

AI4Science: Striking the Best Data-Knowledge Tradeoff

Ju Sun (Computer Sci. & Eng., UMN)

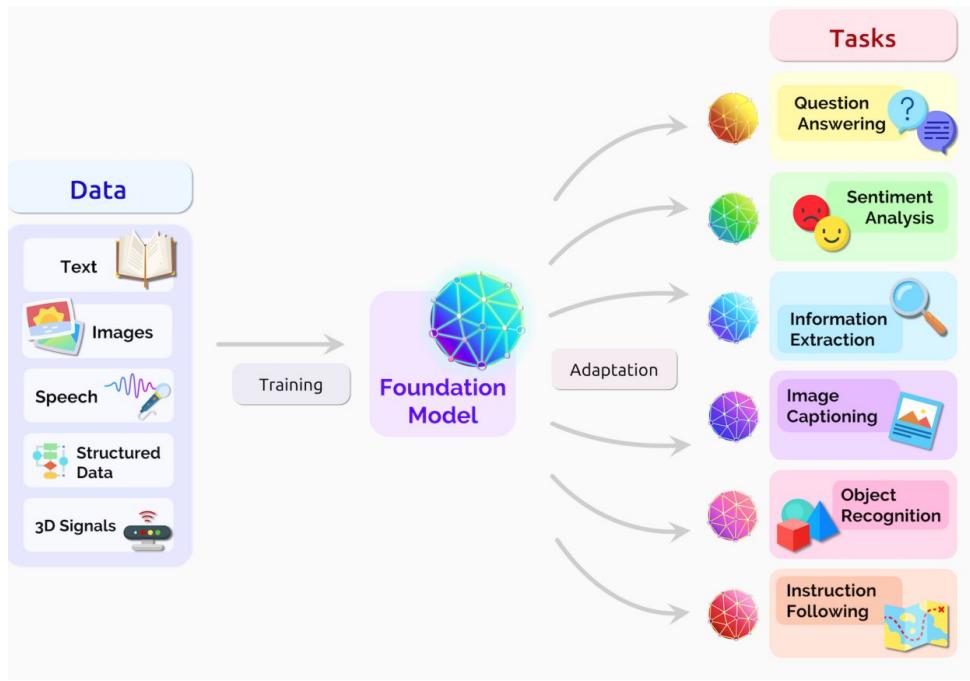
Apr 30, 2024

AI4Science Seminar Series, AWS

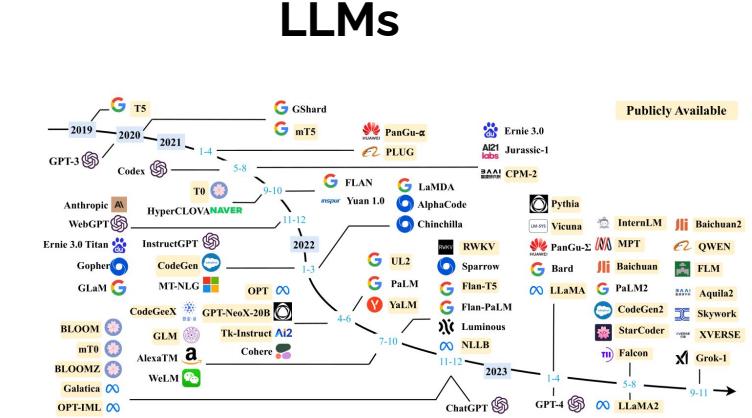


UNIVERSITY OF MINNESOTA
Driven to DiscoverSM

Deep learning models are data monsters



Credit: **On the Opportunities and Risks of Foundation Models**
<https://arxiv.org/abs/2108.07258>



Model	Release Time	Size (B)	Base Model	Adaptation IT	RLHF	Pre-train Data Scale	Latest Data Timestamp	Hardware (GPUs / TPUs)	Training Time	Evaluation ICL CoT
T5 [83]	Oct-2019	11	-	-	-	1T tokens	Apr-2019	1024 TPU v3	-	✓ -
mT5 [83]	Oct-2020	13	-	-	-	1T tokens	-	2048 Ascend 910	-	✓ -
PanGu-α [83]	Apr-2021	13*	-	-	-	1.1TB	-	-	-	✓ -
CPM-2 [83]	Jun-2021	198	-	-	-	2.6TB	-	-	-	✓ -
T0 [28]	Oct-2021	11	T5	✓	-	-	-	512 TPU v3	27 h	✓ -
CodeGen [86]	Mar-2022	16	-	-	-	577B tokens	-	-	-	✓ -
GPT-NeoX-20B [87]	Apr-2022	20	-	-	-	825GB	-	96 40G A100	-	✓ -
Tk-Instruct [88]	Apr-2022	11	T5	✓	-	-	-	256 TPU v3	4 h	✓ -
UL2 [89]	May-2022	20	-	-	-	1T tokens	Apr-2019	512 TPU v4	-	✓ ✓
OPT [90]	May-2022	175	-	-	-	180B tokens	Apr-2019	992 80G A100	-	✓ -
NLLB [91]	Jul-2022	54.5	-	-	-	-	-	-	-	✓ -
CodeGeex [92]	Sep-2022	13	-	-	-	850B tokens	-	1536 Ascend 910	60 d	✓ -
GLM [93]	Oct-2022	130	-	-	-	400B tokens	-	768 40G A100	60 d	✓ -
BLOOM [78]	Oct-2022	11	T5	✓	-	-	-	-	-	✓ ✓
mT0 [94]	Nov-2022	176	-	-	-	366B tokens	-	384 80G A100	105 d	✓ -
Galactica [95]	Nov-2022	120	-	-	-	106B tokens	-	-	-	✓ -
BLOOMZ [95]	Nov-2022	176	BLOOM	✓	-	-	-	-	-	✓ -
OPT-IMI [95]	Dec-2022	175	OPT	✓	-	-	-	-	-	✓ -
LLaMA [96]	Feb-2023	12	-	-	-	1.4T tokens	-	128 40G A100	-	✓ -
Pythia [97]	Apr-2023	65	-	-	-	400B tokens	-	2048 80G A100	21 d	✓ -
CodeGen [98]	May-2023	16	-	-	-	1T tokens	-	512 40G A100	-	✓ ✓
StarCoder [98]	May-2023	15.5	-	-	-	-	-	2000 80G A100	-	✓ -
Baichuan2 [100]	Jul-2023	70	-	✓	✓	2T tokens	-	1024 A800	-	✓ -
Qwen [101]	Sep-2023	13	-	✓	✓	2.6T tokens	-	-	-	✓ -
FLM [102]	Sep-2023	14	-	✓	✓	3.7T tokens	-	192 A800	22 d	✓ -
Skywork [103]	Sep-2023	101	-	✓	✓	311B tokens	-	512 80G A800	-	✓ -
LLaMA2	Oct-2023	13	-	-	-	3.2T tokens	-	-	-	✓ -

Credit: **A Survey of Large Language Models**
<https://arxiv.org/abs/2303.18223>

Towards Geospatial Foundation Models via Continual Pretraining

Matías Mendieta^{1*} Boran Han² Xingjian Shi³ Yi Zhu³ Chen Chen¹

¹ Center for Research in Computer Vision, University of Central Florida

² Amazon Web Services ³ Boson AI



Method	# Images	Epochs	ARP ↑	Time ↓	CO ₂ ↓
ImageNet-22k Sup.	14M	-	0.0	-	-
Sentinel-2 [30]	1.3M	100	-5.83	155.6	22.2
GeoPile	600k	200	0.92	133.3	19.0
GeoPile [†]	600k	200	1.24	133.3	19.0
GeoPile [†]	600k	800	1.45	533.2	76.0
GFM	600k	100	3.31	93.3	13.3

Figure 2. We visualize some datasets with Sentinel-2 (left) and noticeably much lower feature quality (right) across images than that of our Geofine pretraining dataset.

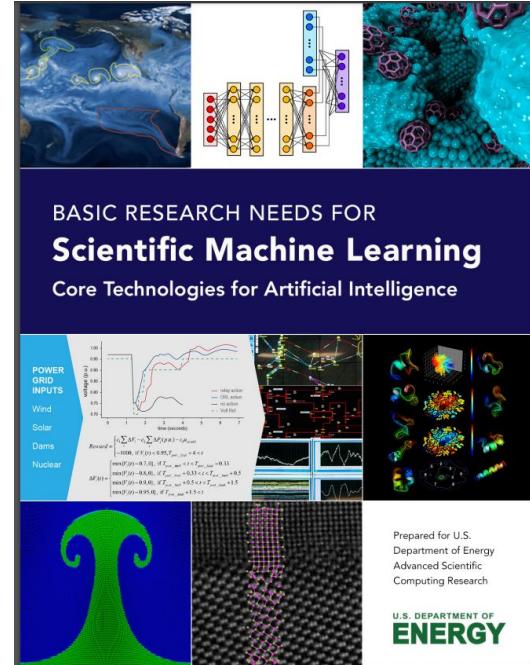
Not all fields are as lucky

Thrust B: How Should Domain Knowledge Be Incorporated into Supervised Machine Learning?

The central question for this thrust is “which knowledge should be leveraged in SciML, and how should this knowledge be included?” Any answers will naturally depend on the SciML task and computational budgets, thus mirroring standard considerations in traditional scientific computing.

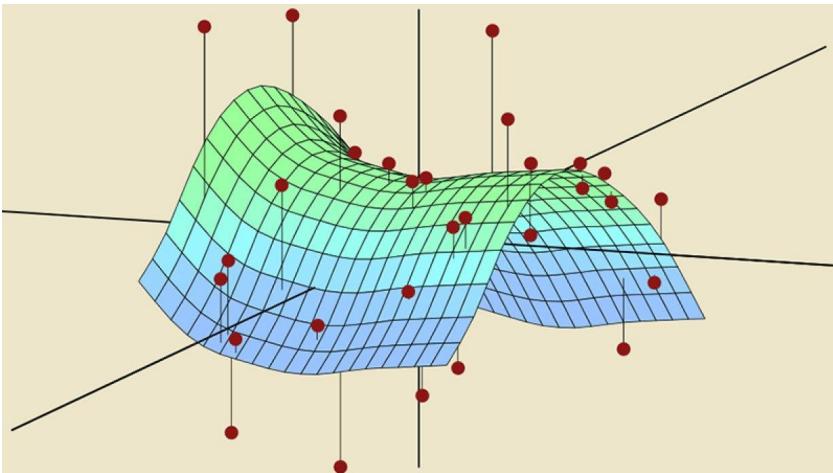
Hard Constraints. One research avenue involves incorporation of domain knowledge through imposition of constraints that cannot be violated. These hard constraints could be enforced during training, replacing what typically is an unconstrained optimization problem with a constrained one. In general, such constraints could involve simulations or highly nonlinear functions of the training parameters. Therefore, there is a need to identify particular cases when constraint qualification conditions can be ensured as these conditions are necessary regularity conditions for constrained optimization [57–59]. Although incorporating constraints during training generally makes maximal use of training data, there may be additional opportunities to employ constraints at the time of prediction (e.g., by projecting predictions onto the region induced by the constraints).

Soft Constraints. A similar avenue for incorporating domain knowledge involves modifying the objective function (soft constraints) used in training. It is understood that ML loss function selection should be guided by the task and data. Therefore, opportunities exist for developing loss functions that incorporate domain knowledge and analyzing the resulting impact on solvability



There's no free lunch!

Supervised learning as data fitting



Typically, #data points we need grow exponentially with respect to dimension (i.e., **curse of dimensionality**)

Knowledge

Small-data AI

Large-data AI

Knowledge ↑

↓ Data



Building in prior knowledge is **crucial** for reducing the data complexity e.g., "convolutional" layers

This talk:

several stories about data-knowledge tradeoffs

- Scientific inverse problems (SIPs)
 - Data-driven (data-rich) methods for SIPs
 - Single-instance (data-poor) methods for SIPs
- Principled computational tool for data-knowledge tradeoffs

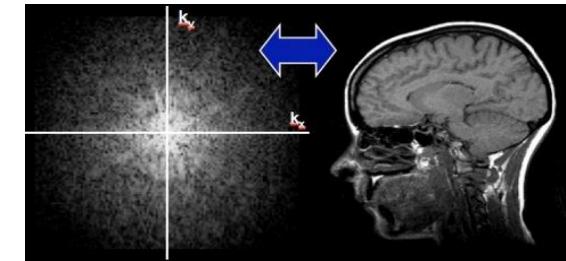
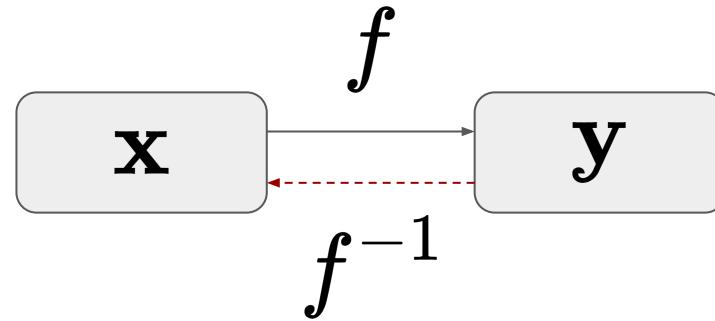
Scientific Inverse Problems

Inverse problems

Inverse problem: given $\mathbf{y} = f(\mathbf{x})$, recover \mathbf{x}



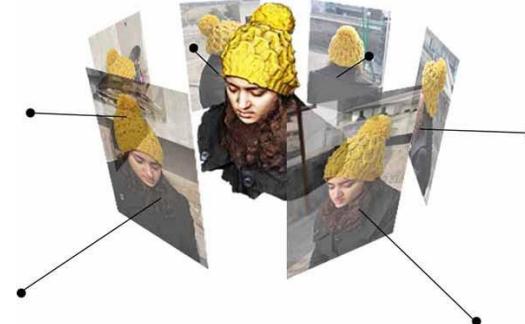
Image denoising



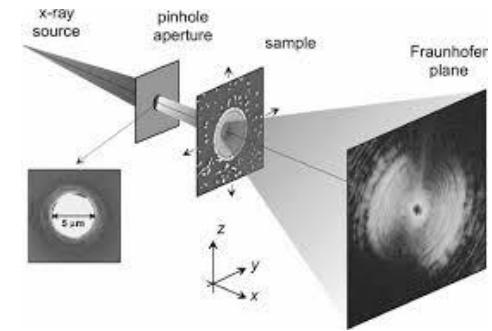
MRI reconstruction



Image super-resolution



3D reconstruction



Coherent diffraction imaging (CDI)

Traditional methods

Inverse problem: given $\mathbf{y} = f(\mathbf{x})$, recover \mathbf{x}

$$\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{R(\mathbf{x})}_{\text{regularizer}}$$

RegFit

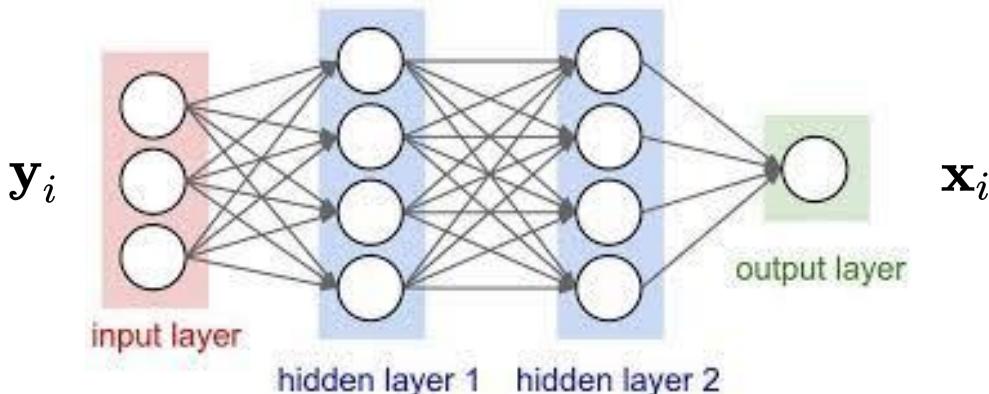
Limitations:

- Which ℓ ? (e.g., unknown/compound noise)
- Which R ? (e.g., structures not amenable to math description)
- Speed

DL methods for SIPs: the radical way

Inverse problem: given $\mathbf{y} = f(\mathbf{x})$, recover \mathbf{x}

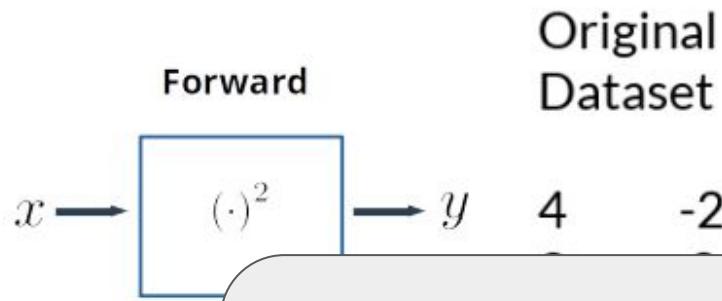
Learn the f^{-1} with a training set $\{(\mathbf{y}_i, \mathbf{x}_i)\}$



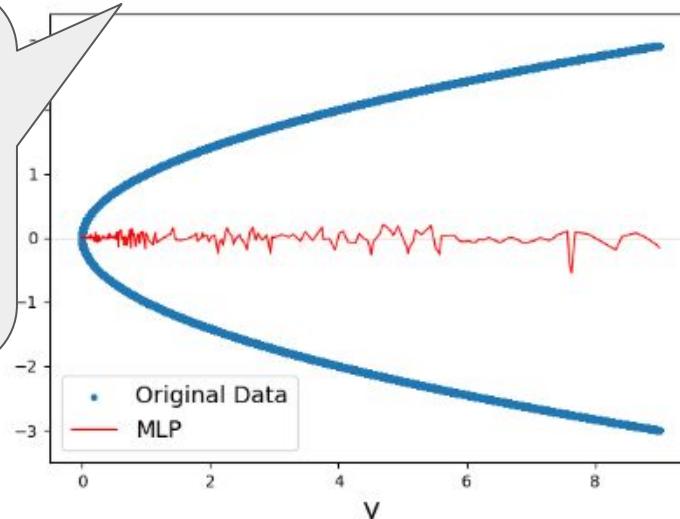
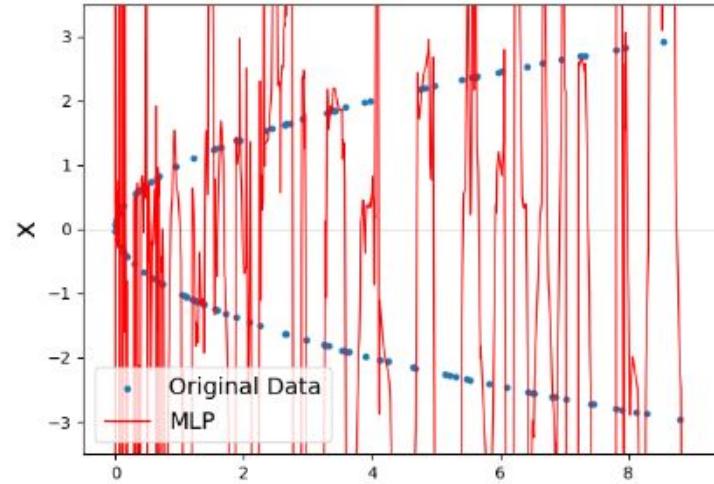
Limitations:

- Wasteful: not using f
- Representative data?
- Not always straightforward
(see, e.g., Tayal et al. **Inverse Problems, Deep Learning, and Symmetry Breaking**.
<https://arxiv.org/abs/2003.09077>)

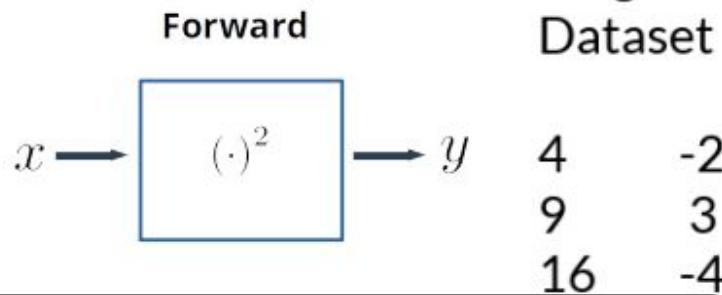
Story I: More could be less



In
More is
less



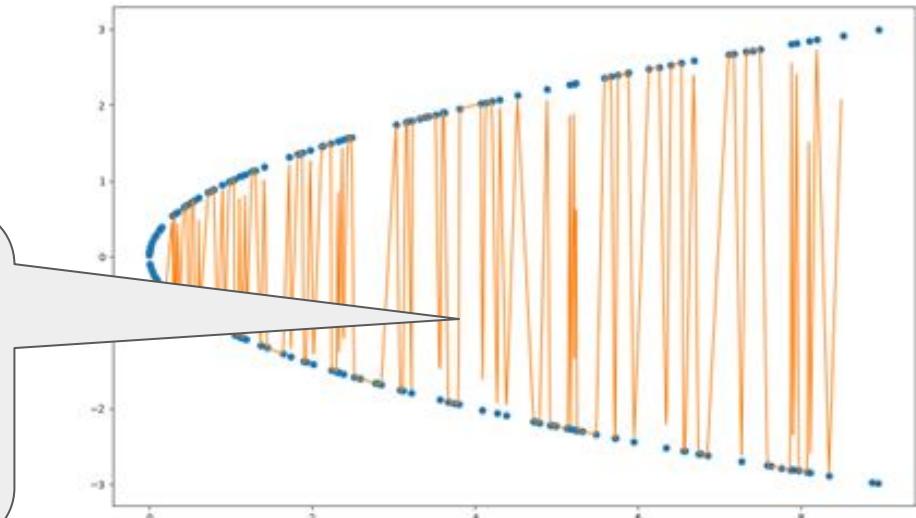
Why “more is less” here?



Highly oscillatory target function to learn by DNNs—difficult

Forward symmetry: $\{+\sqrt{y}, -\sqrt{y}\} \leftrightarrow y$

Implies: on dense training set, very close y 's can map to very far away x 's different by signs



Remedy: symmetry breaking

Forward

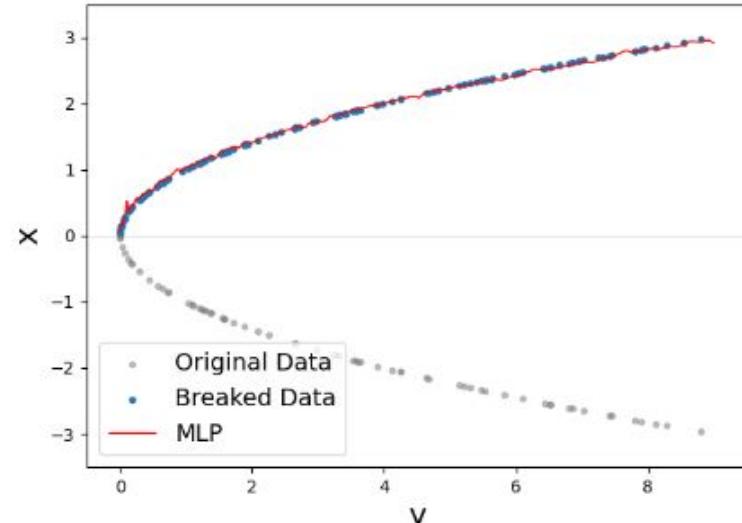
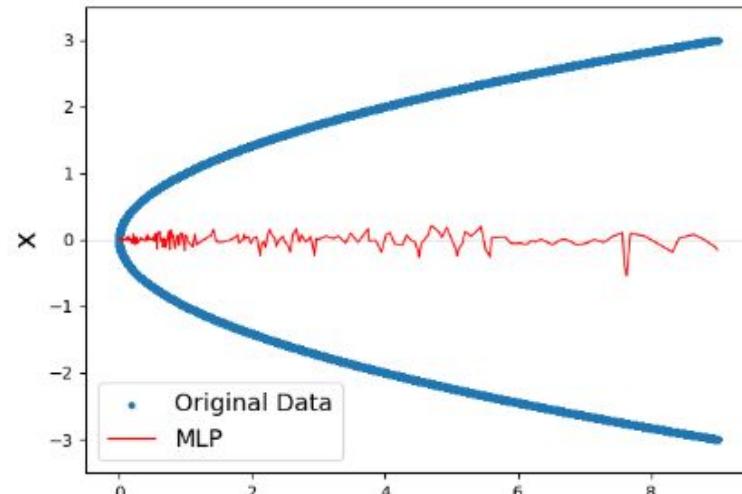
$$x \rightarrow (\cdot)^2 \rightarrow y$$

	Original Dataset	
4	4	-2
9	9	3
16	16	-4
100	100	10
4	4	2
64	64	-8
36	36	6

Inverse

$$y \rightarrow \text{DNN} \rightarrow ?$$

Fix all signs to be positive

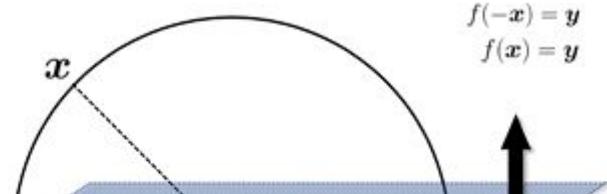


A slightly more complicated example

$$\mathbf{y} = |\mathbf{Ax}|^2 \quad \mathbf{A} : \text{iid Gaussian} \quad (\text{Gaussian phase retrieval})$$

Forward symmetry: global sign

$$\mathbf{y} = |\mathbf{Ax}|^2 = |\mathbf{A}(-\mathbf{x})|^2$$

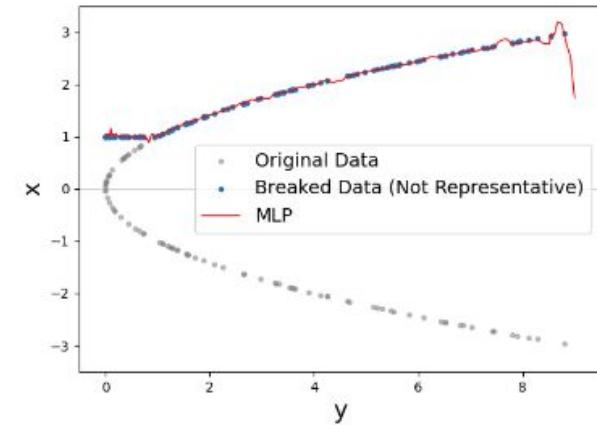
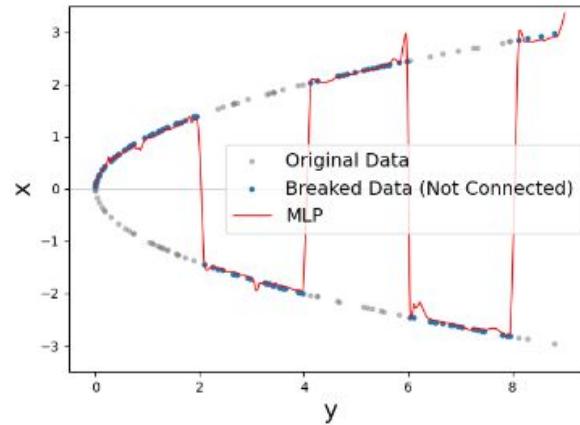
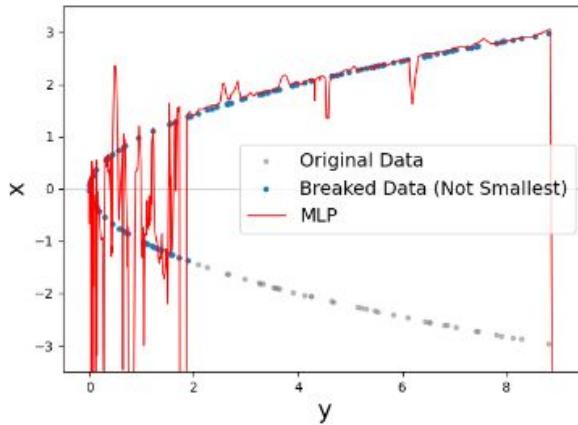


Dim	Sample	After Symmetry Breaking		Before Symmetry Breaking	
		DNN	K-NN	DNN	K-NN
5	2e4	4.08	11.82	85.37	68.26
	5e4	2.20	9.41	90.51	66.58
	1e5	1.30	7.98	96.66	66.18
	1e6	0.37	4.71	122.71	65.08

More is more More is less

Symmetry-breaking principle

Symmetry breaking: a preprocessing step on the training set



Finding the smallest, connected, representative set

What is Wrong with End-to-End Learning for Phase Retrieval?

Wenjie Zhang, Yuxiang Wan, Zhong Zhuang, Ju Sun

For nonlinear inverse problems that are prevalent in imaging science, symmetries in the forward model are common. When data-driven deep learning approaches are used to solve such problems, these intrinsic symmetries can cause substantial learning difficulties. In this paper, we explain how such difficulties arise and, more importantly, how to overcome them by preprocessing the training set before any learning, i.e., symmetry breaking. We take far-field phase retrieval (FFPR), which is central to many areas of scientific imaging, as an example and show that symmetric breaking can substantially improve data-driven learning. We also formulate the mathematical principle of symmetry breaking.

DL methods for SIPs: the middle way

Inverse problem: given $\mathbf{y} = f(\mathbf{x})$, recover \mathbf{x}

$$\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{R(\mathbf{x})}_{\text{regularizer}}$$

RegFit

Recipe: revamp numerical methods for RegFit with **pretrained/trainable DNNs**

DL methods for SIPs: the middle way

Algorithm unrolling

$$\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{R(\mathbf{x})}_{\text{regularizer}}$$

If R proximal friendly

$$\mathbf{x}^{k+1} = \mathcal{P}_R(\mathbf{x}^k - \eta \nabla^\top f(\mathbf{x}^k) \ell'(\mathbf{y}, f(\mathbf{x}^k)))$$

Idea: make \mathcal{P}_R trainable, using $\{(\mathbf{x}_i, \mathbf{y}_i)\}$

E.g.,

$$\ell(\mathbf{y}, f(\mathbf{x})) = \|\mathbf{y} - \mathbf{A} \mathbf{x}\|_2^2$$

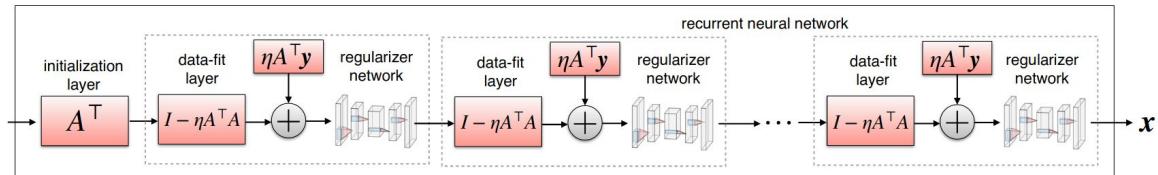


Fig credit: Deep Learning Techniques for Inverse Problems in Imaging <https://arxiv.org/abs/2005.06001>

DL methods for SIPs: the middle way

Using $\{\mathbf{x}_i\}$ only

$$\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{R(\mathbf{x})}_{\text{regularizer}}$$

Plug-and-Play

$$\mathbf{x}^{k+1} = \mathcal{P}_R(\mathbf{x}^k - \eta \nabla^\top f(\mathbf{x}^k) \ell'(\mathbf{y}, f(\mathbf{x}^k)))$$

E.g. replace \mathcal{P}_R with pretrained denoiser

Deep generative models

Pretraining: $\mathbf{x}_i \approx G_\theta(\mathbf{z}_i) \quad \forall i$

Deployment: $\min_{\mathbf{z}} \ell(\mathbf{y}, f \circ G_\theta(\mathbf{z})) + \lambda R \circ G_\theta(\mathbf{z})$

DL methods for SIPs: a survey

Deep Learning Techniques for Inverse Problems in Imaging

Gregory Ongie*, Ajil Jalal†, Christopher A. Metzler‡,
Richard G. Baraniuk§, Alexandros G. Dimakis¶, Rebecca Willett||

April 2020

Abstract

Recent work in machine learning shows that deep neural networks can be used to solve a wide variety of inverse problems arising in computational imaging. We explore the central prevailing themes of this emerging area and present a taxonomy that can be used to categorize different problems and reconstruction methods. Our taxonomy is organized along two central axes: (1) whether or not a forward model is known and to what extent it is used in training and testing, and (2) whether or not the learning is supervised or unsupervised, i.e., whether or not the training relies on access to matched ground truth image and measurement pairs. We also discuss the tradeoffs associated with these different reconstruction approaches, caveats and common failure modes, plus open problems and avenues for future work.

Focuses on **linear**
inverse problems,
i.e., f linear

<https://arxiv.org/abs/2005.06001>

Limitations of middle ways:

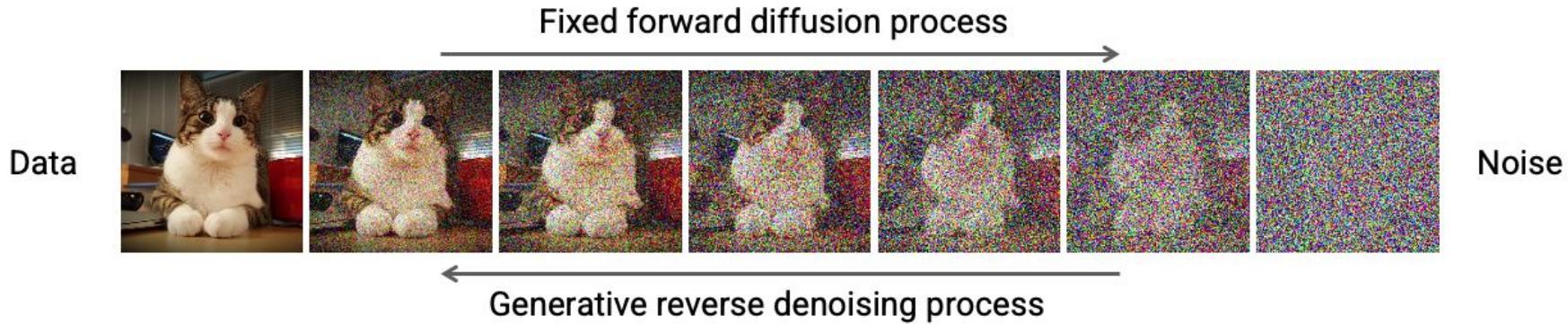
- Representative data?
- Algorithm-sensitive
- Good initialization? (e.g., Manekar et al. **Deep Learning Initialized Phase Retrieval**.
<https://sunju.org/pub/NIPS20-WS-DL4F-PR.pdf>)

Deep generative models

Story II: Don't be too Bayesian

Pretraining: $\mathbf{x}_i \approx G_\theta(\mathbf{z}_i) \quad \forall i$

Deployment: $\min_{\mathbf{z}} \ell(\mathbf{y}, f \circ G_\theta(\mathbf{z})) + \lambda R \circ G_\theta(\mathbf{z})$



How to use pretrained diffusion models for SIPs?

Bayesian thinking

Reverse SDE for DM

$$d\mathbf{x} = \left[-\frac{\beta(t)}{2}\mathbf{x} - \beta(t)\nabla_{\mathbf{x}_t} \log p_t(\mathbf{x}_t) \right] dt + \sqrt{\beta(t)}d\bar{\mathbf{w}}$$

$$\nabla_{\mathbf{x}_t} \log p_t(\mathbf{x}_t | \mathbf{y}) = \nabla_{\mathbf{x}_t} \log p_t(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \log p_t(\mathbf{y} | \mathbf{x}_t)$$

Reverse conditional SDE for SIPs

$$d\mathbf{x} = \left[-\frac{\beta(t)}{2}\mathbf{x} - \beta(t)(\nabla_{\mathbf{x}_t} \log p_t(\mathbf{x}_t) + \nabla_{\mathbf{x}_t} \log p_t(\mathbf{y} | \mathbf{x}_t)) \right] dt + \sqrt{\beta(t)}d\bar{\mathbf{w}}$$

Bayesian thinking: after several approx steps

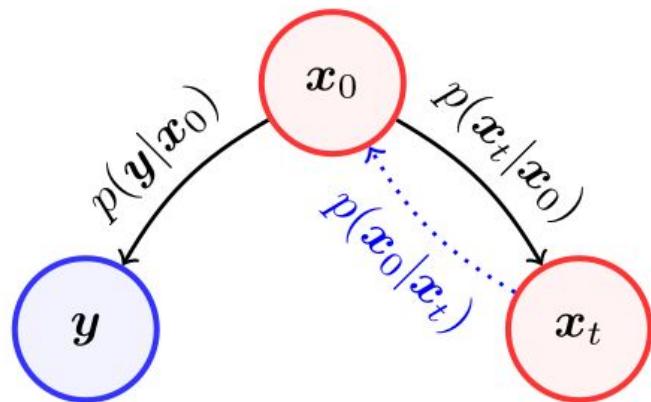


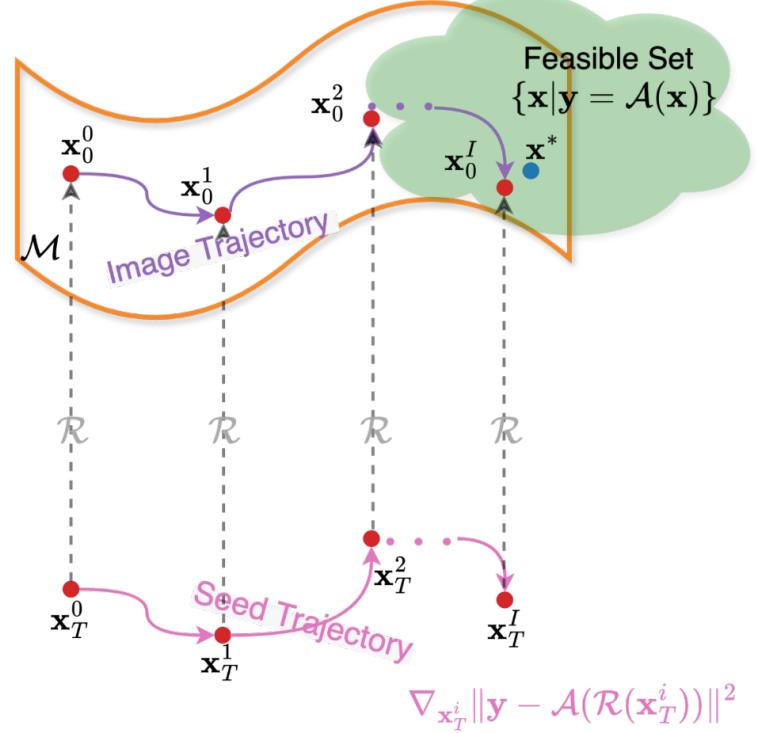
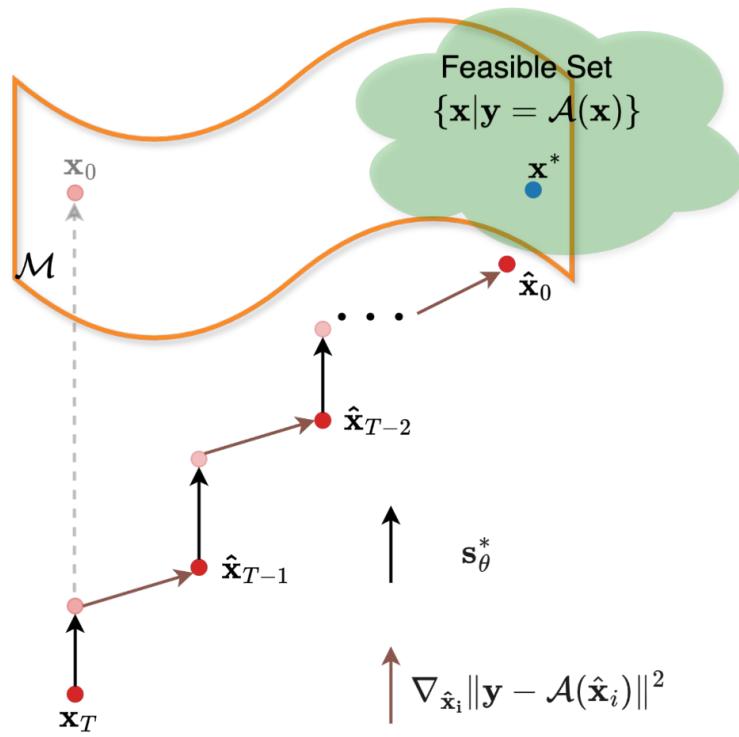
Figure 2: Probabilistic graph. Black solid line: tractable, blue dotted line: intractable in general.

Algorithm 1 DPS - Gaussian

Require: $N, \mathbf{y}, \{\zeta_i\}_{i=1}^N, \{\tilde{\sigma}_i\}_{i=1}^N$

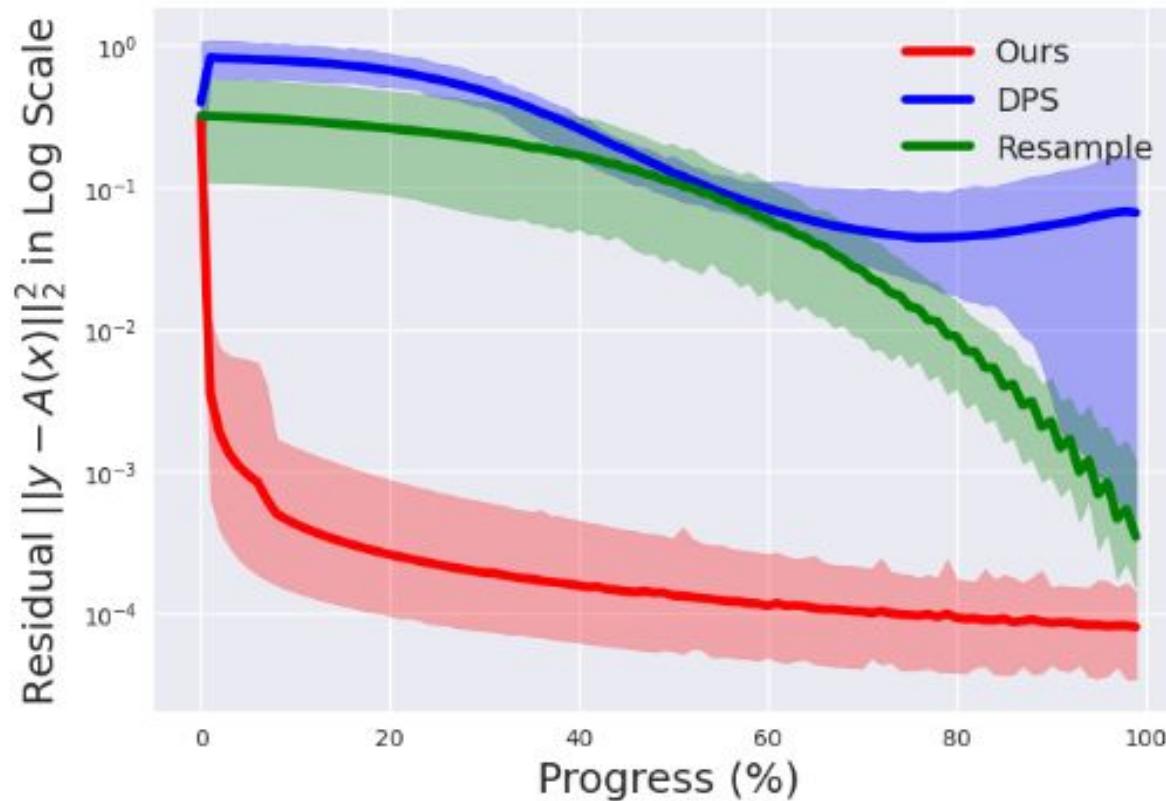
```
1:  $\mathbf{x}_N \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ 
2: for  $i = N - 1$  to 0 do
3:    $\hat{\mathbf{s}} \leftarrow \mathbf{s}_\theta(\mathbf{x}_i, i)$ 
4:    $\hat{\mathbf{x}}_0 \leftarrow \frac{1}{\sqrt{\bar{\alpha}_i}} (\mathbf{x}_i + (1 - \bar{\alpha}_i) \hat{\mathbf{s}})$ 
5:    $\mathbf{z} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ 
6:    $\mathbf{x}'_{i-1} \leftarrow \frac{\sqrt{\alpha_i}(1 - \bar{\alpha}_{i-1})}{1 - \bar{\alpha}_i} \mathbf{x}_i + \frac{\sqrt{\bar{\alpha}_{i-1}} \beta_i}{1 - \bar{\alpha}_i} \hat{\mathbf{x}}_0 + \tilde{\sigma}_i \mathbf{z}$ 
7:    $\mathbf{x}_{i-1} \leftarrow \mathbf{x}'_{i-1} - \zeta_i \nabla_{\mathbf{x}_i} \|\mathbf{y} - \mathcal{A}(\hat{\mathbf{x}}_0)\|_2^2$ 
8: end for
9: return  $\hat{\mathbf{x}}_0$ 
```

Explained in one picture (vs. our plugin idea)



Feasibility crisis

Inverse problem: given $\mathbf{y} = f(\mathbf{x})$, recover \mathbf{x}



Preliminary result on linear SIPs

Table 1: (**Linear IPs**) Quantitative comparison for **super-resolution** and **inpainting** on CelebA [23] and FFHQ [34] with additional Gaussian noise ($\sigma = 0.01$). (**Bold**: best, under: second best, **green**: performance increase, **red**: performance decrease)

	Super-resolution (4×)						Inpainting (Random 70%)					
	CelebA (256 × 256)			FFHQ (256 × 256)			CelebA (256 × 256)			FFHQ (256 × 256)		
	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑
ADMM-PnP [39]	0.217	26.99	0.808	0.229	26.25	0.794	0.091	31.94	0.923	0.104	30.64	0.901
DMPS [40]	<u>0.070</u>	<u>28.89</u>	<u>0.848</u>	0.076	<u>28.03</u>	<u>0.843</u>	0.297	24.52	0.693	0.326	23.31	0.664
DDRM [41]	0.226	26.34	0.754	0.282	25.11	0.731	0.185	26.10	0.712	0.201	25.44	0.722
MCG [40]	0.725	19.88	0.323	0.786	18.20	0.271	1.283	10.16	0.049	1.276	10.37	0.050
ILVR [4]	0.322	21.63	0.603	0.360	20.73	0.570	0.447	15.82	0.484	0.483	15.10	0.450
DPS [18]	0.087	28.32	0.823	0.098	27.44	0.814	0.043	<u>32.24</u>	<u>0.924</u>	0.046	<u>30.95</u>	<u>0.913</u>
ReSample [9]	0.080	28.29	0.819	0.108	25.22	0.773	0.039	30.12	0.904	<u>0.044</u>	27.91	0.884
Ours	0.067	31.25	0.878	<u>0.079</u>	30.25	0.871	0.039	34.03	0.936	0.038	33.01	0.931
Ours vs. Best compe.	-0.003	+2.36	+0.030	+0.003	+2.22	+0.028	-0.000	+1.79	+0.012	-0.006	+2.06	+0.018

Preliminary result on nonlinear SIPs

Table 3: (**Nonlinear IP**) Quantitative comparison for **BID** on CelebA [25] and FFHQ [34] with additional Gaussian noise ($\sigma = 0.01$). (**Bold**: best, under: second best, **green**: performance increase, **red**: performance decrease)

	CelebA (256 × 256)						FFHQ (256 × 256)					
	Motion			Gaussian			Motion			Gaussian		
	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑	LPIPS↓	PSNR↑	SSIM↑
SelfDeblur [43]	0.568	16.59	0.417	0.579	16.55	0.423	0.628	16.33	0.408	0.604	16.22	0.410
DeBlurGANv2 [44]	0.313	20.56	0.613	0.350	24.29	0.743	0.353	19.67	0.581	0.374	23.58	0.726
Stripformer [45]	0.287	22.06	0.644	0.316	25.03	<u>0.747</u>	0.324	21.31	0.613	0.339	<u>24.34</u>	<u>0.728</u>
MPRNet [46]	0.332	20.53	0.620	0.375	22.72	0.698	0.373	19.70	0.590	0.394	22.33	0.685
Pan-DCP [47]	0.606	15.83	0.483	0.653	20.57	0.701	0.616	15.59	0.464	0.667	20.69	0.698
Pan- ℓ_0 [48]	0.631	15.16	0.470	0.654	20.49	0.675	0.642	14.43	0.443	0.669	20.34	0.671
ILVR [41]	0.398	19.23	0.520	0.338	21.20	0.588	0.445	18.33	0.484	0.375	20.45	0.555
BlindDPS [3]	<u>0.164</u>	<u>23.60</u>	<u>0.682</u>	<u>0.173</u>	<u>25.15</u>	0.721	<u>0.185</u>	<u>21.77</u>	<u>0.630</u>	<u>0.193</u>	23.83	0.693
Ours	0.104	29.61	0.825	0.140	28.84	0.795	0.135	27.99	0.794	0.169	28.26	0.811
Ours vs. Best compe.	-0.060	+6.01	+0.143	-0.033	+3.69	+0.048	-0.050	+6.22	+0.164	-0.024	+3.92	+0.083

Am I supposed/allowed to show this?

DMPlug: A Plug-in Method for Solving Inverse Problems with Diffusion Models

Anonymous Author(s)

Affiliation

Address

email

DL methods for SIPS: the **economic** way

Deep image prior (DIP) $\mathbf{x} \approx G_\theta(\mathbf{z})$ G_θ (and \mathbf{z}) trainable

$$\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{R(\mathbf{x})}_{\text{regularizer}}$$

No extra training data!

$$\min_{\theta} \ell(\mathbf{y}, f \circ G_\theta(\mathbf{z})) + \lambda R \circ G_\theta(\mathbf{z})$$

Ulyanov et al. **Deep image prior**. IJCV'20. <https://arxiv.org/abs/1711.10925>

Contrasting: Deep generative models

Pretraining: $\mathbf{x}_i \approx G_\theta(\mathbf{z}_i) \quad \forall i$

Deployment: $\min_{\mathbf{z}} \ell(\mathbf{y}, f \circ G_\theta(\mathbf{z})) + \lambda R \circ G_\theta(\mathbf{z})$

Deep image prior (DIP)

DIP's cousin(s)

$$\mathbf{x} \approx G_\theta(\mathbf{z}) \quad G_\theta \text{ (and } \mathbf{z} \text{) trainable}$$

Idea: (visual) objects as continuous functions

Neural implicit representation (NIR)

$$\mathbf{x} \approx \mathcal{D} \circ \bar{\mathbf{x}} \quad \mathcal{D} : \text{discretization} \quad \bar{\mathbf{x}} : \text{continuous function}$$

Physics-informed neural networks (PINN)

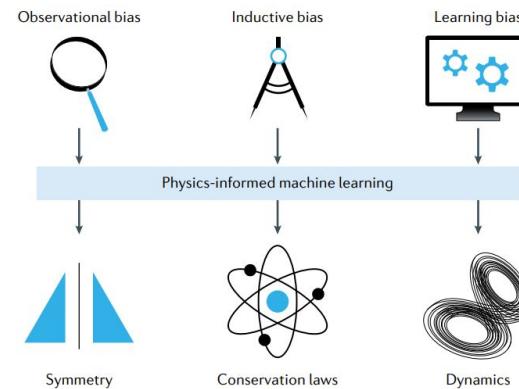
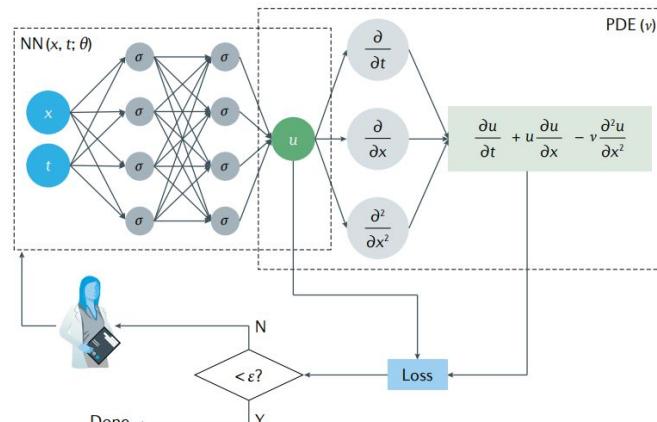
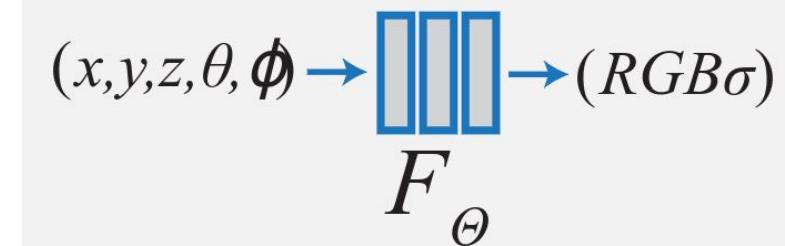
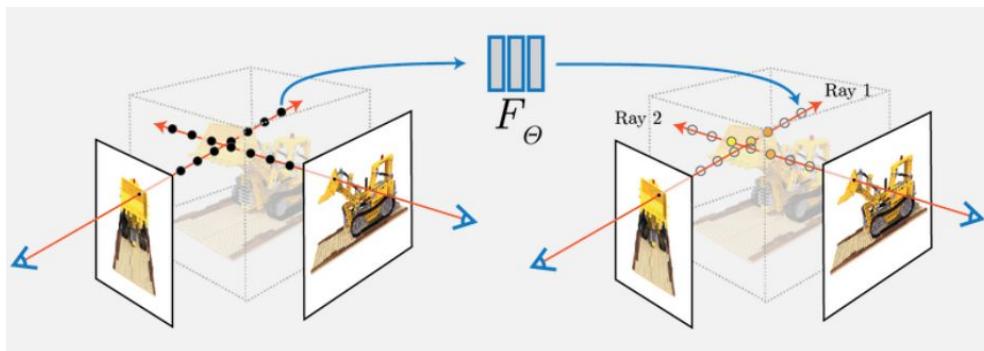
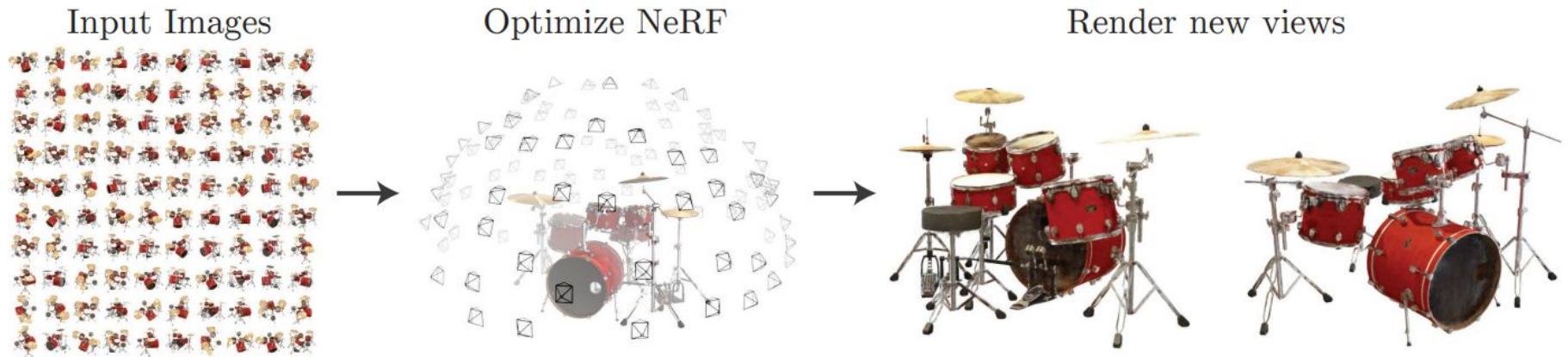


Figure credit: <https://www.nature.com/articles/s42254-021-00314-5>

NIR for 3D rendering and view synthesis



<https://www.matthewtancik.com/nerf>

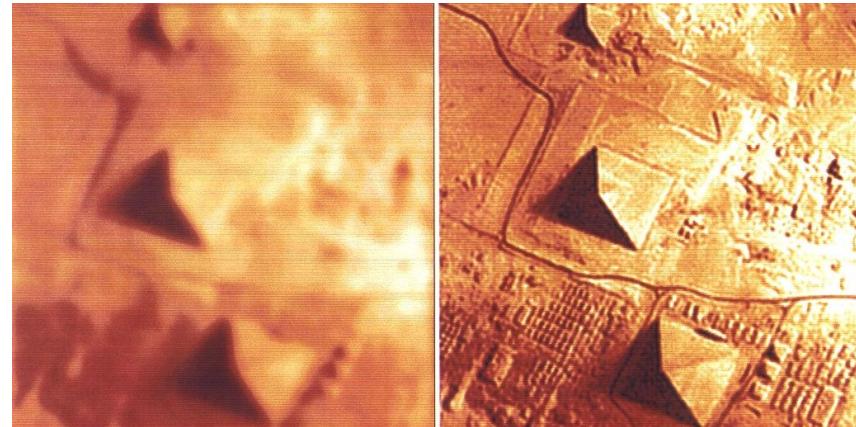
Story III: We benefit from DL even with a single data point

Blind image deblurring (BID)

$$\underbrace{\mathbf{y}}_{\text{blurry and noisy image}} = \overbrace{\mathbf{k}}^{\text{blur kernel}} * \underbrace{\mathbf{x}}_{\text{clean image}} + \overbrace{\mathbf{n}}^{\text{noise}}$$

Given \mathbf{y} ,
recover \mathbf{x} (and/or \mathbf{k})

Also **Blind Deconvolution**



Landmark surveys

- 1996: Kundur and Hatzinakos. **Blind image deconvolution.** <https://doi.org/10.1109/79.489268>
- 2011: Levin et al. **Understanding blind deconvolution algorithms.** <https://doi.org/10.1109/TPAMI.2011.148>
- 2012: Kohler et al. **Recording and playback of camera shake: Benchmarking blind deconvolution with a real-world database.** https://doi.org/10.1007/978-3-642-33786-4_3
- 2016: Lai et al. **A comparative study for single image blind deblurring.** <https://doi.org/10.1109/CVPR.2016.188>
- 2021: Koh et al. **Single image deblurring with neural networks: A comparative survey** <https://doi.org/10.1016/j.cviu.2020.103134>
- 2022: Zhang et al. **Deep image blurring: A survey** <https://doi.org/10.1007/s11263-022-01633-5>

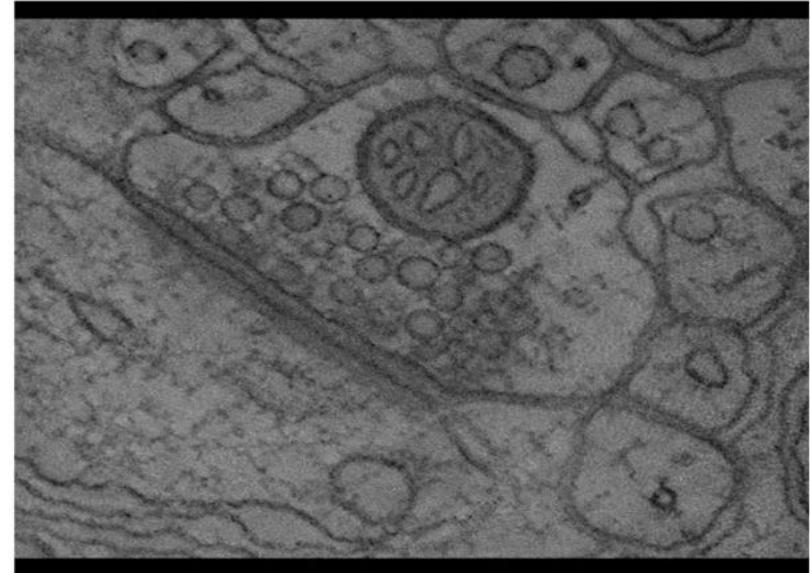
See also: **Awesome Deblurring**

<https://github.com/subeeshvasu/Awesome-Deblurring>

Key challenge of data-driven approach:

obtaining sufficiently expressive data (Koh et al'21. Zhang et al'22)

Untouched practical questions



Key question addressed in this paper: How do we solve blind image deblurring without knowing: (1) the size of the blur kernel, (2) the type and level of noise, and (3) whether it is blur / noise only or both ?

Double DIPs

$$\underbrace{\mathbf{y}}_{\text{blurry and noisy image}} = \underbrace{\mathbf{k}}_{\text{blur kernel}} * \underbrace{\mathbf{x}}_{\text{clean image}} + \underbrace{\mathbf{n}}_{\text{noise}}$$

$$\min_{\mathbf{k}, \mathbf{x}} \underbrace{\ell(\mathbf{y}, \mathbf{k} * \mathbf{x})}_{\text{data fitting}} + \lambda_k \underbrace{R_{\mathbf{k}}(\mathbf{k})}_{\text{regularizing } \mathbf{k}} + \lambda_x \underbrace{R_{\mathbf{x}}(\mathbf{x})}_{\text{regularizing } \mathbf{x}}$$

Idea: parameterize both \mathbf{k} and \mathbf{x} as DIPs

- CNN + CNN (Wang et al'19, <https://doi.ieeecomputersociety.org/10.1109/ICCVW.2019.00127>; Tran et al'21, <https://arxiv.org/abs/2104.00317>)
- MLP + CNN (SelfDeblur; Ren et al'20, <https://arxiv.org/abs/1908.02197>)

Still problematic with

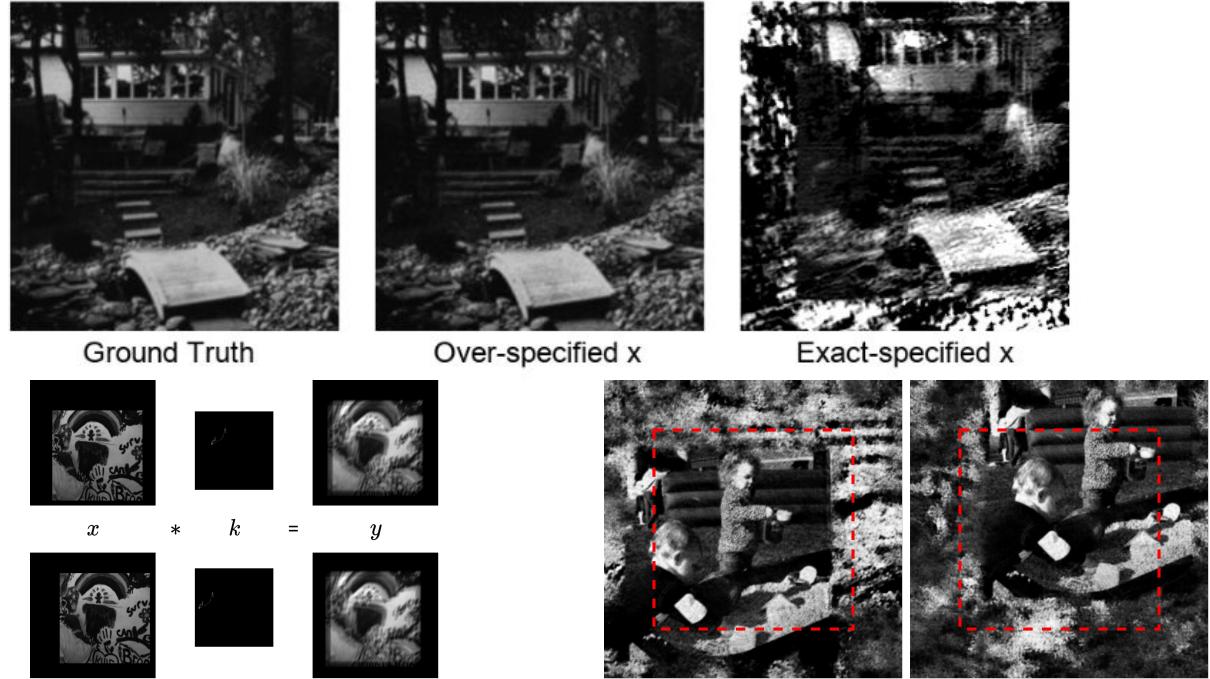
- 1) kernel size over-specification
- 2) substantial noise

A glance of our modifications

Over-specify k
Over-specify x

$k \sim$ half of the image sizes

Handle bounded shift



$$\min_{\theta_k, \theta_x} \|y - G_{\theta_k}(z_k) * G_{\theta_x}(z_x)\|_2^2 + \lambda \frac{\|\nabla G_{\theta_x}(z_x)\|_1}{\|\nabla G_{\theta_x}(z_x)\|_2}$$

ℓ_1/ℓ_2 vs ℓ_1

Table 1: ℓ_1/ℓ_2 vs TV for noise: mean and (std).

	Low Level		High Level	
	PSNR	λ	PSNR	λ
$\frac{L_1}{L_2}$	32.64 (0.69)	0.0001 (0.018)	27.74 (0.23)	0.0002 (0.0019)
TV	31.12 (0.52)	0.002 (0.07)	24.34 (0.78)	0.02 (0.10)

SelfDeblur vs our method



Clean



Blurry and noisy



SelfDeblur



Ours



Clean



Blurry and noisy

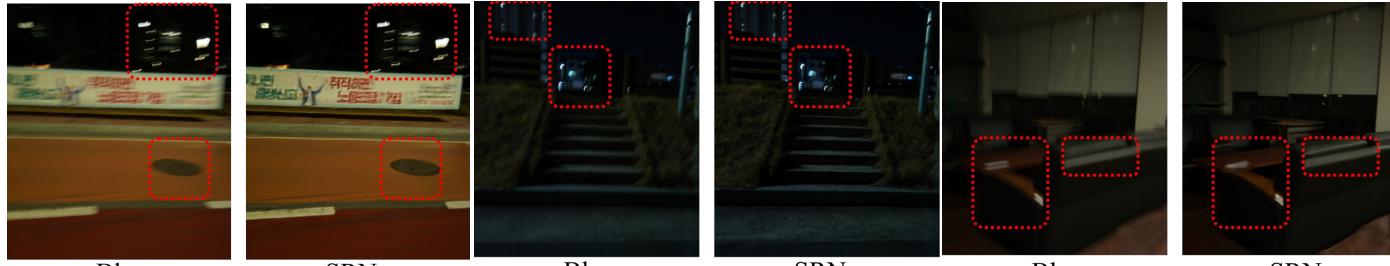


SelfDeblur



Ours

Real world results



Blur

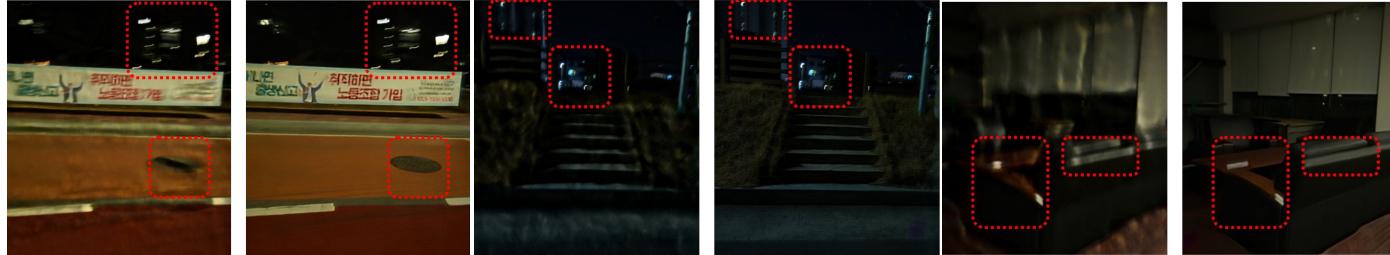
SRN

Blur

SRN

Blur

SRN



ZHANG20

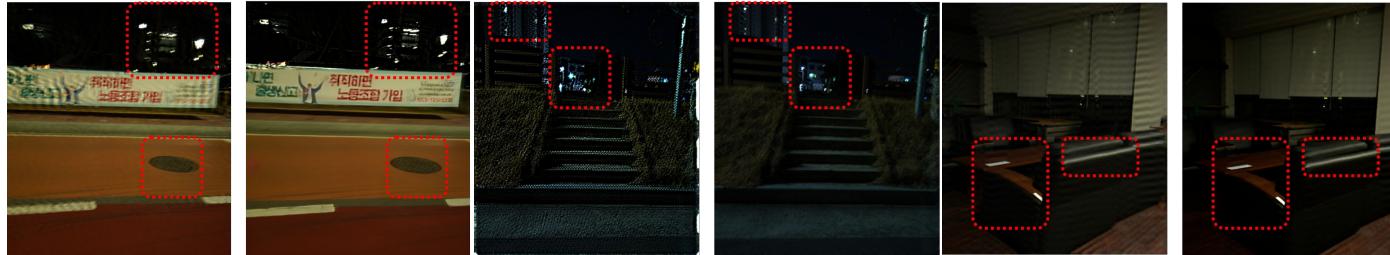
DeblurGAN-v2

ZHANG20

DeblurGAN-v2

ZHANG20

DeblurGAN-v2



SelfDeblur

Our

SelfDeblur

Our

SelfDeblur

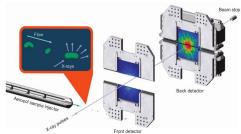
Our

Difficult cases

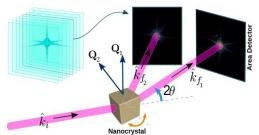
- 1) High depth contrast
- 2) High brightness contrast

**Outperform SOTA
data-driven methods!**

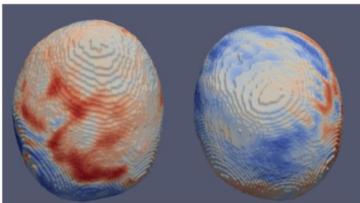
Breakthroughs in imaging



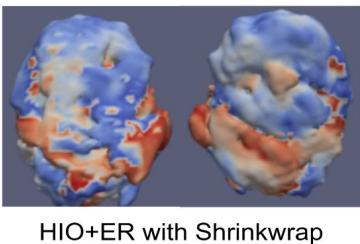
Coherent Diffraction Imaging



Bragg Coherent Diffraction Imaging



Our



HIO+ER with Shrinkwrap

First PR method that won in a double-blind test, and systematic evaluation, beating a 40-years old legacy

Practical Phase Retrieval Using Double Deep Image Priors

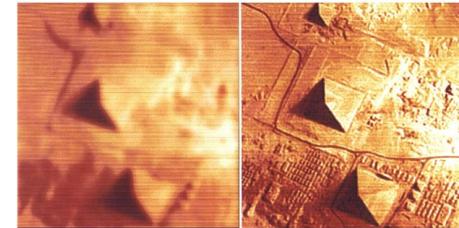
Zhong Zhuang, David Yang, Felix Hofmann, David Barmherzig, Ju Sun

$$\underbrace{\mathbf{y}}_{\text{blurry and noisy image}} = \underbrace{\mathbf{k}}_{\text{blur kernel}} * \underbrace{\mathbf{x}}_{\text{clean image}} + \underbrace{\mathbf{n}}_{\text{noise}}$$

Mostly due to optical deficiencies (e.g., defocus) and motions

Given \mathbf{y} ,
recover \mathbf{x} (and/or \mathbf{k})

Also **Blind Deconvolution**



First BID method that works with unknown kernel size AND substantial noise

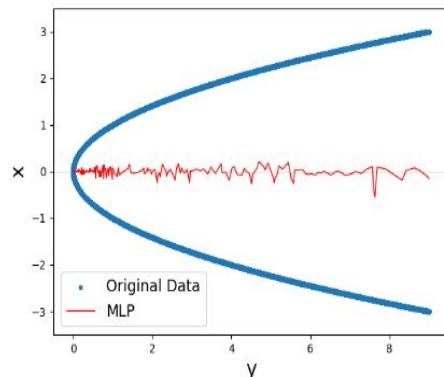
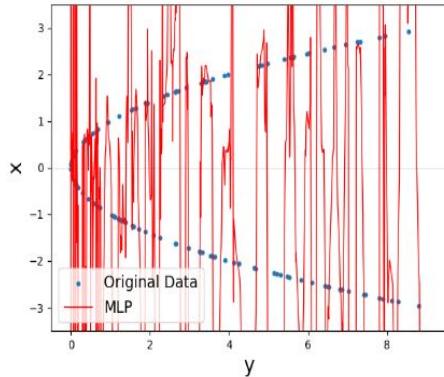
Blind Image Deblurring with Unknown Kernel Size and Substantial Noise

Zhong Zhuang, Taihui Li, Hengkang Wang, Ju Sun

Related papers

- Li et al. **Self-Validation: Early Stopping for Single-Instance Deep Generative Priors** (BMVC'21) <https://arxiv.org/abs/2110.12271>
- Wang et al. **Early Stopping for Deep Image Prior** <https://arxiv.org/abs/2112.06074> (TMLR'23)
- Zhuang et al. **Blind Image Deblurring with Unknown Kernel Size and Substantial Noise.** <https://arxiv.org/abs/2208.09483> (IJCV'24)
- Zhuang et al. **Practical Phase Retrieval Using Double Deep Image Priors.** <https://arxiv.org/abs/2211.00799> (Electronic Imaging'24)
- Li et al. **Deep Random Projector: Toward Efficient Deep Image Prior.** (CVPR'23)

Data-driven methods for SIPs



Story I: More could be less

Single-instance methods for SIPs

Deep image prior (DIP)

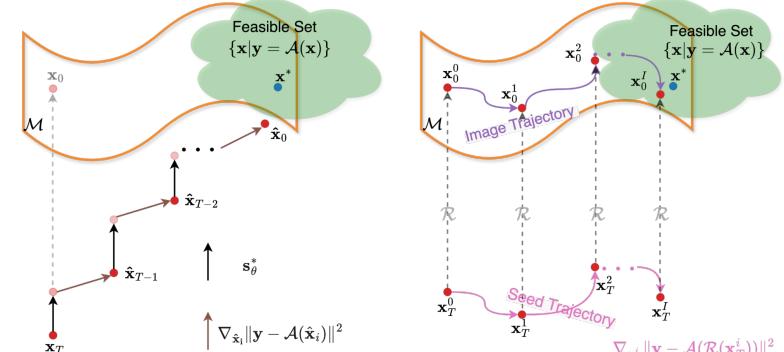
$$\mathbf{x} \approx G_\theta(\mathbf{z}) \quad G_\theta \text{ (and } \mathbf{z} \text{) trainable}$$

$$\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{R(\mathbf{x})}_{\text{regularizer}}$$

$$\min_{\theta} \ell(\mathbf{y}, f \circ G_\theta(\mathbf{z})) + \lambda R \circ G_\theta(\mathbf{z})$$

Story III: Benefit from DL with a single data point

Ulyanov et al. Deep image prior. IJCV20. <https://arxiv.org/abs/1711.10925>



Story II: Don't be too Bayesian

$$\underbrace{\mathbf{y}}_{\text{blurry and noisy image}} = \underbrace{\mathbf{k}}_{\text{blur kernel}} * \underbrace{\mathbf{x}}_{\text{clean image}} + \underbrace{\mathbf{n}}_{\text{noise}}$$

Mostly due to optical deficiencies (e.g., defocus) and motions

Given \mathbf{y} ,
recover \mathbf{x} (and/or \mathbf{k})

Also **Blind Deconvolution**



Principled data-knowledge tradeoff

Knowledge

Building the Future

Thrust B: How Should Domain Knowledge Be Incorporated into Supervised Machine Learning?

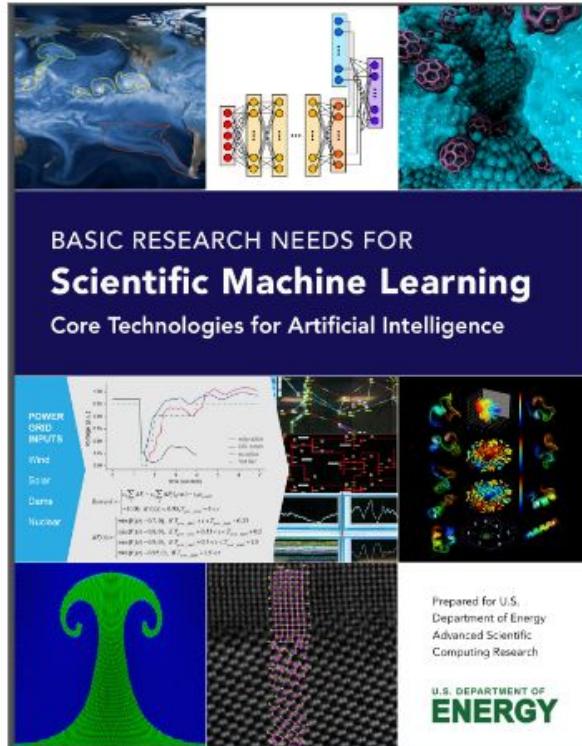
The central question for this thrust is “which knowledge should be leveraged in SciML, and how should this knowledge be included?” Any answers will naturally depend on the SciML task and computational budgets, thus mirroring standard considerations in traditional scientific computing.

Hard Constraints. One research avenue involves incorporation of domain knowledge through imposition of constraints that cannot be violated. These hard constraints could be enforced during training, replacing what typically is an unconstrained optimization problem with a constrained one. In general, such constraints could involve simulations or highly nonlinear functions of the training parameters. Therefore, there is a need to identify particular cases when constraint qualification conditions can be ensured as these conditions are necessary regularity conditions for constrained optimization [57–59]. Although incorporating constraints during training generally makes maximal use of training data, there may be additional opportunities to employ constraints at the time of prediction (e.g., by projecting predictions onto the region induced by the constraints).

Soft Constraints. A similar avenue for incorporating domain knowledge involves modifying the objective function (soft constraints) used in training. It is understood that ML loss function selection should be guided by the task and data. Therefore, opportunities exist for developing loss functions that incorporate domain knowledge and analyzing the resulting impact on solvability

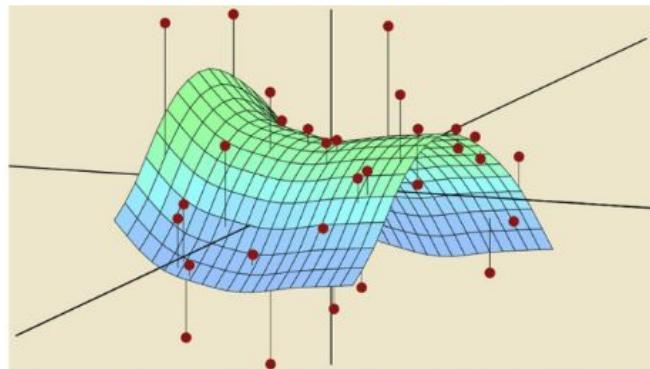
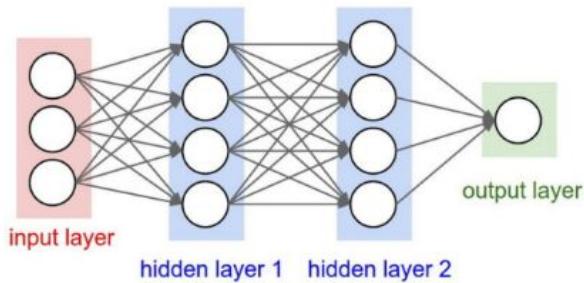
Ref <https://www.osti.gov/servlets/purl/1478744>

Domain-Aware Scientific Machine Learning



When DL meets constraints

Artificial neural networks



used to approximate nonlinear functions

Unconstrained optimization

$$\min_{\boldsymbol{w}'_i s, \boldsymbol{b}'_i s} \frac{1}{n} \sum_{i=1}^n \ell [y_i, \{\text{NN}(\boldsymbol{w}_1, \dots, \boldsymbol{w}_k, \boldsymbol{b}_1, \dots, \boldsymbol{b}_k)\}(\boldsymbol{x}_i)]$$
$$\min_{\boldsymbol{x}} f(\boldsymbol{x})$$

“Solved”

Constrained optimization

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}) \quad \text{s. t. } g(\boldsymbol{x}) \leq \mathbf{0}$$

largely “unsolved”

GAPS

$$\min_{\boldsymbol{x}} f(\boldsymbol{x}) \quad \text{s. t. } g(\boldsymbol{x}) \leq \mathbf{0}$$

largely “unsolved”



An imaginary chat between a PhD student working in deep learning (**DLP**) and a PhD student working in optimization (**OP**)

DLP: Man, I've solved a constrained DL problem recently

OP: Oh, that's a hard problem

DLP: Really? I actually did it

OP: How?

DLP: My problem is $\min_{\boldsymbol{x}} f(\boldsymbol{x}), \text{s.t. } g(\boldsymbol{x}) \leq \mathbf{0}$. I put $g(\boldsymbol{x})$ as a penalty and then call ADAM

OP: Are you sure it works?

DLP: Yes, the performance is improved and my paper is published at ICML

OP: Why don't you try augmented Lagrangian methods?

DLP: No implementation in Pytorch. Is it possible we work out some theory about my method?

OP: I think it's hard. It's not convex

DL with nontrivial constraints: many pitfalls

- **Robustness evaluation**
- Imbalanced learning
- Topology optimization

Deep Learning with Nontrivial Constraints: Methods and Applications

Chuan He¹, Ryan Devera¹, Wenjie Zhang¹, Ying Cui², Zhaosong Lu³ and Ju Sun¹

¹Computer Science and Engineering, University of Minnesota

²Industrial Engineering and Operations Research, University of California, Berkeley

³Industrial and Systems Engineering, University of Minnesota

{he000233, dever120, zhan7867}@umn.edu, yingcui@berkeley.edu, {zhaosong, jusun}@umn.edu

Robustness evaluation: penalty methods for complicated d (perceptual attack)

$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}')) \\ \text{s.t. } & d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n \end{aligned}$$

$d(\mathbf{x}, \mathbf{x}') \doteq \|\phi(\mathbf{x}) - \phi(\mathbf{x}')\|_2$ **perceptual distance**
where $\phi(\mathbf{x}) \doteq [\hat{g}_1(\mathbf{x}), \dots, \hat{g}_L(\mathbf{x})]$

Projection onto the constraint is complicated

Penalty methods

$$\max_{\tilde{\mathbf{x}}} \quad \mathcal{L}(f(\tilde{\mathbf{x}}), y) - \lambda \max \left(0, \|\phi(\tilde{\mathbf{x}}) - \phi(\mathbf{x})\|_2 - \epsilon \right)$$

Solve it for each fixed λ and then increase λ

Algorithm 2 Lagrangian Perceptual Attack (LPA)

```
1: procedure LPA(classifier network  $f(\cdot)$ , LPIPS distance  $d(\cdot, \cdot)$ , input  $\mathbf{x}$ , label  $y$ , bound  $\epsilon$ )
2:    $\lambda \leftarrow 0.01$ 
3:    $\tilde{\mathbf{x}} \leftarrow \mathbf{x} + 0.01 * \mathcal{N}(0, 1)$             $\triangleright$  initialize perturbations with random Gaussian noise
4:   for  $i$  in  $1, \dots, S$  do            $\triangleright$  we use  $S = 5$  iterations to search for the best value of  $\lambda$ 
5:     for  $t$  in  $1, \dots, T$  do            $\triangleright T$  is the number of steps
6:        $\Delta \leftarrow \nabla_{\tilde{\mathbf{x}}} [\mathcal{L}(f(\tilde{\mathbf{x}}), y) - \lambda \max(0, d(\tilde{\mathbf{x}}, \mathbf{x}) - \epsilon)]$             $\triangleright$  take the gradient of (5)
7:        $\hat{\Delta} = \Delta / \|\Delta\|_2$             $\triangleright$  normalize the gradient
8:        $\eta = \epsilon * (0.1)^{t/T}$             $\triangleright$  the step size  $\eta$  decays exponentially
9:        $m \leftarrow d(\tilde{\mathbf{x}}, \tilde{\mathbf{x}} + h\hat{\Delta})/h$             $\triangleright m \approx$  derivative of  $d(\tilde{\mathbf{x}}, \cdot)$  in the direction of  $\hat{\Delta}$ ;  $h = 0.1$ 
10:       $\tilde{\mathbf{x}} \leftarrow \tilde{\mathbf{x}} + (\eta/m)\hat{\Delta}$             $\triangleright$  take a step of size  $\eta$  in LPIPS distance
11:    end for
12:    if  $d(\tilde{\mathbf{x}}, \mathbf{x}) > \epsilon$  then
13:       $\lambda \leftarrow 10\lambda$             $\triangleright$  increase  $\lambda$  if the attack goes outside the bound
14:    end if
15:  end for
16:   $\tilde{\mathbf{x}} \leftarrow \text{PROJECT}(d, \tilde{\mathbf{x}}, \mathbf{x}, \epsilon)$ 
17:  return  $\tilde{\mathbf{x}}$ 
18: end procedure
```

Problem with penalty methods

Method	cross-entropy loss		margin loss	
	Viol. (%) ↓	Att. Succ. (%) ↑	Viol. (%) ↓	Att. Succ. (%) ↑
Fast-LPA	73.8	3.54	41.6	56.8
LPA	0.00	80.5	0.00	97.0
PPGD	5.44	25.5	0.00	38.5
PWCF (ours)	0.62	93.6	0.00	100

LPA, Fast-LPA: penalty methods

PPGD: Projected gradient descent

Penalty methods tend to encounter

large constraint violation (i.e., infeasible solution, known in optimization theory) or **suboptimal solution**

$$\begin{aligned} & \max_{\mathbf{x}'} \ell(\mathbf{y}, f_{\theta}(\mathbf{x}')) \\ \text{s.t. } & d(\mathbf{x}, \mathbf{x}') \leq \varepsilon, \quad \mathbf{x}' \in [0, 1]^n \\ & d(\mathbf{x}, \mathbf{x}') \doteq \|\phi(\mathbf{x}) - \phi(\mathbf{x}')\|_2 \end{aligned}$$

where $\phi(\mathbf{x}) \doteq [\hat{g}_1(\mathbf{x}), \dots, \hat{g}_L(\mathbf{x})]$

PWCF, an optimizer with a principled stopping criterion on **stationarity & feasibility**



Key algorithm

<http://www.timmitchell.com/software/GRANSO/>

Nonconvex, nonsmooth, constrained

$$\min_{\mathbf{x} \in \mathbb{R}^n} f(\mathbf{x}), \quad \text{s.t. } c_i(\mathbf{x}) \leq 0, \quad \forall i \in \mathcal{I}; \quad c_i(\mathbf{x}) = 0, \quad \forall i \in \mathcal{E}.$$

Penalty sequential quadratic programming (P-SQP)

$$\begin{aligned} \min_{d \in \mathbb{R}^n, s \in \mathbb{R}^p} \quad & \mu(f(x_k) + \nabla f(x_k)^T d) + e^T s + \frac{1}{2} d^T H_k d \\ \text{s.t.} \quad & c(x_k) + \nabla c(x_k)^T d \leq s, \quad s \geq 0, \end{aligned}$$

Ref: **Curtis, Frank E., Tim Mitchell, and Michael L. Overton.** "A BFGS-SQP method for nonsmooth, nonconvex, constrained optimization and its evaluation using relative minimization profiles." *Optimization Methods and Software* 32.1 (2017): 148-181.

Algorithm highlights

Steering strategy for the penalty parameter

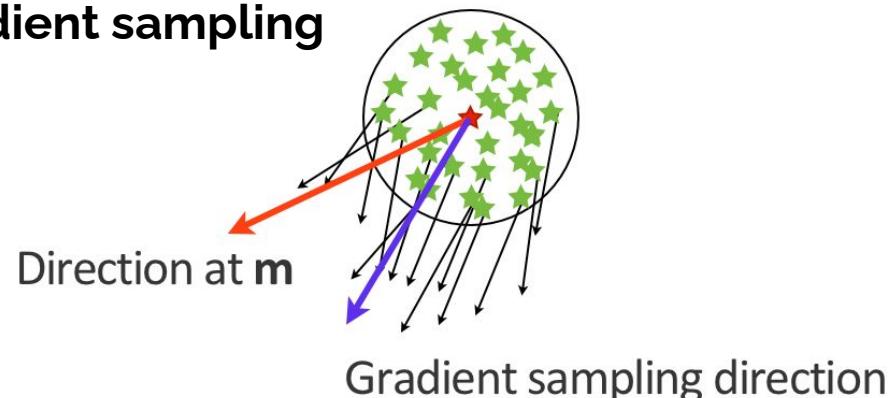
If feasibility improvement is insufficient : $l_\delta(d_k; x_k) < c_v v(x_k)$

Stationarity based on (approximate) gradient sampling

$$G_k := [\nabla f(x^k) \quad \nabla f(x^{k,1}) \quad \cdots \quad \nabla f(x^{k,m})]$$

$$\min_{\lambda \in \mathbb{R}^{m+1}} \frac{1}{2} \|G_k \lambda\|_2^2$$

$$\text{s.t. } \mathbf{1}^T \lambda = 1, \quad \lambda \geq 0$$



Key take-away



- Principled stopping criterion and line search, to obtain a **solution with certificate** (stationarity & feasibility check)
- Quasi-newton style method for fast convergence, i.e.,
reasonable speed and high-precision solution

Ref Curtis, Frank E., Tim Mitchell, and Michael L. Overton. "A BFGS-SQP method for nonsmooth, nonconvex, constrained optimization and its evaluation using relative minimization profiles." Optimization Methods and Software 32.1 (2017): 148-181.



Limitations of GRANSO

```
% Gradient of inner product with respect to A  
f_grad      = imag((conj(Bty)*Cx.')/(y'*x));  
f_grad      = f_grad(:);  
  
% Gradient of inner product with respect to A  
ci_grad     = real((conj(Bty)*Cx.')/(y'*x));  
ci_grad     = ci_grad(:);
```

analytical gradients required

```
p           = size(B,2);  
m           = size(C,1);  
X           = reshape(x,p,m);
```

vector variables only

Lack of Auto-Differentiation

Lack of GPU Support

No native support of tensor variables

⇒ impossible to do deep learning with GRANSO

GRANSO meets PyTorch



NCVX PyGRANSO
Documentation

Search the docs ...

Introduction

Installation

Settings

Examples



NCVX Package

NCVX: A General-Purpose Optimization Solver for Constrained Machine and Deep Learning

Buyun Liang, Tim Mitchell, Ju Sun



Home

$$\min_{\mathbf{x} \in \mathbb{R}^n} f(\mathbf{x}), \text{ s.t. } c_i(\mathbf{x}) \leq 0, \forall i \in \mathcal{I}; c_i(\mathbf{x}) = 0, \forall i \in \mathcal{E}$$

First general-purpose solver for constrained DL problems

Example 1: Support Vector Machine (SVM)

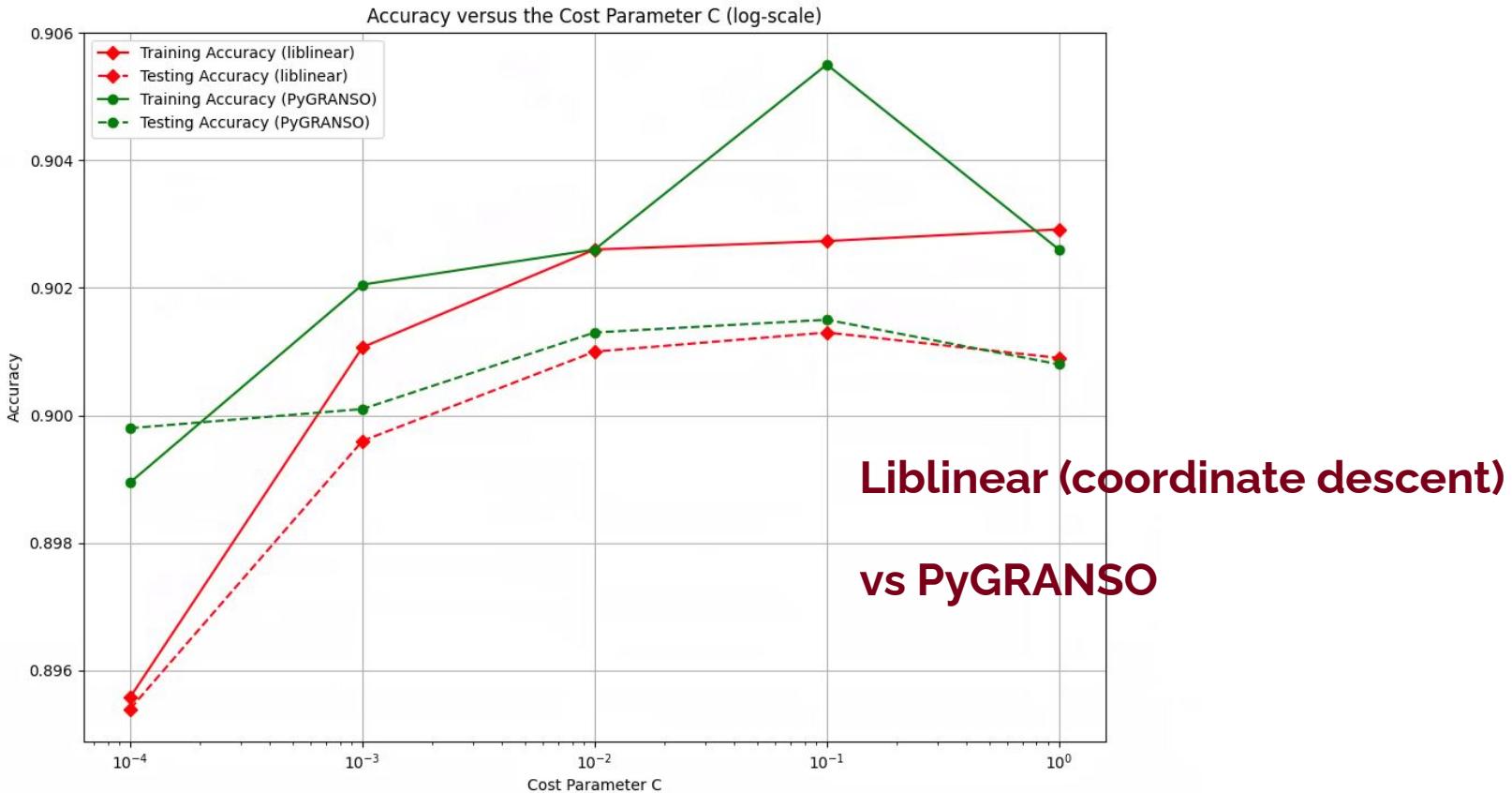
Soft-margin SVM

$$\min_{\mathbf{w}, b, \zeta} \frac{1}{2} \|\mathbf{w}\|^2 + C \sum_{i=1}^n \zeta_i$$

$$\text{s.t. } y_i (\mathbf{w}^\top \mathbf{x}_i + b) \geq 1 - \zeta_i, \quad \zeta_i \geq 0 \quad \forall i = 1, \dots, n$$

```
def comb_fn(X_struct):
    # obtain optimization variables
    w = X_struct.w
    b = X_struct.b
    zeta = X_struct.zeta
    # objective function
    f = 0.5*w.T@w + C*torch.sum(zeta)
    # inequality constraints
    ci = pygransoStruct()
    ci.c1 = 1 - zeta - y*(x@w+b)
    ci.c2 = -zeta
    # equality constraint
    ce = None
    return [f,ci,ce]
# specify optimization variables
var_in = {"w": [d,1], "b": [1,1], "zeta": [n,1]}
# pygranso main algorithm
soln = pygranso(var_in,comb_fn)
```

Binary classification (odd vs even digits) on MNIST dataset



Example 2: Robustness—min formulation

$$\min_{\mathbf{x}'} d(\mathbf{x}, \mathbf{x}')$$

$$\text{s. t. } \max_{\ell \neq c} f_{\boldsymbol{\theta}}^{\ell}(\mathbf{x}') \geq f_{\boldsymbol{\theta}}^c(\mathbf{x}')$$

$$\mathbf{x}' \in [0, 1]^n$$

```
def comb_fn(X_struct):
    # obtain optimization variables
    x_prime = X_struct.x_prime
    # objective function
    f = d(x,x_prime)
    # inequality constraints
    ci = pygransoStruct()
    f_theta_all = f_theta(x_prime)
    fy = f_theta_all[:,y] # true class output
    # output except true class
    fi = torch.hstack((f_theta_all[:, :y], f_theta_all[:, y+1:]))
    ci.c1 = fy - torch.max(fi)
    ci.c2 = -x_prime
    ci.c3 = x_prime-1
    # equality constraint
    ce = None
    return [f,ci,ce]
# specify optimization variable (tensor)
var_in = {"x_prime": list(x.shape)}
# pygranso main algorithm
soln = pygranso(var_in,comb_fn)
```

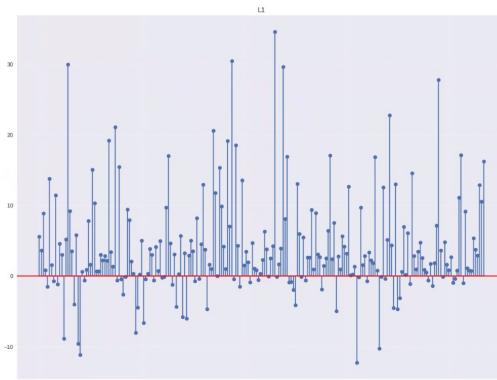
CIFAR10 dataset

Compared with FAB [iterative constraint linearization + projected gradient]

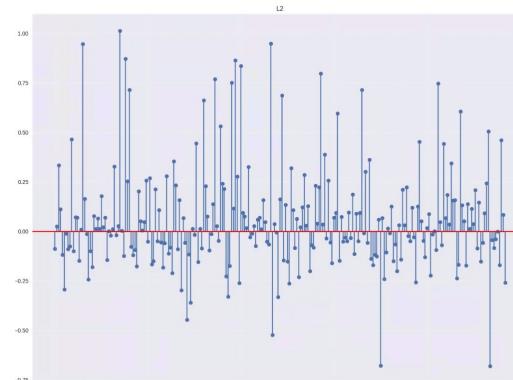
<https://github.com/fra31/auto-attack>

$$\begin{aligned} & \min_{\mathbf{x}'} d(\mathbf{x}, \mathbf{x}') \\ \text{s. t. } & \max_{\ell \neq c} f_{\boldsymbol{\theta}}^{\ell}(\mathbf{x}') \geq f_{\boldsymbol{\theta}}^c(\mathbf{x}') \\ & \mathbf{x}' \in [0, 1]^n \end{aligned}$$

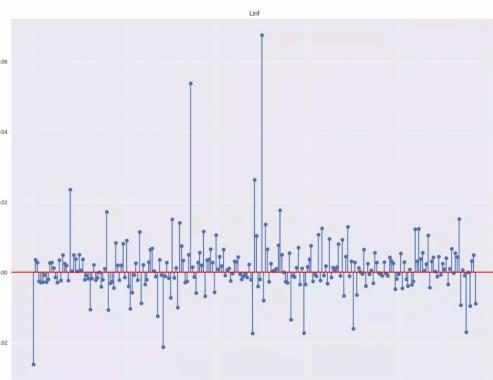
X-axis: image index; Y-axis: PyGRANSO radius - FAB radius



L1 attack



L2 attack



Linf attack

Many
others

<https://ncvx.org/>

NCVX PyGRANSO Documentation

← ⌂ ⌂ ⌂

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Rosenbrock

Eigenvalue Optimization

Dictionary Learning

Nonlinear Feasibility Problem

Sphere Manifold

Trace Optimization

Robust PCA

Generalized LASSO

Logistic Regression

LeNet5

Perceptual Attack

Orthogonal RNN

Highlights

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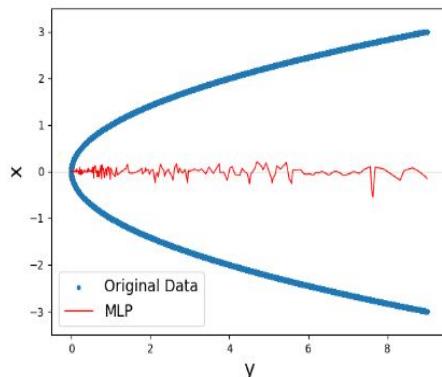
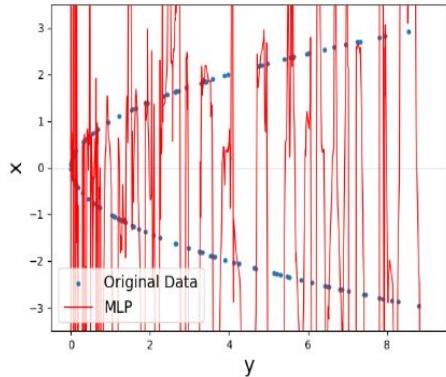
NCVX Package

NCVX (**N**on**C**onVe**X**) is a user-friendly and scalable python software package targeting general nonsmooth NCVX problems with nonsmooth constraints. **NCVX** is being developed by **GLOVEX** at the Department of Computer Science & Engineering, University of Minnesota, Twin Cities.

The initial release of **NCVX** contains the solver **PyGRANSO**, a PyTorch-enabled port of **GRANSO** incorporating auto-differentiation, GPU acceleration, tensor input, and support for new QP solvers. As a highlight, **PyGRANSO** can solve general constrained deep learning problems, the first of its kind.



Data-driven methods for SIPs



Story I: More could be less

Single-instance methods for SIPs

Deep image prior (DIP)

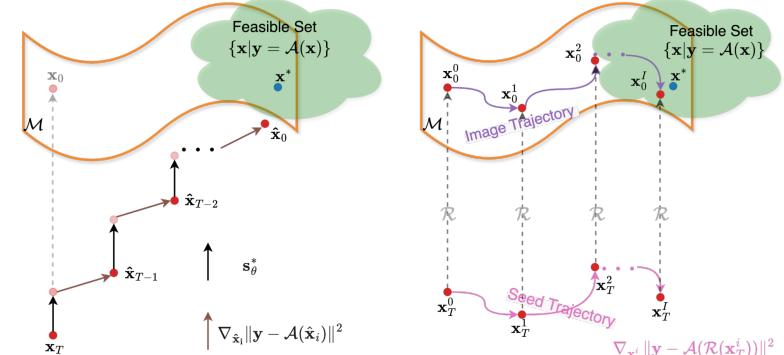
$$\mathbf{x} \approx G_{\theta}(\mathbf{z}) \quad G_{\theta} \text{ (and } \mathbf{z} \text{) trainable}$$

$$\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{R(\mathbf{x})}_{\text{regularizer}}$$

$$\min_{\theta} \ell(\mathbf{y}, f \circ G_{\theta}(\mathbf{z})) + \lambda R \circ G_{\theta}(\mathbf{z})$$

Story III: Benefit from DL with a single data point

Ulyanov et al. Deep image prior. IJCV20. <https://arxiv.org/abs/1711.10925>



Story II: Don't be too Bayesian

$$\underbrace{\mathbf{y}}_{\text{blurry and noisy image}} = \underbrace{\mathbf{k}}_{\text{blur kernel}} * \underbrace{\mathbf{x}}_{\text{clean image}} + \underbrace{\mathbf{n}}_{\text{noise}}$$

Mostly due to optical deficiencies (e.g., defocus) and motions

Given \mathbf{y} ,
recover \mathbf{x} (and/or \mathbf{k})

Also **Blind Deconvolution**



A (the?) tool for DL with nontrivial constraints

GRAVSO + PyTorch



NCVX PyGRANSO
Documentation

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NCVX Package

NCVX: A General-Purpose Optimization Solver for Constrained Machine and Deep Learning

Buyun Liang, Tim Mitchell, Ju Sun



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$$\min_{\mathbf{x} \in \mathbb{R}^n} f(\mathbf{x}), \text{ s.t. } c_i(\mathbf{x}) \leq 0, \forall i \in \mathcal{I}; c_i(\mathbf{x}) = 0, \forall i \in \mathcal{E}$$

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