

From Fully Connected to Convolutional Neural Networks

Ju Sun

Computer Science & Engineering
University of Minnesota, Twin Cities

November 28, 2020

Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

Thanks to the cats

Architectures for classification

Practical tips

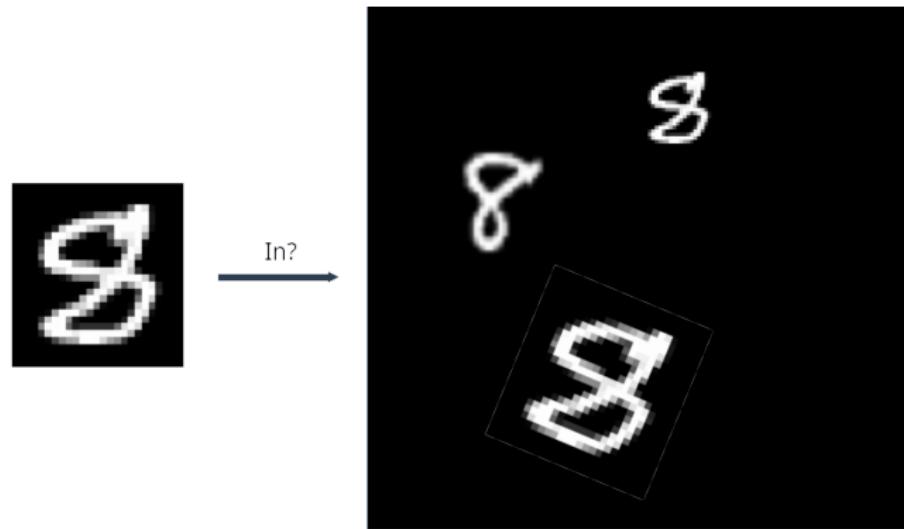
Suggested reading

How to find a pattern in images?



- Each time **inner product** of the original (red) and overlapped (green) patches (i.e., matrices) are taken
- The output matrix is the **correlation**
- Position(s) with the **largest magnitude** is candidate match—**detection**
- Care about the **largest magnitude only** if only interested in Yes/No—**max pooling**

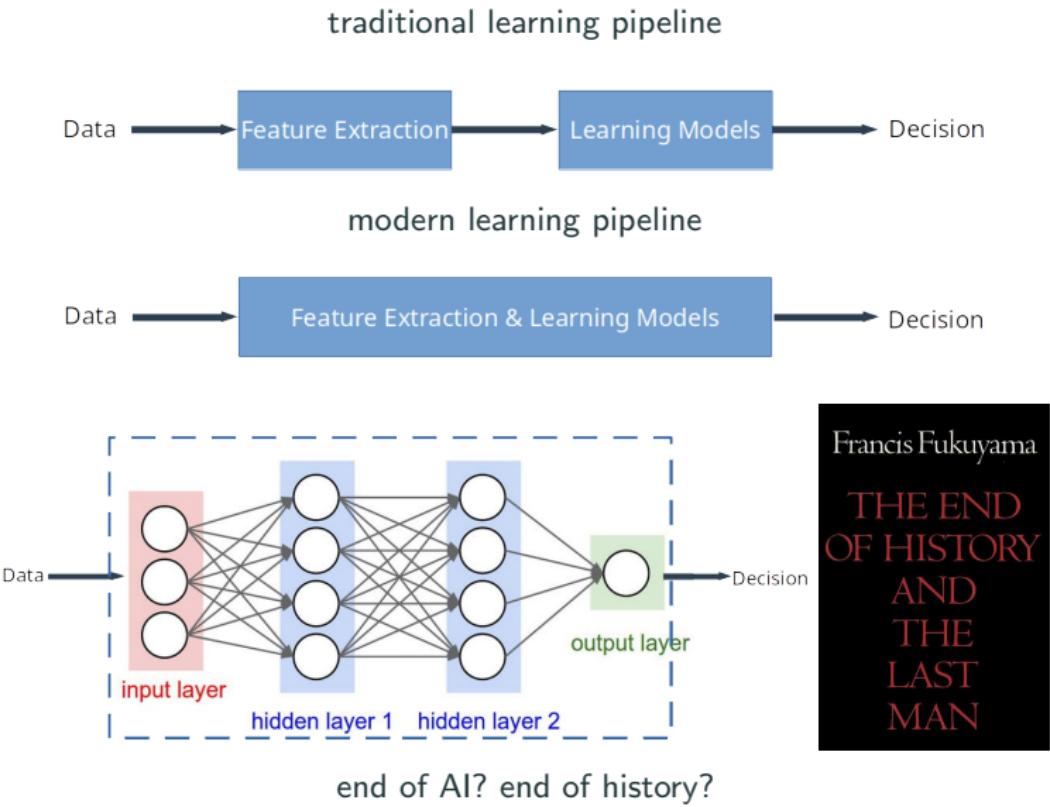
Problem with template matching



It handles the uncertainty about location (i.e., translation), but not

- not rotation or scaling
- local deformation

Feature-based approach!



Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

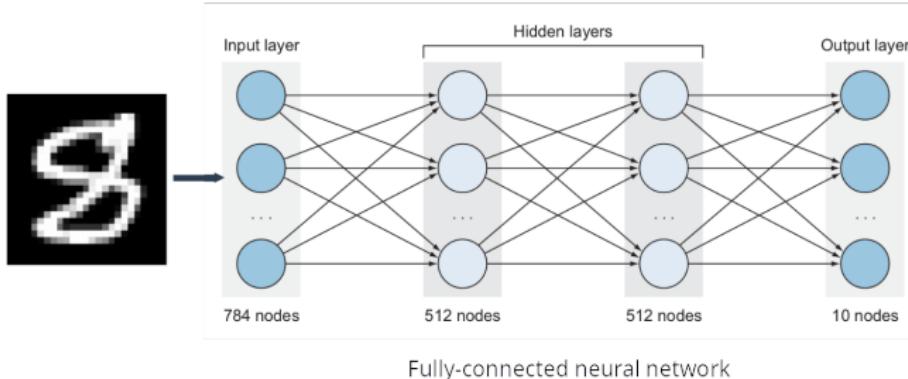
Thanks to the cats

Architectures for classification

Practical tips

Suggested reading

Locality and ordering



Can we learn spatial features easily?

- FCNN treats the input as a vector
- FCNN even insensitive to any universal permutation of the coordinates to all inputs

Invariance



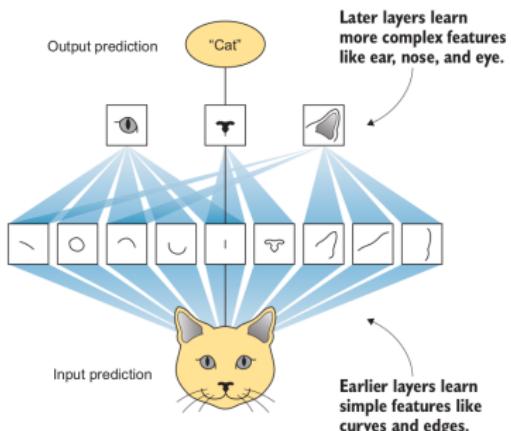
where the pattern is found shouldn't matter much

- In FCNN, all possible translated copies should be available for training
- Similarly for rotation, scaling, local deformation

Ideal neural networks for spatial data

Problems with FCNNs: high **complexity** and lack of **locality** and **invariance**

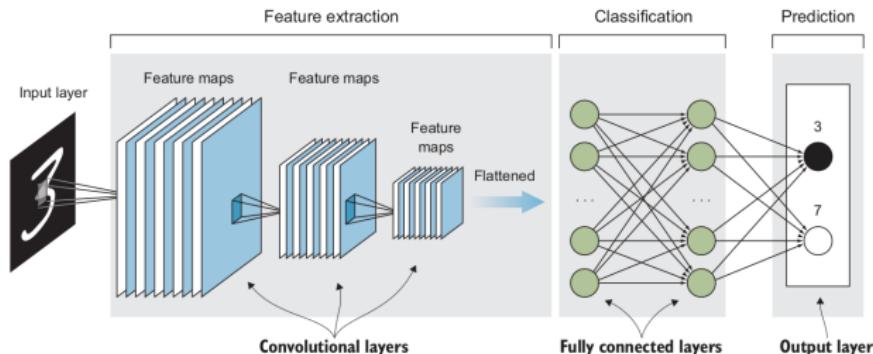
Goal: build these into the architecture directly



- Extracted features invariant to translation, rotation, local deformation
- Low complexity

(Credit: [\[Elgendi, 2020\]](#))

A quick preview of convolutional neural networks (CNN)



(Credit: [Elgendi, 2020])

- Input layer
- **Convolutional layers** for feature extraction
- FC layers for classification
- Output layer for prediction

Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

Thanks to the cats

Architectures for classification

Practical tips

Suggested reading

Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

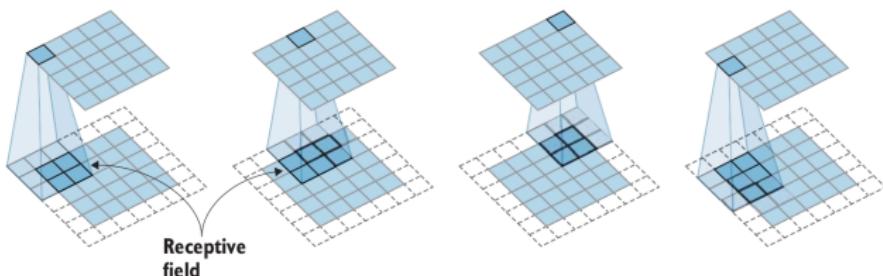
Thanks to the cats

Architectures for classification

Practical tips

Suggested reading

Connection to fully-connected NN



(Credit: [Elgendi, 2020])

input: a whole matrix output: neuron outputs organized into a matrix

- **local/sparse connectivity**: each neuron connects only to its receptive field
- **weight sharing**: all neurons share the same weight pattern

Do we reduce the complexity?

Suppose C_1 input channels and C_2 output channels of size $H \times W$

- # parameters if implementing fully connected layer? $O(C_1C_2H^2W^2)$
- # parameters if implementing convolution of $h \times w$? $O(C_1C_2hw)$

h, w often small constants, e.g., 3 in practice

Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

Thanks to the cats

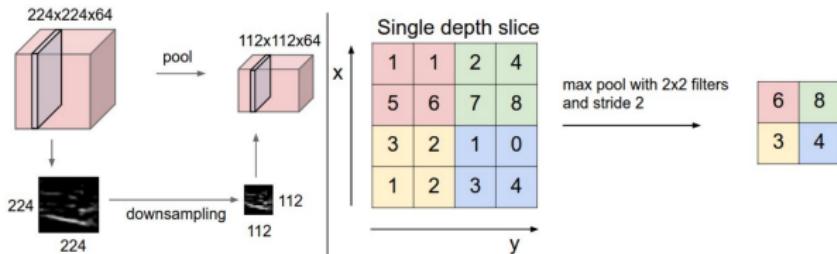
Architectures for classification

Practical tips

Suggested reading

Pooling

Convolution helps to achieve locality, and (much) reduced complexity, what about invariance?



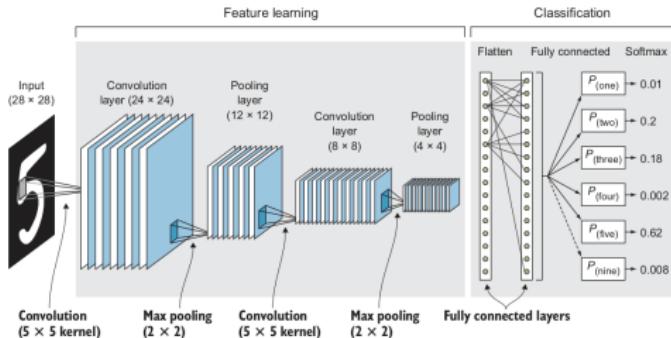
Pooling layer downsamples the volume spatially, independently in each depth slice of the input volume. Left: In this example, the input volume of size $[224 \times 224 \times 64]$ is pooled with filter size 2, stride 2 into output volume of size $[112 \times 112 \times 64]$. Notice that the volume depth is preserved. Right: The most common downsampling operation is max, giving rise to max pooling, here shown with a stride of 2. That is, each max is taken over 4 numbers (little 2×2 square).

(Credit: Stanford CS231N)

- max pooling (i.e., max within the receptive field)
- average pooling (i.e., weighted average within the receptive field)
- strides and receptive field size (often 2/2 or 2/3)

Why pooling?

reduce complexity (with stride ≥ 2)



(Credit: [Elgendi, 2020])

- deeper layer: more filters \implies subsampling avoids explosion in computation
- subsampling keep important features



Figure 3.25 Pooling layers reduce image resolution and keep the image's important features.

(Credit: [Elgendi, 2020])

Why pooling?

(approximate) local translation/distortion invariance

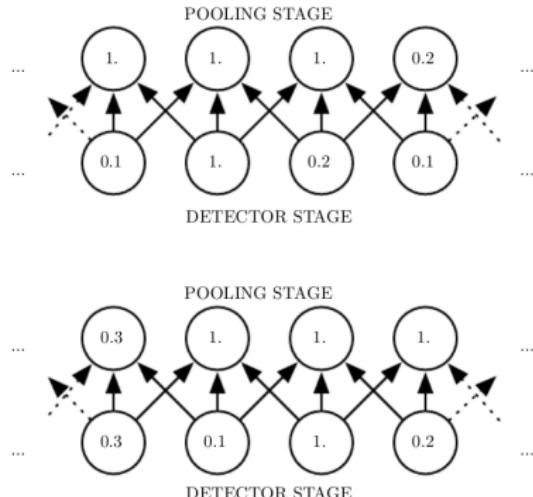
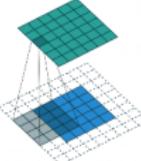
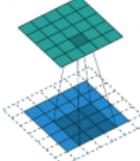
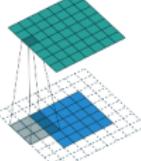
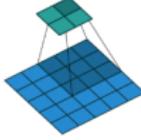
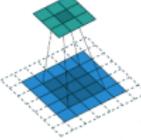
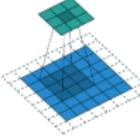


Figure 9.8: Max pooling introduces invariance. (*Top*) A view of the middle of the output of a convolutional layer. The bottom row shows outputs of the nonlinearity. The top row shows the outputs of max pooling, with a stride of one pixel between pooling regions and a pooling region width of three pixels. (*Bottom*) A view of the same network, after the input has been shifted to the right by one pixel. Every value in the bottom row has changed, but only half of the values in the top row have changed, because the max pooling units are only sensitive to the maximum value in the neighborhood, not its exact location.

(Credit: [Goodfellow et al., 2017])

Combine convolution and pooling—convolution with strides

idea: convolution with stride $\geq 2 \approx$ convolution + subsampling

			
No padding, no strides	Arbitrary padding, no strides	Half padding, no strides	Full padding, no strides
			
No padding, strides	Padding, strides	Padding, strides (odd)	

https://github.com/vdumoulin/conv_arithmetic

So use all convolution (with large strides) layers only, no pooling
[Springenberg et al., 2014]

Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

Thanks to the cats

Architectures for classification

Practical tips

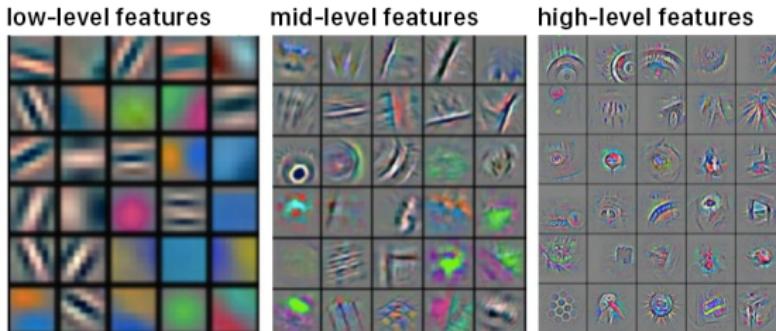
Suggested reading

Why not single layer?

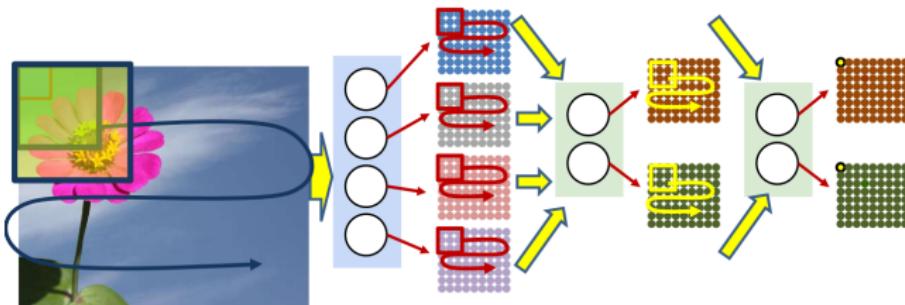


using a one-layer CNN ...

- **efficiency**: one kernel for each variation of 8? for each variation of every object?
- **better**: share kernels across digits or all object categories, but low-level features (edges, corners, etc) likely shareable \implies **form hierarchy**

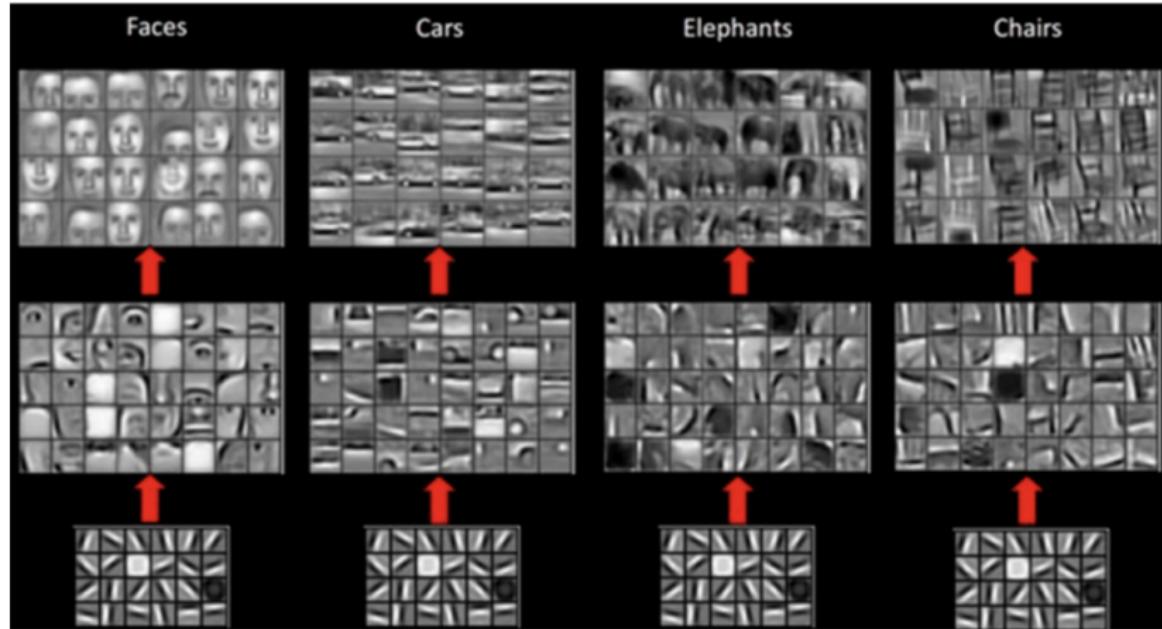


Hierarchical scan



- Later neurons have **increasingly large** effective receptive fields on the input image, i.e., scanning using **composition** of the filters
- composition (with pooling layers or strides) allows local translation and distortion

Examples of learned features



Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

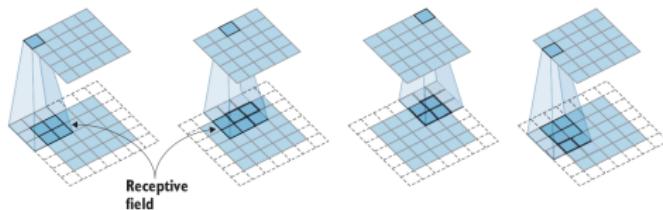
Thanks to the cats

Architectures for classification

Practical tips

Suggested reading

How to compute convolution?



(Credit: [Elgendi, 2020])

- convolution layer is **locally connected, weight-sharing** fully connected layer
- if we vectorize both input and output, the operation can be represented as a **matrix multiplication**

$$\begin{pmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{pmatrix} * \begin{pmatrix} k_1 & k_2 \\ k_3 & k_4 \end{pmatrix} \iff \begin{pmatrix} k_1 & k_2 & 0 & k_3 & k_4 & 0 & 0 & 0 & 0 \\ 0 & k_1 & k_2 & 0 & k_3 & k_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & k_1 & k_2 & 0 & k_3 & k_4 & 0 \\ 0 & 0 & 0 & 0 & k_1 & k_2 & 0 & k_3 & k_4 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \end{pmatrix}$$

so we don't worry about forward and backward pass

More on computation

To compute the convolution

- use (sparse) matrix-vector multiplication (early versions of cuDNN)
- use fast Fourier transform (introduced in later versions of cuDNN)

$$\mathcal{F}(w * x) = \mathcal{F}(w) \odot \mathcal{F}(x)$$

To compute the max-pooling

- forward: simple
- backward? what's $\nabla_x \max(x_1, \dots, x_n)$?

Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

Thanks to the cats

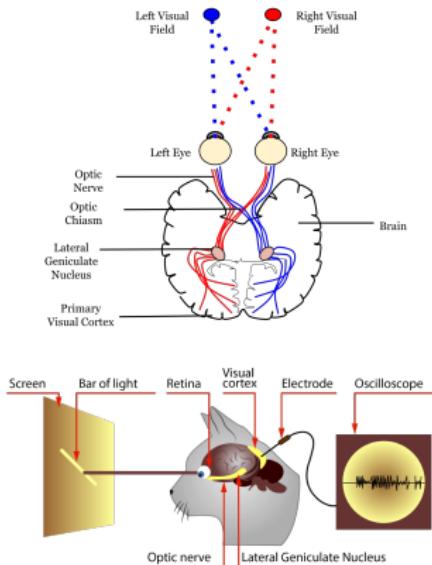
Architectures for classification

Practical tips

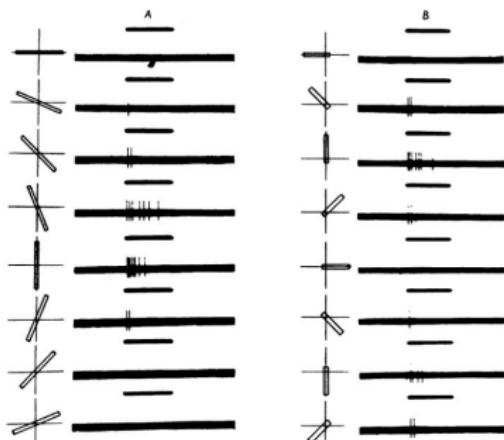
Suggested reading

A brief history of CNN

Hubel and Wiesel 1959 [Hubel and Wiesel, 1959]



focused on the primary visual cortex (V1)



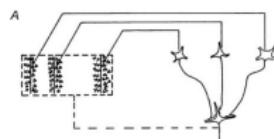
main discovery: directional selectivity of the neurons inside V1, and **local responsiveness** per cell

A brief history of CNN

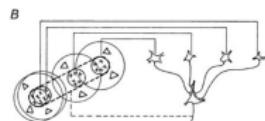
Hubel and Wiesel 1962 [Hubel and Wiesel, 1962]

Two types of cells: *simple S-cells* and *complex C-cells*

- correspond to two levels of processing
- C-cells robust to distortion, but S-cells not



Composition of complex receptive fields from simple cells. The C-cell responds to the largest output from a bank of S-cells to achieve oriented response that is robust to distortion



Transform from circular retinal receptive fields to elongated fields for simple cells. The simple cells are susceptible to fuzziness and noise

- Complex C-cells build from similarly oriented simple cells
 - They “fine-tune” the response of the simple cell
- Show complex buildup – building *more complex patterns* by composing early neural responses
 - Successive transformation through Simple-Complex combination layers

S-cells: conv kernels C-cells: max pooling

A brief history of CNN

Fukushima 1980: Neocognitron [Fukushima, 1980]—unsupervised

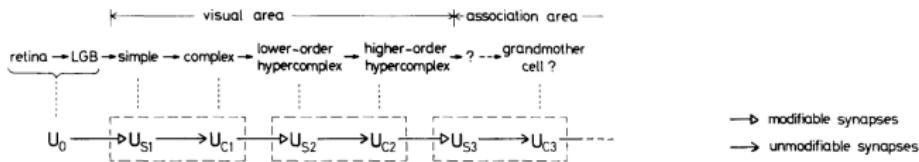
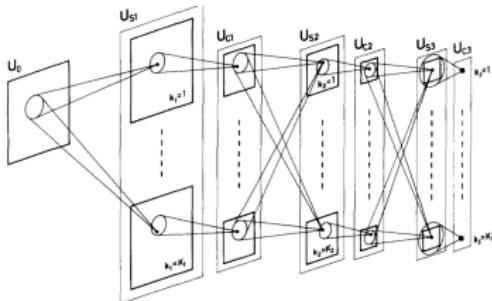
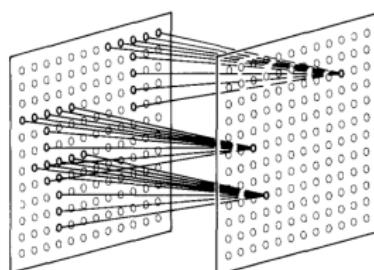


Fig. 1. Correspondence between the hierarchy model by Hubel and Wiesel, and the neural network of the neocognitron

- multi-layers of S-C cells compositions
- only S-cells are learnable



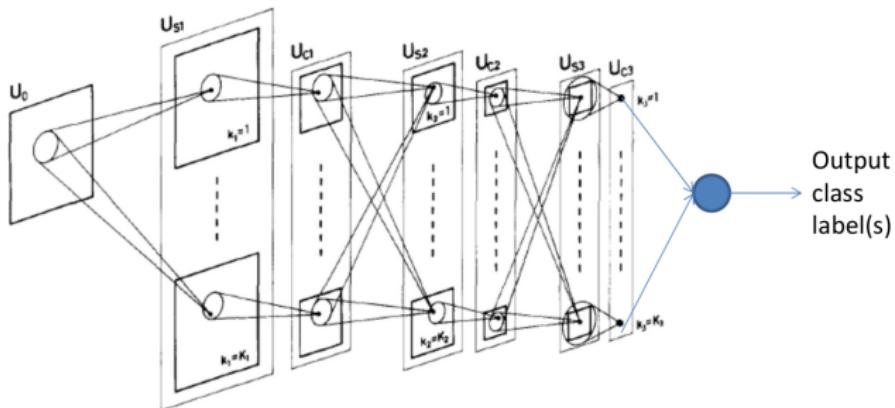
cell planes get smaller but number of planes increase going deeper



S cells have ReLU-like activation, C cells have ReLU+Max like activation

A brief history of CNN

Lecun 1989: supervision added [LeCun et al., 1989, Lecun et al., 1998]



back-propagation used for supervised training for digit recognition

Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

Thanks to the cats

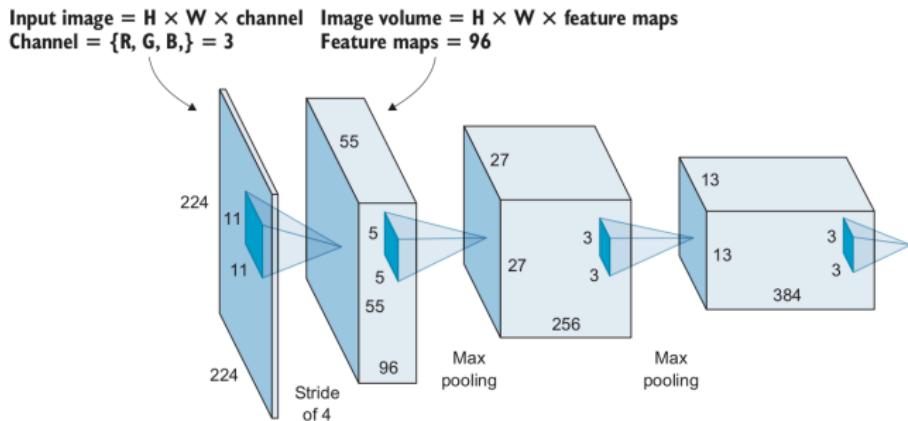
Architectures for classification

Practical tips

Suggested reading

Typical design patterns

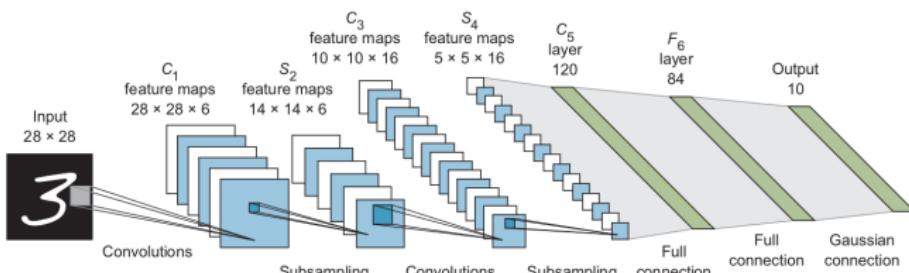
- feature extraction (CONV) + classification (fully connected)
- depth increases (more filters), dimension decreases (subsampling) when moving deeper



(Credit: [Elgendi, 2020])

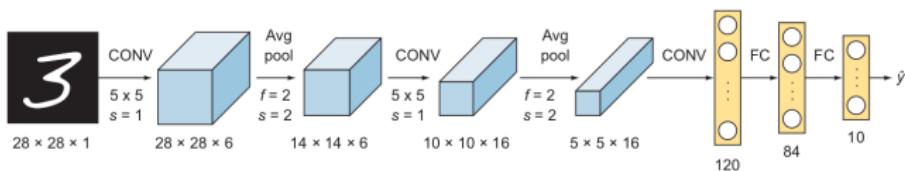
- one or two fully-connected layers for classification

LeNet-5 (1998)



(Credit: [Elgendi, 2020])

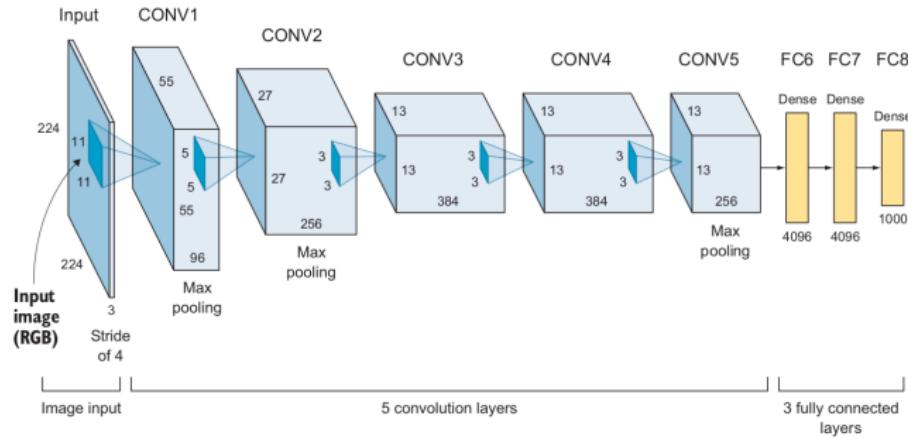
- \tanh used for activation
- 5×5 filters



(Credit: [Elgendi, 2020])

AlexNet (2012)

breakthrough on ImageNet competition in 2012 and impressed the computer vision community



(Credit: [Elgendi, 2020])

- ReLU used for activation
- large filters: 11×11 , 5×5 , 3×3 filters
- dropout used for regularization
- weight decay/regularization

VGG-net (2014)

VGG — Visual Geometry Group (Oxford U.)

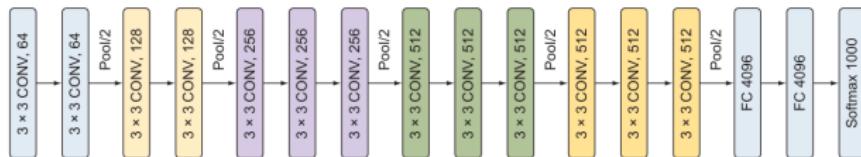


Figure 5.8 VGGNet-16 architecture

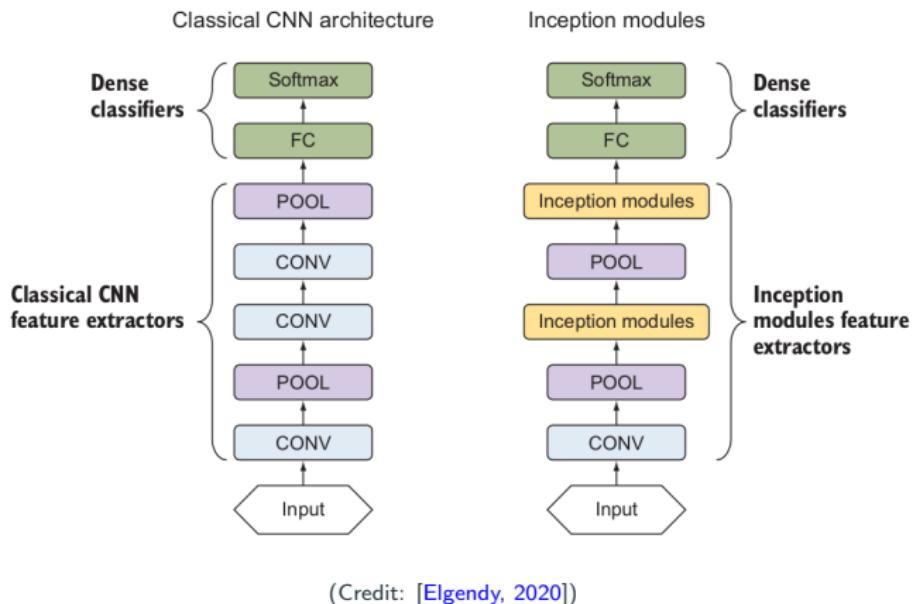
(Credit: [Elgendi, 2020])

- smaller filters (3×3) to make up for large ones in AlexNet. A nice property of convolution:

$$a * (b * c) = (a * b) * c$$

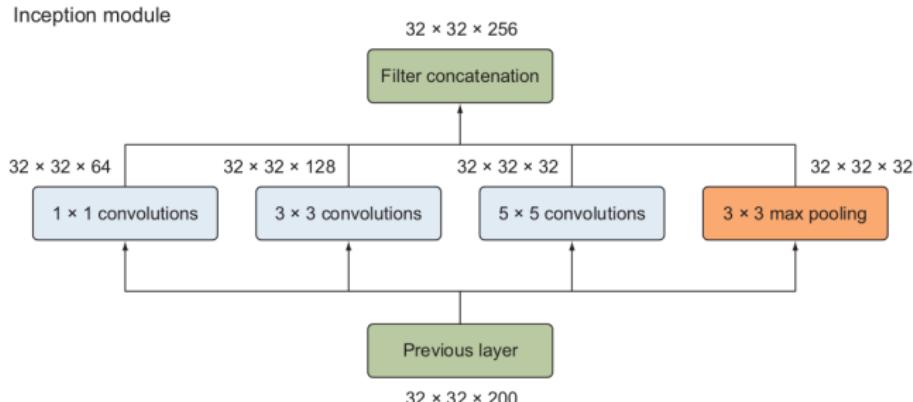
composition of filters covers larger receptive fields

Inception and GoogLeNet (2014)



pack things into **inception modules**

Inception module—basic version



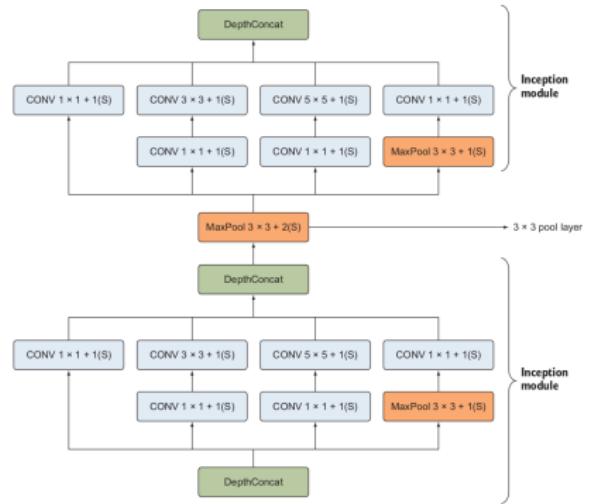
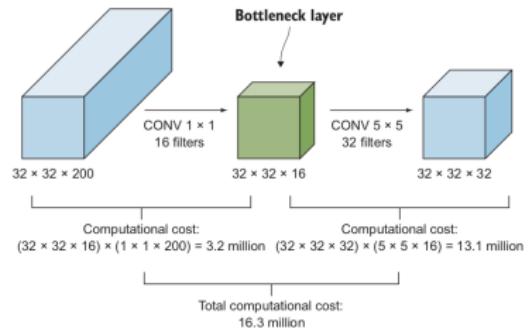
(Credit: [Elgendi, 2020])

idea: apply all filters together and (hopefully) the training process performs the suitable selection/combination itself

- filters can be short-circuited when the values are set to 0

Inception module with dimension reduction

1×1 convolution helps to reduce the #channels \implies saves computation



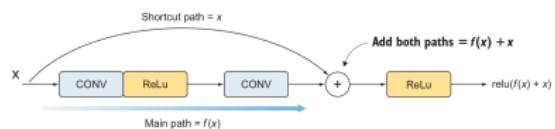
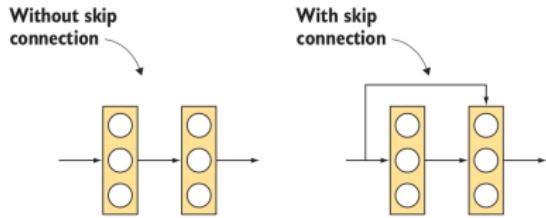
(Credit: [Elgendi, 2020])

(Credit: [Elgendi, 2020])

ResNet (2015)

going really deep...sees performance **degradation**

a solution:



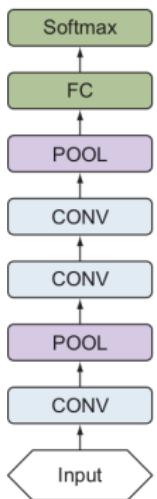
a residual block (Credit: [Elgendi, 2020])

(Credit: [Elgendi, 2020])

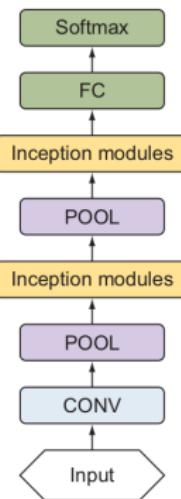
- skip connection
 - * allows short-circuit unnecessary layers—e.g., setting the kernels to zero—and thus avoids performance degradation when adding more layers
 - * mitigates gradient explosion or vanishing— $J_{f+I}(x) = J_f(x) + I$
- batch normalization

Comparison with previous models

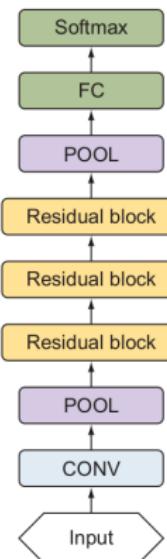
Classical CNN architecture



Inception modules



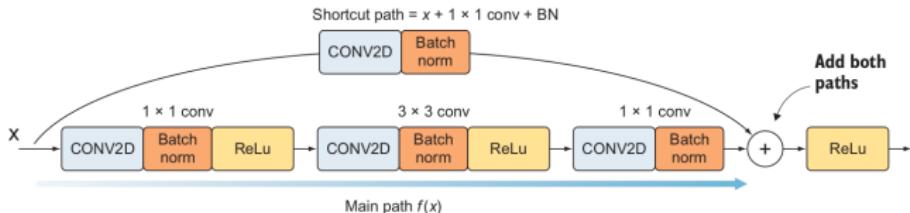
Residual blocks



(Credit: [Elgendi, 2020])

Inside a residual block

Bottleneck residual block with reduce shortcut



(Credit: [Elgendi, 2020])

- no pooling layers
- 1×1 conv before and after 3×3 conv to control #channels and hence computation
- batch normalization (BN) after each conv layer
- 1×1 conv and BN added to the skip connection also to match dim for summation

full details see: https://pytorch.org/hub/pytorch_vision_resnet/

DenseNet (2016)

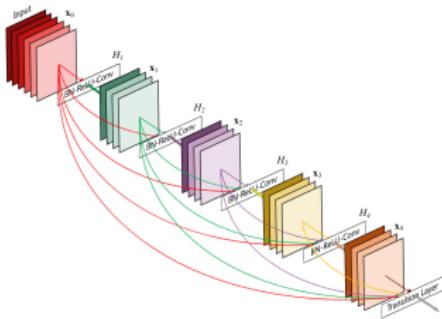


Figure 1: A 5-layer dense block with a growth rate of $k = 4$. Each layer takes all preceding feature-maps as input.

(Credit: [Huang et al., 2016])

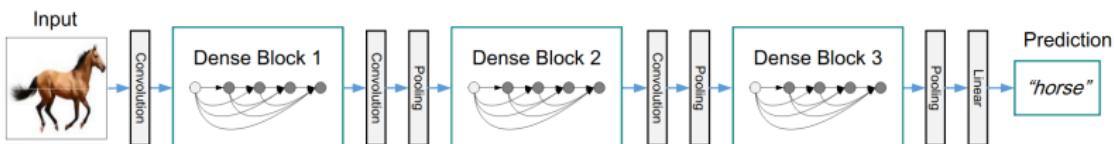


Figure 2: A deep DenseNet with three dense blocks. The layers between two adjacent blocks are referred to as transition layers and change feature-map sizes via convolution and pooling.

(Credit: [Huang et al., 2016])

transition layers adjust the sizes of the feature maps

Other models to watch

on accuracy:

- EfficientNet (2019) [Tan and Le, 2019]
<https://github.com/tensorflow/tpu/tree/master/models/official/efficientnet>
- ResNeXt <https://arxiv.org/abs/1611.05431>

on compact models:

- SqueezeNet <https://arxiv.org/abs/1602.07360>
- ShuffleNet <https://arxiv.org/abs/1807.11164>
- MobileNet <https://arxiv.org/abs/1801.04381>

Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

Thanks to the cats

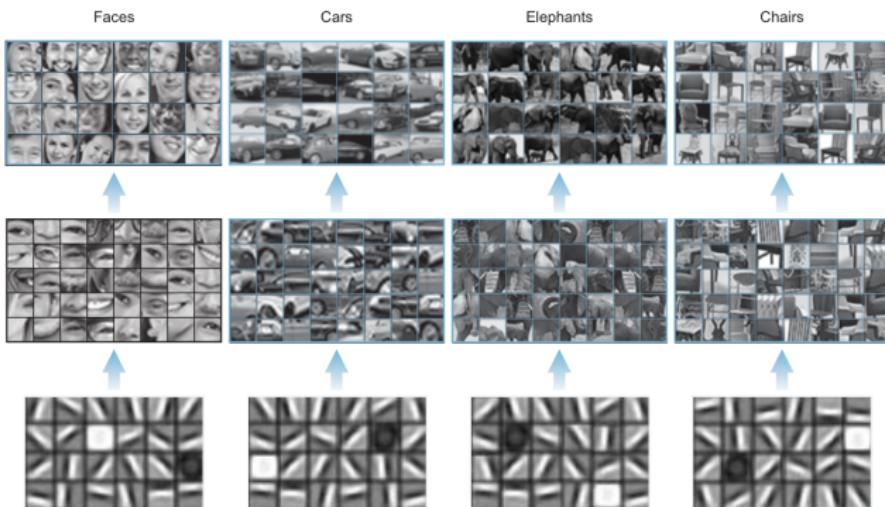
Architectures for classification

Practical tips

Suggested reading

Transfer learning

Recall: (we hope) CNNs learn increasingly complex and semantically meaningful features



(Credit: [[Elgendi, 2020](#)])

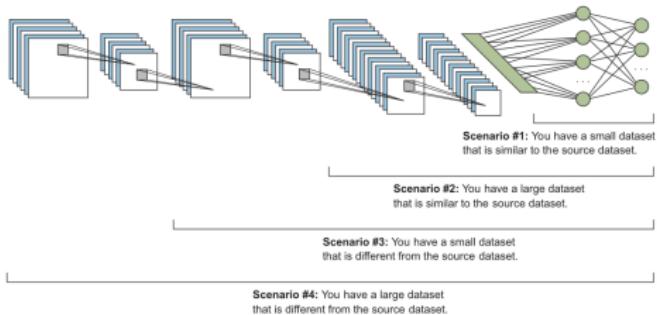
So: early layers trained on a large and diverse dataset, e.g., ImageNet, can be reused. This part is called a **pretrained** model

Transfer learning

source domain: training data for a pre-trained model

target domain: training data for the current model

Scenario	Size of the target data	Similarity of the original and new datasets	Approach
1	Small	Similar	Pretrained network as a feature extractor
2	Large	Similar	Fine-tune through the full network
3	Small	Very different	Fine-tune from activations earlier in the network
4	Large	Very different	Fine-tune through the entire network



underline indicates trainable part (Credit: [Elgendi, 2020])

Pytorch tutorial: https://pytorch.org/tutorials/beginner/transfer_learning_tutorial.html

Stanford notes: <https://cs231n.github.io/transfer-learning/>

Are CNNs only for images?

Recall why CNN? **complexity, locality/ordering, translation-invariance**

These are desired also when processing video, text sequence, times series data, speech data, etc Examples:

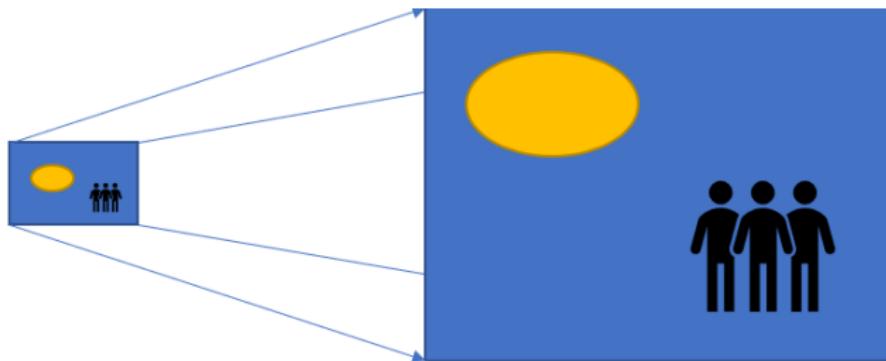
- WaveNet for text-to-speech system
<https://en.wikipedia.org/wiki/WaveNet>
- text classification <https://arxiv.org/abs/1408.5882>
- video analysis [Ji et al., 2013, Karpathy et al., 2014, Huang et al., 2018]
- time series analysis [Yu and Koltun, 2015, Borovskykh et al., 2017]

see also *An Empirical Evaluation of Generic Convolutional and Recurrent Networks for Sequence Modeling* [Bai et al., 2018]

Transposed convolution

convolution with strides: downsampling

transposed convolution: upsampling



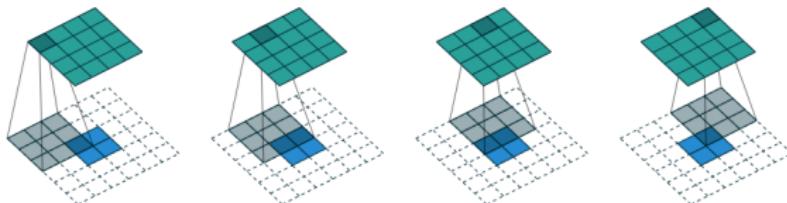
(Credit: <https://naokishibuya.medium.com/>)

often used for segmentation, generation, or other regression—outputs are structured objects such as images, videos, time series, speech, etc

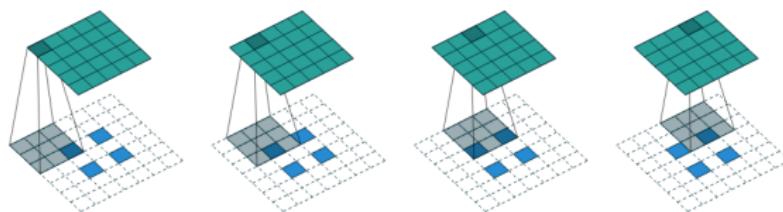
- traditional methods: e.g., nearest neighbor/bilinear/bicubic **interpolation**
- here: interpolation with a **learnable filter**

Transposed convolution

also called **fractionally strided convolutions** or deconvolution (misnomer): zero padding, zero interleaving (when forward stride > 1), and then convolution



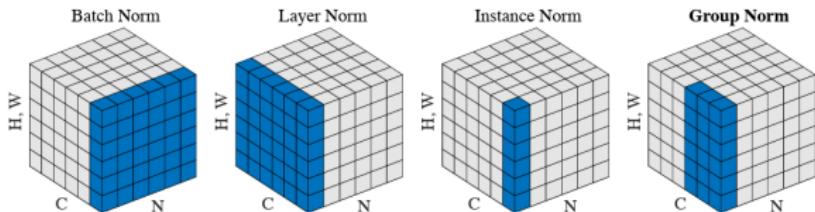
forward stride = 1



forward stride = 2

more details see https://github.com/vdumoulin/conv_arithmetic

Normalization



Normalization methods. Each subplot shows a feature map tensor, with N as the batch axis, C as the channel axis, and (H, W) as the spatial axes. The pixels in blue are normalized by the same mean and variance, computed by aggregating the values of these pixels.

Credit: [\[Wu and He, 2018\]](#)

normalization in different directions/groups of the data tensors

- N is the batch axis
- C is the channel axis
- WH is the per output dimension (1 for fully connected, but 2D for CNNs)

batch normalization is popular, but with **layer/group normalization**:

- small N (batch size) is possible
- simplicity: training/test normalizations are consistent

Outline

Find patterns in an image

Problems with fully connected networks

Components of CNNs

Convolutional layers

Pooling layers

Why multilayers?

Computation

Thanks to the cats

Architectures for classification

Practical tips

Suggested reading

Suggested reading

- Deep Learning for Vision Systems [Elgendi, 2020]
- Convolutional Networks for Images, Speech, and Time-Series [LeCun et al., 1995]
- A guide to convolution arithmetic for deep learning
<https://arxiv.org/abs/1603.07285>
- Gradient-based learning applied to document recognition [Lecun et al., 1998]
- <https://cs231n.github.io/transfer-learning/>

References i

- [Bai et al., 2018] Bai, S., Kolter, J. Z., and Koltun, V. (2018). **An empirical evaluation of generic convolutional and recurrent networks for sequence modeling.** *arXiv:1803.01271*.
- [Borovykh et al., 2017] Borovykh, A., Bohte, S., and Oosterlee, C. W. (2017). **Conditional time series forecasting with convolutional neural networks.** *arXiv:1703.04691*.
- [Elgendi, 2020] Elgendi, M. (2020). **Deep Learning for Vision Systems.** MANNING PUBN.
- [Fukushima, 1980] Fukushima, K. (1980). **Neocognitron: A self-organizing neural network model for a mechanism of pattern recognition unaffected by shift in position.** *Biological Cybernetics*, 36(4):193–202.
- [Goodfellow et al., 2017] Goodfellow, I., Bengio, Y., and Courville, A. (2017). **Deep Learning.** The MIT Press.
- [Huang et al., 2016] Huang, G., Liu, Z., van der Maaten, L., and Weinberger, K. Q. (2016). **Densely connected convolutional networks.** *arXiv:1608.06993*.

References ii

- [Huang et al., 2018] Huang, J., Zhou, W., Zhang, Q., Li, H., and Li, W. (2018). **Video-based sign language recognition without temporal segmentation.** *arXiv:1801.10111*.
- [Hubel and Wiesel, 1959] Hubel, D. H. and Wiesel, T. N. (1959). **Receptive fields of single neurones in the cat's striate cortex.** *The Journal of Physiology*, 148(3):574–591.
- [Hubel and Wiesel, 1962] Hubel, D. H. and Wiesel, T. N. (1962). **Receptive fields, binocular interaction and functional architecture in the cat's visual cortex.** *The Journal of Physiology*, 160(1):106–154.
- [Ji et al., 2013] Ji, S., Xu, W., Yang, M., and Yu, K. (2013). **3d convolutional neural networks for human action recognition.** *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 35(1):221–231.
- [Karpathy et al., 2014] Karpathy, A., Toderici, G., Shetty, S., Leung, T., Sukthankar, R., and Fei-Fei, L. (2014). **Large-scale video classification with convolutional neural networks.** In *2014 IEEE Conference on Computer Vision and Pattern Recognition*. IEEE.

References iii

- [LeCun et al., 1995] LeCun, Y., Bengio, Y., et al. (1995). **Convolutional networks for images, speech, and time series.** *The handbook of brain theory and neural networks*, 3361(10):1995.
- [LeCun et al., 1989] LeCun, Y., Boser, B., Denker, J. S., Henderson, D., Howard, R. E., Hubbard, W., and Jackel, L. D. (1989). **Backpropagation applied to handwritten zip code recognition.** *Neural Computation*, 1(4):541–551.
- [Lecun et al., 1998] Lecun, Y., Bottou, L., Bengio, Y., and Haffner, P. (1998). **Gradient-based learning applied to document recognition.** *Proceedings of the IEEE*, 86(11):2278–2324.
- [Springenberg et al., 2014] Springenberg, J. T., Dosovitskiy, A., Brox, T., and Riedmiller, M. (2014). **Striving for simplicity: The all convolutional net.** *arXiv:1412.6806*.
- [Tan and Le, 2019] Tan, M. and Le, Q. V. (2019). **Efficientnet: Rethinking model scaling for convolutional neural networks.** *arXiv:1905.11946*.
- [Wu and He, 2018] Wu, Y. and He, K. (2018). **Group normalization.** In *Proceedings of the European Conference on Computer Vision (ECCV)*, pages 3–19.

- [Yu and Koltun, 2015] Yu, F. and Koltun, V. (2015). **Multi-scale context aggregation by dilated convolutions.** *arXiv:1511.07122*.