SINGAPORE JUNIOR PHYSICS OLYMPIAD 2017

SPECIAL ROUND

30 June 2017
0900 – 1200

Time Allowed: **THREE Hours**

INSTRUCTIONS

1. This paper contains **10** multiple choice questions, **5** structured questions and **9** printed pages.

2. For the structured questions (100 points):
   - You may use your own approximations and assumptions, where applicable. Make sure you state what they are.
   - Answers which are more **complete** with clear and/or detailed **working** may be awarded bonus points.
   - Open-ended parts of the structured questions may be used as tie breakers, please answer in detail if you have time.
   - Begin each question on a **fresh page**.

3. For the multiple choice questions (20 points):
   - Each of the questions or incomplete statements is followed by five suggested answers or completions. Select the one that is **best** in each case and then write your choice in the answer booklet.
   - **Only** the answer booklet will be marked. However, working for the MCQ questions **may** be marked as tie breakers.

4. Answer **all** questions. Points will **NOT** be deducted for wrong answers.

5. **Scientific calculators** are **allowed** in this test. Graphing calculators are **not** allowed.

6. Please fill in your **name**, **IC number** and **school** on the answer booklet and graph paper **before the competition starts.**

7. A general data sheet is given in page 2. You may detach it **when the competition starts** for easy reference.

GENERAL DATA SHEET

Acceleration due to gravity at Earth surface,	g	$= 9.80$ m s^{-2} = $\|\mathbf{g}\|$
Universal gas constant,	R	$= 8.31$ J mol^{-1} K^{-1}
Vacuum permittivity,	ε_0	$= 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}
Vacuum permeability,	μ_0	$= 4\pi \times 10^{-7}$ T m A^{-1}
Atomic mass unit,	u	$= 1.66 \times 10^{-27}$ kg
Speed of light in vacuum,	c	$= 3.00 \times 10^8$ m s^{-1}
Speed of sound in air,	v_s	$= 340$ m s^{-1}
Charge of electron,	e	$= 1.60 \times 10^{-19}$ C
Planck's constant,	h	$= 6.63 \times 10^{-34}$ J s
Mass of electron,	m_e	$= 9.11 \times 10^{-31}$ kg $= 0.000549u$
Mass of proton,	m_p	$= 1.67 \times 10^{-27}$ kg $= 1.007u$
Mass of deuteron,	m_p	$= 3.34 \times 10^{-27}$ kg $= 2.014u$
Rest mass of alpha particle,	m_α	$= 4.003u$
Boltzmann constant,	k	$= 1.38 \times 10^{-23}$ J K^{-1}
Avogadro's number,	N_A	$= 6.02 \times 10^{23}$ mol^{-1}
Standard atmosphere pressure,	P_0	$= 1.01 \times 10^5$ Pa
Density of water,	ρ_w	$= 1000$ kg m^{-3}
Specific heat (capacity) of water,	c_w	$= 4.19 \times 10^3$ J kg^{-1} K^{-1}
Specific latent heat of vaporization of water,	l_v	$= 2.26 \times 10^6$ J kg^{-1}
Specific latent heat of fusion of water,	l_f	$= 3.34 \times 10^5$ J kg^{-1}
Stefan-Boltzmann constant,	σ	$= 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Radius of the sun,	r_s	= 696,000 km
Distance between sun and earth,	r_{SE}	= 150,000,000 km
Acceleration due to gravity at the sun's surface,	g_s	= 28.02 g
Temperature on the surface of the sun	T_s	= 5780K

Section A: Multiple Choice Questions (20 marks)

Draw the following table on the FIRST page of your answer booklet and fill in the correct answers.

Question	1	2	3	4	5	6	7	8	9	10
Answer										

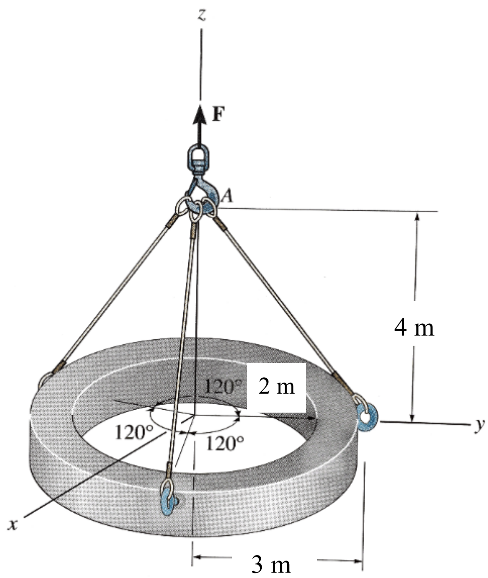
If you wish to show your working/reasoning to any question, please write below the table and indicate the question number clearly.

---

1. Three cables are used to support a ring of mass m in equilibrium, as shown below. The cables are attached to the outer side of the ring, which has outer and inner radii of 3 m and 2 m, respectively. The cables are hooked at a height of 4 m directly above the centre of the ring. The cables are equidistant from each other on the outer side of the ring.

What is the tension in each cable in terms of mg?

A. $mg/3$
B. $2mg/5$
C. $5mg/12$
D. $5mg/9$
E. $3mg/4$



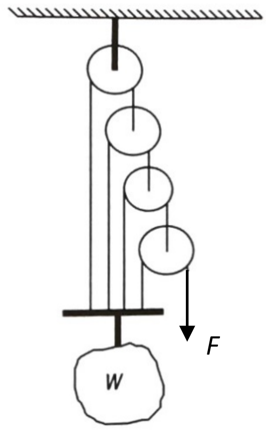
2. Given the gravitational field strength g = 9.8 m s^{-2}, the radius of the Earth R = 6400 km, and the gravitational constant G = 6.7 x 10^{-11} N m^{2} kg^{-2}, the mass of the Earth can be calculated from these data to be approximately

A. 9 x 10^{14} kg
B. 6 x 10^{18} kg
C. 6 x 10^{21} kg
D. 6 x 10^{24} kg
E. There is not enough information to calculate this.

3. The initial velocity of a 5 g particle is (7**i** + 2**j** + 5**k**) m/s. A force of (**i** – **j** + **k**) N acts upon it for 20 s. The final speed of the particle is approximately

A. 10 m/s
B. 14 m/s
C. 206 m/s
D. 4000 m/s
E. 6930 m/s

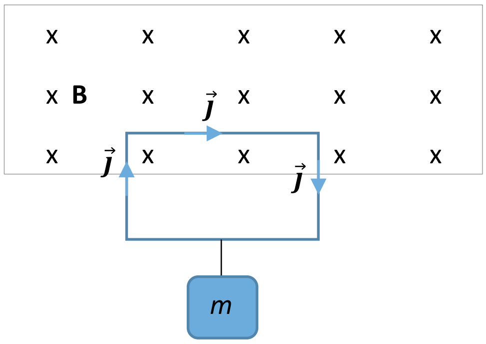
4. In the system of pulleys below, the pulleys and ropes can be assumed to be massless. The ropes are inextensible, and the topmost pulley is fixed to the ceiling. The ropes do not slip on the pulleys and there is no friction on the axles of the pulleys. A force F is applied to the end of the rope, as shown, that keeps the weight W stationary.



What is the ratio $F : W$?

A. 1 : 1
B. 1 : 4
C. 1 : 7
D. 1 : 8
E. 1: 15

5. A rigid wire loop carries a current I that the current density $\vec{j}$ has directions as shown in the figure below. (The battery is not shown for simplicity.) Half of the loop is in a region where a uniform magnetic field B is perpendicular to the plane of the loop, i.e. directed into the page in the figure. A mass m hung from the loop is suspended due to the magnetic force.

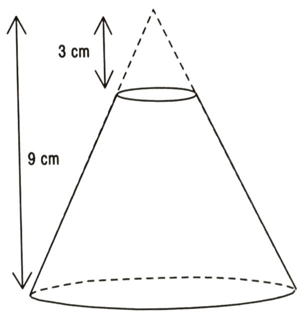


When the current is increased, which of the following statements about the work done by the magnetic force must be true?

A. There is positive work done by the magnetic force.
B. There is negative work done by the magnetic force.
C. There is no work done by the magnetic force.
D. The work done by the magnetic force may be either positive or negative.
E. There is insufficient information to deduce the work done by the magnetic force.

6. The frustrum is a geometric shape made by removing the top of a solid cone. In the figure below, 3 cm of the top of a 9 cm tall solid cone was removed to make a frustrum.

If the centre of mass of a solid cone is located at ¾ of the height from the tip, approximately how high from the base is the centre of mass of the frustrum?



A. 1.3 cm
B. 2.1 cm
C. 2.9 cm
D. 3.2 cm
E. 3.5 cm

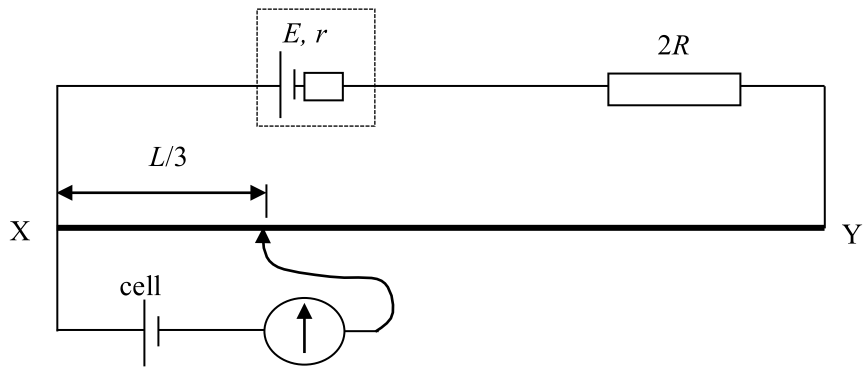
7. A diffraction grating has 500 lines per millimetre and is illuminated normally by monochromatic light of wavelength 600 nm. The total number of bright fringes seen on both sides of the normal, including the central fringe is

A. 5
B. 7
C. 8
D. 9
E. 10

8. A tuning fork of frequency 312 Hz is sounded with another tuning fork of an unknown frequency *f*. Due to interference, 2 beats per second is heard. When some plasticine is added to the prongs of the unknown tuning fork, the beat frequency is reduced. The frequency *f* in Hz is

A. 314
B. 312.5
C. 312.2
D. 311.5
E. 310

9. A potentiometer has a wire XY of length L and resistance R. It is powered by a battery of emf E and internal resistance r in series with a resistor of resistance $2R$. With another cell in the branch circuit, the null point is found to be $L/3$ from X, as shown below.



The emf of the cell is

A. $E/3$
B. $E(2R+r)/3$
C. $ER/(6R+3r)$
D. $ER/(3R+r)$
E. $ER/(9R+3r)$

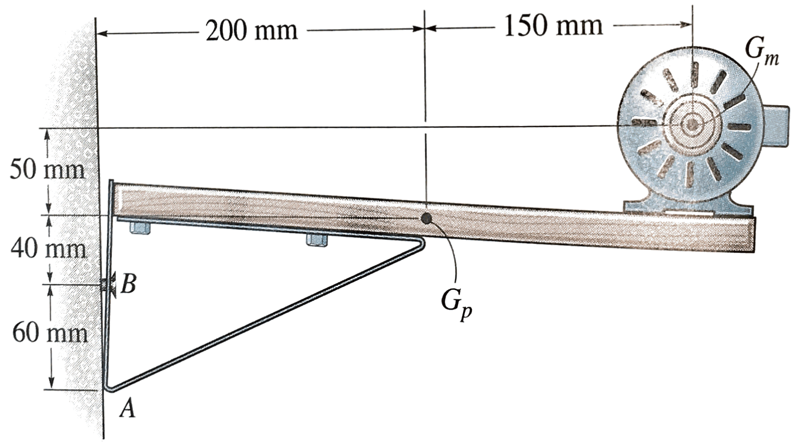
10. Which of the following radioactive samples has the lowest initial decay rate?

A. 1,000,000 mole of ${}^{40}_{19}K$; half-life of 1.25 Giga-years
B. 0.7 mole of ${}^{241}_{94}Pu$; half-life of 4800 days
C. 1.0 mole of ${}^{230}_{90}Th$; half-life of 400 days
D. 0.001 mole of ${}^{228}_{88}Ra$; half-life of 3.5 days
E. 0.004 mole of ${}^{225}_{89}Ac$; half-life of 10 days

Section B: Structured Questions (100 marks)

1.

In the figure shown below, the shelf *supports* the electric motor which has a mass of 15 kg and centre of mass at point G_m. The platform upon which it rests has a mass of 4 kg and its centre of mass is at point G_p. A single bolt B holds the shelf up and the bracket bears against the *smooth* wall at A.



(a) State the conditions for a system to be in (i) translational equilibrium, and (ii) rotational equilibrium.

(b) Sketch a free-body diagram of the forces acting on the shelf, in the situation as shown in the figure. (The shelf comprises of the horizontal wooden board and the triangular frame below.) Provide a legend for the forces you have labelled.

(c) Determine the (i) horizontal, and (ii) vertical forces acting on the shelf at bolt B, and (iii) the force on the shelf at A. You may make use of the dimensions indicated in the figure.

(d) The motor is now switched on. Unfortunately, an eccentric rotation within the motor causes it to vibrate vertically at a single frequency with a small amplitude of less than 1 mm. At one particular frequency, the motor and shelf were observed to be vibrating in phase with an increasing amplitude, causing the bolt to eventually fall off the wall. Explain how this could have occurred.

2.

(a) A guitar string (fixed at both ends) is of length l, mass per unit length μ and tension T.

(i) Write down the relationship between the wavelength λ and length l for the first three harmonics (n =1, 2, 3).

(ii) Explain the terms *pitch* and *timbre*, in terms of the properties of sound waves.

(b) The modern guitar is designed based on the diatonic scale. In this scale, there are 12 semitones between two notes spaced one octave apart, and the ratio of the frequencies of these two notes is equal to 2. The frets of the guitar are then spaced so that adjacent frets on the fingerboard correspond to musical pitches that differ by one semitone. The table below shows some data on the fret number k, and the measured length of the *fifth* guitar string that vibrates when that fret held.

Fret number, k	Length, l_k (cm)
0	61.6
1	58.2
2	54.9
3	51.8
4	49.0
5	46.2
6	43.6

(i) Deduce the relationship between the frequencies of two notes played p frets apart, (e.g. 0^{th} fret and p^{th} fret, or 1^{st} fret and $(p+1)^{th}$ fret, etc).

(ii) Deduce the speed of transverse waves on the *fifth* string of a guitar, which has a fundamental frequency of 110 Hz.

(iii) A student wishes to plot a *straight line* graph showing the relationship between the length of the string l_k, and the fret number k. What could he plot on the vertical and horizontal axes to obtain a straight line graph, and what would be the gradient and y-intercept?

(iv) Plot the graph you suggested in part (iii) on the graph paper provided. Be sure to perform any necessary calculations and display them on a table neatly. Find the gradient and y-intercept of your graph.

(v) If the measurement of length l_k in the table is made by a standard laboratory meter rule, estimate the uncertainty in the measurement of length l_k.

3.

During a rain, raindrops of 3.0 mm in diameter on average, fell from a height of several km above the ground. The drag experienced by a spherical object in air is given by

$$F_{drag} = \frac{1}{2} C_D \rho A v^2,$$

where v is the speed of the object, A is the cross-sectional area of the object, ρ is the density of air, and C_D is the drag coefficient. For a sphere, $C_D = 0.5$ and the density of air is approximately 1.29 kg m^{-3}.

(i) By means of dimensional analysis, show that the drag coefficient must be dimensionless.

(ii) Determine the terminal velocity of the raindrop.

(iii) On average every second, 500 raindrops hit a flat, horizontal roof of area 2.0 m^2. The raindrops do not bounce and the rain does not accumulate on the roof. What is the pressure exerted on the roof?

(iv) For much larger raindrops, the drag coefficient differs from 0.5. Suggest, by giving a simple justification, whether it should be larger or smaller than 0.5.

(v) Explain how an increase in atmospheric temperature may affect the terminal velocity of the raindrop.

4.

A basic astronomical refracting telescope is made from two converging lenses. The lenses consist of the *objective lens* with a long focal length f_1, and an *eyepiece lens* of short focal length f_2. The angular magnification of the telescope is given by $M = f_1 / f_2$. Furthermore, the telescope is considered to be in *normal adjustment* when the final image viewed by the human eye is *formed at infinity*.

(a) With the aid of a ray diagram, explain in detail how such an image may be formed in such a telescope.

(b) Given that f_1= 120 cm and f_2=5 cm, calculate the angular magnification and explain briefly what it means.

(c) The resolving power of the telescope is given by $\theta = 1.22\lambda / D$, where θ is the smallest angle subtended at the telescope by two distant objects which can just be seen separated, λ is the wavelength of light, and D is the diameter of the objective lens.

A student wrote the following statements:
Statement I: The greater the value of θ, the better the resolving power.
Statement II: The larger the angular magnification, the better the resolving power.

Explain whether these statements are true or false. If a statement is not correct, write a true statement involving the same variables mentioned in the original statement.

5.

There are very good reasons why physical activities should not be conducted at noon times in hot weather. When jogging strenuously, an average runner of mass 68 kg and surface area 1.85 m^2 produces energy at a rate of up to 1300 W, 80% of which is converted to heat. The jogger radiates heat, but actually absorbs more from the hot air than he radiates away when the surrounding temperature is higher than his body's.

The total power P radiated by an ideal blackbody is given by $P = \sigma A(T_b^4 - T_s^4)$, where is the A is the surface area, σ is the Stefan-Boltzmann constant (given in the data table), T is the thermodynamic temperature and subscripts b and s refer to the body and surroundings respectively.

At such high levels of activity, the skin's temperature can be elevated to around 33°C instead of the usual 30°C. *In this question, neglect conduction and assume that sweat has the same thermal properties as water*. The only way for the body to get rid of this extra heat is by evaporation of sweat.

(a) Explain how heat is removed through evaporation of sweat.

(b) How much heat per second is produced just by the act of jogging?

(c) How much *net* heat per second does the runner gain just from radiation if the air temperature is 38.0°C ?

(d) What is the *total* amount of excess heat this runner's body must get rid of per second?

(e) How much water must the jogger's body evaporate every minute due to his activity?

(f) If the jogger was not able to get rid of the excess heat, by how much would his body temperature increase above the normal 37°C in 30 minutes of jogging?

-- End of Paper --