Neural Networks: Old and New

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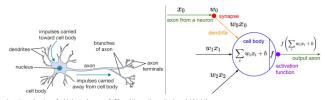
Outline

Start from neurons

Shallow to deep neural networks

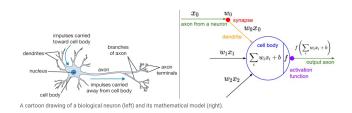
A brief history of Al

Suggested reading



A cartoon drawing of a biological neuron (left) and its mathematical model (right).

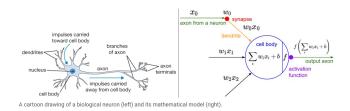
Credit: Stanford CS231N



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Biologically ...

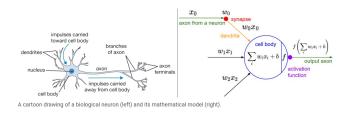
- Each neuron receives signals from its dendrites



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Biologically ...

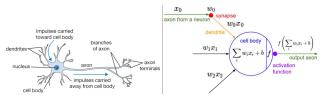
- Each neuron receives signals from its dendrites
- Each neuron outputs signals via its single axon



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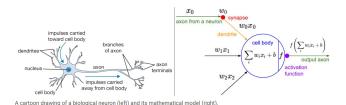
Biologically ...

- Each neuron receives signals from its dendrites
- Each neuron outputs signals via its single axon
- The axon branches out and connects via synapese to dendrites of other neurons



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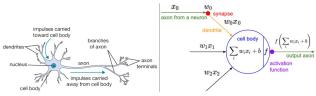
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Mathematically ...

- Each neuron receives x_i 's from its **dendrites**

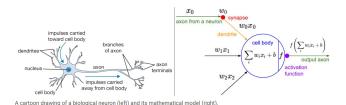


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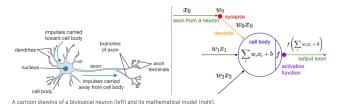
- Each neuron receives x_i 's from its **dendrites**
- x_i 's weighted by w_i 's (synaptic strengths) and summed $\sum_i w_i x_i$



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- The neuron fires only when the combined signal is above a certain threshold: $\sum_i w_i x_i + {\color{red}b}$

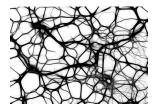


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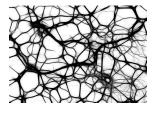
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- The neuron fires only when the combined signal is above a certain threshold: $\sum_i w_i x_i + {\color{black} b}$
- Fire rate is modeled by an **activation function** f, i.e., outputting $f\left(\sum_{i}w_{i}x_{i}+b\right)$

Brain neural networks



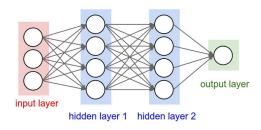
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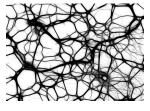


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Artificial neural networks



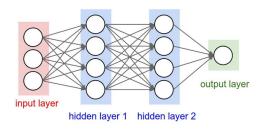
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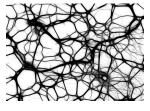
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Why called artificial?

Artificial neural networks

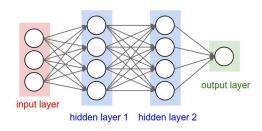


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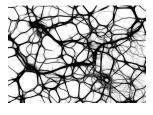
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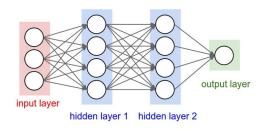
- (Over-)simplification on neural level
- (Over-)simplification on connection level

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Artificial neural networks



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In this course, neural networks are always artificial.

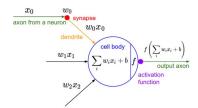
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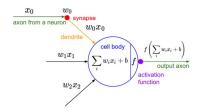
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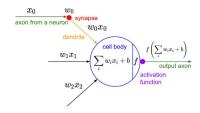
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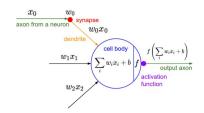


$$f\left(\sum_{i} w_{i}x_{i} + b\right) = f\left(\boldsymbol{w}^{\mathsf{T}}\boldsymbol{x} + b\right)$$



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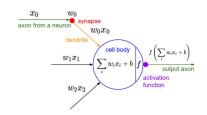
We shall use σ instead of f henceforth.



Examples of activation function σ

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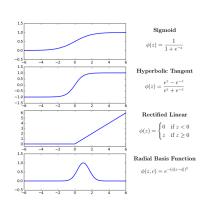
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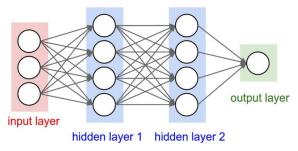


Credit: [Hughes and Correll, 2016]

One neuron: $\sigma\left(\boldsymbol{w}^{\mathsf{T}}\boldsymbol{x}+b\right)$

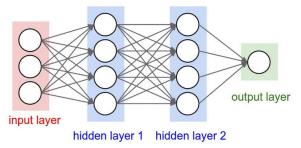
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Neural networks (NN): **structured** organization of artificial neurons



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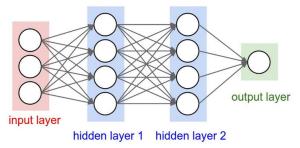
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 $oldsymbol{w}$'s and b's are unknown and need to be learned

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Neural networks (NN): **structured** organization of artificial neurons



 $m{w}$'s and $m{b}$'s are unknown and need to be learned Many models in machine learning $m{are}$ neural networks

Supervised Learning

– Gather training data $\left(oldsymbol{x}_1,oldsymbol{y}_1
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- Find an $f \in \mathcal{H}$ to minimize the average loss

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... known as **empirical risk minimization** (ERM) framework in learning theory

Supervised Learning from NN viewpoint

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$$\boldsymbol{y}_i pprox \left\{ \mathsf{NN}\left(\boldsymbol{w}_1, \dots, \boldsymbol{w}_k, b_1, \dots, b_k \right) \right\} \left(\boldsymbol{x}_i \right) \quad orall_i$$

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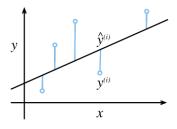
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$$\boldsymbol{y}_{i} \approx \left\{ \mathsf{NN}\left(\boldsymbol{w}_{1}, \ldots, \boldsymbol{w}_{k}, b_{1}, \ldots, b_{k} \right) \right\} \left(\boldsymbol{x}_{i}\right) \quad \forall i$$

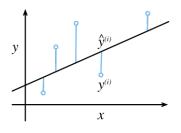
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Linear regression

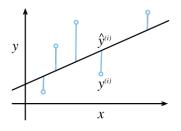


Credit: D2L



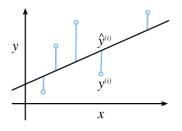
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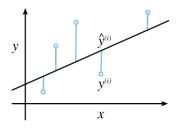
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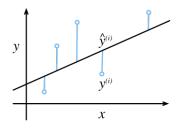
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- Model:
$$y_i \approx {m w}^{\intercal} {m x}_i + b$$

- Loss:
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Optimization:

$$\min_{\boldsymbol{w},b} \ \frac{1}{n} \sum_{i=1}^{n} \|y_i - (\boldsymbol{w}^{\mathsf{T}} \boldsymbol{x}_i + b)\|_2^2$$



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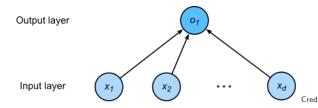
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 σ is the identity function



Frank Rosenblatt

(1928-1971)



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- Data: $(oldsymbol{x}_1, y_1), \ldots, (oldsymbol{x}_n, y_n)$, $oldsymbol{x}_i \in \mathbb{R}^d$, $y_i \in \{+1, -1\}$



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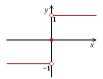




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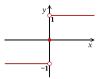


- Loss: $\mathbf{1}\left\{y \neq \hat{y}\right\}$



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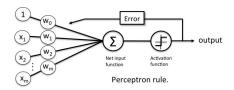
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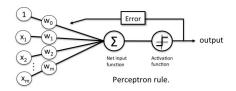
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Perceptron is a single artificial neuron for binary classification

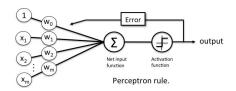


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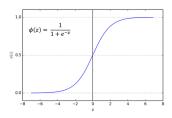
dominated early AI (50's - 70's)

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Logistic regression is similar but with sigmod activiation



– Data: $(x_1,y_1),\ldots,(x_n,y_n)$, $x_i\in\mathbb{R}^d$, $y_i\in\{L_1,\ldots,L_p\}$, i.e., multiclass classification problem

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- Data preprocessing: labels into vectors via one-hot encoding

$$L_k \Longrightarrow [\underbrace{0,\ldots,0}_{k-1\,0's},1,\underbrace{0,\ldots,0}_{n-k\,0's}]^{\mathsf{T}}$$

So: $y_i \Longrightarrow \boldsymbol{y}_i$

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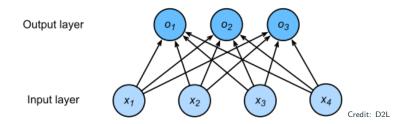
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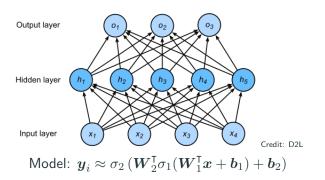
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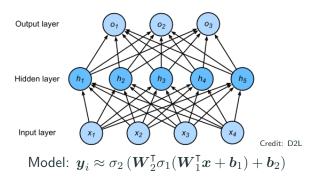
... for multiclass classification



Multilayer perceptrons

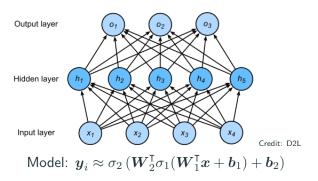


Multilayer perceptrons



Also called feedforward networks or fully-connected networks

Multilayer perceptrons



Also called feedforward networks or fully-connected networks

Modern NNs: many hidden layers (deep), refined connection structure and/or activations

They're all (shallow) NNs

- Linear regression
- Perception and Logistic regression
- Softmax regression
- Multilayer perceptron (feedforward NNs)

They're all (shallow) NNs

- Linear regression
- Perception and Logistic regression
- Softmax regression
- Multilayer perceptron (feedforward NNs)
- Support vector machines (SVM)
- PCA (autoencoder)
- Matrix factorization

see, e.g., Chapter 2 of [Aggarwal, 2018].

Outline

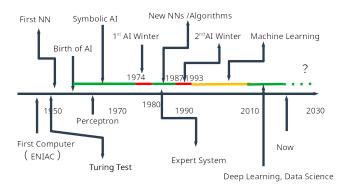
Start from neurons

Shallow to deep neural networks

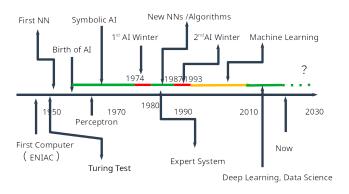
A brief history of AI

Suggested reading

Birth of Al

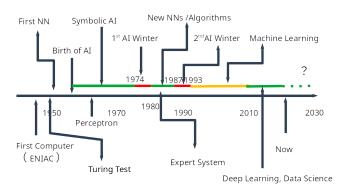


Birth of Al



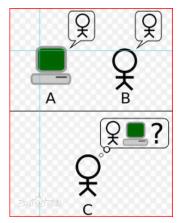
- Crucial precursors: first computer, Turing test

Birth of Al



- Crucial precursors: first computer, Turing test
- 1956: Dartmouth Artificial Intelligence Summer Research
 Project Birth of Al

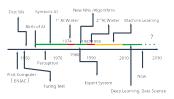
Turing test



Turing Test

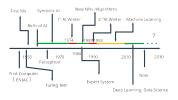


Alan Turing (1912-1954)



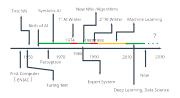


Symbolic AI: based on rules and logic

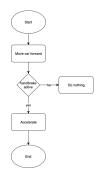


Symbolic AI: based on rules and logic





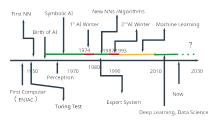
Symbolic AI: based on rules and logic



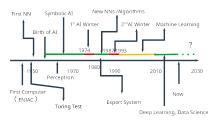


rules for recognizing dogs?

First Al winter



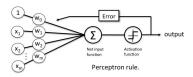
First Al winter





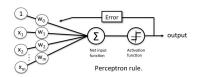
Gartner hype cycle

Perceptron



invented 1962

Perceptron



invented 1962



written in 1969, end of Perceptron era



Marvin Minsky (1927–2016)

Birth of computer vision

MASSACHUSETTS INSTITUTE OF TECHNOLOGY PROJECT MAC

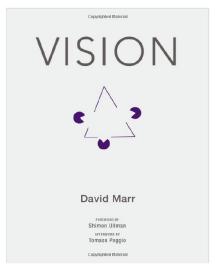
Artificial Intelligence Group Vision Memo. No. 100. July 7, 1966

THE SUMMER VISION PROJE

Seymour Papert

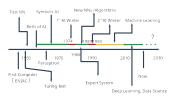
The summer vision project is an attempt to use our summer workers effectively in the construction of a significant part of a visual system. The particular task was chosen partly because it can be segmented into sub-problems which will allow individuals to work independently and yet participate in the construction of a system complex enough to be a real landmark in the development of "pattern recognition".

1966

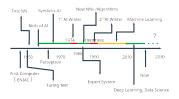


around 1980

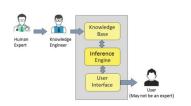
Second golden age



Second golden age



expert system





Can we build comprehensive knowledge bases and know all rules?

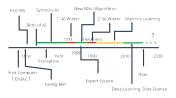
25/31

Big bang in DNNs

Key ingredients of DL have been in place for 25-30 years:

Landmark	Emblem	Epoch
Neocognitron	Fukushima	1980
CNN	Le Cun	mid 1980s'
Backprop	Hinton	mid 1980's
SGD	Le Cun, Bengio etc	mid 1990's
Various	Schmidhuber	mid 1980's
CTF	DARPA etc	mid 1980's

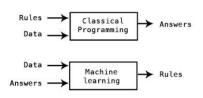
After 2nd Al winter



After 2nd Al winter



Machine learning takes over ...



Golden age of Machine learning

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Starting 1990's
```

Support vector machines (SVM)

Adaboost

Decision trees and random forests

Deep learning

. . .

Outline

Start from neurons

Shallow to deep neural networks

A brief history of AI

Suggested reading

Suggested reading

- Chap 2, Neural Networks and Deep Learning.
- Chap 3-4, Dive into Deep Learning.
- Chap 1, Deep Learning with Python.

References i

[Aggarwal, 2018] Aggarwal, C. C. (2018). **Neural Networks and Deep Learning.** Springer International Publishing.

[Hughes and Correll, 2016] Hughes, D. and Correll, N. (2016). **Distributed machine** learning in materials that couple sensing, actuation, computation and communication. arXiv:1606.03508.