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LECTURE DEMONSTRATION OF FRESNEL DIFFRACTION BY A SLIT AND HALF-PLANE

V. F. Klimkin and G. N. Sankin

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In the present work, lecture demonstration of Fresnel diffraction by a slit and half-plane with the use of modern scientific means of optical experimentation is described. The developed software allows light diffraction to be modeled on a personal computer depending on the wavelength, slit width, and distance from the slit (half-plane) to the screen and simultaneously a graphic method of calculating the field amplitude with the help of the Cornu spiral to be demonstrated. A photodetector system built around a video camera controlled by a personal computer is used to register an actual diffraction pattern. The experimental intensity distributions are compared with the theory. The integrated approach allows the efficiency of comprehension of complex diffraction phenomena in wave optics to be increased.

INTRODUCTION

Diffraction (from the Latin word *diffractus* translated *broken* into English) is a class of phenomena describing the deviation of waves from rectilinear propagation in passing by edges of opaque bodies. Because of diffraction, waves bend around opaque bodies and penetrate into the geometrical shadow zone. Since diffraction is peculiar to any wave motion, the discovery of this phenomenon for light waves by F. Grimaldi and its explanation by O. Fresnel appeared to be the decisive proof of the wave nature of light. Diffraction of particles is the manifestation of the dual nature of microparticles (corpuscular-wave dualism) and can be explained based on the wave principles.

Devices based on the diffraction phenomenon (for example, the Fresnel and Wood lenses) [1–3] are used to amplify the intensity of continuous electromagnetic radiation and to inverse acoustic pulsed wave amplitudes [4, 5] as well as in many spectral (for example, diffraction gratings) and other devices.

Thus, the diffraction phenomena are of considerable importance for comprehension of wave theory and have a number of interesting applications. It should be noted that Fresnel diffraction is a division of wave optics more difficult for mastering by the students and more complex mathematically (when solving its problems). The complexity of this phenomenon calls for an integrated approach to the study of this problem based on numerical modeling, carrying out actual physical experiments, and comparison of the results obtained with the theory. The present work is an example of the development of this approach.

1. BRIEF THEORY

Diffraction by a rectilinear edge

Let a plane monochromatic wave $E_0\exp[i(kz - \omega t)]$ (occupying the region $x < 0$, $z = 0$) be incident normally on the rectilinear edge of an opaque screen. In the case of Fresnel diffraction by the rectilinear edge (Fig. 1), the field amplitude in the observation plane is described by the Fresnel–Kirchhoff formula (in the Fresnel approximation or in the near field) [1–3]:

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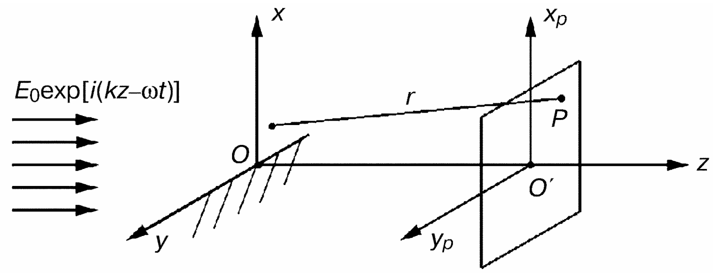

Fig. 1. Scheme of observation of Fresnel diffraction by a rectilinear edge.

$$E(P) = \sqrt{\frac{k}{2\pi iz}} E_0 \exp[i(kz - \omega t)] \int_0^\infty \exp\left[\frac{ik(x - x_p)^2}{2z}\right] dx. \tag{1}$$

It should be noted that diffraction integral (1) is a mathematical form of the Huygens–Fresnel principle. The factor $\sqrt{\frac{k}{2\pi iz}}$ is caused by expansion in cylindrical waves (in the one-dimensional case) [2]. Introducing dimensionless variables

$$\alpha = \sqrt{\frac{k}{\pi z}}(x - x_p) \text{ and } w = -\sqrt{\frac{k}{\pi z}} x_p = -\sqrt{\frac{2}{\lambda z}} x_p, \tag{2}$$

we can write Eq. (1) as

$$E(P) = \frac{E_0}{\sqrt{2i}} \exp[i(kz - \omega t)] \int_w^\infty \exp\left(i\frac{\pi}{2}\alpha^2\right) d\alpha. \tag{3}$$

Thus, a solution of the problem is reduced to the so-called Fresnel integrals

$$C(w) = \int_0^w \cos\left(\frac{\pi}{2}\alpha^2\right) d\alpha \quad \text{and} \quad S(w) = \int_0^w \sin\left(\frac{\pi}{2}\alpha^2\right) d\alpha, \tag{4}$$

calculated by numerical methods. Tables of these integrals are given, for example, in [6]. We note only that

$$C(0) = S(0) = 0,$$

$$C(-w) = -C(w) \text{ and } S(-w) = -S(w), \tag{5}$$

$$C(\pm\infty) = S(\pm\infty) = \pm 1/2.$$

Taking into account Eqs. (4) and (5), we can write for the field at observation point P:

$$E(P) = \frac{E_0}{\sqrt{2i}} \exp[i(kz - \omega t)] \left\{\left[\frac{1}{2} - C(w)\right] + i\left[\frac{1}{2} - S(w)\right]\right\}. \tag{6}$$

From Eq. (6) it follows that the field at point P of the diffraction pattern is determined by the difference between two complex vectors $OO^+ = 1/2 + i1/2$ and $OW = \int_0^w \exp\left(i\frac{\pi}{2}\alpha^2\right)d\alpha = C(w) + iS(w)$. The variable w takes all possible values, and point $W(w)$ traces the curve called the Cornu spiral [1]. We note that point O specifies the origin of rectangular system of coordinates.

The field at point P of the diffraction pattern is determined by the distance from point W of the Cornu spiral to asymptotic points O^+ (+1/2, +1/2). The field intensity at point P is proportional to $|E(P)|^2$ and to within a constant factor is

$$I(P) = \frac{1}{2}I_0\left\{\left[\frac{1}{2} - C(w)\right]^2 + \left[\frac{1}{2} - S(w)\right]^2\right\}. \tag{7}$$

Thus, if the observation point is in the illuminated region ($x_p > 0$, $w < 0$), the end point of the complex vector will move along the left branch of the spiral. The intensity oscillates with the amplitude decreasing with increasing distance of point x_p from the edge of the geometrical shadow zone and asymptotically approaches I_0. At the shadow zone boundary ($x_p = 0$, $w = 0$), we have $I = 1/4I_0$. In the geometrical shadow zone ($x_p < 0$, $w > 0$), the end point of the complex vector will move along the right branch of the spiral. Hence, the intensity here monotonically decreases to zero.

We note that the quantity $\sqrt{\lambda z}$ [see Eq. (2)] characterizes the scale of the diffraction pattern. It determines sizes of the so-called Fresnel zones [1, 2]. The first maximum arises when the observation point is shifted approximately by the width of the first Fresnel zone $\sqrt{\lambda z}$. Hence, the diffraction pattern in the xOz plane (Fig. 1) is formed by a system of bands bent by a parabolic law $x_p \sim \sqrt{n\lambda z}$ $\left(z \sim \frac{1}{n\lambda}x_p^2\right)$. It is shown in Fig. 2*a*. Curves *1-4* are drawn for odd n. A certain deviation of these curves from positions of light bands is observed.

1.2. Diffraction by a slit

For a slit of width d, Fresnel–Kirchhoff integral (1) is taken between the limits $-d/2$ and $+d/2$, and the limits of integral (3) are

$$w_1 = -\sqrt{\frac{2}{\lambda z}}\left(\frac{d}{2} + x_p\right), \; w_2 = \sqrt{\frac{2}{\lambda z}}\left(\frac{d}{2} - x_p\right). \tag{8}$$

The field intensity at point P of the diffraction pattern can be written as follows:

$$I(P) = \frac{1}{2}I_0\left\{[C(w_2) - C(w_1)]^2 + [S(w_2) - S(w_1)]^2\right\}. \tag{9}$$

From Eqs. (8) and (9) it follows that the field intensity at point x_p is determined by the squared distance between the two points of the Cornu spiral with coordinates $C(w_1)$, $S(w_1)$ and $C(w_2)$, $S(w_2)$. For example, if observation point P lies on the z axis ($x_p = 0$ in Fig. 1), these two points are symmetric about the origin of coordinates. As the distance z from the slit (or the slit width d) changes, the field intensity oscillations of variable amplitude are observed at point P. In this case, the intensity maxima and minima approximately coincide with points of intersection of the Cornu spiral with the straight line passing through the spiral foci.

Diffraction patterns from each opaque edge of the slit are overlapped. Light and dark regions lie on intersections of parabolas drawn from each edge by analogy with the rectilinear edge (Fig. 2*b*). Since in the case of diffraction by a rectilinear edge, the amplitude of intensity oscillations in the illuminated region tends to zero with increasing slit width (when z remains constant), the effect of each screen can be considered separately. In conclusion, we note that band coordinates are proportional to $\sqrt{\lambda}$, unlike Fraunhofer diffraction by a slit for which they are proportional to λ [2].

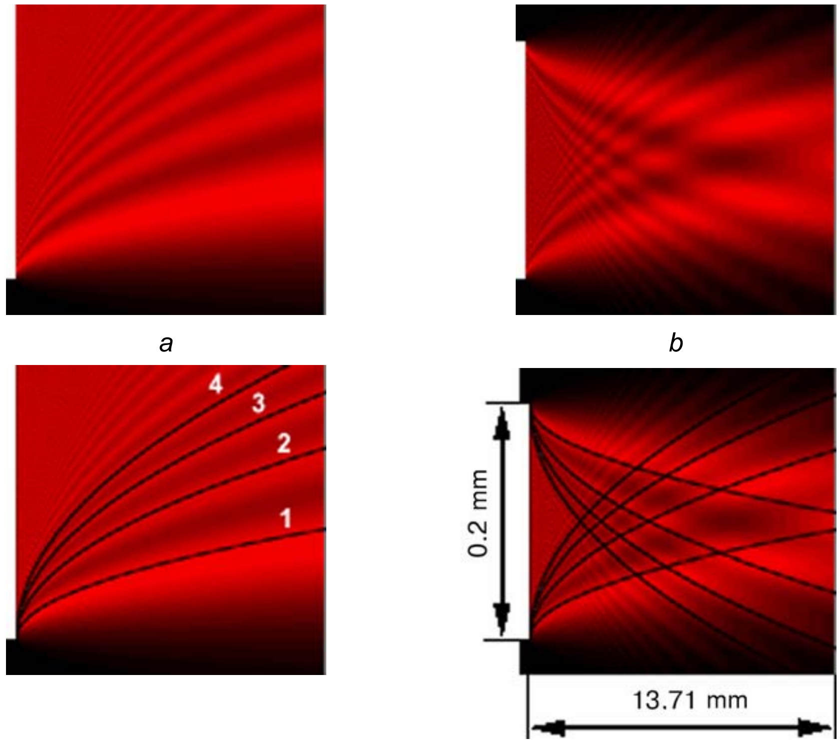

Fig. 2. Band structures in the case of Fresnel diffraction by a rectilinear edge (*a*) and slit (*b*). The light wavelength is $\lambda = 0.6328$ μm. Calculations were carried out by formulas $x_p = \sqrt{\lambda z}$ (curve *1*), $x_p = \sqrt{3\lambda z}$ (curve *2*), $x_p = \sqrt{5\lambda z}$ (curve *3*), and $x_p = \sqrt{7\lambda z}$ (curve *4*).

2. COMPUTER MODELING

The foregoing theoretical background helped us to develop software for lecture demonstration of Fresnel diffraction. Programs "Fresnel diffraction-1" and "Fresnel diffraction-2" were created as appendices to the operational system MS Windows 95/98/NT. They allow light intensity distributions behind a single slit or screen with rectilinear edge to be modeled numerically for normal incidence of coherent radiation.

Figure 3 displays a window of the program "Fresnel diffraction-1." It illustrates the problem formulation (light wavelength, type of the opaque body, and slit width are indicated on panel *1*), results of calculations of the 2-D light diffraction pattern observed to the right of opaque body *2* (in the *xOz* plane in Fig. 1), and the light intensity distribution in the observation plane (its position is shown by straight line *3*). The light wavelength and the distance to observation plane *3* can be changed smoothly with buttons *4* and *5*, respectively. The program allows the teacher to demonstrate the dynamics of the diffraction pattern described subsequently by the geometric optics, Fresnel, and Fraunhofer approximations (curves *6*, *7*, and *8*, respectively) by increasing continuously the distance from the slit to the observation point. This vividly demonstrates that all three approximations characterize the same diffraction process.

Figure 4 shows a window of the program "Fresnel diffraction-2." The results of modeling are displayed on the left of panel *1*. The program allows the light intensity distribution in the observation plane to be investigated in detail depending on the wavelength, slit width, and distance from the slit or rectilinear edge (the parameters are shown on panel *2*) and simultaneously to demonstrate the graphic method of calculating the field amplitude with the help of the Cornu spiral. The light intensity distribution can be analyzed quantitatively with the help of the cursor or marker. Smooth motion of the marker over the intensity distribution curve is carried out with the help of scrolling button *3*. The cursor position is set by the mouse. The position of the marker (cursor) sets a point in the observation plane whose field amplitude is plotted in the

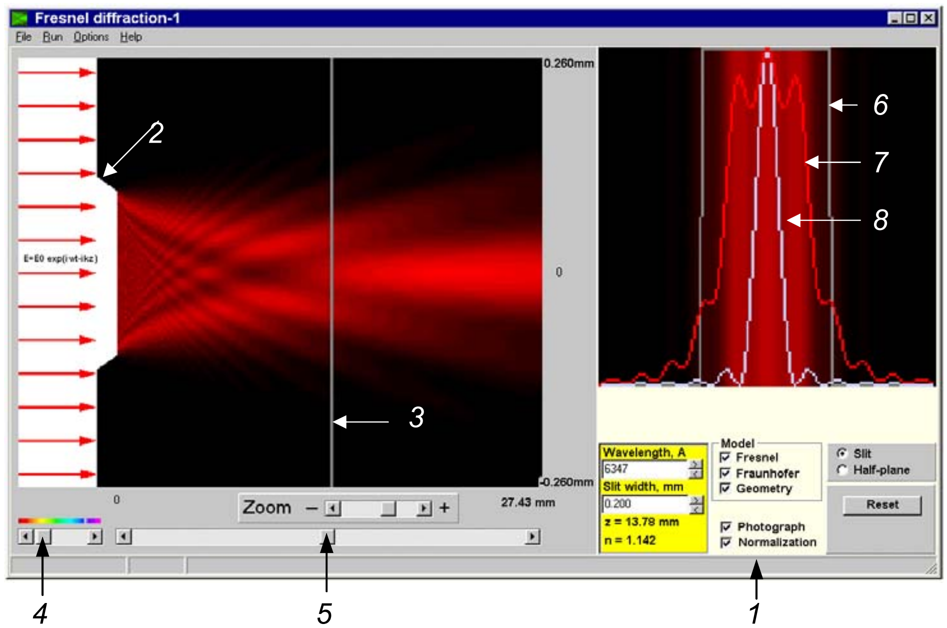

Fig. 3. Window of the program "Fresnel diffraction-1."

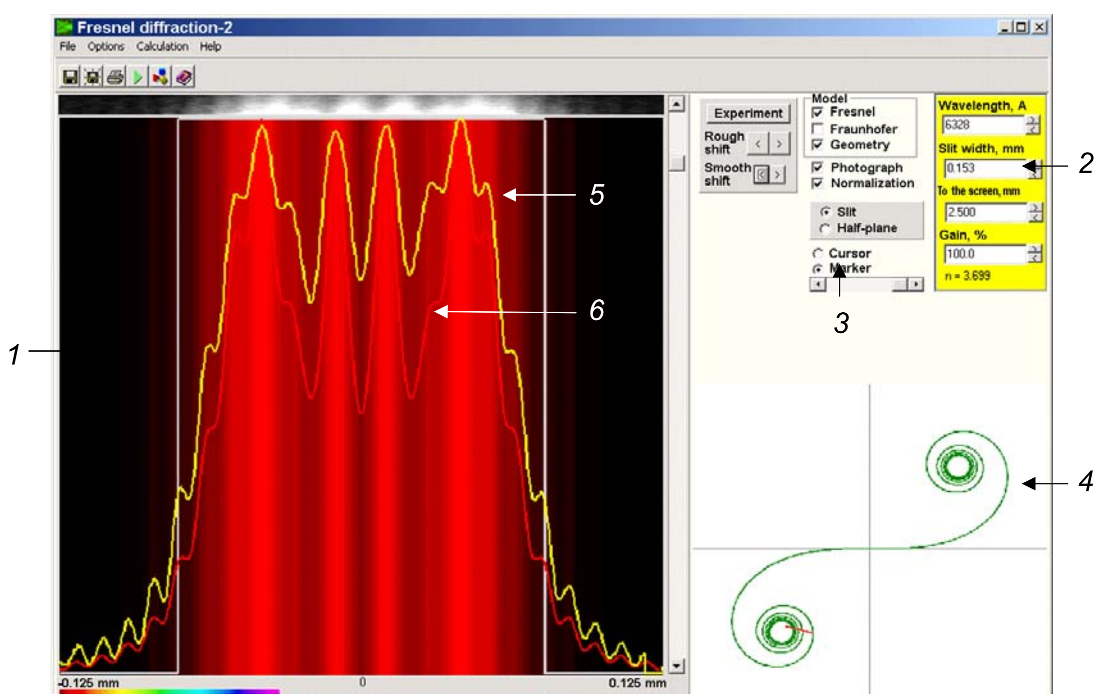

Fig. 4. Window of the program "Fresnel diffraction-2."

Cornu spiral shown to the right of the window on panel *4*. The intensity magnitude is directly proportional to the squared length of this spiral section. The program allows the experimental intensity distribution to be compared with the theory (for example, curves *5* and *6*).

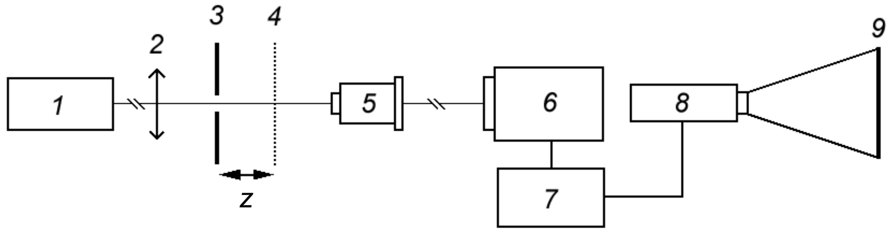

Fig. 5. Block diagram of the setup for lecture experiment comprising He–Ne laser *1*, cylindrical lens *2*, slit *3*, observation plane *4*, microobjective *5*, video camera *6*, personnel computer *7*, multimedia projector *8*, and projection screen *9*.

In the case of a slit of width d, the number of the open Fresnel zones n (on one side from the slit center) calculated by the formula

$$n = \frac{d^2}{4\lambda z}, \tag{10}$$

is displayed on panels *1* and *2* of the corresponding windows of the program.

To show the results of numerical modeling to the lecture audience, the image from the monitor is projected onto a large projection screen with the help of a multimedia projector. The time it took for computer modeling was 5–10 min.

3. LECTURE EXPERIMENT

Figure 5 shows the block diagram of an experimental setup for studying Fresnel diffraction. Coherent light of helium-neon laser *1* is transformed into a vertically diverging beam (to attenuate laser radiation) with the help of cylindrical lens *2* and is then incident on slit *3* having a finely adjustable width. The diffraction pattern is registered with a modern photodetector system comprising video camera *6* controlled by personal computer *7*. To transfer the image from observation plane *4* to the CCD matrix of the video camera, microobjective *5* with a 10-fold magnification and a focal length of about 5 mm was used. The CCD matrix was used to convert electromagnetic radiation into an electric signal. It comprized a file of 768×576 photosensitive eight-bit cells (having a dynamic range of 1: 256) spaced at about 6.5 μm. Its transverse dimension was about 5 mm, and its vertical dimension was 4 mm. The disadvantage of this photodetector system is discrete readout of images. The distance from the object plane to the observation plane of the diffraction pattern z varied from 0 to 20 mm (for slit widths of 0.01–0.5 mm).

Smooth shift of microobjective *5* from the slit (of a preset width) causes the number of intensity maxima and minima in the slit image to decrease and leads to their alternation in the center of the diffraction pattern. The similar phenomena in the Fresnel diffraction zone were also observed when the slit width was smoothly decreased given that the position of the observation plane remained unchanged. Fresnel diffraction by edges of a wide slit and gradual overlap of the diffraction patterns were investigated by narrowing the slit. With further decrease of the slit width, transition from Fresnel to Fraunhofer diffraction was observed.

The developed software enabled us to collect and to process in real time experimental data as well as to display the spatial diffraction pattern and the light intensity distribution in the observation plane on the screen of the monitor and to compare the distributions obtained with the theory. Multimedia projector *8* was used to project the image on projection screen *9* thereby easily demonstrating the experimental results to a large audience.

The compactness of the experimental setup and the high rate of experimental data recording and processing, caused by the direct interface of the recording system with the personal computer, should also be mentioned. This is especially important for the lecture experiment [7].

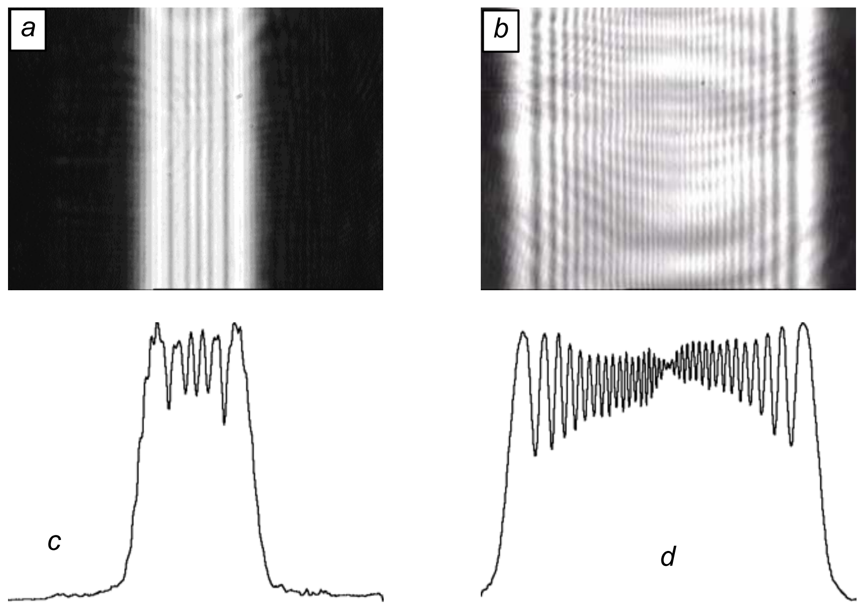

Fig. 6. Fresnel diffraction patterns (*a* and *b*) and light intensity distributions (*c* and *d*) for narrow (*a* and *c*) and wide slits (*b* and *d*) registered with the video camera.

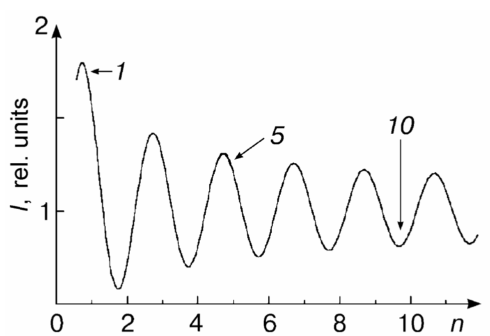

Fig. 7. Dependence of the light intensity in the center of the diffraction pattern for a slit on the number of the open Fresnel zones *n*.

As an example, Fig. 6 shows the Fresnel diffraction patterns for wide and narrow slits and the corresponding light intensity distributions registered with the video camera.

The slit width d or the light wavelength λ can be determined from the diffraction pattern using Eq. (10). To this end, the number of the open Fresnel zones n must be known. In practice, it is better to fix z values (Fig. 5) for which the light intensity in the center of the diffraction pattern is maximum or minimum (has an extremum). This can be easily checked from the intensity distribution by smooth displacement of the observation plane from the slit. In this case, the number of the open Fresnel zones n will not be a natural number. This is caused by unequal areas of the Fresnel zones and leads to the displacement of the focus of the Cornu spiral relative to the origin of coordinates.

Figure 7 shows the dependence of the light intensity in the center of the diffraction pattern (normalized by the incident light intensity) on the number n calculated with the help of the program "Fresnel diffraction-2." It can be seen that intensity maxima (minima) are shifted relative to the odd (even) n by about 0.3 (the data were averaged over 13 realizations) toward smaller values. The serial number of the extremum (for example, Nos. 1, 5, and 10 in Fig. 7) determines the number of light diffraction bands (the number of dark bands is less by unity). If for a given z value (that determines the distance to the observation plane), n^* light bands are seen on the slit image, the number of the open Fresnel zones will be $n = n^* - 0.3$. To simplify calculations by formula (10), we can take advantage of a calculator built in the program. If the scale with a division of 0.1 mm is used to measure z, results will be satisfactory for $z > 2$ mm (when the number of light bands will be 4–8).

The window of the program "Fresnel diffraction-2" displayed in Fig. 4 shows the experimental light intensity distribution (curve *5*) in the case of diffraction by the slit with $d = 150 \pm 5$ μm at $\lambda = 0.63$ μm and $z = 2.5 \pm 0.1$ mm. It can be seen that the number of maxima is 4 (then the number of the open Fresnel zones will be $n = 3.7$).

Calculations under formula (10) yield $d = 153$ μm. Positions of maxima and minima of the corresponding theoretical light intensity distribution (curve *6*) well coincide with those of the experimental distribution. The contrast of the experimental distribution is deteriorated. More detailed calibration of the system demonstrated that the relative error in measuring the slit width did not exceed 5%.

The time it took to carry out the lecture experiment was about 10 min.

In conclusion, we emphasize the following:

1. The integrated approach to lecture demonstrations on diffraction of light (numerical modeling, carrying out an actual physical experiment, and comparison of the results obtained with the theory) allows the students to study more deeply this fundamental physical phenomenon in wave optics.

2. Application of modern photodetector systems and computer support allows the efficiency of lecture experiment to be increased significantly making it not only vivid but also quantitative.

3. Determination of the slit width and the light wavelength from the Fresnel diffraction pattern (comparatively simply and quickly) allows the teacher to demonstrate the feasibility of practical applications of the phenomenon under study.

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