Practical Phase Retrieval Using Double Deep Image Prior(s)

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University of Minnesota
Driven to Discover℠

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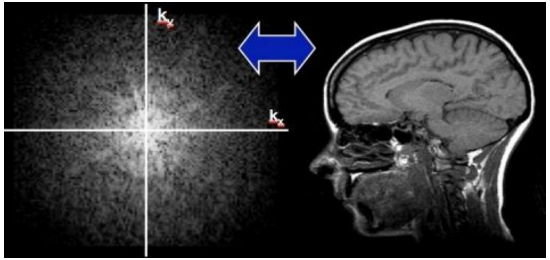
Visual inverse problems

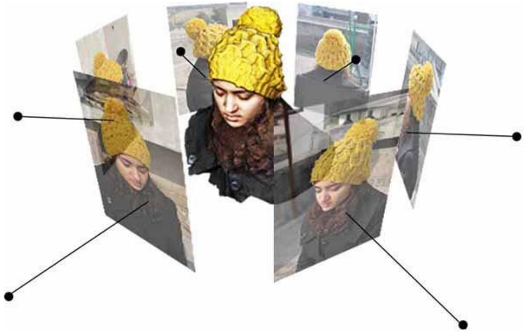
Image denoising

$$\mathbf{x} \underset{f^{-1}}{\overset{f}{\rightleftarrows}} \mathbf{y}$$

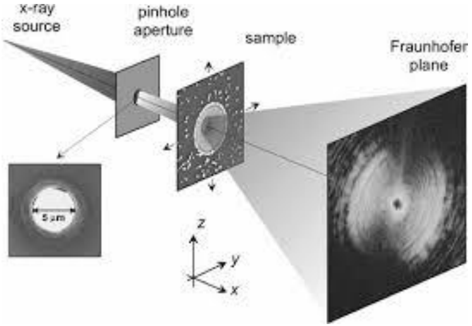


MRI reconstruction

Image super-resolution



3D reconstruction



Coherent diffraction imaging (CDI)

Solving inverse problems by regularized data-fitting

Inverse problem: given $\mathbf{y} = f(\mathbf{x})$, recover $\mathbf{x}$

$$\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{\mathrm{R}(\mathbf{x})}_{\text{regularizer}}$$

RegFit

Limitations:

- Which ℓ ? (e.g., unknown/compound noise)
- Which R ? (e.g., structures not amenable to math description)
- Speed

Plugging in Deep Image Prior (DIP)

Deep image prior (DIP) $\mathbf{x} \approx G_\theta(\mathbf{z})$ G_θ (and $\mathbf{z}$) trainable

$$\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{\mathrm{R}(\mathbf{x})}_{\text{regularizer}}$$

zero-training data!

$$\Downarrow$$

$$\min_{\theta} \ell(\mathbf{y}, f \circ G_\theta(\mathbf{z})) + \lambda R \circ G_\theta(\mathbf{z})$$

Ulyanov et al. **Deep image prior**. IJCV'20. https://arxiv.org/abs/1711.10925

DIP's cousin(s)

Deep image prior (DIP)

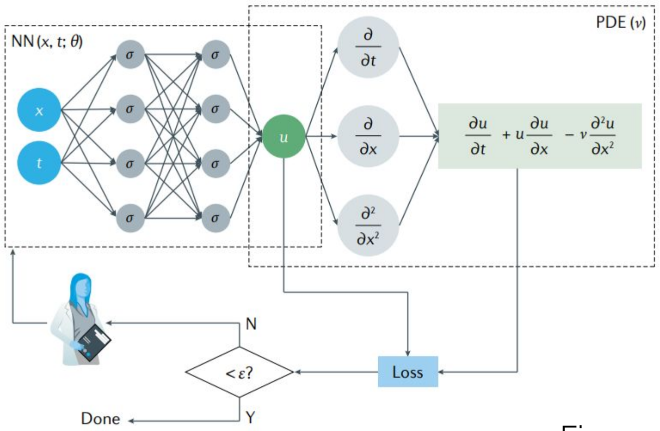
$\mathbf{x} \approx G_\theta(\mathbf{z})$ $\quad G_\theta$ (and $\mathbf{z}$) trainable

<u>Idea</u>: (visual) objects as continuous functions

Neural implicit representation (NIR)

$$\mathbf{x} \approx \mathcal{D} \circ \overline{\mathbf{x}} \qquad \mathcal{D} : \text{discretization} \qquad \overline{\mathbf{x}} : \text{continuous function}$$

Physics-informed neural networks (PINN)



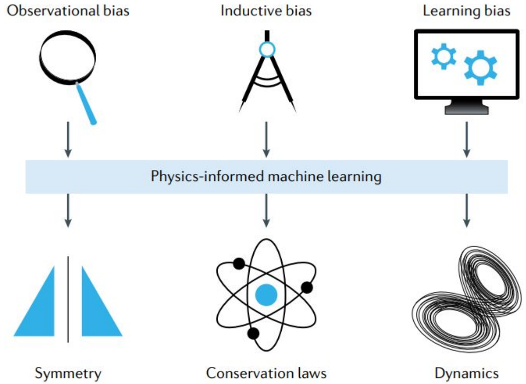
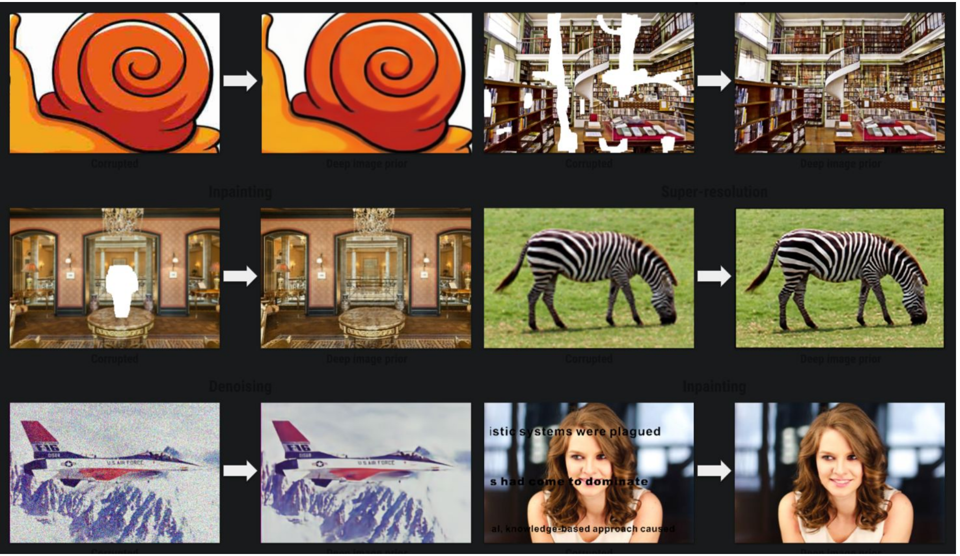


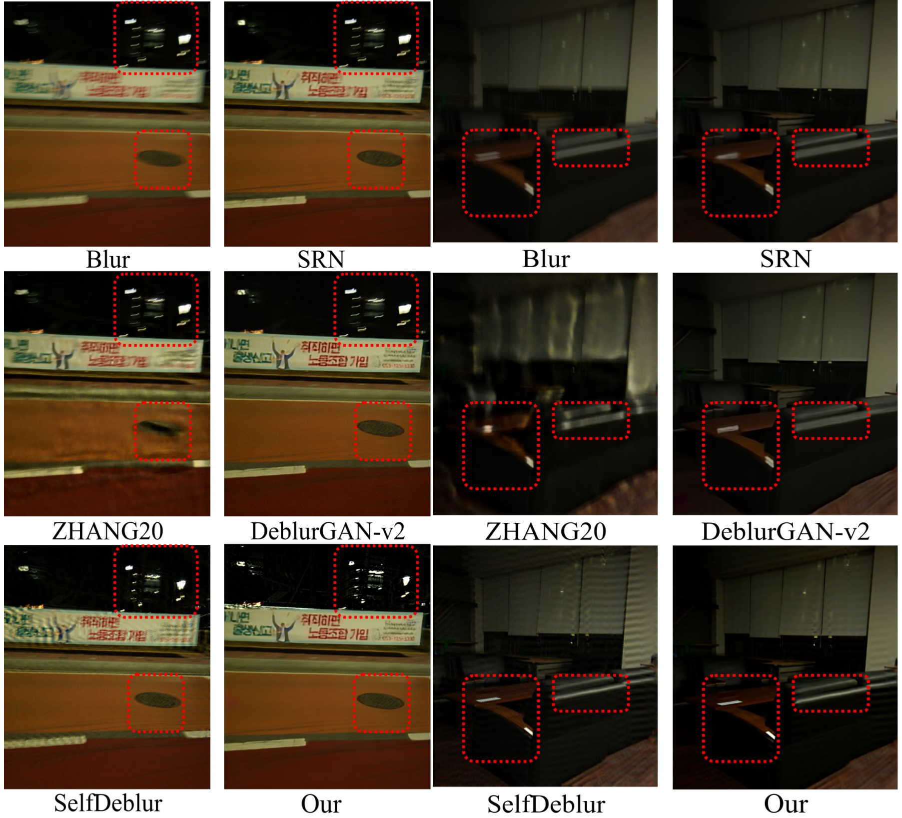
Figure credit: https://www.nature.com/articles/s42254-021-00314-5

Successes of DIP



denoising/inpainting/super-resol/deJEPG/…

https://dmitryulyanov.github.io/deep_image_prior

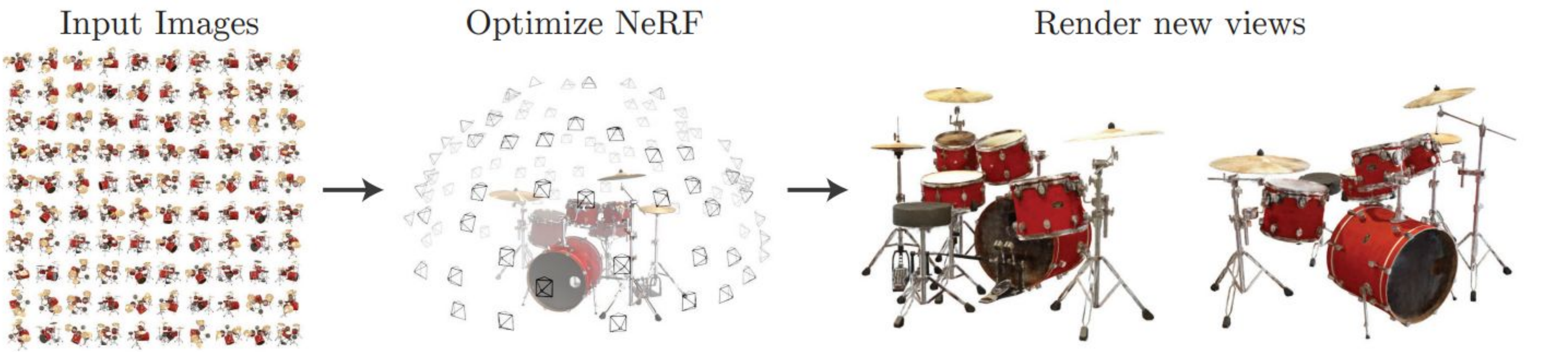


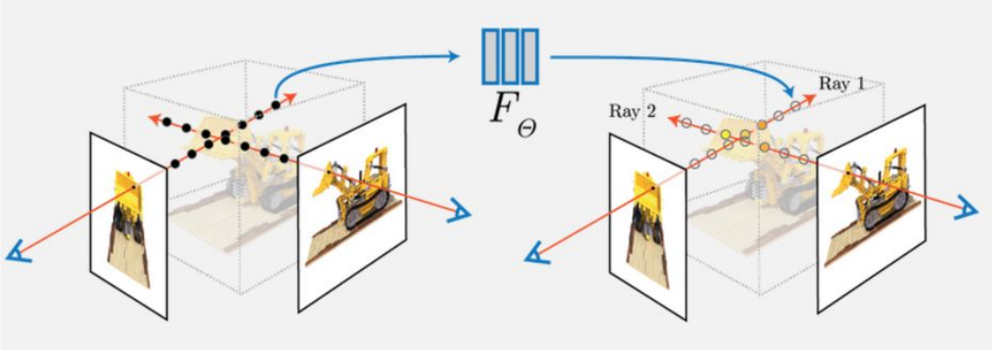
Blind image deblurring (blind deconvolution)

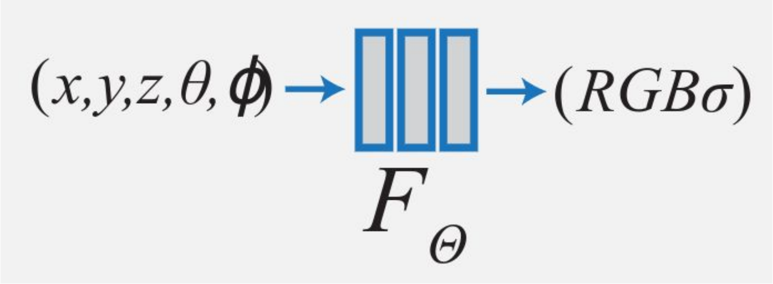
Ren et al. **Neural Blind Deconvolution Using Deep Priors**. CVPR'20. https://arxiv.org/abs/1908.02197

Zhuang et al. **Blind Image Deblurring with Unknown Kernel Size and Substantial Noise**. https://arxiv.org/abs/2208.09483

NIR for 3D rendering and view synthesis







https://www.matthewtancik.com/nerf

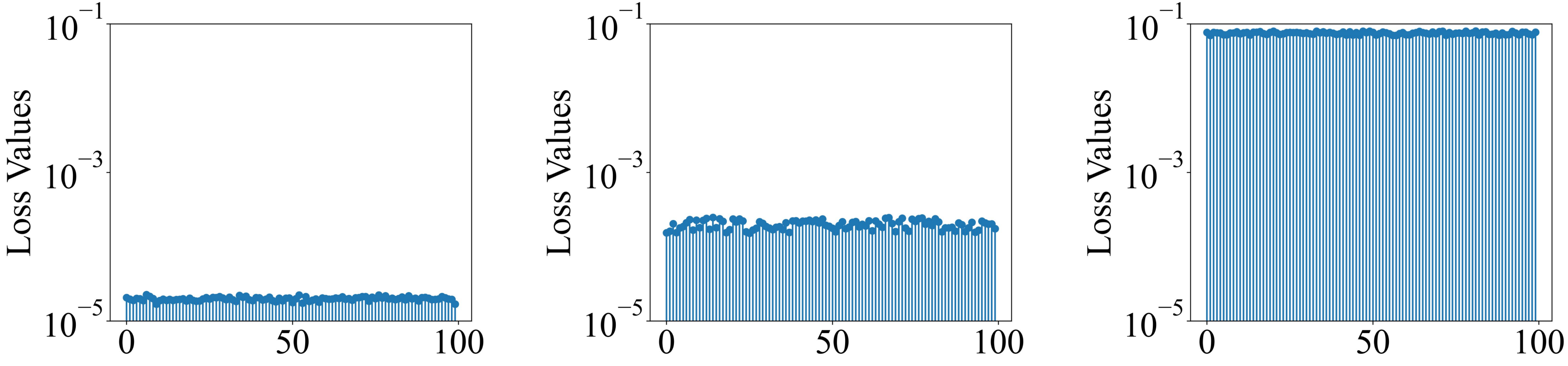
Phase Retrieval

Which phase retrieval?

$$\boldsymbol{Y} = |\mathcal{A}(\boldsymbol{X})|^2 \qquad \textbf{Consider} \qquad \min_{\boldsymbol{X}} \ \|\sqrt{\boldsymbol{Y}} - |\mathcal{A}(\boldsymbol{X})|\|_F^2$$

	Gaussian PR	**Fresnel PR**	**Fraunhofer PR**
$\mathcal{A}(\boldsymbol{X})$	$\{\langle \boldsymbol{G}_i, \boldsymbol{X}\rangle\}_{i=1}^{m}$	$\mathcal{F}(\boldsymbol{X} \odot [e^{i\pi C(m^2+n^2)}]_{m,n})$	$\mathcal{F}(\boldsymbol{X})$
Symmetries	Global phase	Global phase	Shift, flipping, global phase

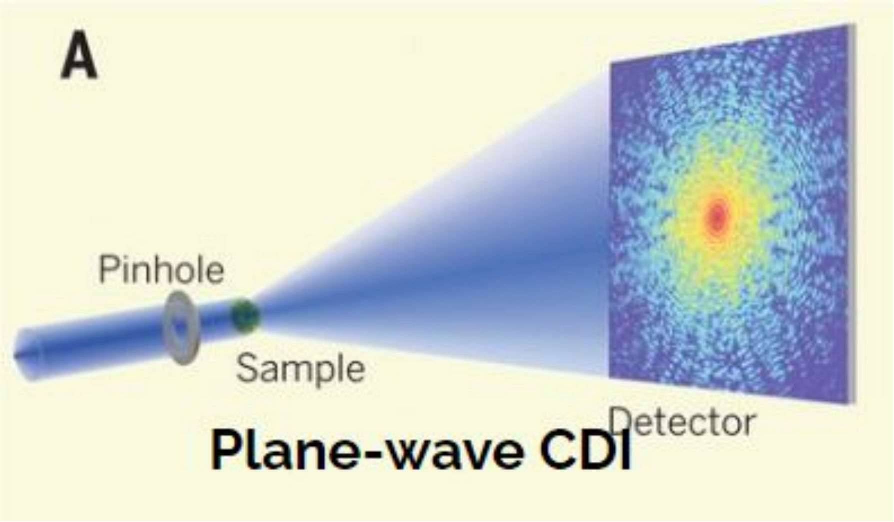
LS-solvable → **un LS-solvable (proliferation of local mins)**

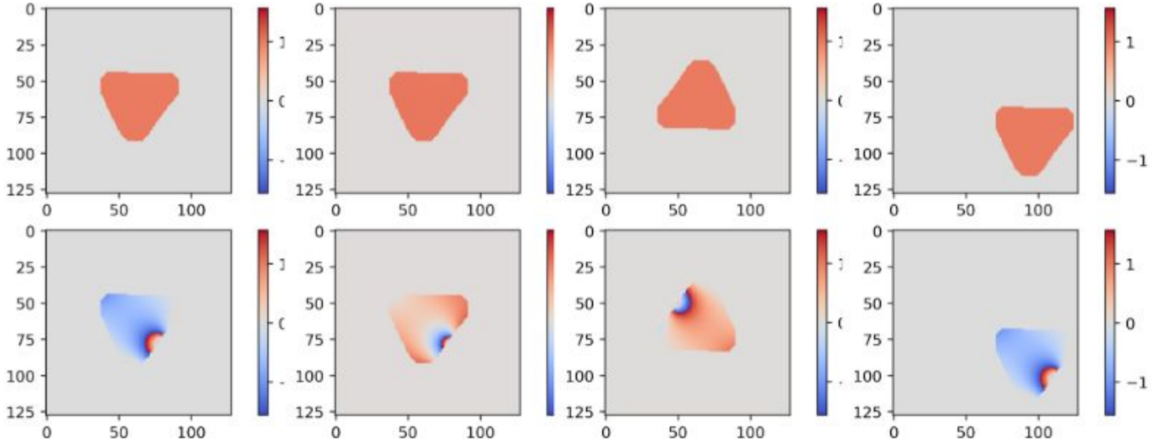


Focus here:

plane-wave CDI (Fraunhofer PR)

$$\boldsymbol{Y} = |\mathcal{F}(\boldsymbol{X})|^2$$





$\mathbf{X}$ complex-valued

Three symmetries:

- **global phase**
- **conjugate flipping**
- **shift**

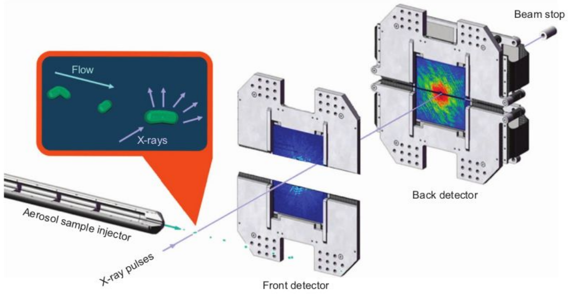
Limitations of SOTA methods on PR

Global issues

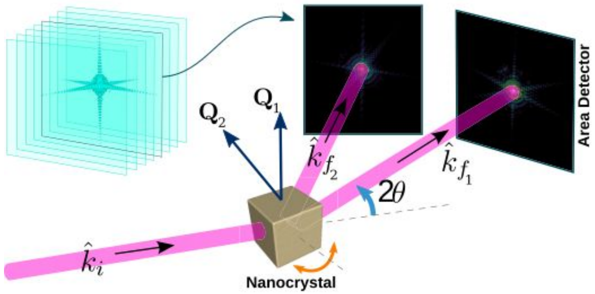
- Sensitivity to initial support estimation
- Sensitivity to **multiple hyperparameters** (e.g., HIO+ER+Shrikwrap)
- Low reconstruction quality (e.g., phases with singularities in BCDI)

Local issues

- Beamstop (i.e., missing data)
- Shot noise



Coherent Diffraction Imaging (CDI)



Bragg Coherent Diffraction Imaging (BCDI)

PR using a single DIP

Deep image prior (DIP) $\mathbf{x} \approx G_\theta(\mathbf{z})$ G_θ (and $\mathbf{z}$) trainable

$$\boxed{\min_{\mathbf{x}} \underbrace{\ell(\mathbf{y}, f(\mathbf{x}))}_{\text{data fitting}} + \lambda \underbrace{\mathrm{R}(\mathbf{x})}_{\text{regularizer}}}$$

$$\Downarrow$$

$$\min_{\theta} \ell(\mathbf{y}, f \circ G_\theta(\mathbf{z})) + \lambda R \circ G_\theta(\mathbf{z})$$

zero-training data!

Ulyanov et al. **Deep image prior**. IJCV'20. https://arxiv.org/abs/1711.10925

$$\min_{\boldsymbol{X} \in \mathbb{C}^{n \times n}} \|\sqrt{\boldsymbol{Y}} - |\mathcal{F}(\boldsymbol{X})|\|_F^2$$

$$\Downarrow$$

$$\min_{\boldsymbol{\theta}} \|\sqrt{\boldsymbol{Y}} - |\mathcal{F} \circ G_{\boldsymbol{\theta}}(\boldsymbol{z})|\|_F^2$$

Double DIPs improve the performance

$$\min_{\boldsymbol{X} \in \mathbb{C}^{n \times n}} \ \|\sqrt{\boldsymbol{Y}} - |\mathcal{F}(\boldsymbol{X})|\|_F^2$$

Reparameterizing X using two DIPs—to reflect the asymmetry in complexity and constraints

$$\boldsymbol{X} = \boldsymbol{X}^{mag} e^{1j * \boldsymbol{X}^{phase}} = G_{\theta_1}^{mag}(z_1)\, e^{1j * G_{\theta_2}^{phase}(z_2)}$$

e.g., for BCDI on crystals, **magnitude known to be uniform**

OR $$\boldsymbol{X} = \boldsymbol{X}^{real} + 1j * \boldsymbol{X}^{imag} = G_{\theta_1}^{real}(z_1) + 1j * G_{\theta_2}^{imag}(z_2)$$

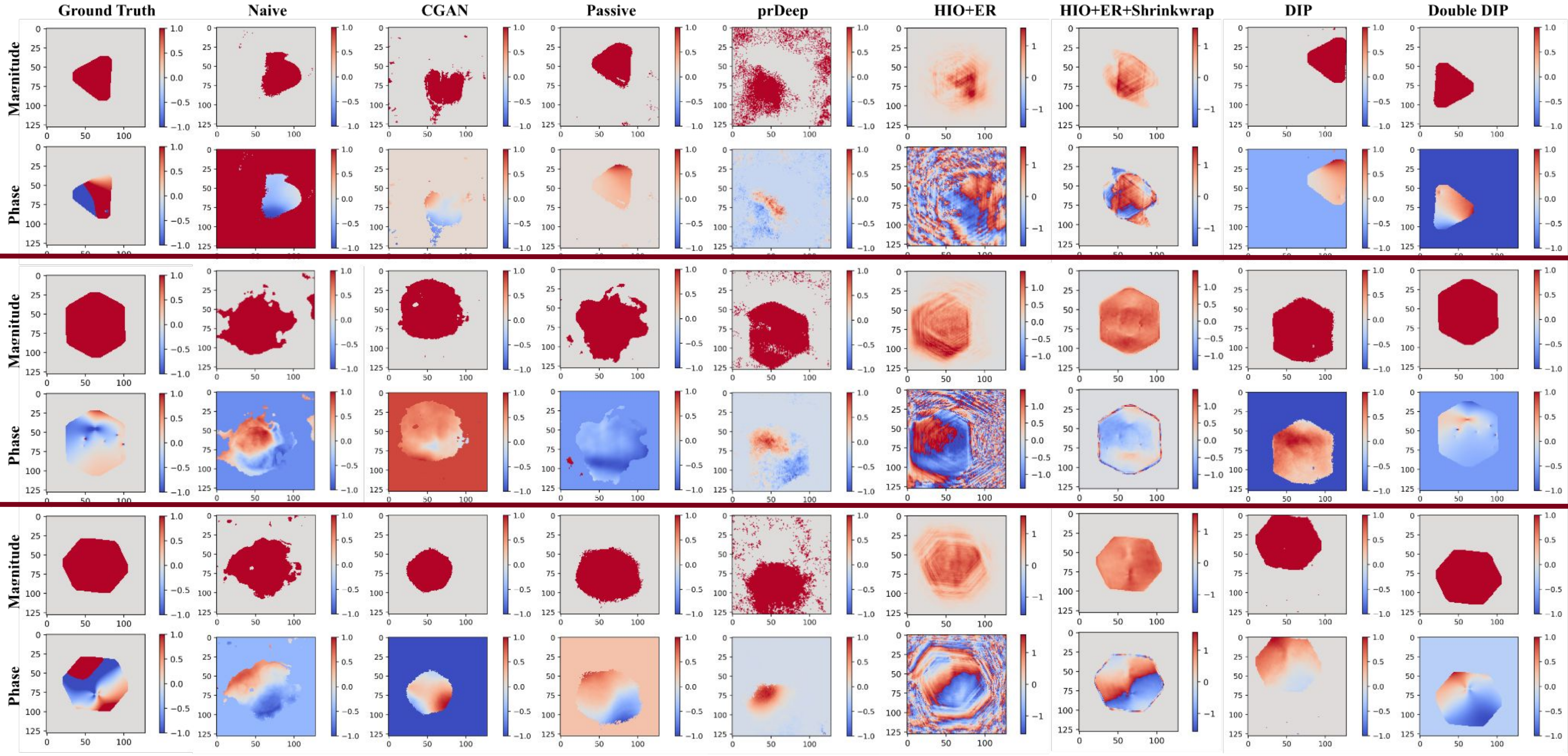
e.g., for CDI on certain bio-specimen, real part **known to be nonnegative**

X half of the size of Y in any dimension: no tight support needed, and information-theoretic limit. No shrinkwrap!

Zhuang et al. **Practical Phase Retrieval Using Double Deep Image Priors** https://arxiv.org/abs/2211.00799

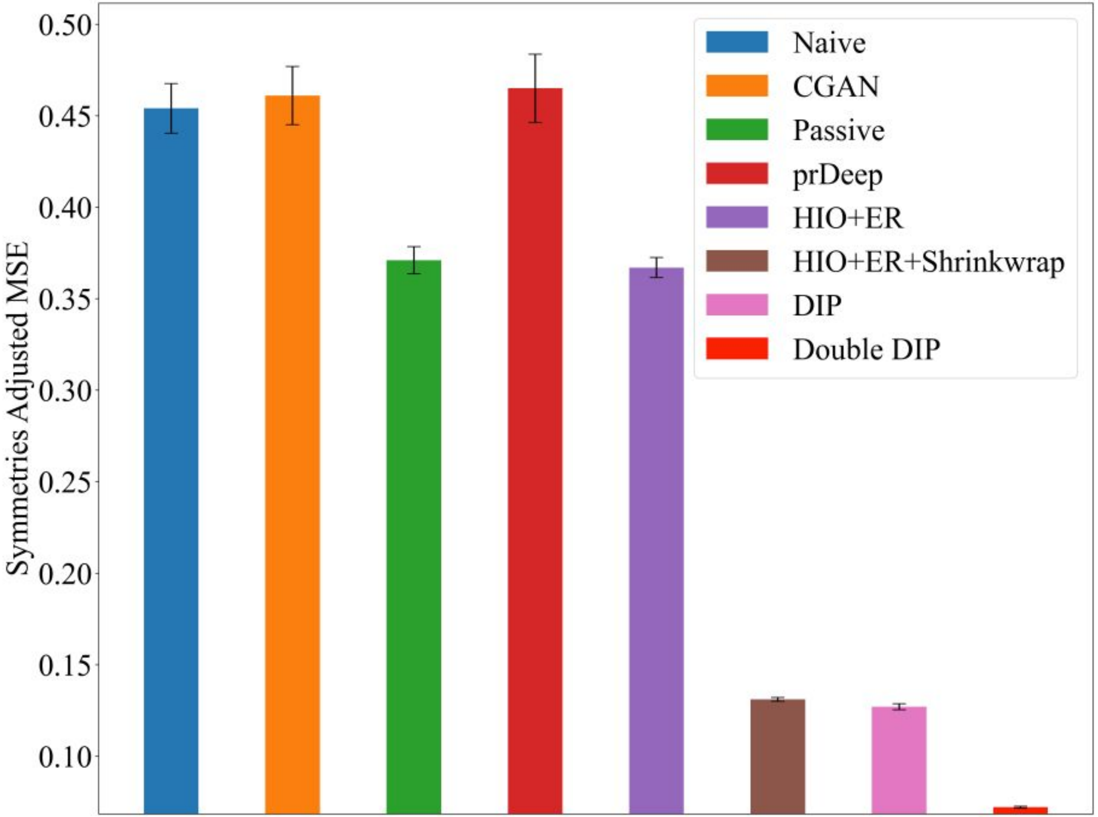
Results on simulated 2D crystal data

No training data!



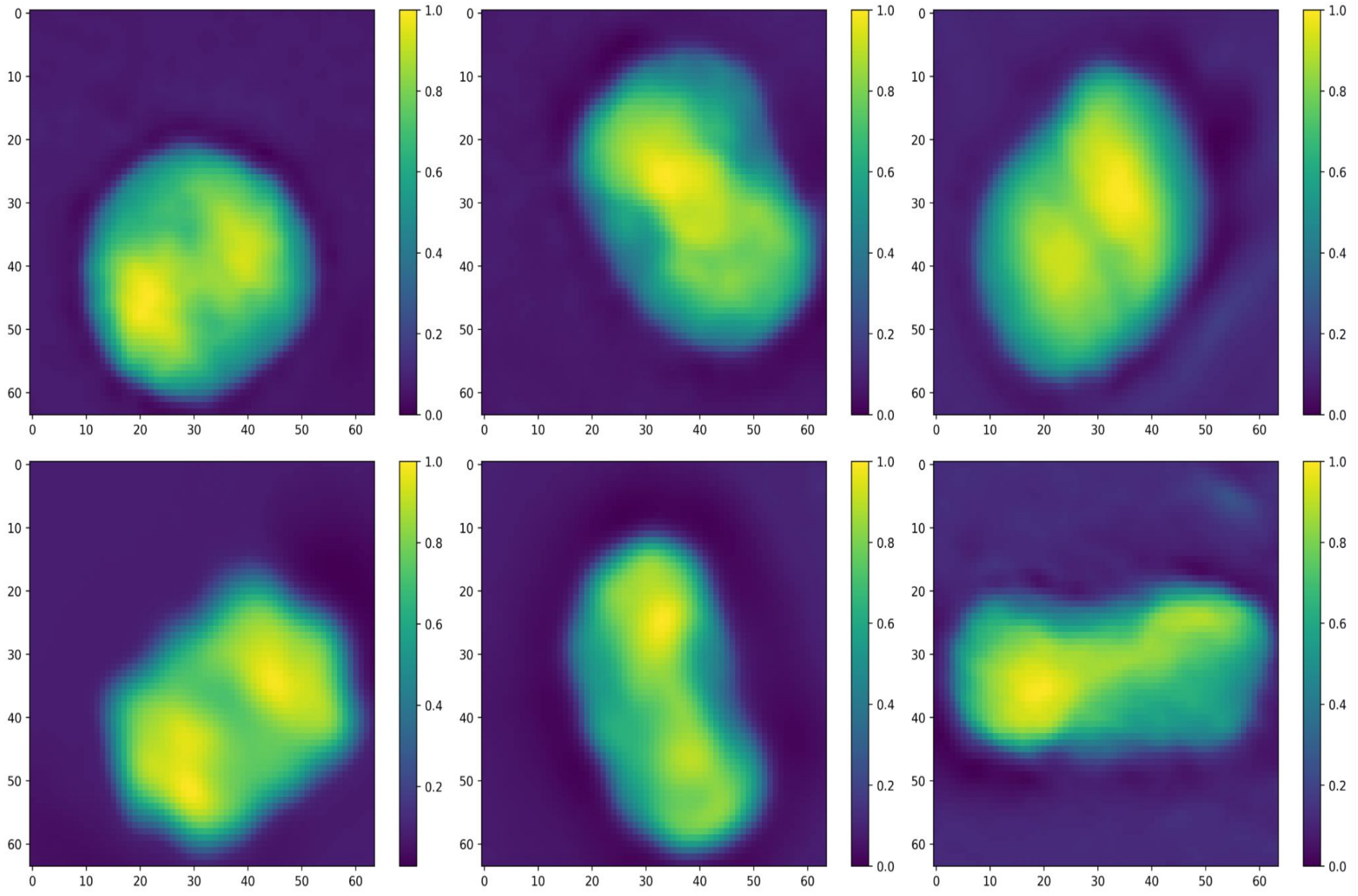
Results on simulated 2D crystal data

Metric:
symmetry-adjusted MSE
Evaluation data:
50 samples

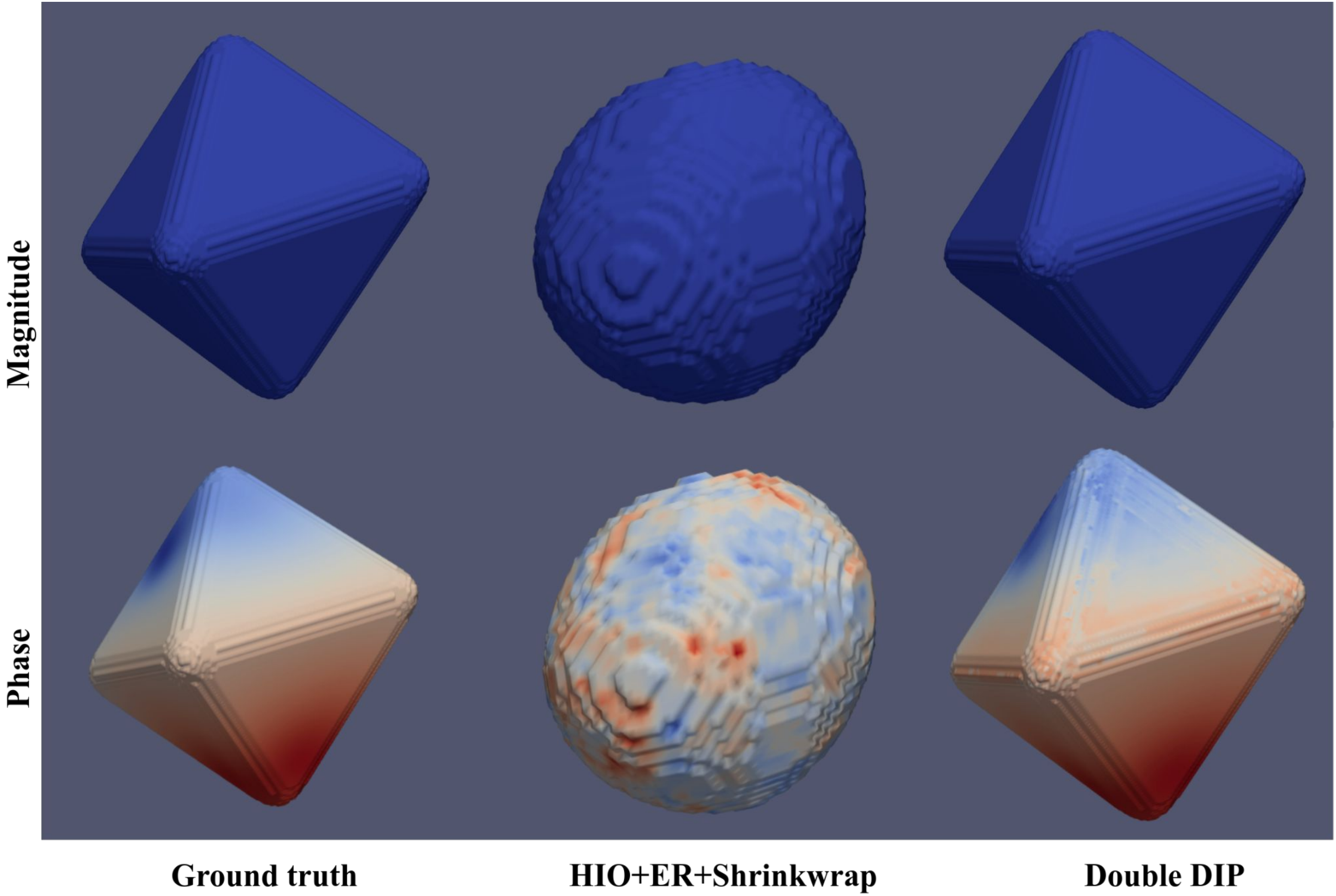


Results on 2D living cell data

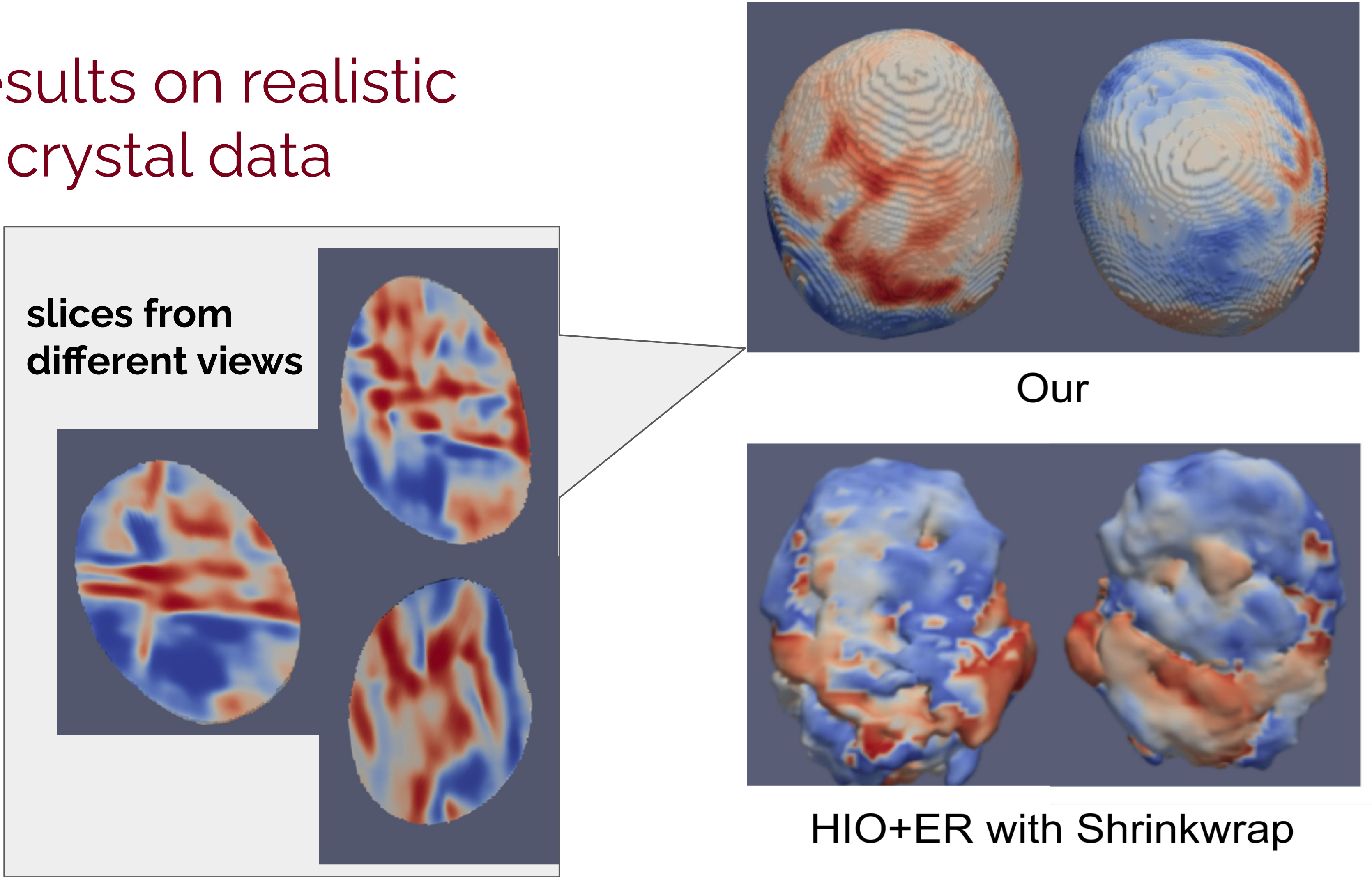
CXIDB 26



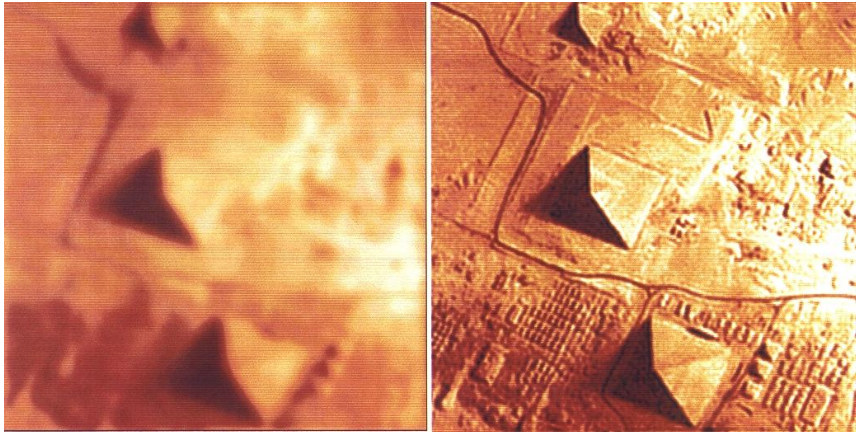
Results on simulated 3D data



Results on realistic 3D crystal data



Blind image deblurring



$$\underbrace{\mathbf{y}}_{\text{blurry and noisy image}} = \overbrace{\mathbf{k}}^{\text{blur kernel}} * \underbrace{\mathbf{x}}_{\text{clean image}} + \overbrace{\mathbf{n}}^{\text{noise}}$$

Also known as **Blind Deconvolution**

Zhuang et al. **Blind Image Deblurring with Unknown Kernel Size and Substantial Noise**. https://arxiv.org/abs/2208.09483

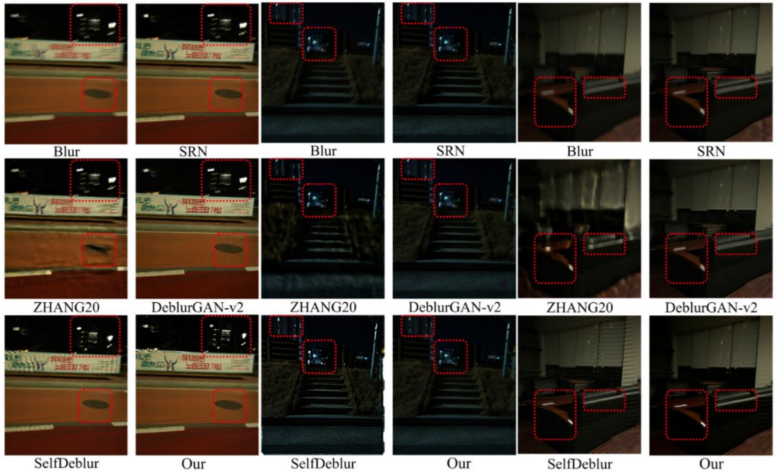
Practical challenges:

1) Unknown kernel size
2) Substantial noise
3) Model stability

$$\min_{\boldsymbol{k},\boldsymbol{x}} \underbrace{\ell(\boldsymbol{y}, \boldsymbol{k} * \boldsymbol{x})}_{\text{data fitting}} + \lambda_{\boldsymbol{k}} \underbrace{R_{\boldsymbol{k}}(\boldsymbol{k})}_{\text{regularizing } \boldsymbol{k}} + \lambda_{\boldsymbol{x}} \underbrace{R_{\boldsymbol{x}}(\boldsymbol{x})}_{\text{regularizing } \boldsymbol{x}}$$

Idea: parameterize both $\mathbf{k}$ and $\mathbf{x}$ as DIPs

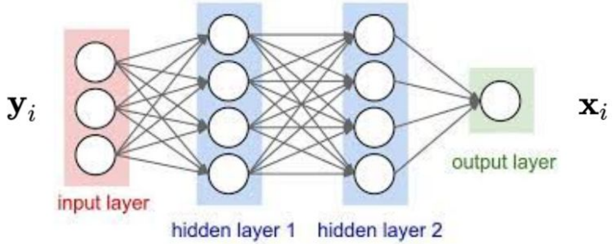
Plus: **several careful modifications**



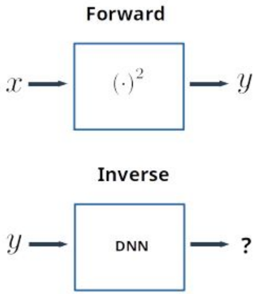
Symmetries issues in data-driven DL methods

Inverse problem: given $\mathbf{y} = f(\mathbf{x})$, recover $\mathbf{x}$

Learn the f^{-1} with a training set $\{(\mathbf{y}_i, \mathbf{x}_i)\}$

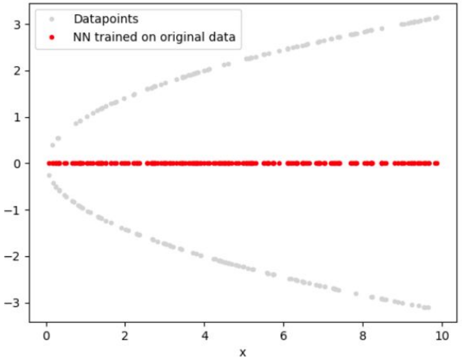


Example: **learning the sqrt function**



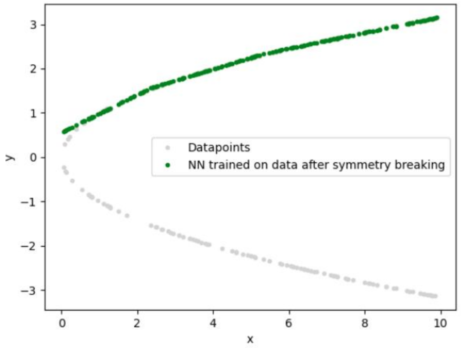
Original Dataset

4	-2
9	3
16	-4
100	10
4	2
64	-8
36	6



Modified Dataset

4	2
9	3
16	4
100	10
4	2
64	8
36	6



Present for any inverse problems with forward symmetries, e.g., all PR problems

- Tayal et al. **Inverse Problems, Deep Learning, and Symmetry Breaking**. https://arxiv.org/abs/2003.09077
- Manekar et al. **Deep Learning Initialized Phase Retrieval.** https://openreview.net/forum?id=gv4l5lfJHP
- Tayal er al. **Unlocking Inverse Problems Using Deep Learning: Breaking Symmetries in Phase Retrieval**. https://openreview.net/forum?id=oyhGIytV1S

Thanks!

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Zhong is on the job market
for a postdoctoral position