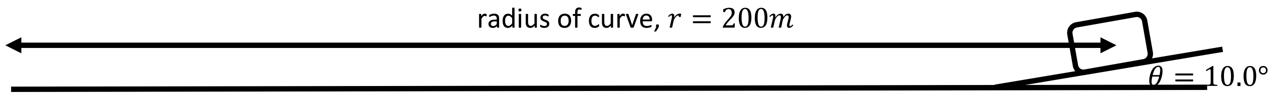
GENERAL INFORMATION SHEET

Acceleration due to gravity at Earth surface,	$g = 9.80 \text{ m s}^{-2} = \lvert\vec{g}\rvert$
Radius of the Earth	$R_E = 6.371 \times 10^6 \text{m}$
Universal gas constant,	$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$
Vacuum permittivity,	$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$
Atomic mass unit,	$u = 1.66 \times 10^{-27} \text{ kg}$
Speed of light in vacuum,	$c = 3.00 \times 10^8 \text{ m s}^{-1}$
Charge of electron,	$e = 1.60 \times 10^{-19} \text{ C}$
Planck's constant,	$h = 6.63 \times 10^{-34} \text{ J s}$
Mass of electron,	$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.000549\,u$
Mass of proton,	$m_p = 1.67 \times 10^{-27} \text{ kg} = 1.007\,u$
Rest mass of alpha particle,	$m_\alpha = 4.003\,u$
Boltzmann constant,	$k = 1.38 \times 10^{-23} \text{ J K}^{-1}$
Avogadro's number,	$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$
Standard atmosphere pressure,	$P_0 = 1.01 \times 10^5 \text{ Pa}$
Density of water,	$\rho_w = 1000 \text{ kg m}^{-3}$
Specific heat (capacity) of water,	$c_w = 4.19 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$
Latent heat of fusion for water,	$L_f = 3.34 \times 10^5 \text{ J kg}^{-1}$
Stefan-Boltzmann constant,	$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Sum of N terms in an arithmetic series,	$\sum_{k=0}^{N-1} a_k = \frac{N(a_0+a_{N-1})}{2}$
Sum of N terms in a geometric series,	$\sum_{k=0}^{N-1} r^k = \frac{1-r^N}{1-r}$
Approximation for square root, for small x	$\sqrt{1+x} \approx 1 + \frac{x}{2}$

1. A vehicle of mass 1200 kg makes a turn in a curve banked at 10.0° **without sliding .** The radius of the curve is 200 m. Assume that friction can be modelled by a **coefficient of friction** which is 0.0500 on an icy road, 0.350 on a rainy day and 0.700 on a dry sunny day and that the vehicle starts to **slide** out of its circular path when the maximum static friction is reached.



(a) **Define** in words and **draw** a free body diagram of the vehicle for each of the 3 situations listed below, label the forces in the diagram.
   i. "Low speed limit"
   ii. "Ideal speed"
   iii. "High speed limit"

(b) For the "high speed limit" situation, **derive** expressions for the
   i. total vertical forces, and
   ii. total horizontal force.

(c) For the "high speed limit" situation on a sunny day, **calculate** the magnitudes of the
   i. total vertical forces, and
   ii. total horizontal force.

(d) Calculate the speed **limits** for the vehicle for the icy road conditions.

(e) MCQ1: How much faster can the vehicle go on a **sunny** day as compared to a **rainy** day (maximum speed on sunny day/maximum speed on a rainy day)?

(A) 1.3
(B) 1.4
(C) 1.5
(D) 1.6
(E) 1.7

(f) **Discuss** the physics if the road was banked at different angles.

Answer

(a) Low speed => friction points outwards, ideal, no friction, high speed friction points inwards. In the stationary reference frame of the road.

(b) (i)Vertical force $R(\cos\theta - \mu\sin\theta) - mg$, (ii) horizontal force $R(\sin\theta + \mu\cos\theta)$

May also "simplify" to get rid of R (since vertical force = 0)

(c) Substituting values (i) 0, (ii) 11800N (almost the same as mg)

(d) (i) Low speed v=15.7ms-1=56.7km/h, (ii) high speed v=21.0ms-1=75.8km/h

(e) High speed situation:

Vertical : $R(\cos\theta - \mu\sin\theta) - mg = 0$

Horizontal : $R(\sin\theta + \mu\cos\theta) = m\frac{v^2}{r}$

$$g\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} = \frac{v^2}{r}$$

$$v = \sqrt{rg\frac{(\sin\theta + \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)}} = 21.1ms^{-1} = 76km/h$$

Low speed situation

Vertical : $R(\cos\theta + \mu\sin\theta) - mg = 0$

Horizontal : $R(\sin\theta - \mu\cos\theta) = m\frac{v^2}{r}$

$$g\frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)} = \frac{v^2}{r}$$

$$v = \sqrt{rg\frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}} = 15.7ms^{-1} = 56km/h$$

(f) MCQ-A:

Substituting in values for $\mu = 0.7$, $v = 159km/h$

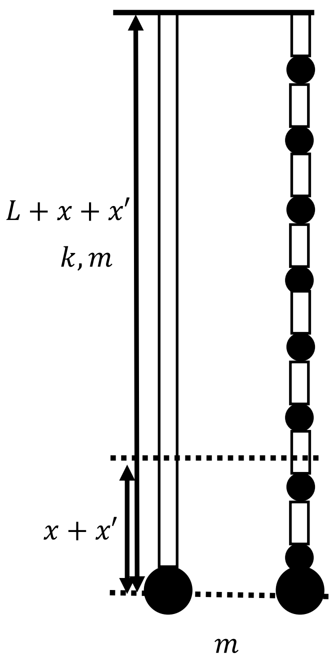
For $= 0.35$, $v = 119km/h$

So ratio is 1.33

(g) Open ended: e.g.s For very small angles, $\sin\theta = 0$ $\cos\theta = 1$, $ratio = \sqrt{\frac{\mu_{sunny}}{\mu_{rainy}}}$ = 1.41….

For no friction, infinite speed is possible with90 degree angle , with dry road, infinite speed is possible with ~50 degree angle …..

2. An ideal, initially **uniform** spring of mass m , unstretched length L and spring constant k stretches by an extension x when hung vertically. An **additional** point mass m is added to the bottom of the spring causing it to extend by a **further** ′ .



(a) Consider the spring as being made up of N smaller massless springs with masses at the end of the spring.

i. **Write** down an expression for the spring constant, k_N , of one of the N springs.

ii. **Explain** your answer.

(b) **Derive** an **expression** for the extension of the whole spring mass system, $x + x'$.

(c) What is the **ratio** of the extension of the bottom 1/3 of the spring with mass $\frac{m}{3}$ kg to that of the top 1/3 of the spring with mass $\frac{m}{3}$ kg ?

(A) 0.17

(B) 0.33

(C) 0.41

(D) 0.59

(E) 0.64

(d) **Calculate** the extension of the middle 1/3 of the spring.

(e) **Discuss** the physics of the above vertically hung spring mass system when the point mass at the bottom is pulled down a little and suddenly let go.

(a) (i) $k_N = Nk$

(ii)Consider a massless spring , spring constant k ,with a mass m attached to the bottom

The extension is $\mathrm{x} = \frac{\mathrm{mg}}{\mathrm{k}}$

If we divide the massless spring into N segments, then each segment should extend by the same amount as each other. As they are connected one after the other, the total extension must equal the extension before it was divided. Also since the spring is massless, the tension must be the same on each spring. Therefore the extension of each spring is x/N and the force is mg, therefore the spring constant is $mg/(x/N) = Nk$.

(b) Consider if we split it into N segments, spring constant Nk followed by mass m/N and assume N is large

$$x = \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} \frac{\left(m + i\frac{m}{N}\right)g}{Nk} = \frac{mg}{k} + \frac{mg}{N^2k}\sum_{i=1}^{N} i = \frac{mg}{k} + \frac{mg}{N^2k}\frac{N(1+N)}{2} = \frac{3mg}{2k}$$

Or from general round, without the additional mass the extension is $mg/2k$ so with the additional mass m, the extension is $3mg/2k$

(c) Answer E:

The total extension starting from the bottom and adding upwards is

Sum for the bottom 1/3

$$x = \sum_{i=1}^{N/3} x_i = \sum_{i=1}^{N/3} \frac{\left(m + i\frac{m}{N}\right)g}{Nk} = \frac{mg}{3k} + \frac{mg}{N^2k}\sum_{i=1}^{N/3} i \approx \frac{mg}{3k} + \frac{mg}{k}\frac{1}{18} = \frac{7mg}{18k}$$

Top 1/3

$$x = \sum_{i=2N/3}^{N} x_i = \sum_{i=2N/3}^{N} \frac{\left(m + i\frac{m}{N}\right)g}{Nk} = \frac{mg}{3k} + \frac{mg}{N^2k}\sum_{i=2N/3}^{N} i \approx \frac{mg}{3k} + \frac{mg}{N^2k}\frac{N/3\,(2N/3+N)}{2} = \frac{11mg}{18k}$$

Ratio is thus 7/11~0.64

(d) Extension = $\left(\frac{3}{2} - \frac{11}{18} - \frac{7}{18}\right)\frac{mg}{k} = \frac{mg}{2k}$

(e) Open ended. there will be some oscillation but there is no clear frequency , Also wave, but wave speed changes as you go up the spring sqrt(T/m) …

3. As a model for a ping-pong ball made to bounce vertically from a paddle, consider a surface moving up and down sinusoidally with an **amplitude** of $A = 0.05$ m and **frequency** of $f = 2$ Hz. A light ball, constrained to move only in the vertical direction, **bounces to the same height** h above the location of the bounce/collision **each and every time**. Assume air resistance is negligible. When the ball was dropped on the motionless paddle, it **loses** 19% of its kinetic energy.

(a) **Derive** an **expression** for the time, T_b , it takes from one bounce to the next.

(b) **Calculate** the coefficient of restitution, e for the ball bouncing from the surface.

(c) **Derive** an **expression** for the speed, v_s, at which the surface must move for the ball to have the **same speed** v_b just after each bounce as it had just before the bounce.

(d) MCQ 3: What is the **maximum** possible height h to which the ball can bounce?

(A) 2 m

(B) 5 m

(C) 7 m

(D) 10 m

(E) 14 m

(e) **Explain** your answer to part d in detail.

(f) **Discuss** other possible regular motion of the ball without changing the given parameters.

(a) Since the time it takes from one bounce to the next, , is twice the time it takes to fall. Return time may be calculated from $h = \frac{1}{2} g \left(\frac{t}{2}\right)^2$

$$T_b = \sqrt{\frac{8h}{g}}$$

(b) Coefficient of restitution $e = \frac{relative\ velocity\ after\ collision}{relative\ velocity\ before\ collision} = \sqrt{\frac{KE_f}{KE_i}} = \sqrt{1 - 0.19} = 0.9$

(c) $e = \frac{relative\ velocity\ after\ collision}{relative\ velocity\ before\ collision}$

$$v_b - v_s = e(v_b + v_s)$$
$$v_s = \left(\frac{1-e}{1+e}\right) v_b = \frac{1}{19} v_b$$

(d) Answer:B

(e) In reality the situation is quite unstable. However, we consider the ideal situation where the ball gets maximum energy if we hit it when the surface is moving upwards at the highest speed. However, it also needs to return in a time which is an integer multiple of the period of the surface. So repeatable bounces are only possible when the correct velocity coupled with the correct return time.

It must also be an integer multiple of the period $t = 2\sqrt{\frac{2h}{g}} = nT = \frac{n}{f}$

$$h = n^2 \frac{gT^2}{8}$$

Velocity of the ball at the bottom may be calculated from conservation of energy

$v_0 = \sqrt{2gh} = n\frac{gT}{2}$

maximum velocity of the surface $v_{max} = \frac{2\pi}{T} A = 0.628$

So

$$v = n\left(\frac{1-e}{1+e}\right)\frac{gT}{2} = n(0.129) = 0.516$$

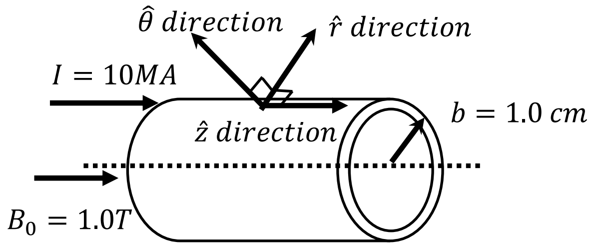
If n=4, can get v closest to the maximum.

Substituting in the values, $h = 4.9m$ above the point of bouncing. The point of bouncing is close to the equilibrium position of the surface.

Guess and check is also acceptable.(but not considered as good as an answer which explains the physics in more detail)

(f) Change n bounce from lower speed “position” of surface.

4. A constant 10 MA current, I, flows along a long, thin perfect conducting hollow cylinder. Initially, the outer radius of the cylinder, b, is 1.0 cm and the inner radius is a. A uniform, external magnetic field B_0 of 1.0 T is applied along the axis of the cylinder. For calculations, you may assume that the thickness of the cylinder is negligible.



(a) **Write down** an **expression** for the magnetic field at the surface of the cylinder.

(b) Assuming that the current density is uniform, **sketch** well **labelled graphs** of the following against the distance from the center r in the range $r = a$ to $r = b$. (labels should numerical values or expressions wherever possible)

i. Current enclosed at r

ii. Magnitude of magnetic field at r

iii. Force acting on a thin shell of current between r and $r + \Delta r$, where $\Delta r \ll b - a$ and is a constant.

(c) MCQ 4: What is the **pressure** acting on the whole curved surface of the cylinder?

(A) 0 Pa

(B) 0.8 MPa

(C) 0.16 GPa

(D) 16 GPa

(E) 32 GPa

(d) **Explain** your answer in part (c).

(e) The cylinder is flexible and allowed to move.

i. **Explain** what happens to the area, magnetic field and magnetic flux within the cylinder as the cylinder moves (e.g expands or contracts) as time passes.

ii. What is the radius of the cylinder at equilibrium?

Answer:

(a) , Magnetic field of a long, straight conductor + external field $B = \frac{\mu_0 I}{2\pi r}\hat{\theta} + B_0\hat{z} \approx \frac{\mu_0 I}{2\pi r}\hat{\theta}$

(b) (i) almost straight line with slight upwards curve graph, maximum 10MA, min 0, (ii) similar curve, take into account the applied magnetic field.(iii) force curve also similar. A force curve flat 0 is also acceptable as students may not be familiar with cylindrical coordinates. (Full credit is given for straight linse of the right trend with appropriate labels)

(c) Answer: D

(d) Assuming B and I are at the same location, $F = BIL = \frac{\mu_0 I^2 L}{2\pi r}$ only the component of B perpendicular to L is important.

Actually I produces B and thus via a similar argument for KE, the force is actually half that of the above calculated $F = BIL = \frac{\mu_0 I^2 L}{4\pi r}$

$$P = \frac{F}{2\pi r L} = \frac{\mu_0 I^2}{2(2\pi r)^2}$$

(note: the graphs they plot previously are hints to this)

Alternatively, for students who can do calculus, Consider a thin cylindrical shell at position r from the axis, within the cylinder carrying current $\Delta I(r)$. The force acting on this shell is $\Delta F = B(r)\Delta I(r)L = \frac{\mu_0 I(r)}{2\pi r}\Delta I(r)L$

The total force acting on all the currents will be $\int_0^F dF = \int_0^I \frac{\mu_0 I}{2\pi r} L dI$

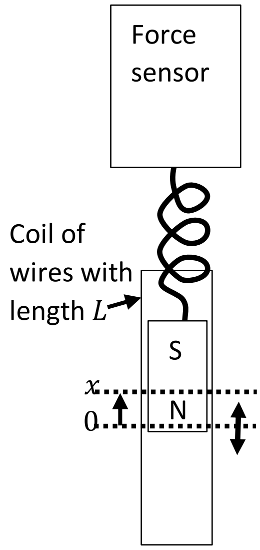
Integrating $F = \int_0^I \frac{\mu_0 I}{2\pi r} L dI = \frac{\mu_0 I^2 L}{4\pi r}$

(e) (i) changing magnetic flux induces current to flow in the cylinder. By Lenz law it will oppose the change. Since cylinder is assumed to be perfect conductor, magnetic flux does not change. (ii) at equilibrium magnetic field inside is equal to magnetic field outside so no force. For magnetic flux to be constant , B field inside the cylinder increases as area decreases. Furthermore outside magnetic field increases as radius decreases.

$\frac{\mu_0 I}{2\pi r} = B_0 \frac{b^2}{r^2}$

$$r = \frac{2\pi B_0 b^2}{\mu_0 I} = 50\mu m$$

5. A bar magnet is attached to the end of a non-magnetic spring, with spring constant $484Nm^{-1}$, as shown in the diagram. The other end of the spring is attached to a fixed force sensor which was **zeroed** when the magnet was at its equilibrium position, before being zeroed, the force sensor shows 4.8N. The magnet is right at the center of a coil of wires when the magnet is in its equilibrium position. Assume the spring has negligible mass and that friction is negligible. The coil has length L_C and the magnet is has length $L_m \approx L_C/4$. The spring is stretched by $x = 1\ cm < L_m$ and released at time $t = 0$ such that it performs **simple harmonic motion** in the coil. When the magnet was initially moved from far above the coil to inside the center of the coil a plot of the EMF vs time was obtained. The area under this curve was found to be -0.003Vs.



(a) Using the same x scale **sketch** well labelled graphs of (labels should give **numerical values** and **expressions** where possible)

(i) magnetic flux, $\emptyset$ through the coil of wires against the position x of the magnet, where x represents the distance between the center of mass of the magnet above its equilibrium position , and

(ii) the slope of above graph ,$\frac{d\emptyset}{dx}$ against x.

(b) Using the same time scale and leaving some space for part (c), sketch well labelled graphs of

(i) force against time,

(ii) velocity of the magnet, dx/dt, against time, and

(iii) displacement of the magnet, x, against time.

(c) **Sketch** a well **labelled** graph of EMF vs time below the graph in part (b) using the same time scale. Show how the various features of the EMF curve corresponds to the features on the force curve.

(d) MCQ 5: Which of the following statement is **true**?

(A) The magnitude of the EMF is maximum only when the force is zero.

(B) The magnitude of the EMF is maximum only when the magnitude of the force is maximum.

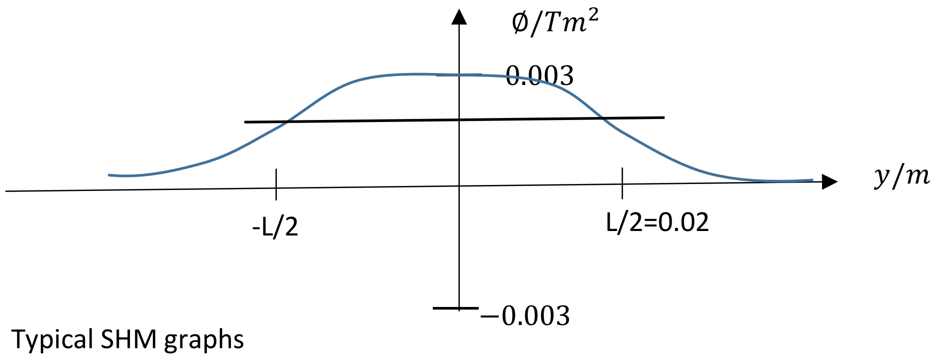
(C) The magnitude of the EMF is zero only when the magnitude of the force is maximum.

(D) The magnitude of the EMF is zero only when the force is zero.

(E) None of the above statements are true.

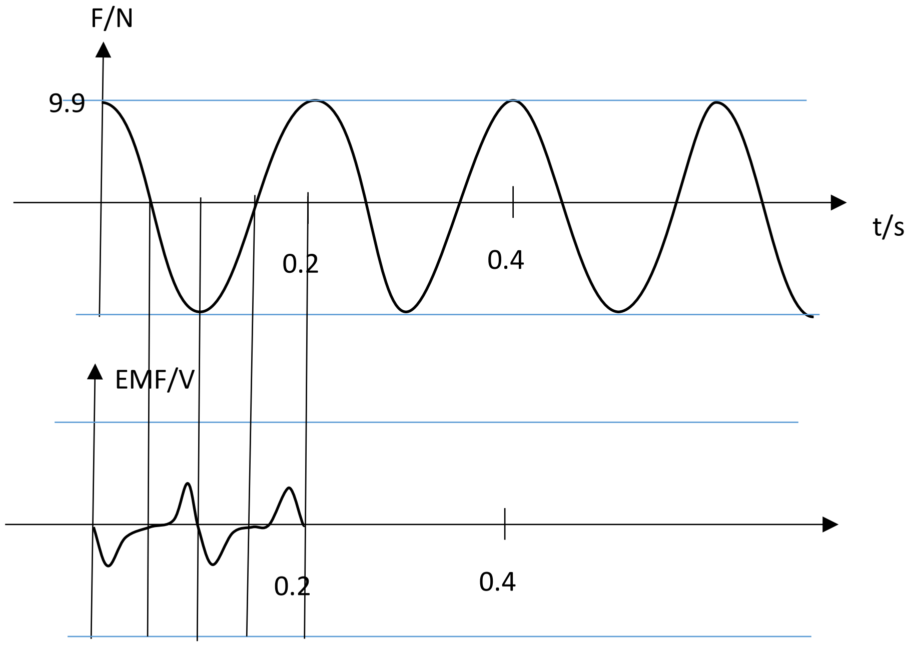
(e) **Discuss** the effect of connecting a resistor with small resistance across the coil on the measured voltage.

(a) Flux vs y "The area under this curve was found to be -0.003Vs. " this defines the sign of flux above the coil as flux = -area under the curve.



(b) Typical SHM graphs

(c) EMF depends on how flux changes in time and depends on both the factors,(i) how flux changes with position and (i) how magnets position changes with time. Prior graphs (a-ii, b-ii and b-iii) are hints towards this.



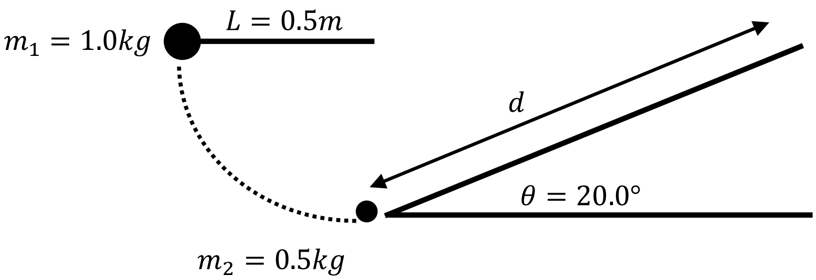
(d) Answer E. see above graph marks are also awarded for labeling magnitudes of force velocity, displacement and time.

(e) Open ended. Some examples to discuss (i) damping, (ii) if resistance is small as compared to resistance of coil (iii), if small compared to the reactance due inductance of coil.

https://www.researchgate.net/publication/228415690_A_datalogger_demonstration_of_electromagnetic_induction_with_a_falling_oscillating_and_swinging_magnet

6. A 1.0 kg point mass is attached to a taut string of length 0.5 m as a pendulum. The mass and string are initially held horizontally. When the mass is released, it swings down and strikes a 0.5 kg point mass at the bottom of its trajectory in an elastic collision. The 0.5 kg mass then rolls up an incline of 20.0°. What distance, d, does the point mass travel up the slope?

(A) 0.89 m
(B) 1.8 m
(C) 2.6 m
(D) 4.2 m
(E) 5.2 m



Answer: C

$$\frac{1}{2}m_1u^2 = m_1gL$$
$$u = \sqrt{2gL}$$

$$\frac{1}{2}m_1u^2 = \frac{1}{2}m_1v_1^{\ 2} + \frac{1}{2}m_2v_2^{\ 2}$$
$$m_1u = m_1v_1 + m_2v_2$$
$$\therefore v_2 = \frac{2m_1}{m_1+m_2}u = \frac{2m_1}{m_1+m_2}\sqrt{2gL}$$

$$\frac{1}{2}m_2v_2^{\ 2} = m_2gh$$
$$h = \frac{v_2^{\ 2}}{2g} = \frac{2gL}{2g}\left(\frac{2m_1}{m_1+m_2}\right)^2 = L\left(\frac{2m_1}{m_1+m_2}\right)^2 = (0.5)\left(\frac{2.0}{1.5}\right)^2$$
$$d = \frac{h}{\sin\theta} = \frac{8.0}{9.0\sin 20^\circ} = 2.6\text{m}$$

7. A point mass is launched at 30.0 m s^{-1}, at an angle of 45.0° above a flat horizontal plane. Upon the impact of the particle with the plane, the point mass bounces off and moves forward at the same angle to the horizontal, but loses 1/4 of its kinetic energy. What is the total horizontal distance that the mass travels before coming to rest?

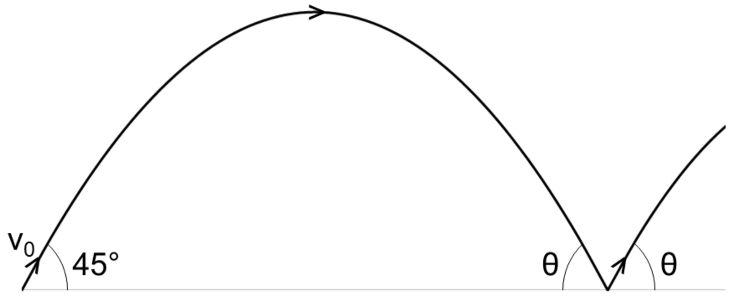
(A) 91 m

(B) 183 m

(C) 210 m

(D) 367 m

(E) 685 m



Answer: D

Initial velocity after n^{th} bounce $v_n = v_0\left(\sqrt{\frac{3}{4}}\right)^n$

Distance travelled between n^{th} and $(n+1)^{th}$ bounce

$$d_n = \frac{v_n^{\,2}}{g} = \frac{v_0^{\,2}}{g}\left(\frac{3}{4}\right)^n$$

Total distance travelled as n approaches ∞

$$\sum_{n=0}^{\infty} d_n = \frac{v_0^{\,2}}{g}\sum_{n=0}^{\infty}\left(\frac{3}{4}\right)^n = \frac{v_0^{\,2}}{g}\left(\frac{1}{1-\frac{3}{4}}\right) = \frac{4v_0^{\,2}}{g}$$

$$\frac{4v_0^{\,2}}{g} = \frac{4(30.0)^2}{9.81} = 367\text{m}$$

8. 2 point charges of mass 1.0×10^{-3} kg and equal charge q are separately hung on strings of length 1.0 m connected to the same point. The angle between the strings is 30.0°. What is the charge q?

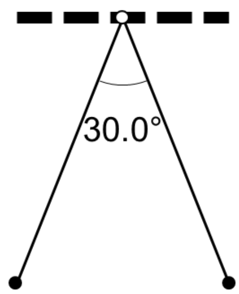
(A) 7.9×10^{-8} C

(B) 1.4×10^{-7} C

(C) 2.8×10^{-7} C

(D) 7.9×10^{-7} C

(E) 8.9×10^{-6} C



Answer: C

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{\left(2L\sin\frac{\theta}{2}\right)^2} = mg\tan\frac{\theta}{2}$$

9. An ideal monatomic gas is initially at pressure $p_A = 1.00 \times 10^5$ Pa and volume 0.500 m^3 (state A). It is isovolumetrically heated to 800 K (state B), then allowed to isothermally expand to volume $2.00\ m^3$ (state C). It is then isovolumetrically cooled back to 300 K (state D) and isothermally compressed to 0.500 m^3, returning to its initial state A. What is the pressure of the gas at state C, p_C ?

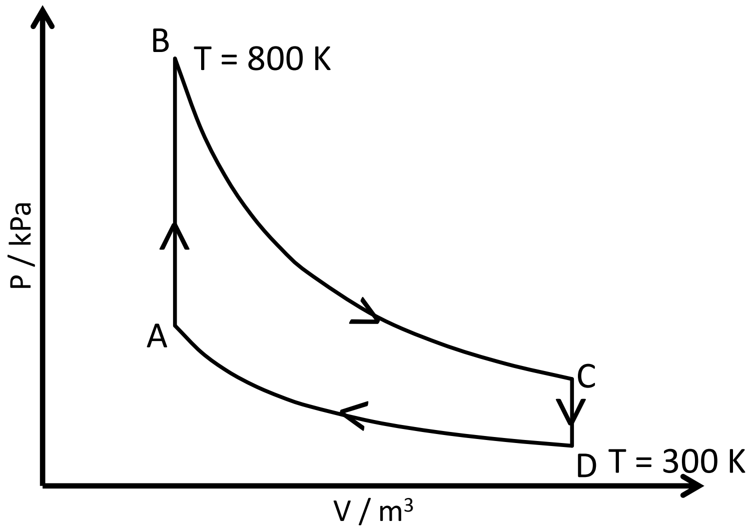
(A) $p_C = 25\ kPa$

(B) $p_C = 66.7\ kPa$

(C) $p_C = 133\ kPa$

(D) $p_C = 267\ kPa$

(E) $p_C = 300\ kPa$



Answer: B

$$p_D V_D = p_A V_A$$

$$p_D = p_A \frac{V_A}{V_D} = (100)\left(\frac{0.500}{2.00}\right) = 25\text{kPa}$$

$$\frac{p_C}{T_C} = \frac{p_D}{T_D}$$

$$p_C = p_D \frac{T_C}{T_D} = (25)\left(\frac{800}{300}\right) = 66.7\text{kPa}$$

10. Wind turbines are used to generate electricity in some countries. An example is the Brooklyn wind turbine in Wellington which has 20m length blades which has a 225kW capacity. What is the area swept out by the blades required to get the same power from wind in Singapore as compared to Wellington New Zealand. You may assume that wind speed in Wellington is about $8ms^{-1}$, the wind speed in Singapore is about $2ms^{-1}$ and that the efficiency of the wind turbines are comparable.

(A) less than 2500 m^2

(B) between 2500 m^2 to 10,000 m^2

(C) between 10,000 m^2 to 40,000 m^2

(D) between 40,000 m^2 to 0.16 km^2

(E) more than 0.16 km^2

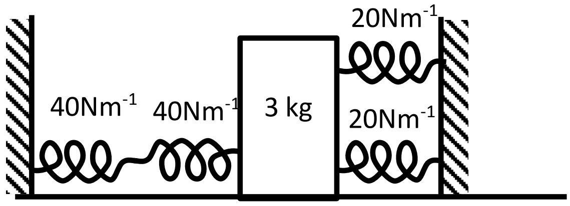
Answer: D

KE of wind is proportional to mv^2 where v is the wind speed. However the mass per unit time flowing past the wind turbine is also proportional to density, v & area of the turbine. So the power is proportional to Av^3 To get the same power ,since v^3 drops 64 time, A must increase 64 times. The area of the Brooklyn wind turbine is $1250m^2$ So the required area is $80{,}000m^2$¬.

However practically this area can be achieved by increasing the number of wind turbines instead of building one giant wind turbine. It should be noted that it will not be feasible to build huge wind turbines from the engineering point of view .Also it is not economically feasible to build 64 wind turbines just to get 225kW capacity. For comparison, the 5 liter v8 engine of the 2015 Ford Mustang GT puts out more than 300kW.

http://www.channelnewsasia.com/news/singapore/from-bacteria-to-wind/2375836.html

11. A mass $m_0 = 3.00\ kg$ is connected by 4 horizontal springs stretched between two rigid walls **as shown in the diagram**. The spring constants of the springs are $20.0\ N\ m^{-1}$ and $40.0\ N\ m^{-1}$. The mass is then displaced slightly to the right and allowed to oscillate in the horizontal direction. Assume friction is negligible. What is the frequency of the oscillations?



(A) 3.65 Hz
(B) 2.58 Hz
(C) 0.712 Hz
(D) 0.581 Hz
(E) 0.411 Hz

Answer: C

$$F = -60.0(x)$$

$$a = -\frac{60.0}{3.0}x$$

$$\omega = \sqrt{\frac{60.0}{3.0}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{60.0}{3.0}} = 0.712\text{Hz}$$

12. An **additional mass** m is added to the oscillator in the previous problem and the period changed by 10^{-8} %. What is the mass?

(A) 0.60 μg

(B) 0.42 μg

(C) 0.35 μg

(D) 0.30 μg

(E) 0.17 μg

Answer: A

$$\frac{T_2}{T_1} = \frac{\sqrt{\frac{m_0 + m}{k}}}{\sqrt{\frac{m_0}{k}}} = \sqrt{1 + \frac{m}{m_0}}$$

Need this step as calculator may get it wrong otherwise.

$$\frac{m}{2m_0} = \frac{T_2}{T_1} - 1 = 10^{-10}$$

$$m = 6 \times 10^{-10}\ kg$$

Note: This situation in this question is not really practical here but it is possible to get oscillators with very high frequency in which case even a small change in mass is detectable.

13. A satellite orbits the Earth at 10,400 km from the surface of the earth, at the equator, in the same sense as the earth's rotation. It was directly above Singapore at some time, how much **time** must elapse before it is directly over Singapore again? Assume the satellite travels in a circular orbit,

(A) 12 hrs

(B) 8 hrs

(C) 6 hrs

(D) 4 hrs

(E) 3 hrs

Answer: B

$$\omega_{sat}^2 r_{sat} = \frac{GM}{r_{sat}^2}$$

$$\omega_{sat} = \sqrt{\frac{GM}{r_{sat}^3}} = \sqrt{\frac{gR_E^2}{r_{sat}^3}}$$

$$t = \frac{2\pi}{\omega_{sat} - \omega_E} = 8hrs$$

14. An A.C generator operates at 12V and a constant frequency of 15.9hz. The generator has a rotating magnet inside a coil which generates the EMF. The coil has an internal resistance of 0.01 ohm and internal inductance of 0.01H. Which of the components when connected as a load to the generator will achieve the **most power dissipated** in the **external** resistor?

(A) 0.05 Ohm resistor in series with 0.01F capacitor

(B) 0.04 Ohm resistor in series with 0.01H inductor

(C) 0.03 Ohm resistor

(D) 0.02 Ohm resistor in parallel with 0.01F capacitor

(E) 0.01 Ohm resistor in parallel with 0.01H inductor

Answer: A

Angular frequency is 100 Hz, 0.01H and 0.01F capacitance has reactance of 1ohm. If we calculate each one, will take some time.

However it's quite easy if you notice that in B the reactance from inductance and capacitance cancels.

15. Water enters the pipe shown in the diagram at 3.0 ms-1. The radius of the pipe at the point of entry is 15.0 cm. The pipe then narrows to a radius of 12.0 cm. Assume that atmospheric pressure is constant at 100 kPa, **calculate** the **height difference** h in the water levels on the left and right as indicated?

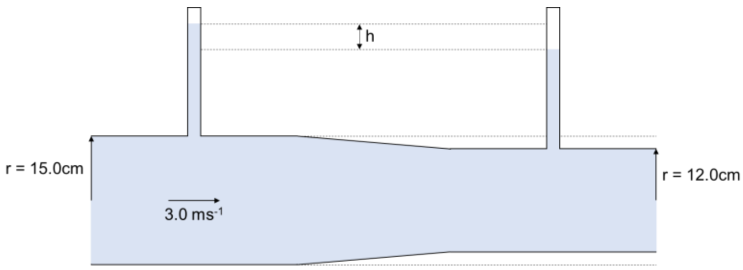
(A) 0.18 m

(B) 0.21 m

(C) 0.24 m

(D) 0.27 m

(E) 0.30 m



Answer: E

$$A_1 v_1 = A_2 v_2$$

$$\frac{1}{2}\rho v_1^{\ 2} = \frac{1}{2}\rho v_2^{\ 2} + \Delta P$$

$$\Delta P = \frac{1}{2}\rho(v_1^{\ 2} - v_2^{\ 2})$$

$$= \frac{1}{2}\rho v_1^{\ 2}\left(1 - \frac{A_1^{\ 2}}{A_2^{\ 2}}\right)$$

$$y = \frac{\Delta P}{\rho g} = \frac{v_1^{\ 2}}{2g}\left(1 - \frac{r_1^{\ 4}}{r_2^{\ 4}}\right) = 0.271\text{m}$$

$$h = y + 3.0\ \text{cm} = 0.301\text{m}$$

16. A copper sphere of radius 1.00 cm is placed into 50.0 cm^3 of water. The copper sphere is initially at 25.0°C while the water is initially at 40.0°C. Assuming no heat transfer with the environment, **what is the final temperature** of the water and copper sphere? Heat capacity of copper = 0.385 J/(g K), density of copper = 8.96 g/cm^3.

(A) 39.0°C

(B) 37.5°C

(C) 35.4°C

(D) 34.7°C

(E) 30.8°C

Answer: A

$$m_{Cu} = \frac{4\pi}{3} r^3 \rho = 37.53\text{g}$$

$$m_{Cu} C_{Cu} (T - 25) = m_w C_w (40 - T)$$

$$T = \frac{40 m_w C_w + 25 m_{Cu} C_{Cu}}{m_{Cu} C_{Cu} + m_w C_w}$$

$$= \frac{40(50)(4.19) + 25(37.53)(0.385)}{(37.53)(0.385) + (50)(4.19)} = 39.0^\circ\text{C}$$

17. A 50kg woman climbs 280m up to the top of the UOB Plaza One. She drinks 250g cup of crushed ice and 80g of water at 0° C. What is the **ratio** of work done against gravity to heat required to heat up the drink to 37° C?

(A) 100:1

(B) 10:1

(C) 1:1

(D) 1:10

(E) 1:100

Answer: C

Work done $W = mgh = 137200J$

Heat required $Q = L_f m_{ice} + c_w m_{total} \Delta T = 334 \times 250 + 4.19 \times 330 \times 37 = 135000J$

Note: this may lead to the misconception that drinking ice water is as good as exercise ☺

18. **Unpolarised** light of intensity I_0 passes through an ideal polarizing filter that has its axis vertically oriented. The polarized light then passes through 3 additional ideal polarizers, with polarizing axes at 30°, 60° and 90° to the vertical in the order that the light passes through them. What is the intensity of the light exiting the last polarizer?

(A) 0

(B) $\frac{3\sqrt{3}}{16} I_0$

(C) $\frac{3\sqrt{3}}{8} I_0$

(D) $\frac{27}{64} I_0$

(E) $\frac{27}{128} I_0$

Answer: E

After passing through the first filter, the intensity is $\frac{1}{2} I_0$

With each subsequent filter, the 30° change in axis results in the intensity being multiplied by $\cos^2 30^\circ = \frac{3}{4}$

Hence, the exiting light has an intensity of $\frac{27}{128} I_0$

19. A transparent vacuum tube with two electrodes, one made of potassium, K, work function 2.3eV, and the other of platinum, Pt, work function 6.3eV, is connected to a variable voltage power supply and voltmeter and ammeter as shown in the figure. The terminal of the power supply connected to the **Pt plate is grounded** and defined to be 0V. UV light of 345 nm wavelength shines on **both electrodes**. Current **flow starts** when the potential at the K electrode, _____. (Hint: Batteries have two different electrodes to produce a potential difference across the electrodes)

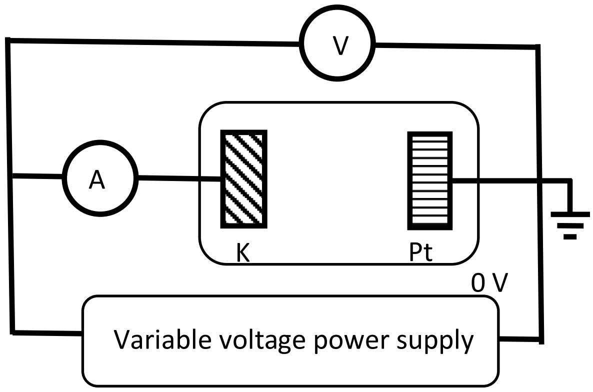
(A) V is anything except 0

(B) $V < +1.3V$

(C) $V > +2.3V$

(D) $V < -2.7V$

(E) None of the above



Answer: D

Note how ground is connected i.e. how 0V is defined and $eV_s = h\nu - \varphi_{collector}$ see https://www.researchgate.net/publication/231117927_The_photoelectric_effect_Experimental_confirmation_concerning_a_widespread_misconception_in_the_theory

20. It is possible to fuse two nuclei of deuterium, ${}_{1}^{2}H$ together to produce helium-3, a neutron and some energy i.e. ${}_{2}^{3}He + n^{0} + 3.27\text{MeV}$. Consider the situation where a deuteron with 0.10 MeV kinetic energy fuses with a stationary deuterium nucleus. **Calculate** the **ratio** of maximum helium-3 energy to minimum? (Hint: consider a frame of reference where the center of mass is stationary)

(A) 2.4

(B) 2.7

(C) 3.0

(D) 3.3

(E) 3.6

Answer: A.

Consider the case where they start off with no KE and momentum. By conservation of momentum, He and n have equal momentum. Since $E = p^2/2m$, n will have 3 times more energy than He, and therefore in this case than He has 0.8175MeV.

In the CM frame, total initial kinetic energy is 0.05MeV, so He has 0.830MeV, and vHE=0.024c. velocity of CM = 0.005c

Convert to lab frame lowest energy is when He is moving in opposite direction as initial i.e. v=0.024c-0.005c D and maximum energy is when He is moving in same direction as initial D i.e v=0.024c+0.005c.

----End of Paper---