# Quantum Error Correction

Huichen Sun

# Outline

# An example of QEC

Three questions to address

1. Quantum non-cloning
2. Measurement could destroy the qubit information
3. Error happens in a continuous way

# An example of QEC

Bit flip error: X
$X|0\rangle=|1\rangle$, $X|1\rangle=|0\rangle$
Repeating in computational basis, bit flip error can be fixed using parity checking qubit (1,2) and (2,3)
$|000\rangle \rightarrow |001\rangle$, Parity check: (1, -1), bit flip in the 3rd qubit
$|000\rangle \rightarrow |110\rangle$, Parity check: (1, -1), bit flip in the 3rd qubit?

Phase flip error: Z
$Z|0\rangle=|0\rangle$, $Z|1\rangle=-|1\rangle$
$Z|+\rangle=|-\rangle$, $Z|-\rangle=|+\rangle$
Repeating in $|\pm\rangle$ can fix phase flip error

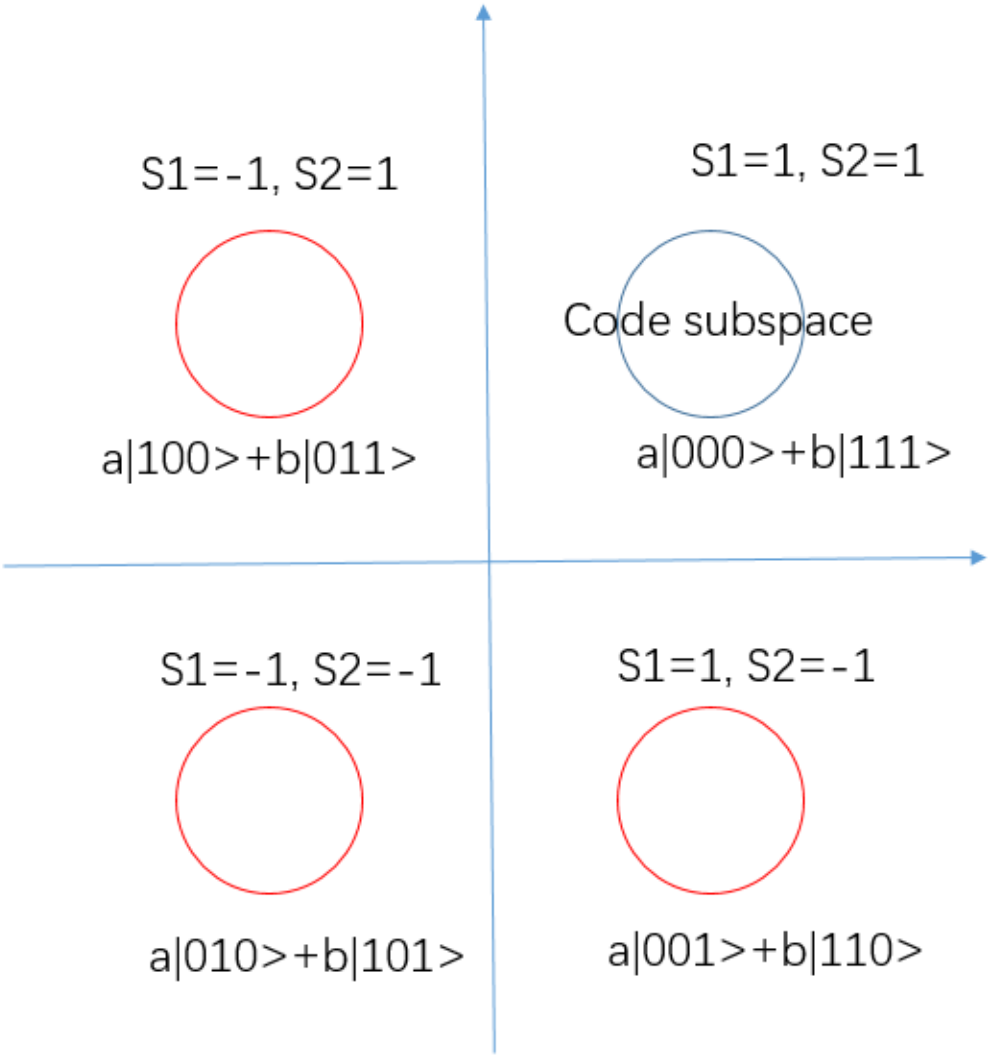
$|\overline{0}\rangle = (|000\rangle+|111\rangle)\otimes(|000\rangle+|111\rangle)\otimes(|000\rangle+|111\rangle)$
$|\overline{1}\rangle = (|000\rangle-|111\rangle)\otimes(|000\rangle-|111\rangle)\otimes(|000\rangle-|111\rangle)$

# Quantum error correcting criteria

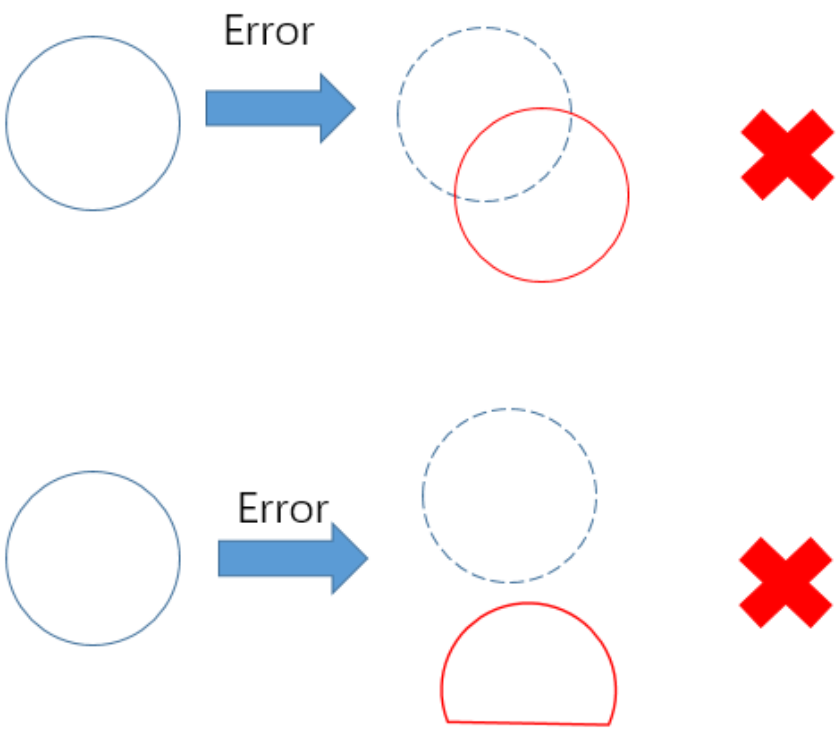
Error model: $\mathcal{E}[\rho]=\Sigma_i E_i \rho\, E_i^\dagger$
QEC criteria: $\langle \phi_i | E_k^\dagger E_l | \phi_j \rangle = C_{kl}\delta_{ij}$, $\{C_{kl}\}$ is a Hermitian

1. Encode information into the code subspace
2. perform logical operation
3. measure error syndrome and correct error repeatedly
4. decode and measurement

# Quantum error correcting criteria



# Quantum error correcting criteria



# Fault-Tolerant Quantum Computation

Steps needed in quantum computation

1. State Preparation
2. Quantum Gates
3. Measurement
4. Wait

1. Each step has a very low error rates
2. Each step does not make the situation worse in error propagation (fault tolerant)

# Fault-Tolerant Quantum Computation

Threshold Theorem: If we have a QEC code, a fault tolerant protocol, there exists a threshold error probability $p_{\text{th}}$. A ideal circuit C can be realized with at most error $\epsilon$ in output, with at most Polylog(|C|/$\epsilon$) qubits and time steps.
Method: Concatenating QEC codes

# Outlook

Surface codes: only requiring near-neighbouring interaction on a 2D plane
$p_{\text{th}} \approx 1\%$, $p_{\text{gate}} \approx 99.9\%$, $N_{\text{logic}} \approx 1000$
Factoring a 2000-bit number requires $N_{\text{Shor}}(2000) \approx 10^7$ physical qubits

A. G. Fowler et al., “Surface codes: Towards practical large-scale quantum computation”, Phys. Rev. A 86, 032324 (2012).
R. Barends et al., “Superconducting quantum circuits at the surface code threshold for fault tolerance”, Nature 508, 500 (2014).

Cat codes: encode qubits into Fock states of photons

# References

[1] J. D. Hidary, “Quantum Computing: An applied Approach”.
[2] D. Bacon, Lecture notes for CSE 599d - Quantum Computing
[3] D. Gottesman, “An Introduction to Quantum Error Correction and Fault-Tolerant Quantum Computation”, ArXiv:0904.2557 [Quant-Ph] (2009).

# Thank you!