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ADDITIONAL MATERIAL AVAILABLE (SEE FULL REFERRAL LETTER):

High-Spin Structure of ⁹⁶Nb: A New Level Scheme and Shell Model Insights

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High-spin states in 96 Nb have been investigated by using in-beam γ ray spectroscopy with the 82 Se(18 O,p3n) 96 Nb reaction at beam energies of 82 and 88 MeV. Particle- γ - γ coincidence measurements employing CsI and HPGe arrays were utilized to enhance selectivity for Nb isotopic products. The decay sequences of previously known states have been extended to $I=(25^-)$ through the identification of fourteen new γ rays. The spin and parity of most γ rays in 96 Nb have been assigned for the first time using Angular Distribution of γ rays (ADO) and polarization analysis. The level structures in 96 Nb have been interpreted through shell model calculations employing the GWBXG and SNET interactions. The results indicate that the inclusion of proton core excitation relative to the Z=38 subshell closure is crucial for accurately describing the experimental high-spin states at $I \sim (18-25)\hbar$ with excitation energies above 6.2 MeV.

PACS numbers: 23.20.Lv, 25.70.Jj, 21.60.Cs, 27.60.+j

I. INTRODUCTION

Nuclei in the neutron-rich $A \approx 100$ region are generally challenging to be populated using fusion-evaporation reactions with stable beam-target combinations. Nuclear structure information for these nuclei is typically obtained through fusion-fission methods. However, this technique presents challenges in assigning spin and parity to excited states due to poor statistics and the lack of initial directional correlation information for the fission products. Therefore, it is proposed to utilize beam-target combinations with the highest neutron-to-proton ratios, such as $^{18}\text{O+}^{82}\text{Se}$, and employ particle discrimination techniques to investigate reaction channels for specific neutron-rich isotopes near the stability line, aiming to explore their high-spin level structures.

Recent comprehensive studies by our collaborative research team have significantly advanced our understanding of the excited states of niobium isotopes, particularly in $^{90-94}$ Nb [1–8], utilizing γ -ray spectroscopy and shell-model calculations. Our findings for 90 Nb [1, 8] demonstrate that both positive-and negative-parity states are accurately described by proton excitations within the fpg orbitals, combined with a single neutron hole in the $g_{9/2}$ orbital. The (16⁺) state at 7035.8

keV is attributed to a neutron excitation across the N = 50shell closure and a proton excitation in the $f_{5/2}$ orbital. For ⁹¹Nb [2, 5], high-spin states arise from proton excitations spanning the $1p_{3/2}$, $1f_{5/2}$, and $1p_{1/2}$ orbitals to the $1g_{9/2}$ orbital, along with neutron excitations from the $1g_{9/2}$ orbital to the $1d_{5/2}$ or $1g_{7/2}$ orbitals, facilitated by the breaking of the Z = 38 and N = 50 core structures. In ⁹²Nb [3, 6], the highspin states around $I \sim 18 - 20\hbar$ with excitation energies above 8.5 MeV can be fully described only when accounting for proton core excitation relative to the Z = 38 subshell closure and neutron particle-hole excitation for the N = 50 shell closure. In ⁹³Nb [4], low-lying states are predominantly driven by proton excitations across the Z = 40 subshell to the $1g_{9/2}$ orbital, while moderate-spin states involve neutron excitations across the N = 56 subshell. For high-spin states above 9.1 MeV, neutron excitation across the N = 50 shell closure becomes the dominant factor. In 94 Nb [7], the neutron $1h_{11/2}$ orbital play a significant role in understanding the structure. These studies offer deeper insights into the high-spin states of niobium isotopes, highlighting the importance of proton and neutron excitations across shell gaps and core excitations in determining their nuclear structure.

The odd-odd nucleus ⁹⁶Nb is located on the neutron-rich side of the stability line, specifically three neutrons away from the stable isotope ⁹³Nb. It is expected that an investigation into the high-spin states of ⁹⁶Nb will provide deeper and more valuable insights into the single-particle excitations of both

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valence protons and neutrons, as well as the mechanism of core breaking.

The low-lying levels of 96 Nb have been previously investigated through (p, n γ) reactions [9, 10], yielding results for excitation energies up to 1.6 MeV. Higher excited states of this nucleus, with excitation energies reaching 2.96 MeV, were subsequently explored using one- and two-nucleon transfer reactions, including (t, α) [11], (3 He, t) [12, 13], (α , d) [14]. The high-spin structure of this nucleus was further examined through fusion-fission and multinucleon binary grazing reactions involving 24 Mg + 173 Yb [15], 23 Na + 176 Yb [15], 36 S + 176 Yb [16], 96 Zr + 124 Sn [16]. However, most excited states have not been assigned experimental spin and parity in the previous studies. Additionally, ambiguities regarding the placement of certain γ transitions persist in the level scheme [16].

This paper presents newly identified high-spin states in the ⁹⁶Nb nucleus. The spin and parity of all states have been determined and assigned based on directional correlation and polarization measurements. Additionally, the ⁹⁶Nb level scheme has been benchmarked against shell-model calculations using two effective interactions.

II. EXPERIMENTS

The experiment was performed at the HI-13 tandem accelerator in the China Institute of Atomic Energy (CIAE). Excited states of ⁹⁶Nb were populated via the heavy-ion fusionevaporation reaction 82 Se(18 O, p3n) 96 Nb at beam energies of 82 and 88 MeV. The target was a 0.85-mg/cm²-thick isotopically enriched ⁸²Se metallic foil with a 4.45-mg/cm² Au backing to stop the recoiling nuclei. The γ rays emitted from the evaporation residues were detected using the Conjoint Gamma Array in China (CGAC), as shown in Fig. 1. The CGAC is composed of 12 Clover-type HPGe detectors and 28 n-type coaxial HPGe detectors, totaling 40 detectors, all equipped with BGO anti-Compton shielding. This array consists of 76 individual detection modules, each demonstrating an energy resolution between 2.0 keV and 2.8 keV at the 1332 keV. During the experiment, a total of twenty-nine Comptonsuppressed HPGe detectors were arranged, with nine (five clover HPGe detectors) positioned at 90°, seven at 60°, eight at 120° , and six at 149° . The identification of in-beam γ transitions from charged particle channels has been to utilize coincidences between γ rays and light charged particles (p and α) to separate the transitions of interest from the much more intense background from xn channels. The evaporated charged particles were recorded using an approximately 4π charged particle detector array, which consists of 128 pieces of CsI crystals. The electronic signals generated from the interaction of γ rays and charged particles with the detectors were collected and processed with a general-purpose digital data acquisition system (GDDAQ), housed in two compact PCI/PXI crates and consisting of two 14-bit 250-MHz Pixie-16 modules and fourteen 14-bit 100-MHz Pixie-16 modules [17, 18].

All HPGe detectors were calibrated for energy and efficiency using the standard energy calibration γ -rays from the



FIG. 1: Conjoint Gamma Array in China (CGAC).

decay of 60 Co, 133 Ba and 152 Eu radioactive sources. A total of $1.0 \times 10^8~p$ - γ - γ coincidence data were accumulated in event-by-event mode. The recorded $\gamma - \gamma$ coincidence events were sorted into two-dimensional $E_{\gamma} - E_{\gamma}$ symmetric matrix, gated by the number of protons corresponding to Nb reaction channels, and then analyzed using the software package RAD-WARE [19].

To obtain information on the multipolarity of γ rays, two asymmetric matrices were constructed. The first axis (y axis) utilized the γ rays detected at all angles, while the second axis (x axis) included the γ rays detected at 149° and 90°, respectively. The angular distribution asymmetry (ADO) ratios, defined as $R_{\rm ADO}(\gamma) = I_{\gamma}(149^{\circ})/I_{\gamma}(90^{\circ})$, were derived from the γ ray coincidence intensities observed by the detectors at either 149° and 90°, utilizing gates set on the all-angles axis (y axis). For the present detector geometry, the typical $R_{\rm ADO}$ value for stretched quadrupole (or $\Delta I = 0$ dipole) transitions was approximately \sim 1.6, while for stretched pure dipole transitions, it was around 0.8.

In addition to ADO ratios, the unambiguous determination of multipolarities for γ transitions can be definitively derived from linear polarization measurements of observed γ -ray transitions. These measurements were conducted in coincidence using a clover detector, which served as a Compton polarimeter [20–22]. Data matrices were constructed with one axis denoting the energy recorded by all detectors, while the other axis corresponded to energy scattered either perpendicularly or parallel to the beam axis. The count of perpendicular N_{\perp} and parallel N_{\parallel} scatters for a specific γ -ray was ascertained by projecting the gated spectra. The asymmetry parameter (A_P) was subsequently calculated from these spectra applying the appropriate formula:

$$A_P = \frac{aN_{\perp} - N_{||}}{aN_{||} + N_{||}} \tag{1}$$

The value of a, representing the asymmetry of the clover array, was determined using data from the 133 Ba and 152 Eu radioactive sources.

The polarization (P) of γ radiation, the polarization sensitivity (Q), and the polarization asymmetry (A) are related through the equation $(P = \frac{A}{O})$. The Q values for the array were adopted from Ref. [7]. A positive polarization asymmetry indicates an electric nature (stretched E1, E2) or non-stretched M1 transition, while a negative value characterizes a magnetic nature (stretched M1, M2) or non-stretched E1 transition. A value close to zero signifies the occurrence of a mixed transition. The current statistics for ⁹⁶Nb, which is derived from the weak p-reaction channel, allows for the determination of several linear polarization parameters for the strong transitions, as outlined in Table I. These findings offer crucial insights into parity changes and the electromagnetic nature of the linking transitions. Moreover, corroborating evidence from crossover or parallel transitions provides additional support for spin and multipolarity assignments.

III. THE LEVEL SCHEME

The level scheme of ⁹⁶Nb deduced in the present work is shown in Fig. 2. The placement of γ rays in the level scheme is based on the $\gamma - \gamma$ coincidence relations, γ -ray intensities, R_{ADO} values and polarization (P) measurements. The γ -ray intensities were determined from the total projection of the $p - \gamma - \gamma$ coincidence matrix. Spin and parity assignments are on the basis of $\gamma - \gamma$ directional correlations and deexcitation modes, or referenced from Ref [15, 16] for the ground and first excited states. Compared with the results reported in Refs [15, 16], the level scheme of ⁹⁶Nb was extended significantly in the present work. Fourteen new γ rays and eleven new states were identified and assigned in the present level scheme. The energies, relative intensities, polarization P and ADO ratios of γ rays as well as the spin and parity I^{π} assignments of levels are summarized in Table I. The typical coincidence γ ray spectra gated on the transitions of 96 Nb are shown in Figs. 3. Detailed experimental results are provided below.

Previous studies [9–16] have assigned spin-parities of 6⁺ and (7⁺) to the ground state and first excited state of ⁹⁶Nb, respectively. However, the spin and parity of the state above the (7^+) level at 222 keV were not determined in these works. The ADO and polarization analysis conducted in this study provides an opportunity to assign the spin and parity for highspin states in 96 Nb. The R_{ADO} and P values for the 222keV transition suggest a M1+E2 character. Consequently, the first excited state, depopulated by the 222-keV transition, has been confirmed to have a spin-parity of 7⁺, thus supporting all prior tentative assignments. The experimentally measured R_{ADO} and polarization values for the 1164-keV transition are 0.54(8) and -0.10(22), respectively, indicative of a mixed multipolarity M1+E2 transition with a spin change of $\Delta I = 1$. The 1386-keV level has been conclusively assigned a spin-parity of $I^{\pi} = 8^{+}$. The observed R_{ADO} and polarization values of 0.91(36) and -0.22(64) for the 1656-keV transition support a mixed multipolarity M1+E2 transition. Consequently, the 1878-keV level, deexcitation via the 1656-keV transition, is determined to possess a spin-parities of $I^{\pi} = 8^{+}$. The 1527keV transition has R_{ADO} and polarization values determined experimentally as 0.85(11) and 0.50(16) respectively, suggesting a pure E1 transition. This leads to the assignment of $I^{\pi} = 8^{-}$ for the 1749-keV level.

For the positive-parity part of the level scheme, most of the γ -rays reported in the previous studies [15, 16] were confirmed. As illustrated in Fig. 3 (a), nearly all transitions between these positive-parity states are observable. However, the 1343-keV transition identified by Brown et al. [16] in the positive sequence was not observed in the present study. Moreover, the 780- and 1480-keV transitions do not coincide with the 852-keV transition. Nevertheless, the 852-keV transition was observed in the γ -ray coincidence spectrum gated on the 918-keV and 745-keV transitions. The intensity of the 852-keV transition is stronger than that of the 918-keV transition. Consequently, the 852- and 918-keV transitions are positioned above the (10⁺) state at 2131 keV, as shown in Fig. 2. Based on the experimentally measured R_{ADO} values of 1.50(30) and 0.88(27), the transitions at 852 keV and 918 keV are identified as (E2) and (M1+E2) multipolarity transitions, respectively. Consequently, the spin-parity assignments for the levels at 2983 keV and 3901 keV have been determined to be (12^+) and (13^+) , respectively. While the polarization analysis for the 745-keV γ -ray was inconclusive, the significantly enhanced R_{ADO} parameter of 1.36(36) strongly supports an E2 multipolarity, indicating a spin change of $\Delta I = 2$. In parallel, the 825-keV transition shows R_{ADO} characteristics that are consistent with a stretched $\Delta I = 2$ transition. Furthermore, the newly observed 833-keV transition is illustrated in Fig. 3 (a). The latest R_{ADO} values and the established transition selection rules suggest that the spin and parity assignments for the 2964-, 2921-, 3051-, 3611-, and 4391-keV levels are likely (12^+) , (11^+) , (11^+) , (12^+) , and (14^+) , respectively.

Regarding the negative-parity states, all the γ -rays below the 4545-keV level reported in previous studies [15, 16] have been verified, as shown in Fig. 3 (a)-(d). The analysis of the new R_{ADO} values and the corresponding transition selection rules points to the spin-parity assignments of (9⁻), (10⁻), (10⁻), (11⁻), (12⁻) and (13⁻) for the 1944-, 2377-, 2750-, 2954-, 3263-, and 3847-keV levels, respectively. With experimentally determined R_{ADO} and polarization values of 1.94(57) and 0.08(56) respectively, the 801-keV transition is consistent with an *E2* multipolarity transition characterized by a spin change of $\Delta I = 2$. Therefore, it has been determined that the 4064-keV level possesses a spin-parity of $I^{\pi} = 14^{-}$. According to the newly obtained R_{ADO} value, the spin and parity of the 4545-keV level is likely (15⁻).

The negative-parity states above the 4545-keV level have been newly identified. Thirteen new transitions, including six cascade $\Delta I=2$ transitions, have been added, extending the excitation energy up to 10398 keV. The placement of these γ transitions was determined through analysis of individual γ - γ coincidence spectra and their respective intensities. Additional confirmation for the placement of these γ rays within the sequence was obtained by examining the energy sums. As demonstrated in Fig. 3 (a)-(d), all thirteen new transitions are visible in the gated spectrum. These new transitions primarily decay into the 15⁻ state via the 456-keV and 597-keV transitions. Experimental results indicate that the 1055-keV transitions.

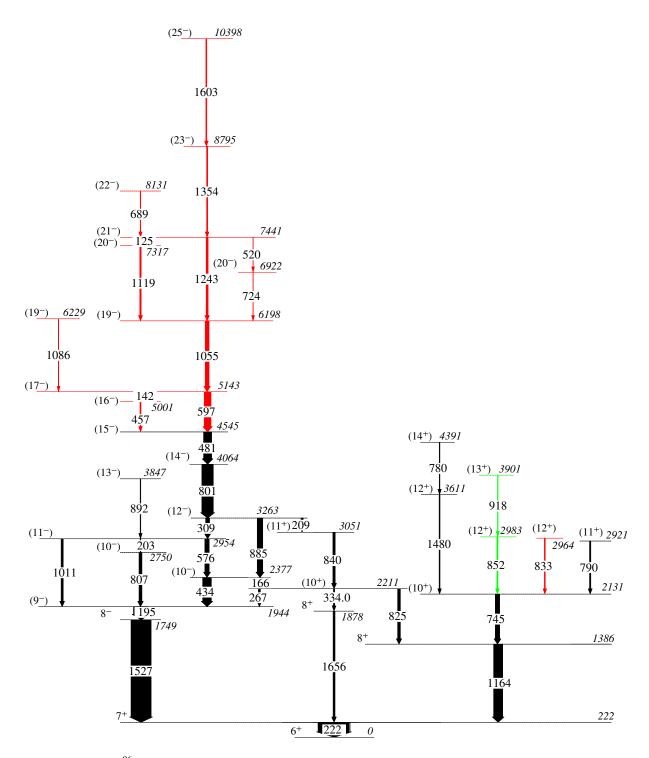


FIG. 2: The level scheme of 96 Nb established from the present work is shown. The energies of the observed excited states and γ -ray transitions are given in keV. The thicknesses of the arrows are roughly proportional to the γ -ray intensity listed in Table I. The new and rearranged γ rays are marked in red and green, respectively.

tion has an R_{ADO} value of 2.34(57), which is consistent with an E2 multipolarity transition. The observed R_{ADO} value of 1.72(54) for the 1243-keV transition support an E2 multipolarity transition with a spin change of $\Delta I = 2$. Furthermore, considering the dipole nature of the 457-, 142-, 724-, 520-, 1119-, 125-, and 689-keV transitions and the quadrupole

nature of the 597-, 1055-, 1086-, 1243-, 1354-, and 1603-keV transitions, the assignments of $I^{\pi}=(16^-),(17^-),(19^-),(19^-),(20^-),(20^-),(21^-),(22^-),(23^-),$ and (25^-) for the 5001-, 5143-, 6198-,6229-, 6922-, 7317-, 7441-, 8131-, 8795-, and 10398-keV levels, respectively, were suggested for the negative-parity sequence.

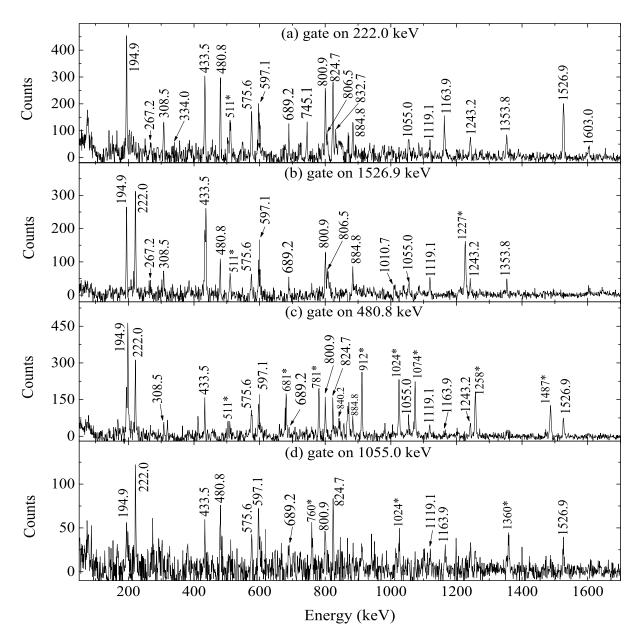


FIG. 3: Representative spectrums obtained by gating on (a) 222.0 keV, (b) 1526.9 keV, (c) 480.8 keV, and (d) 1055.0 keV transitions in the symmetric γ - γ coincidence matrix, in coincidence with the protons detected in the CsI ball. The main coincidence γ rays are labeled with its energies in keV. Transitions marked with asterisks, as well as those not labeled by energy, are attributed to contaminant lines close to the gating energies.

IV. DISCUSSION

The level scheme depicted in Fig. 2 reveals several high-energy γ transitions around 1.5 MeV, indicating nucleon excitation across a significant energy (shell) gap. For instance, the high-energy γ transitions, such as (19⁻) (6198 keV) \rightarrow (17⁻) (5143 keV), (21⁻) (7441 keV) \rightarrow (19⁻) (6198 keV), (23⁻) (8795 keV) \rightarrow (21⁻) (7441 keV), and (25⁻) (10398 keV) \rightarrow 23⁻ (8795 keV), could be attributed to proton excitation across the Z=38 gap. These transitions suggest changes in intrinsic structure due to such excitations. To verify this hypothesis, calculations were performed incorporat-

ing the proton excitations across the Z=38 subshell gap. The isotones 93 Sr [24] and 97 Mo [25], along with the isotopes $^{90-97,99}$ Nb [1–8, 16, 24, 26], were systematically investigated using the shell model. These studies demonstrate that promoting protons from the pf orbits into the g orbit is sufficient to describe the low-spin states. For high-spin states, it is proposed to consider proton or neutron core excitation through the Z=38 or N=50 subshell or shell closure.

To elucidate the decay sequences observed in ⁹⁶Nb, we conducted shell-model calculations using the nushellx computational framework [27], employing two different interactions. Specifically, the theoretical approach utilizes the GWB model

TABLE I: Level excitation energy E_x , energies E_{γ} , relative intensities, R_{ADO} ratios, polarization P, initial- and final-state spin-parities, and multipolarities of γ -ray transitions assigned to 96 Nb in the present work.

$\overline{E_{x} \text{ (keV)}}$	E _γ (keV)	$I_{\gamma}{}^{a}$	R_{ADO}	Polarization(P)	$I_i^{\pi} \rightarrow I_f^{\pi b}$	Mult.
222.0	222.0(2)	155	0.67(11)	-0.51(22)	$7^{+} \rightarrow 6^{+}$	M1+E2
1385.9	1163.9(3)	45.3(61)	0.54(8)	-0.10(22)	$8^+ \rightarrow 7^+$	M1+E2
1748.9	1526.9(2)	100.0(44)	0.85(11)	0.50(16)	$8^- o 7^+$	<i>E</i> 1
1877.5	1655.5(7)	9.7(40)	0.91(36)	-0.22(64)	$8^+ \rightarrow 7^+$	M1 + E2
1943.8	194.9(2)	72.5	0.53(8)		$9^{(-)} ightarrow (8^-)$	(M1 + E2)
2131.0	745.1(2)	21.8(24)	1.36(36)		$10^{(+)} \to (8^+)$	E2
2211.0	267.2(2)	9.6(42)			$(10^+) \to 9^{(-)}$	(E1)
	334.0(4)	6.2(9)			$(10^+) \to (8^+)$	(E2)
	824.7(4)	14.0(55)	1.78(43)		$(10^+) \to (8^+)$	(E2)
2377.3	165.5(4)	7.8(16)			$(10^-) \to (10^+)$	(E1)
	433.5(2)	43.4(35)	0.83(15)		$(10^-) \to 9^{(-)}$	(M1 + E2)
2750.3	806.5(4)	14.6(36)	0.83(16)		$(10^-) \to 9^{(-)}$	(M1 + E2)
2920.8	789.8(7)	6.6(17)	1.09(53)		$(11^+) \to 10^{(+)}$	(M1+E2)
2954.5	202.7(5)	3.6(6)	0.58(8)		$(11^{-}) \rightarrow (10^{-})$	(M1+E2)
	575.6(2)	21.7(79)	1.17(27)		$(11^{-}) \rightarrow (10^{-})$	(M1+E2)
	1010.7(5)	4.8(18)	1.54(45)		$(11^{-}) \rightarrow 9^{(-)}$	(E2)
2963.7	832.7(3)	4.6(29)	1.72(64)		$(12^+) \rightarrow 10^{(+)}$	(E2)
2982.8	851.8(5)	5.8(20)	1.50(30)		,	(E2)
3051.2	840.2(7)	11.6(28)	1.09(37)		$(11^+) \to (10^+)$	(M1+E2)
3263.0	208.6(3)	10.8(18)	0.59(35)		$(12^{-}) \rightarrow (11^{+})$	(E1)
	308.5(2)	19.8(33)	0.82(48)		$(12^{-1}) \rightarrow (11^{-1})$	(M1 + E2)
	884.8(2)	27.6(32)	1.62(34)		$(12^{-}) \rightarrow (10^{-})$	(E2)
3611.2	1480.2(2)	4.8(14)	1.54(20)		$(12^+) \to 10^{(+)}$	(E2)
3846.9	892.4(7)	3.4(7)	1.38(38)		$(13^{-}) \rightarrow (11^{-})$	(E2)
3900.9	918.1(5)	2.9(19)	0.88(27)			(M1 + E2)
4063.9	800.9(2)	57.0(54)	1.94(57)	0.08(56)	$(14^-) \to (12^-)$	E2
4391.4	780.2(2)	3.5(7)	1.86(82)		$(14^+) \to (12^+)$	(E2)
4544.7	480.8(2)	40.1(36)	1.14(17)		$(15^-) \to (14^-)$	(M1 + E2)
5001.4	456.7(4)	5.1(4)	0.38(21)		$(16^-) \to (15^-)$	(M1 + E2)
5143.1	141.7(4)	3.5(15)	0.56(28)		$(17^{-}) \rightarrow (16^{-})$	(M1 + E2)
	597.1(3)	34.0(62)			$(17^-) \to (15^-)$	(E2)
6198.1	1055.0(4)	19.4(42)	2.34(57)		$(19^{-}) \rightarrow (17^{-})$	(E2)
6229.0	1085.9(2)	1.0(1)	0.79(32)		$(18^{-}) \to (17^{-})$	(M1+E2)
6922.0	723.9(4)	1.3(1)	1.19(35)		$(20^{-}) \rightarrow (19^{-})$	(M1+E2)
7317.2	1119.1(2)	7.1(19)	1.06(46)		$(20^{-}) \rightarrow (19^{-})$	(M1+E2)
7441.3	124.9(6)	1.8(2)			$(21^-) \to (20^-)$	(M1 + E2)
	519.8(2)	<1			$(21^-) \to (20^-)$	(M1 + E2)
0.4.2.0. 7	1243.2(7)	10.0(13)	1.72(54)		$(21^{-}) \to (19^{-})$	(E2)
8130.5	689.2(4)	3.1(6)	1.00(18)		$(22^{-}) \to (21^{-})$	(M1+E2)
8795.1	1353.8(4)	5.5(18)	1.51(49)		$(23^{-}) \to (21^{-})$	(M1+E2)
10398.1	1603.0(4)	4.9(19)	1.61(57)		$(25^-) \to (23^-)$	(E2)

^a Intensities are normalized to the 1526.9-keV transition with I_{γ} =100.

space in conjunction with the GWBXG effective interaction, comprising four proton orbitals $(1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 1g_{9/2})$ and six neutron orbits $(2p_{1/2}, 1g_{9/2}, 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2})$ relative to the doubly magic 66 Ni core. As established in prior studies [28–32], the single-particle energies (SPEs) relative to the 66 Ni core were parametrized as follows: $\varepsilon^{\pi}_{1f_{5/2}}$ =-5.322 MeV, $\varepsilon^{\pi}_{2p_{3/2}}$ =-6.144 MeV, $\varepsilon^{\pi}_{2p_{1/2}}$ =-3.941 MeV, $\varepsilon^{\pi}_{1g_{9/2}}$ =-1.250 MeV, $\varepsilon^{\nu}_{2p_{1/2}}$ =-0.696 MeV, $\varepsilon^{\nu}_{1g_{9/2}}$ =-2.597 MeV, $\varepsilon^{\nu}_{1g_{7/2}}$ =5.159 MeV, $\varepsilon^{\nu}_{2d_{5/2}}$ =1.830 MeV, $\varepsilon^{\nu}_{2d_{3/2}}$ =4.261 MeV, and $\varepsilon^{\nu}_{3s_{1/2}}$ =1.741 MeV. These SPE parameters, combined with optimized resid-

ual interaction strengths derived from systematic fitting procedures, were subsequently used in the configuration-interaction calculations to determine nuclear level energies.

The nucleus 96 Nb contains 13 valence protons and 17 valence neutrons within the considered configuration space. Due to the extensive dimensions of the valence space, truncations were implemented in our calculations, as referenced in Refs. [7, 33]. Four protons were allowed to be excited from the lower-energy $1f_{5/2}$ and $2p_{3/2}$ orbitals to the higher-energy $2p_{1/2}$ and $1g_{9/2}$ orbitals, crossing the Z=38 subshell gap. In contrast, neutron excitations from lower-energy orbitals to

^b Excitation energies of initial E_i and final E_f states.

SNE

GWB

Exp.

OWD	Exp.	SINE	
14 ⁺ 3482	1 <u>4</u> ⁺ <u>439</u> 1 1 <u>3</u> ⁺ <u>390</u> 1 1 <u>2</u> ⁺ <u>36</u> 11	$ \begin{array}{rrr} 14^{+} & 4067 \\ 12^{+} & 3859 \\ 13^{+} & 3838 \end{array} $	
13 ⁺ 3002 12 ⁺ 2809 12 ⁺ 2670 12 ⁺ 2406 11 ⁺ 2285 11 ⁺ 2095 10 ⁺ 1811	$ \begin{array}{ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{rrr} 12^{+} & 2976 \\ \hline 11^{+} & 2927 \\ 12^{+} & 2614 \\ \underline{11^{+}} & 2463 \\ 10^{+} & 2455 \end{array} $ $ 10^{+} & 1833 $	
8 ⁺ 1165 10 ⁺ 1117	<u>8+</u> <u>13</u> 86	8 ⁺ 1204 8 ⁺ 819	
$ \begin{array}{ccc} 8^{+} & 402 \\ 7^{+} & 170 \\ 6^{+} & 0 \end{array} $	$\frac{7^{+}}{6^{+}}$ $\frac{222}{0}$	7 ⁺ 137 6 ⁺ 0	

Evn

GWR

SNF

FIG. 4: Comparison of experimental excitation energies in $^{96}{\rm Nb}$ (π =+) with shell-model predictions with the model spaces GWB and SNE.

higher-energy orbitals across the N = 50 shell gap were not permitted.

Based on shell-model calculations using the GWBXG interaction, the high-spin states in $^{96}{\rm Nb}$ identified in this study can primarily be attributed to four distinct mechanisms: (a) excitation of either one proton or a proton pair from the fully occupied $2p_{1/2}$ orbit to the $1g_{9/2}$ orbit; (b) excitation of protons from the fully filled $1f_{5/2},\,2p_{3/2}$ subshells to the $1g_{9/2}$ orbit; (c) valence neutron excitation from the $2d_{5/2}$ orbit to the $1g_{7/2}$ orbit; and (d) neutron excitation from the $2d_{5/2}$ orbit to the $3s_{1/2}$ orbit. These excitation mechanisms have been used to elucidate the observed states of $^{96}{\rm Nb}.$

The calculated energy levels of ⁹⁶Nb are compared with the experimental data in Figs. 4 and 5. To investigate the structural properties of the positive- and negative-parity sequences, the primary components of the wave functions for each state are detailed in Tables II and III. These components are characterized by one to three dominant configurations that significantly contribute to the overall wave function.

The $I^{\pi} = 6^+$ ground state is predominantly generated by the coupling of one $1g_{9/2}$ proton with an unpaired neutron in the $2d_{5/2}$ orbital. The configuration dominated by a single

O 11 2	-	
	25 ⁻ 10398	
25 ⁻ 10136		
		25- 9376
	22- 0705	
	23- 8795	
23- 8395		
22- 8167	22 ⁻ 8131	
		23- 7709
	21^{-} 7441	22 <u>- 72</u> 78
2 <u>1</u> - 7231	20	221218
$ \begin{array}{ccc} 21^{-} & 7231 \\ 20^{-} & 7039 \\ \hline 20^{-} & 6965 \end{array} $	20- 6922	21- 6795
20 0,00		$ \begin{array}{cccccccccccccccccccccccccccccccccccc$
19- 6348		1 <u>9</u> 6658
19 ⁻ 6129	19^{-} 6229 19^{-} 6198	
	19 0198	
17- 5207	$ \begin{array}{rrr} 17^{-} & 5143 \\ 16^{-} & 5001 \end{array} $	
16- 4822	16 5001	
	15- 4545	17- 4644
	13.13	
	14- 4064	1 <u>6</u> - <u>41</u> 44
	13- 3847	
		1 <u>5</u> - <u>36</u> 68
14^{-} 3441 13^{-} 3246	12- 3263	13- 3269
13 3240		
10- 2888	11^{-} 2955 10^{-} 2750	1 <u>4</u> - <u>29</u> 29
11-12- 2526 2500	10 2750	
$11^{-12^{-}} 2526 2500$ $10^{-9^{-}} 2477 2490$	1 <u>0</u> - <u>237</u> 7	1 <u>1</u> - <u>23</u> 60 12- <u>2249</u>
8- 2058	0- 101:	12 ⁻ 2249 10 ⁻ 2049
	9 ⁻ 1944 8 ⁻ 1749	
	3 1/17	$\frac{8^{-}}{9^{-}}$ $\frac{1773}{1688}$
		1 <u>0</u> - <u>14</u> 02

FIG. 5: Comparison of experimental excitation energies in $^{96}{\rm Nb}$ (π =-) with shell-model predictions with the model spaces GWB and SNE.

 $1g_{9/2}$ proton coupled to a valence neutron in the $1g_{7/2}$ orbital predominantly results in the first excited state with $I^{\pi}=7^+$. The calculated excitation energies for the positive parity states with spins $I^{\pi}=6^+$ and $I^{\pi}=7^+$ are in reasonable agreement with experimental results. For the $I^{\pi}=8^+_1$ to $I^{\pi}=14^+_1$ states, the calculated energies are 400 to 1014 keV lower than the experimental values. However, the differences between the calculated energy gaps and observed γ transitions range from 30 to 584 keV. Specifically, the differences for the transitions $10^+_1 \rightarrow 8^+_1$ and $14^+_1 \rightarrow 12^+_3$ are just 30 and 107 keV, respectively. The configurations $\pi(1g_{9/2}) \otimes v(1g_{7/2}/2d_{5/2})^5$ and $\pi(1g_{9/2})^3 \otimes v(1g_{7/2}/2d_{5/2})^5$ play a dominant role in the formation of the $I^{\pi}=8^+_1$ to $I^{\pi}=14^+$ excited state.

The calculated lowest-lying $8_1^-, 9_1^-, 10_1^-$, and 10_2^- states are 100-500 keV higher than the experimental ones. In contrast, all other negative parity states are predicted to be at the same or lower energies. The calculations predict that the yrast 8_1^- to 16_1^- states share the configuration $\pi[(2p_{1/2})^1(1g_{9/2})^2] \otimes \nu(1g_{7/2}/2d_{5/2})^5$. However, the 10_2^- state involves the excitation of two protons from the $1f_{5/2}$ and $2p_{1/2}$ into $1g_{9/2}$ orbital. Additionally, a pair of neutrons is removed from the $2d_{5/2}$ orbital and transferred to the $3s_{1/2}$ orbital.

The 19^-_1 to 23^-_1 states, except for 19^-_2 , involve the excitation of one proton across the shell gap at Z=38 into the $1g_{9/2}$ orbital. Consequently, a gap of approximately 1-1.5 MeV is predicted for the transition $19^-_1 \rightarrow 17^-_1$, $19^-_2 \rightarrow 17^-_1$, $20^-_2 \rightarrow 19^-_1$, $21^-_1 \rightarrow 19^-_1$, and $23^-_1 \rightarrow 21^-_1$. Notably, 20^-_1 and 22^-_1 states involve one neutron excited from the $2d_{5/2}$ or $1g_{7/2}$ orbital to the $3s_{1/2}$ orbital. The primary configuration of the states from 19^-_1 to 23^-_1 is $\pi[(1f_{5/2})^{-1}(1g_{9/2})^4] \otimes \nu(1g_{7/2}/2d_{5/2})^5$ or $\pi[(1f_{5/2})^{-1}(1g_{9/2})^4] \otimes \nu[(1g_{7/2}/2d_{5/2})^4(3s_{1/2})^1]$, as shown in Table II. The larger gap spacing (1603 keV) between the 25^-_1 and 23^-_1 states is likely due to the excitation of a pair of $1f_{5/2}$ protons into the $1g_{9/2}$ orbital, crossing the Z=38 subshell closure.

The structure of the ⁹⁶Nb nucleus is expected to be influenced by variations in neutron number, particularly as the high- $j \, 1h_{11/2}$ orbital begin to fill in Nb isotopes with increasing neutron number. Previous studies have indicated the impact of the $1h_{11/2}$ neutron orbital on the level structure of isotopes in this mass region, including Rb [34], Sr [35-39], Y [40], Zr [41-43], and Nb [7]. Consequently, shell model calculations for 96Nb were performed utilizing the SNET interaction and an expanded SNE configuration model space, which encompasses eight proton orbits $(1f_{5/2}, 2p_{3/2}, 2p_{1/2},$ $1g_{9/2}$, $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$) and nine neutron orbits $(1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 1g_{9/2}, 1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2})$ above an inert ⁵⁶Ni core. The single-particle energies utilized in this interaction were established as follows: $\varepsilon_{1f_{5/2}}^{\pi}$ =0.525 MeV, $\varepsilon^{\pi}_{2p_{3/2}}$ =1.228 MeV, $\varepsilon^{\pi}_{2p_{1/2}}$ =5.106 MeV, $\varepsilon^{\pi}_{1g_{9/2}}$ =5.518 MeV, $\varepsilon^{\pi}_{1g_{7/2}}$ =20.656 MeV, $\varepsilon^{\pi}_{2d_{3/2}}$ =20.016 MeV, $\varepsilon^{\pi}_{3s_{1/2}}$ =6.895 MeV, $\varepsilon^{v}_{1f_{5/2}}$ =0 MeV, $\varepsilon^{v}_{2p_{3/2}}$ =0 MeV, $\varepsilon^{v}_{2p_{1/2}}$ =0 MeV, $\varepsilon^{v}_{1g_{9/2}}$ =0 MeV, $\varepsilon^{v}_{1g_{9/2}}$ =0 MeV, $\varepsilon^{v}_{2d_{5/2}}$ =2.313 MeV, $\varepsilon^{v}_{2d_{3/2}}$ =3.440 MeV, $\varepsilon^{v}_{3s_{1/2}}$ =1.532 MeV, and $\varepsilon^{v}_{1h_{11/2}}$ =-0.589 MeV [44]. To en-

TABLE II: Main partitions of the wave functions for 96 Nb within the GWB model space. $\pi \otimes v$ represents $\pi(1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 1g_{9/2}) \otimes v(1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2})$

$\overline{E_{calc} (\text{keV})}$	J_i^{π}	Co	onfigurations $(\pi \otimes v)$
	ı	%	Leading config.
0	6_{1}^{+} 7_{1}^{+}	66.3	$(6,4,2,1) \otimes (0,5,0,0)$
170	7_1^+	11.9	$(6,4,0,3) \otimes (1,4,0,0)$
		10.1	$(6,4,0,3) \otimes (1,2,0,2)$
		7.8	$(4,4,2,3) \otimes (1,4,0,0)$
402	8_{1}^{+}	25.1	$(6,4,2,1) \otimes (1,4,0,0)$
	•	10.7	$(6,4,0,3) \otimes (1,4,0,0)$
1165	8_{2}^{+} 10_{1}^{+}	35.2	$(6,4,2,1) \otimes (1,4,0,0)$
1117	10^{+}_{1}	36.3	$(6,4,2,1) \otimes (1,4,0,0)$
	1	7.5	$(6,4,0,3) \otimes (1,4,0,0)$
		6.7	$(6,4,2,1) \otimes (1,3,0,1)$
1811	10^{+}_{2}	28.9	$(6,4,2,1) \otimes (1,4,0,0)$
2095	10_{2}^{+} 11_{1}^{+}	12.1	$(6,4,2,1) \otimes (1,4,0,0)$
	1	8.0	$(6,4,0,3) \otimes (1,3,0,1)$
		6.5	$(6,4,0,3) \otimes (1,4,0,0)$
2285	112+	20.9	$(6,4,2,1) \otimes (1,4,0,0)$
2406	11_{2}^{+} 12_{1}^{+}	47.1	$(6,4,2,1) \otimes (1,4,0,0)$
	1	7.8	$(6,4,0,3) \otimes (1,4,0,0)$
		5.7	$(6,2,2,3) \otimes (1,4,0,0)$
2670	12^{+}_{2}	9.3	$(6,4,2,1) \otimes (1,4,0,0)$
20,0	1-2	4.8	$(6,4,0,3) \otimes (1,4,0,0)$
2809	12^{+}_{3}	16.8	$(6,4,0,3) \otimes (2,3,0,0)$
2007	123	9.6	$(4,4,2,3) \otimes (2,3,0,0)$
3002	131+	17.8	$(6,4,0,3) \otimes (2,3,0,0)$
3002	131	9.4	$(4,4,2,3) \otimes (2,3,0,0)$
3482	14_{1}^{+}	19.5	$(6,4,0,3) \otimes (1,4,0,0)$
3402	171	8.0	$(4,4,2,3) \otimes (1,4,0,0)$
2058	8_{1}^{-}	25.9	$(4,4,2,3) \otimes (1,4,0,0)$ $(6,4,1,2) \otimes (1,4,0,0)$
2036	°1	7.9	$(5,4,2,2) \otimes (1,4,0,0)$ $(5,4,2,2) \otimes (1,4,0,0)$
2477	9_{1}^{-}	19.9	$(5,4,2,2) \otimes (1,4,0,0)$ $(6,4,1,2) \otimes (1,4,0,0)$
2411	1	6.5	$(6,4,1,2) \otimes (1,4,0,0)$ $(6,4,1,2) \otimes (1,3,0,1)$
2490	10_{1}^{-}	37.0	$(6,4,1,2) \otimes (1,3,0,1)$ $(6,4,1,2) \otimes (1,4,0,0)$
2490	101	8.1	$(6,4,1,2) \otimes (1,4,0,0)$ $(6,4,1,2) \otimes (0,4,1,0)$
2888	10_{2}^{-}	9.6	$(5,4,0,4) \otimes (1,2,0,2)$
2000	102	7.4	$(5,4,0,4) \otimes (1,2,0,2)$ $(5,4,0,4) \otimes (1,4,0,0)$
2500	11 ₁	37.2	$(5,4,0,4) \otimes (1,4,0,0)$ $(6,4,1,2) \otimes (1,4,0,0)$
2300	111	17.9	$(6,3,2,2) \otimes (1,4,0,0)$
2526	12-	52.5	$(6,4,1,2) \otimes (1,4,0,0)$
2320	12_{1}^{-}	10.1	$(6,4,1,2) \otimes (1,4,0,0)$ $(6,4,1,2) \otimes (1,2,0,2)$
3246	12-	36.5	
3240	13_	15.0	$(6,4,1,2) \otimes (1,4,0,0)$
		17.9	$(6,4,1,2) \otimes (1,3,0,1)$
2441	1.4=	12.1	$(6,3,2,2) \otimes (1,4,0,0)$
3441	14_{1}^{-}	48.6	$(6,4,1,2) \otimes (1,4,0,0)$
4500	15-	18.4	$(6,4,1,2) \otimes (1,3,0,1)$
4588	15_{1}^{-}	36.7	$(6,4,1,2) \otimes (1,4,0,0)$
		11.8	$(5,4,2,2) \otimes (1,4,0,0)$
4000	1.6-	9.3	$(6,4,1,2) \otimes (1,3,0,1)$
4822	16_{1}^{-}	49.0	$(6,4,1,2) \otimes (1,4,0,0)$
5005	15-	11.7	$(6,4,1,2) \otimes (1,2,0,2)$
5207	17_{1}^{-}	63.4	$(6,4,1,2) \otimes (2,3,0,0)$
6129	19_{1}^{-}	21.1	$(5,4,0,4) \otimes (2,3,0,0)$
		15.5	$(4,4,1,4) \otimes (2,3,0,0)$
6348	19_{2}^{-}	58.1	$(6,4,1,2) \otimes (2,3,0,0)$
		14.9	$(6,4,1,2) \otimes (2,2,0,1)$
6965	20_{1}^{-}	12.5	$(5,4,0,4) \otimes (2,2,0,1)$
		8.1	$(4,4,1,4) \otimes (2,2,0,1)$

Table II. (Continued)

$\overline{E_{calc}}$ (keV)	J_i^{π}	Configurations $(\pi \bigotimes v)$		
		%	Leading config.	
7039	20_{2}^{-}	16.8	$(5,4,0,4) \otimes (2,3,0,0)$	
	-	9.4	$(5,4,0,4) \otimes (2,2,0,1)$	
		8.7	$(4,4,1,4) \otimes (2,3,0,0)$	
7231	21_{1}^{-}	21.6	$(4,4,1,4) \otimes (2,3,0,0)$	
	•	19.7	$(5,4,0,4) \otimes (2,3,0,0)$	
8167	22_{1}^{-}	14.1	$(5,4,0,4) \otimes (2,2,0,1)$	
	•	10.4	$(5,4,0,4) \otimes (2,3,0,0)$	
8395	23_{1}^{-}	22.9	$(5,4,0,4) \otimes (2,3,0,0)$	
	•	17.6	$(5,4,0,4) \otimes (2,1,0,2)$	
		15.9	$(4,4,1,4) \otimes (2,3,0,0)$	
10136	25_{1}^{-}	54.1	$(4,4,1,4) \otimes (2,3,0,0)$	
		10.2	$(4,3,2,4) \otimes (2,3,0,0)$	
			· · · · · · · · · · · · · · · · · · ·	

sure computational tractability, consistent truncation schemes were systematically implemented across the model space. Specifically, proton excitations were limited to a maximum of four particles crossing the Z=38 subshell closure, while neutron excitations across the N=50 major shell gap were strictly prohibited in low-energy orbitals. Five neutrons are allowed to be excited from the $2d_{5/2}$ orbital to the $1g_{7/2}$, $2d_{3/2}$ and $3s_{1/2}$ orbitals. Furthermore, the configuration space allowed one neutron to occupy the high-spin $1h_{11/2}$ orbital.

Similarly, based on shell-model calculations using the SNET interaction, the high-spin states in 96 Nb identified in this study can primarily be attributed to three distinct mechanisms: (a) the excitation of a proton pair from the fully occupied $2p_{1/2}$ orbital to the $1g_{9/2}$ orbital; (b) the excitation of valence neutrons from the $2d_{5/2}$ orbital to the $1g_{7/2}$ orbital; and (c) the excitation of neutrons from the $2d_{5/2}$ orbital to the $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ orbits.

As shown in Fig. 4, the calculated excitation energies for the positive parity states are in reasonable agreement with the experimental results. The first discrepancy is that the energy gap between the 10_2^+ and 11_1^+ is not observed in the calculation results, with the difference being just 8 keV. Additionally, the difference between the 12_1^+ and 12_2^+ states is much larger than the experimental ones. The primary contributions to the 6_1^+ to 13_1^+ states obtained in the shell model arise from one proton in the $1g_{9/2}$ orbital coupled with five neutrons in the $2d_{5/2}$, $1g_{7/2}$, $2d_{3/2}$, and $3s_{1/2}$ orbitals. The calculations predict that the 14_1^+ state includes excitation of one proton from the $2p_{1/2}$ orbital into the $1g_{9/2}$ orbital, as well as the excitation of one neutron from the $1g_{7/2}$ or $2d_{5/2}$ orbital to the $1h_{11/2}$ orbital.

The calculated energy gaps between the negative states largely reproduce the energies of the observed γ -rays, with differences ranging from approximately 64 to 719 keV. It should be noted that the sequences of calculated 19^-_2 and 20^-_1 , 13^-_1 and 14^-_1 , 11^-_1 and 12^-_1 , 9^-_1 and 10^-_1 , as well as 8^-_1 and 9^-_1 are reversed compared to the experimental results. The calculated 8^-_1 to 14^-_1 states predominantly involve $\pi(1g_{9/2})^1 \otimes \nu[(2d_{5/2})^{-1}(1h_{11/2})^1]$ configurations. The 15^-_1 and 16^-_1 states share the same proton configuration as the

TABLE III: Main partitions of the wave functions for 96 Nb within the SNE model space. $\pi \otimes \nu$ represents $\pi(1f_{5/2}, 2p_{3/2}, 2p_{1/2}, 1g_{9/2}) \otimes \nu(1g_{7/2}, 2d_{5/2}, 2d_{3/2}, 3s_{1/2}, 1h_{11/2})$

${F$, $(\text{lea}V)$	J_i^{π}		Configurations ($\pi \otimes \nu$)
$\overline{E_{calc}}$ (keV)	$J_i^{}$	-%	Configurations $(\pi \bigotimes v)$ Leading config.
0	6.+	53.9	$\frac{(6,4,2,1)\otimes(0,5,0,0,0)}{(6,4,2,1)\otimes(0,5,0,0,0)}$
137	$6_{1}^{+} \ 7_{1}^{+} \ 8_{1}^{+} \ 8_{2}^{+}$	53.3	$(6,4,2,1) \otimes (0,5,0,0,0)$
819	8+	65.6	$(6,4,2,1) \otimes (0,4,0,1,0)$
1204	87	39.2	$(6,4,2,1) \otimes (1,4,0,0,0)$
	-2	9.7	$(6,4,0,3) \otimes (1,4,0,0,0)$
1833	10^{+}_{1}	38.9	$(6,4,2,1) \otimes (1,4,0,0,0)$
	1	16.5	$(6,4,2,1) \otimes (1,3,0,1,0)$
2455	10^{+}_{2}	26.1	$(6,4,2,1) \otimes (0,4,1,0,0)$
	2	22.1	$(6,4,2,1) \otimes (1,3,0,1,0)$
2463	11_{1}^{+}	53.2	$(6,4,2,1) \otimes (1,4,0,0,0)$
		10.5	$(6,4,0,3) \otimes (1,4,0,0,0)$
2927	11_{2}^{+}	43.6	$(6,4,2,1) \otimes (1,3,0,1,0)$
		10.5	$(6,4,0,3) \otimes (1,3,0,1,0)$
2614	12_{1}^{+}	58.1	$(6,4,2,1) \otimes (1,4,0,0,0)$
	1	10.7	$(6,4,0,3) \otimes (1,4,0,0,0)$
2976	12^{+}_{2}	58.5	$(6,4,2,1) \otimes (1,3,0,1,0)$
20.70		12.0	$(6,4,0,3) \otimes (1,3,0,1,0)$
3859	12_{3}^{+}	15.1	$(6,4,2,1) \otimes (1,3,0,1,0)$
		14.2	$(6,4,2,1) \otimes (2,3,0,0,0)$
2020	12+	12.7	$(6,4,0,3) \otimes (2,3,0,0,0)$
3838	13_{1}^{+}	25.2	$(6,4,2,1) \otimes (1,3,0,1,0)$
4067	1.4+	12.5	$(6,4,2,1) \otimes (2,3,0,0,0)$
4067	14_{1}^{+}	34.1 8.1	$(6,4,1,2) \otimes (0,4,0,0,1)$ $(6,4,1,2) \otimes (0,2,0,2,1)$
		7.4	$(6,4,1,2) \otimes (0,2,0,2,1)$ $(6,4,1,2) \otimes (2,2,0,0,1)$
1773	8_{1}^{-}	43.3	$(6,4,1,2) \otimes (2,2,0,0,1)$ $(6,4,2,1) \otimes (0,4,0,0,1)$
1773	01	6.1	$(6,4,0,3) \otimes (0,4,0,0,1)$
1688	9 ₁	44.7	$(6,4,2,1) \otimes (0,4,0,0,1)$
1000	1	11.3	$(6,4,2,1) \otimes (0,3,0,1,1)$
1402	10_{1}^{-}	39.1	$(6,4,2,1) \otimes (0,4,0,0,1)$
	- 1	7.5	$(6,4,2,1) \otimes (0,2,0,2,1)$
		6.3	$(6,4,0,3) \otimes (0,4,0,0,1)$
2049	10_{2}^{-}	43.0	$(6,4,2,1) \otimes (0,4,0,0,1)$
	_	15.3	$(6,4,2,1) \otimes (0,3,0,1,1)$
2360	11_{1}^{-}	37.5	$(6,4,2,1) \otimes (0,4,0,0,1)$
	•	21.3	$(6,4,2,1) \otimes (0,3,0,1,1)$
2249	12_{1}^{-}	32.4	$(6,4,2,1) \otimes (0,4,0,0,1)$
		19.4	$(6,4,2,1) \otimes (0,3,0,1,1)$
3269	13_{1}^{-}	32.2	$(6,4,2,1) \otimes (0,4,0,0,1)$
2929	14_{1}^{-}	30.8	$(6,4,2,1) \otimes (0,4,0,0,1)$
		24.5	$(6,4,2,1) \otimes (0,3,0,1,1)$
3668	15_{1}^{-}	32.2	$(6,4,2,1) \otimes (1,3,0,0,1)$
		12.0	$(6,4,0,3) \otimes (1,3,0,0,1)$
41.4.4	17=	7.5	$(6,4,2,1) \otimes (1,2,0,1,1)$
4144	16_{1}^{-}	34.3	$(6,4,2,1) \otimes (1,3,0,0,1)$
1611	17-	13.3	$(6,4,0,3) \otimes (1,3,0,0,1)$
4644	17_{1}^{-}	25.3	$(6,4,2,1) \otimes (1,2,0,1,1)$
6404	10-	22.8	$(6,4,2,1) \otimes (1,3,0,0,1)$
6404	19_{1}^{-}	18.4 12.0	$(6,4,0,3) \otimes (1,3,0,0,1)$
6712	19_{2}^{-}	12.0	$(6,4,0,3) \otimes (1,2,0,1,1)$ $(6,4,2,1) \otimes (1,2,1,0,1)$
0/12	192	10.3	$(6,4,2,1) \otimes (1,2,1,0,1)$ $(6,4,2,1) \otimes (2,2,0,0,1)$
		10.3	$(6,4,2,1) \otimes (2,2,0,0,1)$ $(6,4,0,3) \otimes (2,2,0,0,1)$
6658	20_{1}^{-}	24.1	$(6,4,0,3) \otimes (2,2,0,0,1)$ $(6,4,0,3) \otimes (2,2,0,0,1)$
2000	1	12.8	$(6,4,2,1) \otimes (2,2,0,0,1)$
			(,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,

Table III. (Continued)

$\overline{E_{calc} \text{ (keV)}}$	J_i^{π}	C	Configurations $(\pi \otimes v)$		
		%	Leading config.		
6780	20_{2}^{-}	17.6	$(6,4,0,3) \otimes (0,3,0,1,1)$		
	-	12.5	$(6,4,0,3) \otimes (0,4,0,0,1)$		
6795	21_{1}^{-}	29.9	$(6,4,0,3) \otimes (1,3,0,0,1)$		
		9.1	$(6,4,0,3) \otimes (1,2,0,1,1)$		
7278	22_{1}^{-}	30.8	$(6,4,0,3) \otimes (1,3,0,0,1)$		
		5.7	$(6,4,0,3) \otimes (1,2,0,1,1)$		
		5.5	$(4,4,2,3) \otimes (1,3,0,0,1)$		
7709	23_{1}^{-}	30.2	$(6,4,0,3) \otimes (1,2,0,1,1)$		
	1	16.0	$(6,4,0,3) \otimes (1,3,0,0,1)$		
9376	25^{-}_{1}	28.2	$(6,4,0,3) \otimes (2,2,0,0,1)$		
		18.1	$(6,4,0,3) \otimes (1,2,1,0,1)$		

lower-lying negative parity states but involve neutron excitation from the $2d_{5/2}$ orbital to the $1g_{7/2}$ orbital. Except for the 19_2^- state, the 19_1^- to 25_1^- states involve the excitation of two protons from the $2p_{1/2}$ orbital to the $1g_{9/2}$ orbital. The calculated results in the SNE space show that there is no prediction of proton excitation across the shell gap at Z=38.

The overall agreement between the observed states and those predicted by shell model calculations suggests that excitations across the Z=38 subshell gap do not play a significant role up to $I \sim 17\hbar$. However, the higher-spin levels above $I \sim 17\hbar$ are dominated by the excitation of protons over the shell gap at Z=38 into the $1g_{9/2}$ orbital, as shown in Table II. Although the overall agreement between the calculated and measured levels is still far from satisfactory, it is noteworthy that calculations with the GWBXG interaction reproduce the positive parity states well, while those with the SNET interaction match the negative parity states effectively. Our calculations with any single interaction cannot accurately reproduce both positive and negative parity states simultaneously. The observed discrepancies likely arise from outdated interaction models within this mass region, necessitating the recalibration of two-body matrix elements and single-particle energy parameters [25, 41, 45–47]. We believe that the current dataset will assist in developing improved effective interactions tailored for modeling high-spin nuclear states within this regime.

V. CONCLUSIONS

High-spin states of ⁹⁶Nb were populated via the ⁸²Se(¹⁸O,p3n)⁹⁶Nb reaction at beam energies of 82 and 88 MeV. The previously established level scheme of 96Nb has been significantly expanded with the identification of fourteen new γ rays. Based on the beam-target combinations with the highest neutron-to-proton ratios, most of the γ rays have been assigned spin and parity using ADO and polarization analysis. Shell-model calculations were conducted using two distinct interactions and model spaces to analyze the experimental data for ⁹⁶Nb. Despite the overall agreement being insufficient, the GWBXG interaction notably aligns with positiveparity states, whereas SNET corresponds better to negativeparity configurations. Current calculations show that no single interaction can precisely reproduce both parity systems simultaneously. The present calculations provide a partial interpretation of the ⁹⁶Nb spectra. While we cannot definitively assign the observed energy levels, both model spaces indicate that the level sequences primarily reflect single-particle behavior. Valence nucleon excitations outside the ⁸⁸Sr core dominate the lower and intermediate levels, whereas highangular-momentum states are driven mainly by proton excitations across the Z=38 subshell. To improve the predictive accuracy of the shell model, it is essential to develop optimized effective interactions for characterizing high-spin nuclear configurations in heavier nuclei.

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