg. The radius of the orbit of the satellite is changed from $R = 4\ R_p$ to $R = 2R_p$. Use the figure to determine the change in orbital speed of the satellite.

D5 CIE J85/II/9 (part)

The curve in Fig. 5 shows the way in which the gravitational potential energy of a body of mass m in the field of the Earth depends on r, the distance from the centre of the Earth, for values of r greater than the Earth's radius R_E.

(a) What does the gradient of the tangent to the curve at $r = R_E$ represent?

The body referred to above is a rocket which is projected vertically upwards from the Earth. At a certain distance R from the centre of the Earth, the total energy of the rocket (i.e. its gravitational potential energy plus its kinetic energy) may be represented by a point on the line PQ. Five points A, B, C, D, E have been marked on this line. Which point (or points) could represent the total energy of the rocket

(b) if it were momentarily at rest at the top of its trajectory,

(c) if it were falling towards the Earth,

(d) if it were moving away from the Earth, with sufficient energy to reach an infinite distance?

In each case, explain briefly how you arrive at your answer.

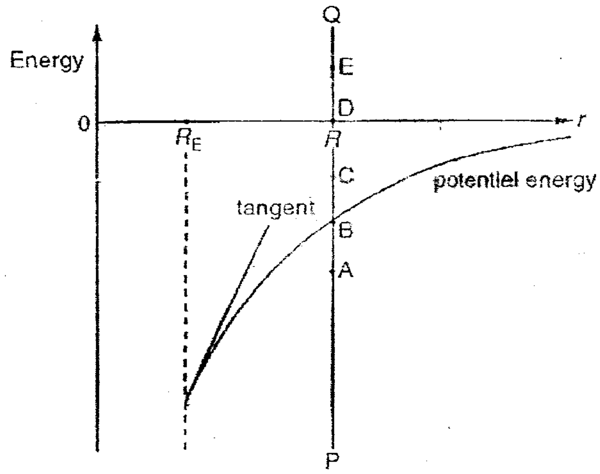


Fig. 5

D6 CIE J87/II/8 (part)

A point mass m is at a distance r from the centre of the Earth.

(a) Write down an expression, in terms of m, r, the Earth's mass m_E and the gravitational constant G, for the gravitational potential energy V of the mass. (Consider only values of r greater than the Earth's radius. Mass of Moon = 7.4×10^{22} kg; mass of Earth = 6.0×10^{24} kg.)

Certain meteorites (tektites) found on Earth have a composition identical with that of lunar granite. It is thought that they may be debris from a volcanic eruption on the Moon. Fig. 6, which is not to scale, shows how the gravitational potential between the surface of the Moon and the surface of the Earth varies along the line connecting their centres. At the point P the gravitational potential is at a maximum.

(b) State how the resultant gravitational force on the tektite at any point between the Moon and the Earth could be deduced from Fig. 6.

(c) When a tektite is at P the gravitational forces on it due to Moon and Earth are F_M and F_E respectively. State the relation which applies between F_M and F_E. Hence find the values of X/Y, where X and Y are the distances of P from the centre of the Moon and the centre of the Earth respectively.

(d) If a tektite is to reach the Earth, it must be ejected from the volcano on the Moon with a certain minimum speed V_0. Making use of appropriate values from Fig. 6, find this speed. Explain your reasoning.

(e) Discuss very briefly whether a tektite will reach the Earth's surface with a speed less than, equal to or greater than the speed of projection. (Neglect atmospheric resistance.)

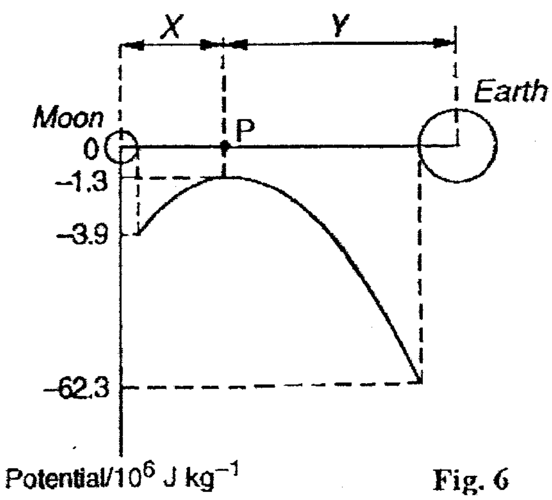


Fig. 6

Numerical solutions

S2) 7.22×10^{8} J
S3) (a) -3.0×10^{6} J kg^{-1}, (b) 4.5×10^{9} J
S4) - 0.8 MJ
S5) 5.8×10^{7} m s^{-1}

D1) (a) -2.5×10^{-10} J (b) -4.3×10^{-10} J
D2) (a) 1.48 h (b) 7.8×10^{3} m s^{-1} (c) 6.4×10^{9} J
D3) 9.9×10^{12} J
D4) (d) (iii) 518 m s^{-1}
D6) (c) $X/Y = 0.11$; (d) 2.3×10^{3} m s^{-1}

D5)

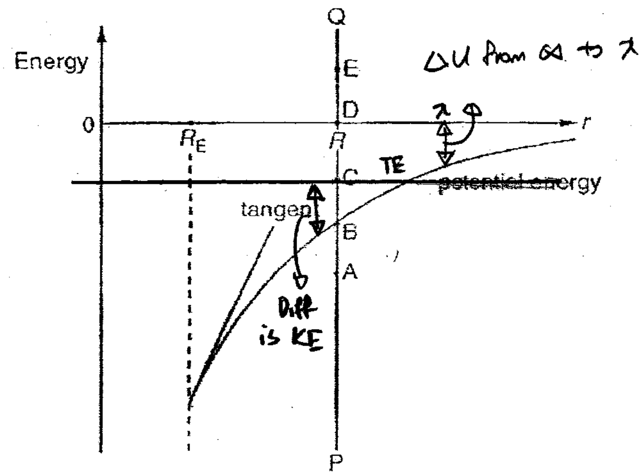


Fig. 5

(b) if it were momentarily at rest at the top of its trajectory, B

(c) if it were falling towards the Earth, C

(d) if it were moving away from the Earth, with sufficient energy to reach an infinite distance? D, E

In each case, explain briefly how you arrive at your answer.

At ∞, TE = KE + 0