ZEE-B – Zeeman-Effekt (Variant B)
Auswertung

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19. August 2021

Teilversuch 1: Vermessung des Magnetfeldes

Fehler $\Delta B = \pm 10\,\mathrm{mT}$

I/A	1,070	2,096	2,995	4,153	5,300	6,033	7,09	8,03	9,06	9,48
$\Delta I/\mathrm{A}$	0,005	0,005	0,010	0,020	0,010	0,010	0,01	0,01	0,01	0,01
B/mT	671	1298	1910	2700	3460	3950	4630	5240	5750	5970

Als Hintergrund haben wir zwei Messungen:

Messung	Hintergrund
Davor	$(0{,}11 \pm 0{,}02)\,\mathrm{mT}$
Danach	$(1{,}13 \pm 0{,}02)\,\mathrm{mT}$

Da diese Hintergrundwerte deutlich unter der Unsicherheit ΔB liegt, vernächlässigen wir den Hintergrund.

Wir führen nun eine Kurveanpassung zu $B = mI + c$ mittels `gnuplot` durch (siehe Appendix A):

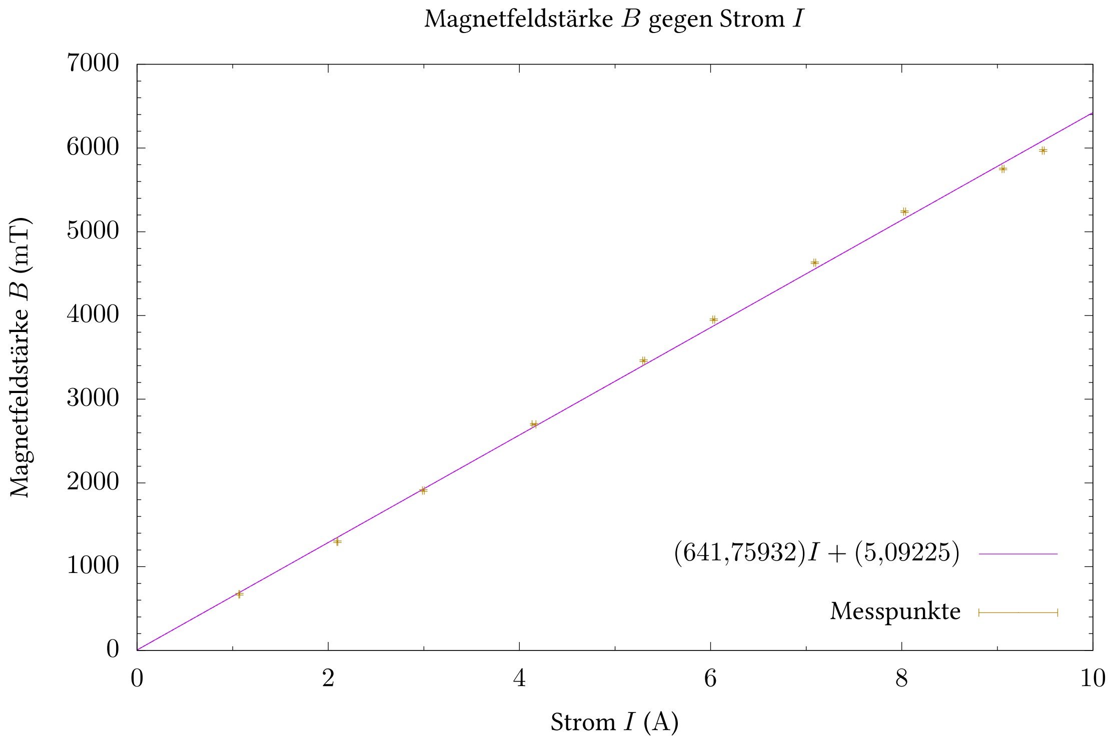


Abbildung 1.1: Magnetfeldstärke gegen Strom
$\left(\chi^2_{\text{red}} = 39{,}1889 \text{ (klein gegen Werten)} \Rightarrow \text{Gute Anpassung}\right)$

Als Endergebnis erhalten wir:

Variable	Roh	Gerundet
m	$(641{,}759 \pm 8{,}077)\,\mathrm{mT\,A^{-1}}$	$(641 \pm 9)\,\mathrm{mT\,A^{-1}}$
c	$(5{,}09 \pm 49{,}33)\,\mathrm{mT}$	$(5 \pm 50)\,\mathrm{mT}$

Da 0 im Fehlerintervall von c liegt, ist die Kurveanpassung auch vernünftig. Für die Kalibrierung von Strom zu Magnetfeldstärke dient also die folgende Formel:

$$B/\mathrm{mT} = 641 \times I + 5 \tag{1.1}$$

$$\begin{aligned}\Delta B/\mathrm{mT} &= \sqrt{\left(\frac{\partial B}{\partial m}\Delta m\right)^2 + \left(\frac{\partial B}{\partial I}\Delta I\right)^2 + \left(\frac{\partial B}{\partial c}\Delta c\right)^2} \\ &= \sqrt{(I\Delta m)^2 + (m\Delta I)^2 + (\Delta c)^2} \\ &= \sqrt{81I^2 + 410881(\Delta I)^2 + 2500}\end{aligned} \tag{1.2}$$

Teilversuch 2: Kalibrierung des Linsensystems

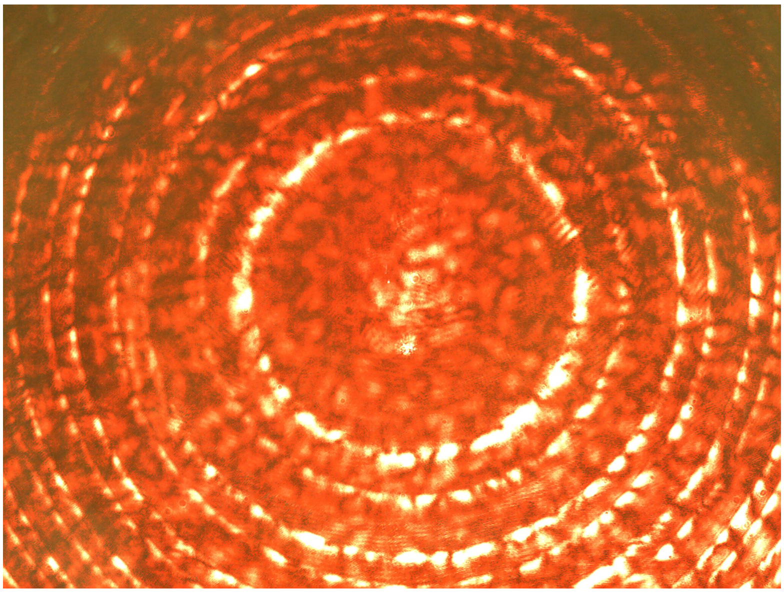


Abbildung 2.1: Interferenzringe mit Laserpointer

Teilversuch 3: Qualitative Betrachtung des Spektrums von Cadmium

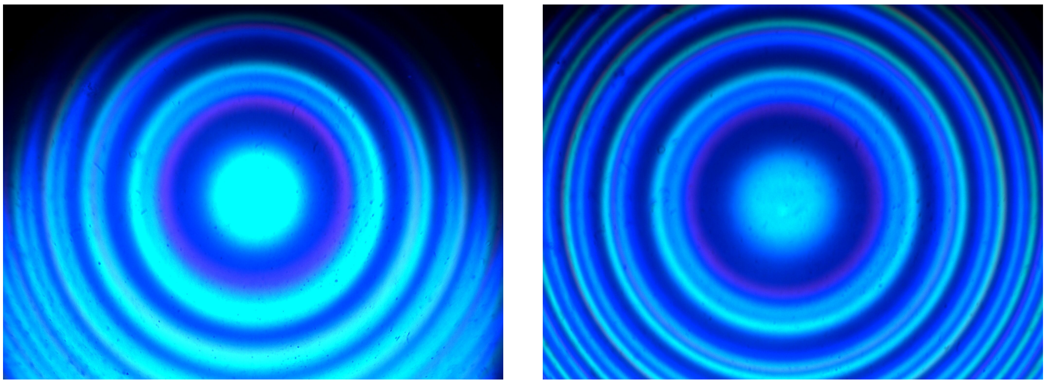


Abbildung 3.1: Interferenzringe mit Cd-Lampe. Vor Justierung (Links). Nach Justierung (Rechts)

Es ist zu bemerken, dass ohne Kamera ist das Interferenzmuster schwer zu sehen. Laut Abbildung 1 der Anleitung gibt es nur 5 Übergängen, die im sichtbaren Bereich liegt. Wir nehmen nun an, dass die Kamera auch nur Licht im sichtbaren Bereich abbilden kann.

Die Zuordnung ist somit:

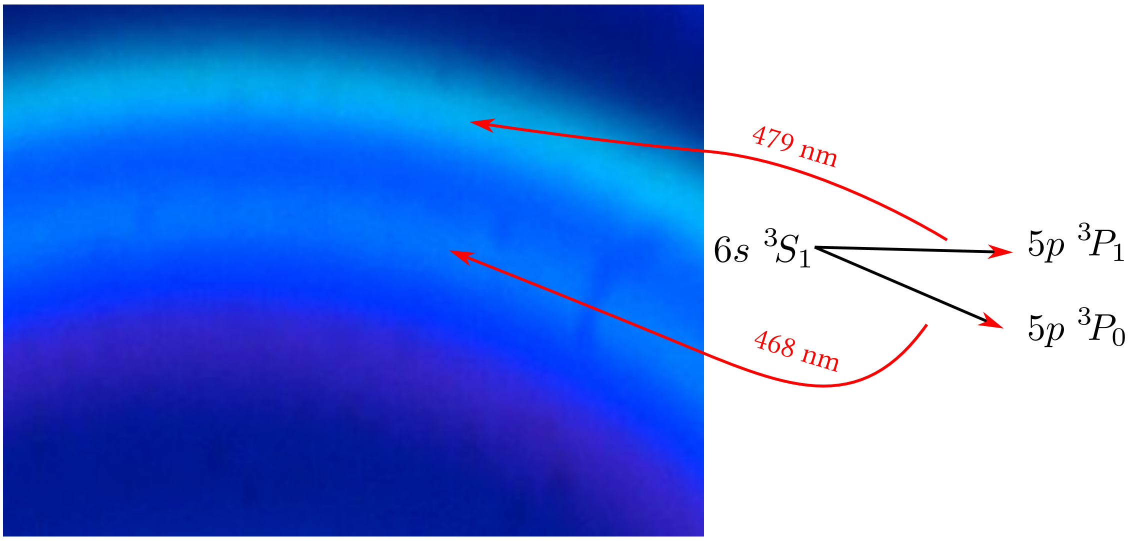


Abbildung 3.2: Zuordnen der sichtbaren Emissionslinien

Die andere sichtbare Linien (508,59 nm, 515,47 nm, 643,85 nm) sind wahrscheinlich zu schwach, um in diesem Bild zu sehen. Man sieht hier auch zusätzlich eine lila Emissionslinie. Sie liegt vermutlich im unsichtbaren UV Bereich ($300\,\mathrm{nm} < \lambda < 450\,\mathrm{nm}$). Da es mehrere Emissionslinie in diesem Bereich liegt, lässt diese Linie nicht so gut zuordnen.

Teilversuch 4: Quantitative Vermessung des normalen Zeeman-Effekts (transversale Beobachtung)

Es gab am Anfang eine Überbeleuchtung:

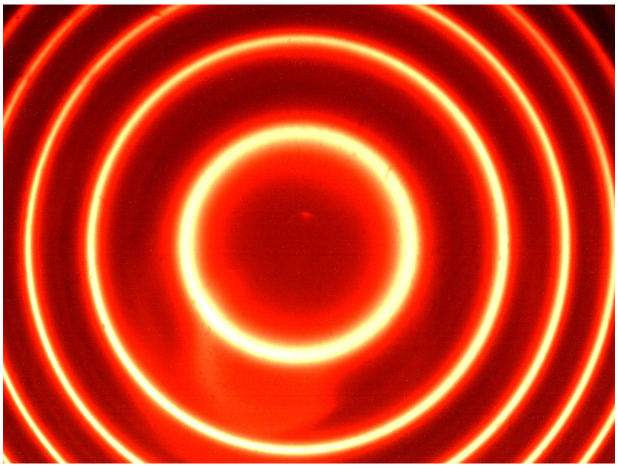


Abbildung 4.1: Überbeleuchtete Interferenzringe von rote Emissionslinie

Nach Anpassung der Beleuchtung im Program.

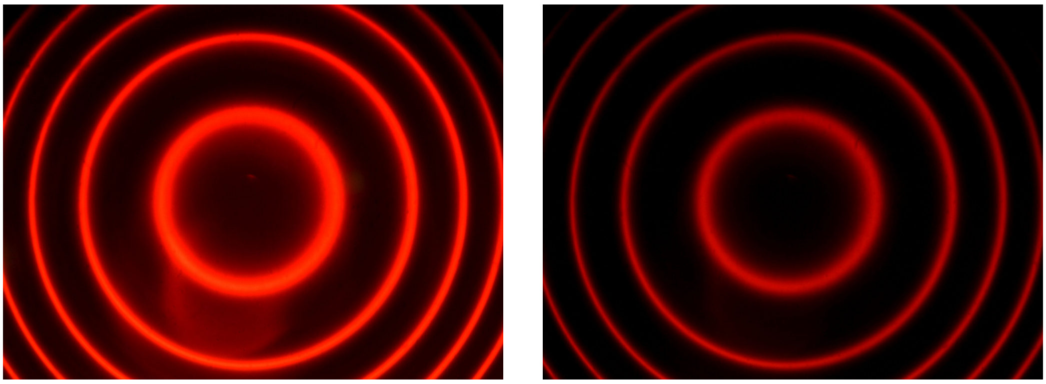


Abbildung 4.2: Interferenzringe von rote Emissionslinie. Ohne Polarisationsfilter (Links). Mit Polarisationsfilter (Rechts)

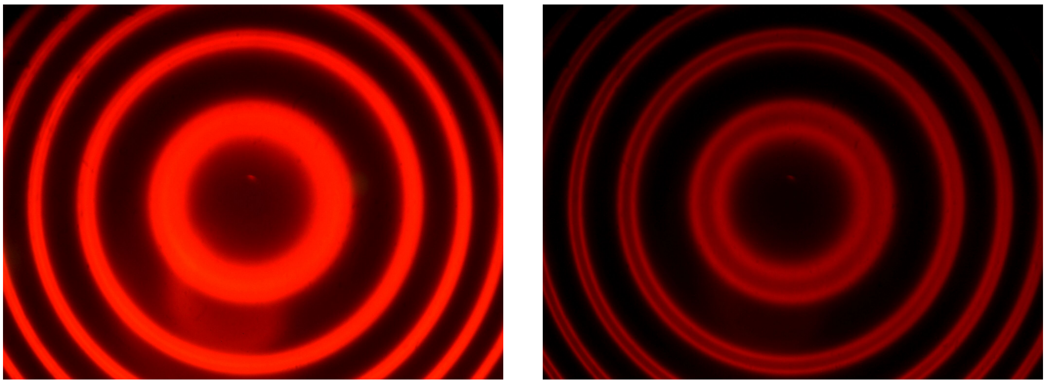


Abbildung 4.3: Interferenzringe von rote Emissionslinie im Magnetfeld $B \approx 2$ - 3A. Ohne Polarisationsfilter (Links). Mit Polarisationsfilter (Rechts)

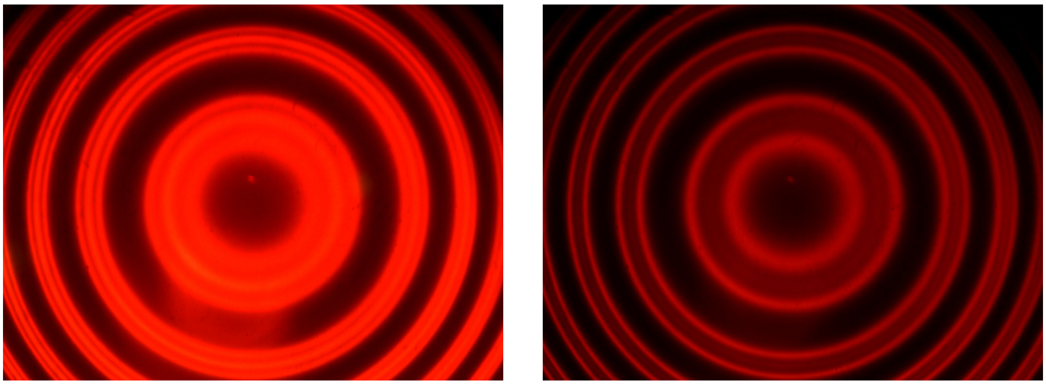
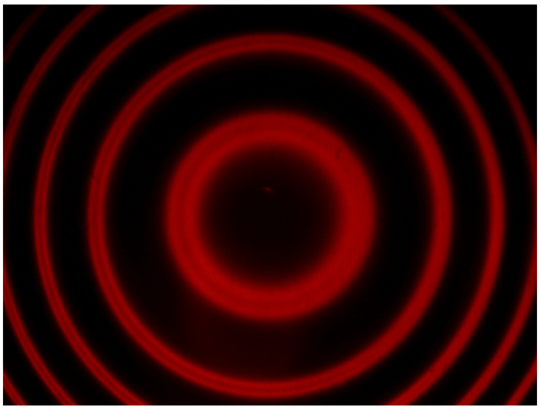
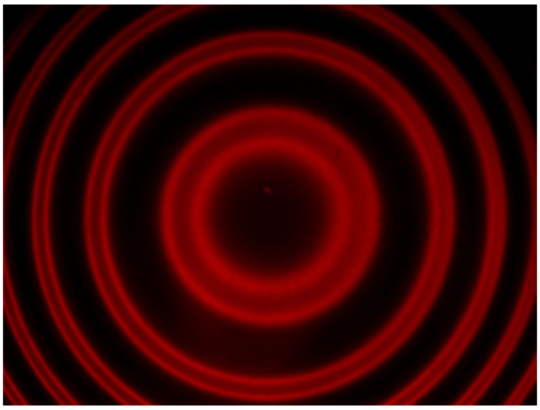


Abbildung 4.4: Interferenzringe von rote Emissionslinie im Magnetfeld $B \approx 6$ A. Ohne Polarisationsfilter (Links). Mit Polarisationsfilter (Rechts)

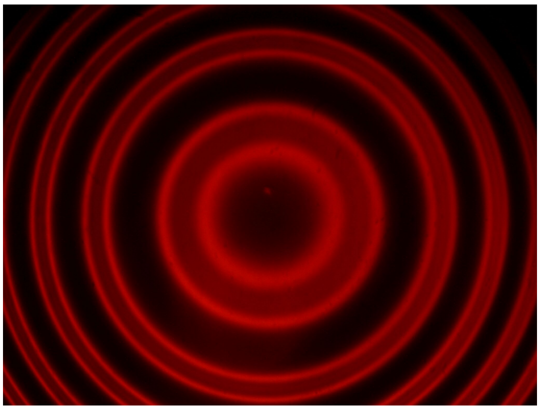
Für die eigentliche Messung:



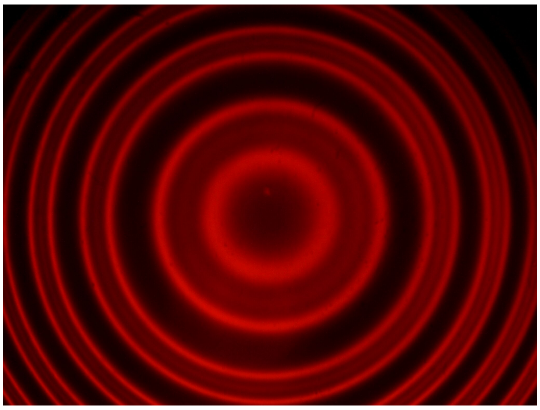
(a) $I = (2{,}495 \pm 0{,}005)$ A



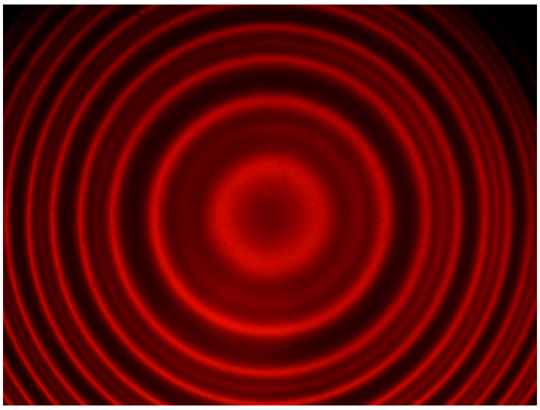
(b) $I = (4{,}190 \pm 0{,}010)$ A



(c) $I = (5{,}662 \pm 0{,}010)$ A



(d) $I = (7{,}01 \pm 0{,}01)$ A



(e) $I = (8{,}78 \pm 0{,}01)$ A

Abbildung 4.5: Messungen

Da die Messreihe zu lang ist, wird sie hier nicht wieder formatiert. Sie finden die Messreihe im Laborprotokoll unter Teilversuch 4. Alle Rechnungen für r_m^2 und $\Delta r_m^2 = 2r_m(\Delta r_m)$ werden direkt in `gnuplot` berechnet und somit hier nicht weiter beschrieben.

Wir führe nun die benötigte Kurveanpassungen zu $r_m^2 = mp + c$ durch. Der p-Achsenschnittpunkt p_0 ist somit gegeben durch:

$$p_0 = -\frac{c}{m} \tag{4.1}$$

$$\Delta p_0 = |p_0|\sqrt{\left(\frac{\Delta c}{c}\right)^2 + \left(\frac{\Delta m}{m}\right)^2} \tag{4.2}$$

und im `gnuplot` direkt berechnet.

Für λ_-:

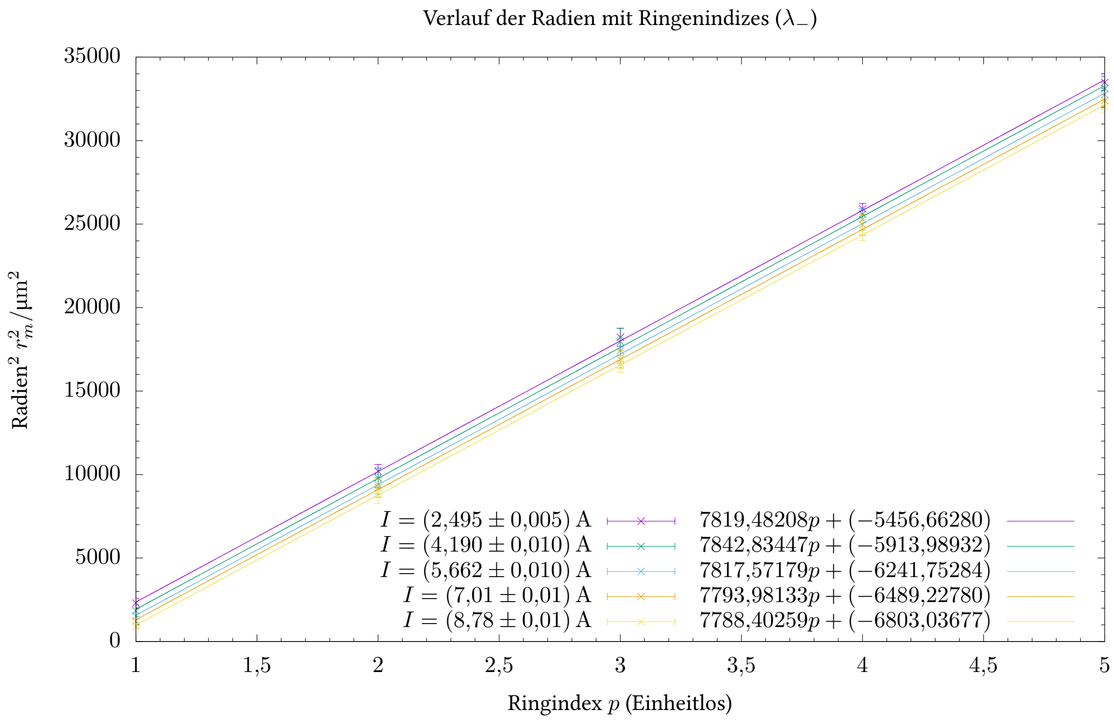


Abbildung 4.6: Verlauf der Ringradien

Strom I/A	$m/\mu m^2$	$c/\mu m^2$	p_0	χ^2_{red}
$2{,}495 \pm 0{,}005$	$7819{,}482\,08 \pm 41{,}582\,14$	$-5456{,}662\,80 \pm 135{,}037\,26$	$0{,}697\,83 \pm 0{,}017\,66$	$0{,}160\,85$
$4{,}190 \pm 0{,}010$	$7842{,}834\,47 \pm 54{,}297\,14$	$-5913{,}989\,32 \pm 192{,}436\,57$	$0{,}754\,06 \pm 0{,}025\,09$	$0{,}223\,24$
$5{,}662 \pm 0{,}010$	$7817{,}571\,79 \pm 40{,}499\,64$	$-6241{,}752\,84 \pm 142{,}581\,67$	$0{,}798\,43 \pm 0{,}018\,70$	$0{,}101\,89$
$7{,}01 \pm 0{,}01$	$7793{,}981\,33 \pm 42{,}734\,37$	$-6489{,}227\,80 \pm 151{,}477\,92$	$0{,}832\,59 \pm 0{,}019\,96$	$0{,}106\,81$
$8{,}78 \pm 0{,}01$	$7788{,}402\,59 \pm 46{,}191\,94$	$-6803{,}036\,77 \pm 163{,}696\,71$	$0{,}873\,48 \pm 0{,}021\,65$	$0{,}125\,85$

Die kleine χ^2_{red}'s zeigt eine gute Kurveanpassung.

Für λ_+:

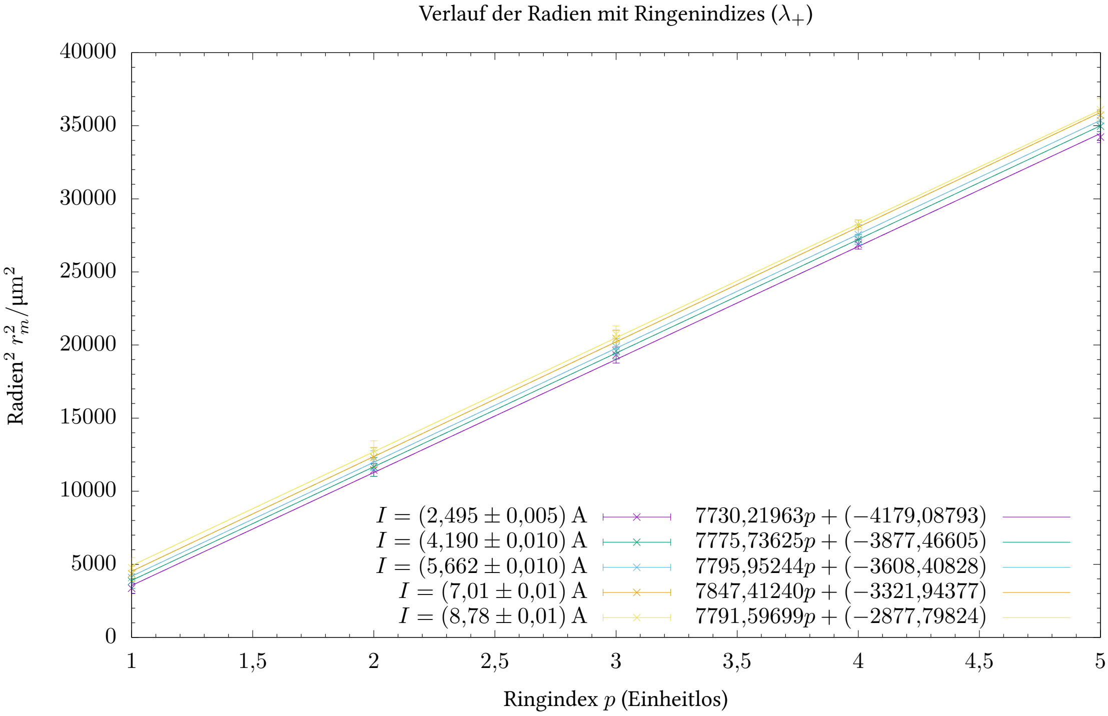


Abbildung 4.7: Verlauf der Ringradien

Strom I/A	$m/\mu m^2$	$c/\mu m^2$	p_0	χ^2_{red}
$2{,}495 \pm 0{,}005$	$7730{,}219\,63 \pm 78{,}433\,01$	$-4179{,}087\,93 \pm 267{,}597\,15$	$0{,}540\,62 \pm 0{,}035\,05$	$0{,}473\,01$
$4{,}190 \pm 0{,}010$	$7775{,}736\,25 \pm 24{,}729\,55$	$-3877{,}466\,05 \pm 93{,}088\,47$	$0{,}498\,66 \pm 0{,}012\,08$	$0{,}030\,54$
$5{,}662 \pm 0{,}010$	$7795{,}952\,44 \pm 27{,}190\,18$	$-3608{,}408\,28 \pm 100{,}572\,27$	$0{,}462\,86 \pm 0{,}013\,00$	$0{,}033\,90$
$7{,}01 \pm 0{,}01$	$7847{,}412\,40 \pm 73{,}536\,70$	$-3321{,}943\,77 \pm 279{,}185\,66$	$0{,}423\,32 \pm 0{,}035\,80$	$0{,}249\,68$
$8{,}78 \pm 0{,}01$	$7791{,}596\,99 \pm 60{,}488\,53$	$-2877{,}798\,24 \pm 213{,}595\,18$	$0{,}369\,35 \pm 0{,}027\,56$	$0{,}089\,22$

Die kleine χ^2_{red}'s zeigt eine gute Kurveanpassung.

Die Quellcodes finden Sie im Appendix B.1.

Aus Gleichungen (34) und (35) der Anleitung ist:

$$\Delta p_0 = p_0^+ - p_0^- = -2dn(k^+ - k^-) = 2dn\Delta k \tag{4.3}$$

$$\implies \Delta k = \frac{\Delta p_0}{2dn} \tag{4.4}$$

Wir bezeichnen Δk und ΔP_0 als K und P, sodass keine Verwechselungsgefahr bei der Berechnung der Unsicherheiten entsteht:

$$K = \frac{P}{2dn} \qquad \Rightarrow \qquad \Delta K = \frac{\Delta P}{2dn} \tag{4.5}$$

Dabei gilt:

$$\Delta P = \sqrt{\left(\Delta p_0^+\right)^2 + \left(\Delta p_0^-\right)^2} \tag{4.6}$$

Es ist hier zu bemerken, dass λ_+ und λ_- während des Versuchs vertauscht waren. K ist somit:

$$K = \frac{p_0^- - p_0^+}{2dn} \qquad \Delta K = \frac{\sqrt{\left(\Delta p_0^+\right)^2 + \left(\Delta p_0^-\right)^2}}{2dn} \tag{4.7}$$

wobei p_0^+ und p_0^- die experimentelle $+$ und $-$ sind. Wegen zeitliche Gründen sind die Auswertung hier nicht neu gemacht.

Wir runden nun alle Werten entsprechend die gewöhnte Rundungsregeln und wandeln die Stromwerte in Magnetfeldstärke gemäß Gleichungen (1.1) und (1.2). Alle Rechnungen erfolgt im Libreoffice Calc. Die m und c-Werten werden hier nicht gerundet, da wir sowieso die Werte direkt im `gnuplot` auswerten.

Gegeben sei $d = 3 \cdot 10^{-3}$ m und $n = 1{,}45$:

I/A	P_0^-	P_0^+	B/T	K/m^{-1}
$2{,}495 \pm 0{,}005$	$0{,}698 \pm 0{,}018$	$0{,}54 \pm 0{,}04$	$1{,}60 \pm 0{,}06$	18 ± 6
$4{,}190 \pm 0{,}010$	$0{,}754 \pm 0{,}026$	$0{,}499 \pm 0{,}013$	$2{,}69 \pm 0{,}07$	29 ± 4
$5{,}662 \pm 0{,}010$	$0{,}798 \pm 0{,}019$	$0{,}463 \pm 0{,}014$	$3{,}63 \pm 0{,}08$	$38{,}5 \pm 2{,}8$
$7{,}01 \pm 0{,}01$	$0{,}833 \pm 0{,}020$	$0{,}42 \pm 0{,}04$	$4{,}50 \pm 0{,}09$	47 ± 6
$8{,}78 \pm 0{,}01$	$0{,}873 \pm 0{,}022$	$0{,}369 \pm 0{,}028$	$5{,}63 \pm 0{,}10$	58 ± 5

Damit mit Appendix B.2:

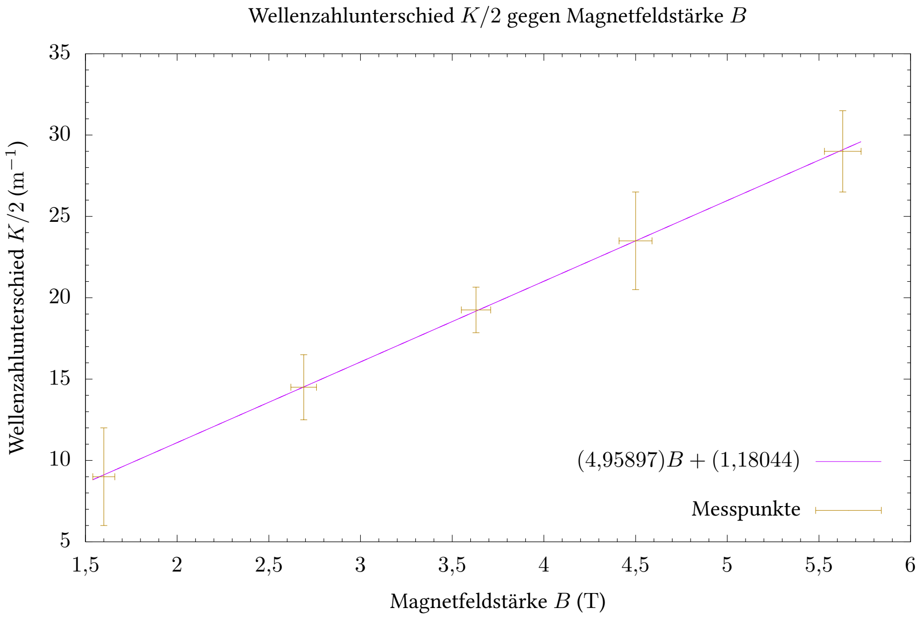


Abbildung 4.8: Gute Kurveanpassung $K = mB + c$, $\chi^2_{\text{red}} = 0.0017626 << 1$

Daraus erhalten wir: $m = (4{,}958\,97 \pm 0{,}035\,95)\,\mathrm{T}^{-1}\,\mathrm{m} = (4{,}96 \pm 0{,}04)\,\mathrm{T}^{-1}\,\mathrm{m}$ und $c = (1{,}180 \pm 0{,}135)\,\mathrm{m}^{-1} = (1{,}18 \pm 0{,}14)\,\mathrm{m}^{-1}$.

Damit ist $g_{1\to 2} = 0{,}1060 \pm 0{,}0009$ nach Gleichung (21) der Anleitung, was sich mit unseren theoretisch erwarteten Wert $-1, 0, 1$ signifikant unterscheidet.

A gnuplot Quellcode zur Auswertung von Teilversuch 1

```
#!/usr/bin/env gnuplot

set term epslatex color size 6in, 4in
set output "tv1.tex"
set decimalsign locale 'de_DE.UTF-8'

set title "Magnetfeldstärke $B$ gegen Strom $I$"
set ylabel "Magnetfeldstärke $B$ ($\\si{\\milli\\tesla}$)"
set xlabel "Strom $I$ ($\\si{\\ampere}$)"

set mxtics
set mytics
set samples 10000

f(x) = m*x + c

# (x, y, xdelta, ydelta)
fit f(x) "tv1.dat" u 1:2:3:4 xyerrors via m,c

set xrange [0:10]

# Linien
set key bottom right spacing 2

titel = "$(".gprintf("%.5f", m).")I + (".gprintf("%.5f", c).")$"
plot f(x) title titel lc rgb 'dark-magenta', \
    "tv1.dat" u 1:2:3:4 with xyerrorbars title "Messpunkte" pointtype 0 lc rgb 'dark-goldenrod'
```

mit tv1.dat:

```
# I/A   B/mT   delta I delta B
1,070    671   0,005   10
2,096   1298   0,005   10
2,995   1910   0,010   10
4,153   2700   0,020   10
5,300   3460   0,010   10
6,033   3950   0,010   10
7,09    4630   0,01    10
8,03    5240   0,01    10
9,06    5750   0,01    10
9,48    5970   0,01    10
```

Rohausgabe:

```
After 4 iterations the fit converged.
final sum of squares of residuals : 313.511
rel. change during last iteration : -4.72691e-06

degrees of freedom    (FIT_NDF)                        : 8
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 6.2601
variance of residuals (reduced chisquare) = WSSR/ndf   : 39.1889
p-value of the Chisq distribution (FIT_P)              : 0

```

```
Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 641.759          +/- 8.077        (1.259%)
c               = 5.09225          +/- 49.33        (968.7%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.880  1.000
```

B gnuplot Quellcodes zur Auswertung von Teilversuch 4

B.1 Ringradien gegen Ringenindizes

```
#!/usr/bin/env gnuplot
# Version >= 5.2

lambdaminus = 1 # bzw. 0

if (lambdaminus) {
    set term epslatex color size 7in, 4.5in
    set output "tv4-1-minus.tex"

    set title "Verlauf der Radien mit Ringenindizes ($\\lambda_-$)"
} else {
    set term epslatex color size 7in, 4.5in
    set output "tv4-1-plus.tex"

    set title "Verlauf der Radien mit Ringenindizes ($\\lambda_+$)"
}

set decimalsign ","

set ylabel "Radien$^2$ $r_m^2/\\si{\\micro\\meter\\squared}$"
set xlabel "Ringindex $p$ (Einheitlos)"

set mxtics
set mytics
set samples 10000

f(x) = m*x + c # Linear fit

array A_m[5]
array A_m_err[5]
array A_c[5]
array A_c_err[5]
array chisq[5]
array titel[5]
array input_mp[5]
```

```
array titel_mp[5]
array A_p0[5]
array A_p0_err[5]

# http://gnuplot.info/demo_5.4/array.1.gnu
array strom[5]     = ["2.495", "4.190", "5.662", "7.01", "8.78"]
array strom_err[5] = ["5", "10", "10", "1", "1"]

# https://stackoverflow.com/a/17884635
do for [t=1:5] {
    inp = "tv4-".t.".dat"
    input_mp[t] = inp
    titel_mp[t] = "$I = \\SI{".strom[t]."(".strom_err[t].")}{\\ampere}$"

    m = 1; c = 1;
    if (lambdaminus) {
        fit f(x) inp u 1:($2*$2):(2*$2*$3) yerrors via m,c
    } else {
        fit f(x) inp u 1:($4*$4):(2*$4*$5) yerrors via m,c
    }

    A_m[t] = m
    A_m_err[t] = m_err
    A_c[t] = c
    A_c_err[t] = c_err
    chisq[t] = FIT_STDFIT**2
    titel[t] = "$".gprintf("%.5f", m)."p + (".gprintf("%.5f", c).")$"

    A_p0[t] = -c/m
    A_p0_err[t] = abs(A_p0[t]) * sqrt((c_err/c)**2 + (m_err/m)**2)
}

set key bottom right vertical maxrows 5 width -7


if (lambdaminus) {
    plot for [i=1:5] input_mp[i] u 1:($2*$2):(2*$2*$3) with yerrorbars title titel_mp[i] pointtype 77 lc i, for [i=1:5] A_m[i]*x+A_c[i] title titel[i] lc i
} else {
    plot for [i=1:5] input_mp[i] u 1:($4*$4):(2*$4*$5) with yerrorbars title titel_mp[i] pointtype 77 lc i, for [i=1:5] A_m[i]*x+A_c[i] title titel[i] lc i
}

print ""
if (lambdaminus) { print "lambda-" } else { print "lambda+" }

# Raw data output
```

```
print A_m
print A_m_err

# LaTeX table output
print "\\toprule"
print "Strom $I/\\si{\\ampere}$ & $m/\\si{\\micro\\meter\\squared}$ & $c/\\si{\\micro\\meter\\squared}$ & $p_0$ & $\\chi^2_\\text{red}$ \\\\"
print "\\midrule"
do for [t=1:5] {
    print "\t\\num{".strom[t]."(".strom_err[t].")} & \\num{".gprintf("%.5f", A_m[t])."(".gprintf("%.0f", A_m_err[t]*10**5).")} & \\num{".gprintf("%.5f", A_c[t])."(".gprintf("%.0f", A_c_err[t]*10**5).")}"." & \\num{".gprintf("%.5f", A_p0[t])."(".gprintf("%.0f", A_p0_err[t]*10**5).")} & \\num{".gprintf("%.5f", chisq[t])."} \\\\"
}
print "\\bottomrule"
print ""

# Raw data output in table form
print "# Nr\tm/um^2 \tm_err/um^2\tc/um^2 \t c_err/um^2 \t p \t p_err"
do for [t=1:5] {
    print "".t."\t".sprintf("%.10f", A_m[t])."\t".sprintf("%.10f", A_m_err[t])."\t".sprintf("%.10f", A_c[t])."\t".sprintf("%.10f", A_c_err[t])."\t".sprintf("%.10f", A_p0[t])."\t".sprintf("%.10f", A_p0_err[t])
}
```

mit

tv4-1.dat:

```
#   lambda -     lambda +
# p r/um    dr  r/um    dr
1   48,30   3   58,70   3
2   101,59  2,5 107,02  2,5
3   135,36  2   139,63  2
4   161,72  1,5 164,65  1,5
5   183,72  1,5 185,69  1,5
```

tv4-3.dat:

```
#   lambda -     lambda +
# p r/um    dr  r/um    dr
1   39,26   5   64,70   4
2   97,88   3   110,70  3
3   132,59  3   141,91  2
4   159,59  2   166,79  2
5   181,31  1,5 188,77  1,5
```

tv4-2.dat:

```
#   lambda -     lambda +
# p r/um    dr  r/um    dr
1   43,85   4   62,07   4
2   99,09   3   108,51  3
3   134,89  3   140,23  2
4   160,43  1,5 165,42  1,5
5   182,71  1,5 187,02  1,5
```

tv4-4.dat:

```
#   lambda -     lambda +
# p r/um    dr  r/um    dr
1   35,45   6   66,38   4
2   96,26   3   111,61  3
3   130,98  2   143,36  2
4   158,46  2   168,27  1,5
5   180,88  1,5 189,16  1,5
```

tv4-5.dat:

```
#   lambda -    lambda +
# p r/um    dr  r/um    dr
1   30,79   7   69,88   5
2   94,15   3   113,33  3
3   129,67  2   144,56  2
4   157,19  2   168,98  1,5
5   179,64  1,5 190,10  2
```

Rohausgabe: λ_-:

```
iter      chisq       delta/lim  lambda   m             c
   0 1.6686639557e+04   0.00e+00  6.69e-03    1.000000e+00   1.000000e+00
   1 4.5514983573e+02  -3.57e+06  6.69e-04    5.626171e+03   3.729550e+02
   2 1.3254968490e+00  -3.42e+07  6.69e-05    7.733635e+03  -5.147936e+03
   3 4.8256122798e-01  -1.75e+05  6.69e-06    7.819436e+03  -5.456495e+03
   4 4.8256097818e-01  -5.18e-02  6.69e-07    7.819482e+03  -5.456663e+03
iter      chisq       delta/lim  lambda   m             c

After 4 iterations the fit converged.
final sum of squares of residuals : 0.482561
rel. change during last iteration : -5.17652e-07

degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.401066
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.160854
p-value of the Chisq distribution (FIT_P)              : 0.922706

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 7819.48          +/- 41.58        (0.5318%)
c               = -5456.66         +/- 135          (2.475%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.879  1.000
iter      chisq       delta/lim  lambda   m             c
   0 1.5474967614e+04   0.00e+00  6.39e-03    1.000000e+00   1.000000e+00
   1 3.8640256958e+02  -3.90e+06  6.39e-04    5.622902e+03   4.773145e+02
   2 1.8368825165e+00  -2.09e+07  6.39e-05    7.729605e+03  -5.474338e+03
   3 6.6970959016e-01  -1.74e+05  6.39e-06    7.842753e+03  -5.913672e+03
   4 6.6970898273e-01  -9.07e-02  6.39e-07    7.842834e+03  -5.913989e+03
iter      chisq       delta/lim  lambda   m             c

After 4 iterations the fit converged.
final sum of squares of residuals : 0.669709
rel. change during last iteration : -9.07016e-07

degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.472479
```

```
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.223236
p-value of the Chisq distribution (FIT_P)              : 0.880304

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 7842.83          +/- 54.3         (0.6923%)
c               = -5913.99         +/- 192.4        (3.254%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.894  1.000
iter      chisq       delta/lim  lambda   m             c
   0 1.0526830137e+04   0.00e+00  5.33e-03    1.000000e+00   1.000000e+00
   1 3.0771158033e+02  -3.32e+06  5.33e-04    5.601429e+03   2.472810e+02
   2 1.0177570375e+00  -3.01e+07  5.33e-05    7.721799e+03  -5.865193e+03
   3 3.0567896612e-01  -2.33e+05  5.33e-06    7.817514e+03  -6.241526e+03
   4 3.0567870771e-01  -8.45e-02  5.33e-07    7.817572e+03  -6.241753e+03
iter      chisq       delta/lim  lambda   m             c

After 4 iterations the fit converged.
final sum of squares of residuals : 0.305679
rel. change during last iteration : -8.45339e-07

degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.319207
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.101893
p-value of the Chisq distribution (FIT_P)              : 0.958957

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 7817.57          +/- 40.5         (0.5181%)
c               = -6241.75         +/- 142.6        (2.284%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.874  1.000
iter      chisq       delta/lim  lambda   m             c
   0 1.0976687949e+04   0.00e+00  5.54e-03    1.000000e+00   1.000000e+00
   1 3.1709119045e+02  -3.36e+06  5.54e-04    5.484630e+03   3.421966e+02
   2 1.2906238259e+00  -2.45e+07  5.54e-05    7.676945e+03  -6.033082e+03
   3 3.2044487748e-01  -3.03e+05  5.54e-06    7.793900e+03  -6.488908e+03
   4 3.2044440168e-01  -1.48e-01  5.54e-07    7.793981e+03  -6.489228e+03
iter      chisq       delta/lim  lambda   m             c

After 4 iterations the fit converged.
final sum of squares of residuals : 0.320444
rel. change during last iteration : -1.48482e-06
```

```

degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.326825
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.106815
p-value of the Chisq distribution (FIT_P)              : 0.956139

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 7793.98          +/- 42.73        (0.5483%)
c               = -6489.23         +/- 151.5        (2.334%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.891  1.000
iter      chisq       delta/lim  lambda   m             c
   0 1.0836599662e+04   0.00e+00  5.57e-03    1.000000e+00   1.000000e+00
   1 3.3516021190e+02  -3.13e+06  5.57e-04    5.418597e+03   2.913066e+02
   2 1.4395131509e+00  -2.32e+07  5.57e-05    7.666430e+03  -6.327888e+03
   3 3.7754029575e-01  -2.81e+05  5.57e-06    7.788317e+03  -6.802702e+03
   4 3.7753977043e-01  -1.39e-01  5.57e-07    7.788403e+03  -6.803037e+03
iter      chisq       delta/lim  lambda   m             c

After 4 iterations the fit converged.
final sum of squares of residuals : 0.37754
rel. change during last iteration : -1.39141e-06

degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.354749
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.125847
p-value of the Chisq distribution (FIT_P)              : 0.944842

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 7788.4           +/- 46.19        (0.5931%)
c               = -6803.04         +/- 163.7        (2.406%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.892  1.000

lambda-
[7819.48207698864,7842.83446531683,7817.57179454737,7793.98133299567,7788.40258574259]

[41.582142626232,54.2971415193017,40.4996415277695,42.7343688524447,46.1919447295457]

\toprule
```

```
Strom $I/\si{\ampere}$ & $m/\si{\micro\meter\squared}$ & $c/\si{\micro\meter\squared}$ & $p_0$ & $\chi^2_\text{red}$ \\
\midrule
        \num{2.495(5)} & \num{7819,48208(4158214)} & \num{-5456,66280(13503726)} & \num{0,69783(1766)} & \num{0,16085} \\
        \num{4.190(10)} & \num{7842,83447(5429714)} & \num{-5913,98932(19243657)} & \num{0,75406(2509)} & \num{0,22324} \\
        \num{5.662(10)} & \num{7817,57179(4049964)} & \num{-6241,75284(14258167)} & \num{0,79843(1870)} & \num{0,10189} \\
        \num{7.01(1)} & \num{7793,98133(4273437)} & \num{-6489,22780(15147792)} & \num{0,83259(1996)} & \num{0,10681} \\
        \num{8.78(1)} & \num{7788,40259(4619194)} & \num{-6803,03677(16369671)} & \num{0,87348(2165)} & \num{0,12585} \\
\bottomrule

# Nr        m/um^2         m_err/um^2        c/um^2          c_err/um^2          p          p_err
1        7819.4820769886        41.5821426262        -5456.6628032539        135.0372595466
2        7842.8344653168        54.2971415193        -5913.9893218755        192.4365696268
3        7817.5717945474        40.4996415278        -6241.7528405377        142.5816729242
4        7793.9813329957        42.7343688524        -6489.2277952663        151.4779180213
5        7788.4025857426        46.1919447295        -6803.0367673382        163.6967074291
```

λ_+:

```
iter      chisq       delta/lim  lambda   m             c
   0 1.7290333963e+04   0.00e+00  6.51e-03    1.000000e+00   1.000000e+00
   1 3.1739184179e+02  -5.35e+06  6.51e-04    5.803695e+03   7.564391e+02
   2 2.1264580056e+00  -1.48e+07  6.51e-05    7.642359e+03  -3.852195e+03
   3 1.4190371838e+00  -4.99e+04  6.51e-06    7.730160e+03  -4.178863e+03
   4 1.4190368504e+00  -2.35e-02  6.51e-07    7.730220e+03  -4.179088e+03
iter      chisq       delta/lim  lambda   m             c

After 4 iterations the fit converged.
final sum of squares of residuals : 1.41904
rel. change during last iteration : -2.34937e-07

degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.687759
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.473012
p-value of the Chisq distribution (FIT_P)              : 0.701079

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 7730.22          +/- 78.43        (1.015%)
```

```
c               = -4179.09         +/- 267.6        (6.403%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.896  1.000
iter      chisq       delta/lim  lambda   m             c
   0 1.7151333909e+04   0.00e+00  6.28e-03    1.000000e+00   1.000000e+00
   1 2.3309007856e+02  -7.26e+06  6.28e-04    5.929880e+03   1.026471e+03
   2 1.1034685800e+00  -2.10e+07  6.28e-05    7.641657e+03  -3.341991e+03
   3 9.1627152079e-02  -1.10e+06  6.28e-06    7.775578e+03  -3.876831e+03
   4 9.1625731428e-02  -1.55e+00  6.28e-07    7.775736e+03  -3.877466e+03
   * 9.1625731429e-02   2.82e-09  6.28e-06    7.775736e+03  -3.877466e+03
   5 9.1625731428e-02  -3.33e-10  6.28e-07    7.775736e+03  -3.877466e+03
iter      chisq       delta/lim  lambda   m             c

After 5 iterations the fit converged.
final sum of squares of residuals : 0.0916257
rel. change during last iteration : -3.33216e-15

degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.174762
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.0305419
p-value of the Chisq distribution (FIT_P)              : 0.992823

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 7775.74          +/- 24.73        (0.318%)
c               = -3877.47         +/- 93.09        (2.401%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.930  1.000
iter      chisq       delta/lim  lambda   m             c
   0 1.2196569842e+04   0.00e+00  5.23e-03    1.000000e+00   1.000000e+00
   1 1.7571218655e+02  -6.84e+06  5.23e-04    6.038880e+03   9.227500e+02
   2 5.6628399866e-01  -3.09e+07  5.23e-05    7.703106e+03  -3.236405e+03
   3 1.0169424967e-01  -4.57e+05  5.23e-06    7.795873e+03  -3.608087e+03
   4 1.0169390349e-01  -3.40e-01  5.23e-07    7.795952e+03  -3.608408e+03
iter      chisq       delta/lim  lambda   m             c

After 4 iterations the fit converged.
final sum of squares of residuals : 0.101694
rel. change during last iteration : -3.40417e-06

degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.184114
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.033898
```

```
p-value of the Chisq distribution (FIT_P)              : 0.991633

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 7795.95          +/- 27.19        (0.3488%)
c               = -3608.41         +/- 100.6        (2.787%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.905  1.000
iter      chisq       delta/lim  lambda   m             c
   0 1.7668396390e+04   0.00e+00  6.18e-03    1.000000e+00   1.000000e+00
   1 2.1600955001e+02  -8.08e+06  6.18e-04    6.101670e+03   1.126831e+03
   2 1.6023889616e+00  -1.34e+07  6.18e-05    7.718917e+03  -2.806139e+03
   3 7.4905507827e-01  -1.14e+05  6.18e-06    7.847251e+03  -3.321293e+03
   4 7.4905371928e-01  -1.81e-01  6.18e-07    7.847412e+03  -3.321944e+03
iter      chisq       delta/lim  lambda   m             c

After 4 iterations the fit converged.
final sum of squares of residuals : 0.749054
rel. change during last iteration : -1.81428e-06

degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.499684
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.249685
p-value of the Chisq distribution (FIT_P)              : 0.86161

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 7847.41          +/- 73.54        (0.9371%)
c               = -3321.94         +/- 279.2        (8.404%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.933  1.000
iter      chisq       delta/lim  lambda   m             c
   0 1.1006598138e+04   0.00e+00  4.91e-03    1.000000e+00   1.000000e+00
   1 1.2855999927e+02  -8.46e+06  4.91e-04    6.030397e+03   1.275157e+03
   2 7.6302629260e-01  -1.67e+07  4.91e-05    7.655422e+03  -2.374837e+03
   3 2.6765082271e-01  -1.85e+05  4.91e-06    7.791418e+03  -2.877135e+03
   4 2.6764996182e-01  -3.22e-01  4.91e-07    7.791597e+03  -2.877798e+03
iter      chisq       delta/lim  lambda   m             c

After 4 iterations the fit converged.
final sum of squares of residuals : 0.26765
rel. change during last iteration : -3.21646e-06
```

```
degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.298692
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.0892167
p-value of the Chisq distribution (FIT_P)              : 0.965993

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 7791.6           +/- 60.49        (0.7763%)
c               = -2877.8          +/- 213.6        (7.422%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.944  1.000

lambda+
[7730.21963446915,7775.73624531868,7795.95243793266,7847.41240195937,7791.596990915]

[78.4330147740594,24.7295464306261,27.190181003849,73.5366995800329,60.4885315766415]

\toprule
Strom $I/\si{\ampere}$ & $m/\si{\micro\meter\squared}$ &
↪  $c/\si{\micro\meter\squared}$ & $p_0$ & $\chi^2_\text{red}$ \\
\midrule
        \num{2.495(5)} & \num{7730,21963(7843301)} &
        ↪  \num{-4179,08793(26759715)} & \num{0,54062(3505)} & \num{0,47301}
        ↪  \\
        \num{4.190(10)} & \num{7775,73625(2472955)} &
        ↪  \num{-3877,46605(9308847)} & \num{0,49866(1208)} & \num{0,03054}
        ↪  \\
        \num{5.662(10)} & \num{7795,95244(2719018)} &
        ↪  \num{-3608,40828(10057227)} & \num{0,46286(1300)} & \num{0,03390}
        ↪  \\
        \num{7.01(1)} & \num{7847,41240(7353670)} &
        ↪  \num{-3321,94377(27918566)} & \num{0,42332(3580)} & \num{0,24968}
        ↪  \\
        \num{8.78(1)} & \num{7791,59699(6048853)} &
        ↪  \num{-2877,79824(21359518)} & \num{0,36935(2756)} & \num{0,08922}
        ↪  \\
\bottomrule

# Nr        m/um^2          m_err/um^2          c/um^2            c_err/um^2
↪            p           p_err
1         7730.2196344692         78.4330147741         -4179.0879252342         267.5971535148
2         7775.7362453187         24.7295464306         -3877.4660484435         93.0884721173
3         7795.9524379327         27.1901810038         -3608.4082794282         100.5722674516
4         7847.4124019594         73.5366995800         -3321.9437703410         279.1856585427
5         7791.5969909150         60.4885315766         -2877.7982375139         213.5951810253
```

B.2 gnuplot Quellcode zur Auswertung von $g_{1\to 2}$

```
#!/usr/bin/env gnuplot

set term epslatex color size 6in, 4in
set output "tv4-B-K.tex"
set decimalsign locale 'de_DE.UTF-8'

set title "Wellenzahlunterschied $K/2$ gegen Magnetfeldstärke $B$"
set ylabel "Wellenzahlunterschied $K/2$ ($\\si{\\per\\meter}$)"
set xlabel "Magnetfeldstärke $B$ ($\\si{\\tesla}$)"

set mxtics
set mytics
set samples 10000

f(x) = m*x + c

# (x, y, xdelta, ydelta)
fit f(x) "tv4-B-K.dat" u 1:($3/2):2:($4/2) xyerrors via m,c

# Linien
set key bottom right spacing 2

titel = "$(".gprintf("%.5f", m).")B + (".gprintf("%.5f", c).")$"
plot f(x) title titel lc rgb 'dark-magenta', \
    "tv4-B-K.dat" u 1:($3/2):2:($4/2) with xyerrorbars title "Messpunkte"
    ↪ pointtype 0 lc rgb 'dark-goldenrod'
```

mit tv4-B-K.dat:

```
# B/T   delB    K    del K
1,60    0,06    18   6
2,69    0,07    29   4
3,63    0,08    38,5   2,8
4,50    0,09    47   6
5,63    0,10    58   5
```

Rohausgabe:

```
iter      chisq       delta/lim  lambda   m             c
   0 2.5833233603e+02   0.00e+00  1.31e+00    1.000000e+00   1.000000e+00
   1 2.1302132977e+00  -1.20e+07  1.31e-01    4.426298e+00   1.833777e+00
   2 6.6462394827e-03  -3.20e+07  1.31e-02    4.928417e+00   1.298707e+00
   3 5.2878052841e-03  -2.57e+04  1.31e-03    4.958905e+00   1.180707e+00
   4 5.2877921906e-03  -2.48e-01  1.31e-04    4.958969e+00   1.180442e+00
iter      chisq       delta/lim  lambda   m             c

After 4 iterations the fit converged.
final sum of squares of residuals : 0.00528779
rel. change during last iteration : -2.47619e-06

degrees of freedom    (FIT_NDF)                        : 3
rms of residuals      (FIT_STDFIT) = sqrt(WSSR/ndf)    : 0.0419833
```

```
variance of residuals (reduced chisquare) = WSSR/ndf   : 0.0017626
p-value of the Chisq distribution (FIT_P)              : 0.999898

Final set of parameters            Asymptotic Standard Error
=======================            ==========================
m               = 4.95897          +/- 0.03595      (0.725%)
c               = 1.18044          +/- 0.135        (11.44%)

correlation matrix of the fit parameters:
                m      c
m               1.000
c              -0.955  1.000
```