Personal Statement

JingAn Sun | https://jingansun.netlify.app/

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1 Why I choose theoretical physics

When I first entered university, I liked mathematics better. The rigor logic of math fascinated me, so I learned some courses in Department of Mathematics, including mathematical analysis and advanced algebra when I was a freshman. This did take me a lot of time, but it was very helpful. I won the first prize in National Mathematics Competition for College Students. Until the second semester of my sophomore year, I learned the first theoretical physics course—Classical Mechanics. In Classical Mechanics, the Lagrangian contains all the information of a mechanical system, and there is no need to do complicated force analysis; Hamilton Mechanics is even linked to quantum mechanics. This interconnected nature of physics is very attractive to me. Further study made me realize that physical theories are not independent of the objective world. Many theories originate from tracing the roots of experimental phenomena and can predict certain phenomena. From this perspective, I think physics describes the objective world,and the physical theory becomes more charming thanks to the introduction of mathematical tools. It makes me determine to study physics further, and choose the theoretical direction. A pen, some paper, a computer and a cup of coffee would be my favorite work mode.

2 Course learning situation

I have taken basic courses in physics, including Atomic Physics, Mathematical Physics Methods, Theoretical Physics Volume I, Theoretical Physics Volume II, I achived A-level grades in these courses. But I know that if I want to study theoretical physics further, the knowledge is not enough, so I taught myself Statistical Physics II, and Quantum MechanicsII and Solid State Physics which I am studying in this semester. In addition, I am studying Particle Physics, which may lay foundation for the study of Quantum Field Theory. Therefore, I believe that I have a solid theoretical foundation and the basic qualities to become a physics graduate student.

3 Research Experiences

3.1 Particle Flow Network

In high-energy physics we collide two particles, which is called a collision event. One collision event will produce a series of secondary particles. The flows consist of these particles are called Jets. The detectors arranged around the collider, such as calorimeters and position detectors, can only record the information of the final particles. What we need to do is to reconstruct the internal processes from these last-level information. It is very meaningful beacuse it enables us to verify the correctness of physical theories, such as the quark model and Higgges mechanism, even can help us discover new particles and new physics.

How do we use machine learning methods? The information recorded by the detector includes particle energy, momentum, three-dimensional coordinates, etc. We view it as a multi-dimensional image, so CNN or DNN can be used. Also the jet is a variable length time series, which is suitable for RNN. However, the training results of these models are not very satisfactory, and the classification accuracy are all less than 90%.

We constructed PFN(Particle Flow Network).Suppose an event with Mparticles, noted $\{p_i\}_{i=1}^{M}$， $p_i \in R^d$, which is a vector contains particle features (energy, momentum, three-dimensional coordinates, etc). We use the following function to approximate any observable quantity：

$$\hat{O}(\{p_1, \dots p_M\}) = F\left(\sum_{i=1}^{M} \Phi(p_i)\right) \tag{3.1}$$

where $\Phi : R^d \to R^l$maps each particla to latent space,$F : R^l \to Y$is a continuous

Main Courses and Ref		
Courses	Grade	Ref
Advanced Mathematics A I	A	Advanced Mathematics A I
Advanced Mathematics A II	A	Advanced Mathematics A II
Atomic Physics	A	Yang Fujia Atomic Physics
Nuclear Physics	A-	Lu Xiting Nuclear Physics Yang Fujia Nuclear Physics
Theoretical Physics Volume I	A-	Jin S.N Ma Y.L Theoretical Mechanics Wang Z.C Thermal Statistical Physics
Theoretical Physics Volume II	A	Guo Shuohong Electrodynamics Qian Bochu Quantum Mechanics Su Rukeng Quantum Mechanics
Engineering Mathematics A	A-	Xie Q.H Advanced Algebra D.P.Bertsekas Introduction to Probability
Methods of Mathematical Physics A	A-	Hu S.Z Methods of Mathematical Physics
Principles of Mathematical Analysis	B+	W.R Principle of Mathematical Analysis
Differential Equations	B+	Lou H.W Ordinary Differential Equations
Introduction to Stochastic Processes	B+	Fang Z.B,Liao B.Q Stochastic Processes
Solid State Physics	Waiting for exam	Huang Kun Solid State Physics Ashcroft Mermin Solid State Physics
Particle Physics	Waiting for exam	D.G Introduction to Elementary Particles Mark Thomson Modern Particle Physics
Quamtum MechanicsII	Audit	Yuli V. N Advanced Quantum Mechanics J.Sakurai Modern Quantum Mechanics Ka Xinglin Advanced Quantum Mechanics
Statistical PhysicsII	Audit	Su Rukeng Statistical Physics

Table 1: Main Courses and Ref

function. The network is as fig 1. Based on this network, the Z boson decay mode is

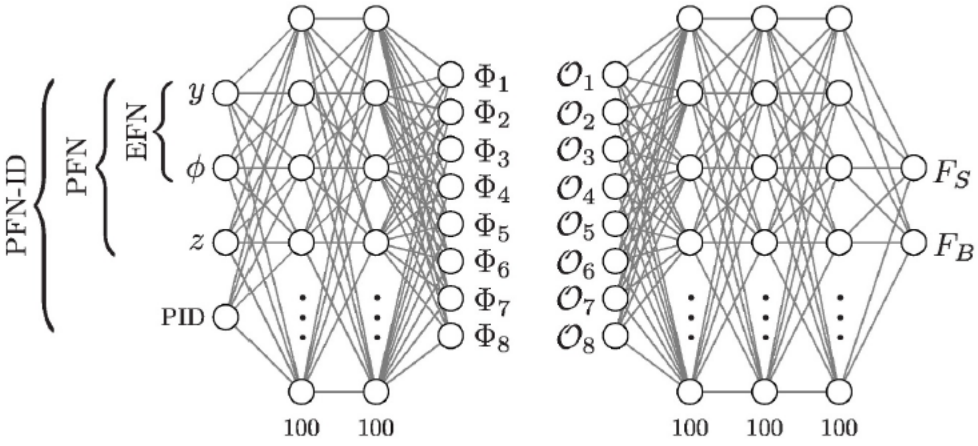

Fig 1: PFN

classified into three categories, and the classification accuracy reaches 95%.

In particle physics, the measurement of R value directly corresponds to whether quarks have color charge. It is defined as:

$$R = \frac{\Gamma_{Z \to qq}}{\Gamma_{Z \to hardons}} \tag{3.2}$$

According to QED, the level of collision energy determines whether the reaction is forbidden, so in experiments we will see that the R value increases stepwise as the energy increases. The cross section of the reaction(Z to positive and negative quarks) is proportional to Q^2, where Q is the quantum number of quark taste freedom, but quarks also have three colors.Finally, $R = 3\sum Q^2$.

Next, we explore how to increase the error of the R value according to the confusion matrix given by PFN, and expect the error of the R value to reach 10e-9, which is more accurate than the traditional method.So this more accurately verifies the theory that quarks have color charge.Remarkably,this is a new method to measure R value.

3.2 GCM many-body calculation

In ultra cold atoms, atoms can be well bound and the coupling strength between atoms can be manipulated. Based on this, we can better study many-body systems. But the many-body problem is very complicated and numerical calculations will consume computing resources, so we need a simple and accurate method for calculations. DOI:10.1103/PhysRevLett.108.045301, this article assumes that atoms are in the harmonic trap and the interaction is *ldelta* function, that is, consider such a Hamiltonian:

$$H = -\frac{1}{2}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + \sum_{i=1}^{N}\frac{x_i^2}{2} + g\sum_{i<j}\delta(x_i - x_j) \tag{3.3}$$

Where g is the coupling strength.

When only the relative motion part is considered, the total wave function of the many-body system is constructed according to the analytical solution of the two-body, which is the parabolic cylinder function. And then we can calculate the energy of the system by the variational principle. This method is used when there are few bodies (less than 6-body), which is very close to the results of numerical calculations, but in higher dimensions, the calculation becomes complicated and difficult to generalize.

We use GCM (Generate Coordinate Method) which is introduced in the nuclear cluster theory to describe the collective motion of nuclei. This method introduces auxiliary parameters, constructs a multi-body wave function according to the symmetry requirements of the system, and then adds them according to the weighted overlap to obtain the system wave function. Finally, by principle of variation, we get the Hill-Wheeler equation and the problem is transformed into a generalized eigenvalue problem.

Set $\boldsymbol{\alpha}$ as a parameter, namely generator coordinates, $\Phi(\boldsymbol{x}, \boldsymbol{\alpha})$ is the corresponding many-body wave function, $f(\boldsymbol{\alpha})$ is Weight. So the system wave function is:

$$\Psi(\boldsymbol{x}) = \int d\boldsymbol{\alpha}\Phi(\boldsymbol{x}, \boldsymbol{\alpha})f(\boldsymbol{\alpha}) \tag{3.4}$$

By vartional principle, we get Hill-Wheeler equation:

$$\int d\boldsymbol{\alpha'}[H(\boldsymbol{\alpha}, \boldsymbol{\alpha'}) - EN(\boldsymbol{\alpha}, \boldsymbol{\alpha'})]f(\alpha') = 0 \tag{3.5}$$

where

$$H(\alpha,\alpha') = \int \Phi^*(\boldsymbol{x},\boldsymbol{\alpha}) H \Phi(\boldsymbol{x},\boldsymbol{\alpha'}) d\boldsymbol{x} \tag{3.6}$$

$$N(\alpha,\alpha') = \int \Phi^*(\boldsymbol{x},\boldsymbol{\alpha}) \Phi(\boldsymbol{x},\boldsymbol{\alpha'}) d\boldsymbol{x} \tag{3.7}$$

where H is the system Hamiltonian, E is the total energy of the system (ground state)

We still consider the above Hamiltonian, and suppose the single-particle trial wave function is:

$$\phi(x) = e^{-\alpha(x-\beta)^2} \tag{3.8}$$

Design system wave function

$$\Phi(x_1, x_2, \dots x_N)_{\alpha\beta} = \prod_{i=1}^{N} e^{-\alpha_i(x-\beta_i)^2} \tag{3.9}$$

According to the principle of variation, the Hill-Wheeler equation is transformed into a generalized eigenvalue problem in the case of discrete parameters:

$$H\boldsymbol{\Phi} = EN\boldsymbol{\Phi} \tag{3.10}$$

$$H_{\alpha\beta,\gamma\lambda} = \langle \Phi_{\alpha\beta} | H | \Phi_{\gamma\lambda} \rangle \qquad N_{\alpha\beta,\gamma\lambda} = \langle \Phi_{\alpha\beta} | \Phi_{\gamma\lambda} \rangle \tag{3.11}$$

What we need to do is to solve the generalized eigenvalue problem.

In Fig2, we observe an excellent agreement between the energy calculated using GCM and the results from the article as a function of g. However, the trial wave function I selected is simple and the calculation is simple. What we need to calculate are only two terms and other terms are their permutations or substitution of parameters.

- Nomal Kernel:

$$N = \int e^{-\alpha(x-s)^2} e^{-\beta(x-t)^2} dx \tag{3.12}$$

- H kernel:

$$H = \int e^{-\alpha(x-s)^2} \Big(-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{1}{2}x^2 \Big) e^{-\beta(x-t)^2} dx \tag{3.13}$$

So it is easy to generalize to high-dimensional situations (that is, accurate results can still be obtained if N is at least greater than 10)

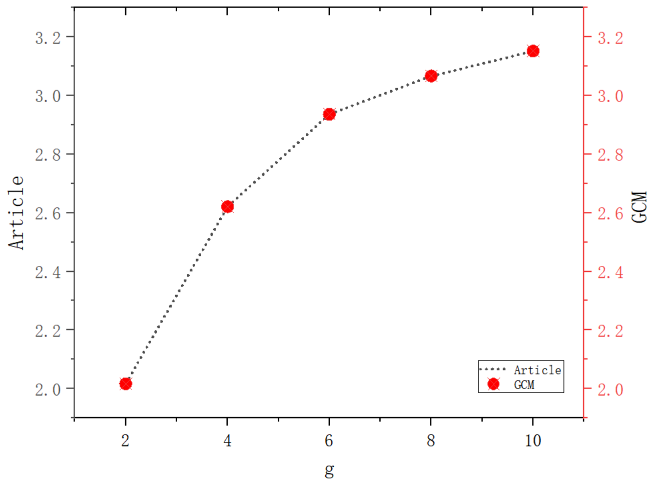

Fig 2: Total energies as a function of the interaction strength g in article (black lines) and GCM calculations (red marks) , for three particles

4 Research Interests

I prefer to engage in research on cold atom physics. I learned about this direction when I was studying statistical physics. When we put the atom in an extremely low temperature environment, the wavelength of the thermal de Broglie wave of the atom would be greater than the mean inter-particle distance and the quantum effect would be very significant, thus presenting novel quantum states of matter, such as BEC, superflow, superconductivity, etc. The cold atoms system is also a good platform for studying many-body problems, because the coupling strength of the interaction is controllable in experiments. What I am doing now is to use the Generate Coordinate method to solve the ground state energy of the bose system with interaction.

In addition, cold atoms are also used to simulate gauge fields to verify the theory of particle physics in condensed matter systems. Alse thanks to that cold atoms can be precisely manipulated, it can be used to study quantum information and quantum computing. In short, I think the cold atom physics is of great research significance.

5 Research Plan and Future Vision

Professional Course Study Course learning is the cornerstone of academic research. I will work hard to study graduate courses, such as quantum field theory, solid theory, etc. read a wide range of academic papers to have a deep understanding of these basic knowledge from many angles, which will lay a good foundation for academic research.

Project research First of all, I want to engage in cold atom physics research. If I am lucky enough to be admitted, I will further communicate with my supervisor, read reviews , and learn the basic knowledge of this field. At the same time, I hope that I can follow a small project, because studying will be more efficient while doing research. I think I would have mastered the basic methods of research and had an overview of the research field by that time. Next, I will follow my supervisor or by myself to challenge more difficult problems, trying to make some contributation to theoretical development at this field. I will also pay attention to the intersection of disciplines. The machine learning project gives me some experience in applying machine learning to physics. I will also apply the machine learning methods to research projects, or to understand machine learning from the perspective of physics.

Life Academic research will inevitably encounter difficulties, which requires adjustment in life. If I encountered difficulties in my studies when I was a undergraduate, I would communicate and share with my classmates. Chatting with classmates can always relieve stress. I also take exercise to relax. My favorite sport is running, because the process of running is always quiet. After a few kilometers, the sweat will make me feel particularly comfortable.

Thank you for reading my personal statement!