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Obtaining a Potential Game Through Pricing, to Achieve Optimal Resource Management in Non-cooperative Cognitive Radio Networks

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Abstract Optimal resource management in a cognitive radio network has been studied using the game theory. Based on personal interests, users select their own desired utility function and compete for channel and power selection. This non-cooperative approach is controlled through an appropriate pricing method. We have shown that if the utility function in a cooperative potential game is used as the pricing function in a non-cooperative network, the game governing the non-cooperative network will also become potential and will thus converge to Nash equilibrium. The existence of selfish users will cause the network to be unstable, the one which has presumptively been designed with users' cooperation. Besides, it decreases resource utilization gain. Using the recommended pricing has been shown to equilibrate the network. The equilibrium points also enjoy some optimality criteria such as Pareto optimality. By conducting simulations and studying parameters like sum-rate of network and its total interference, it is shown that resource utilization will also approaches to optimal states.

Keywords Cognitive radio network · Pricing · Distributed resource access · Non-cooperative users · Potential games · Pareto optimality

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1 Introduction

One of the important abilities of cognitive radios (CRs) is the continuous observation of the spectrum bands and the possibility of finding a band fit for establishing the intended communication link. A cognitive radio will thus be capable of using a spectrum band whose application license is held by another user (provided that it will not disturb the licensed users). For a typical CR network, the Signal to Interference + Noise Ratio (SINR) seen by any user should be high enough. Besides, the *interference temperature*, defined as total interference generated in frequency band of interest by unlicensed users, should not exceed the permissible level.

In this paper, management and optimization of a typical non-cooperative cognitive radio network is studied by means of the game theory. Using this mathematical tool, many types of cognitive radio network taxonomies and relative problems are modelled, analyzed and optimized [1, 2].

In a non-cooperative network, users are not necessarily required to comply with a control center or establish their link through it, and they may only follow a distributed resource access algorithm, which means they are almost free to choose their strategy by only considering their personal preferences. This work is dedicated to study on a network in which, all users have same rights, and the network lacks any licensed radio. Such networks are called CR dedicated networks or *Spectrum Commons* [3, 4].

A detailed clarification about different taxonomies of these kinds of CR networks is offered in [3], and some implementing problems are mentioned. Some key features of the management protocol of such a network are specified in [4]. In [5], a MC Protocol presented, which is based on the exchange of RTS/CTS packets similar to IEEE.802.11 standard. The authors have introduced utility functions, which if observed by the network users in their process of strategy selection, they'd be induced to react cooperatively. A similar utility function is also introduced by the authors in [6]. However, one of the drawbacks of the schemes presented in [5] and [6] is the imposing of the utility function type on the network users which requires high level of cooperation between users and thus is not implementable in a non-cooperative network.

In distributed resource access networks, pricing is used as a tool to decrease the selfish behavior of users. A linear pricing method has been employed in [7] and [8]. The advantages of this pricing method are mentioned to be its understandable meaning and capability to be implemented in centralized or distributed manners. By calculating an appropriate pricing factor in [8], suboptimal Nash Equilibriums (NEs) are achieved regarding Pareto optimality criterion. A more advanced pricing method is the adaptive pricing in which the pricing factor for each user is calculated separately and thus the pricing factor of a user would be differing from those of others [9]. These price calculations are performed in order to converge to the NEs with desired optimality characteristics. This method helps the equilibriums of the game to be closer to Pareto frontier. The proposed algorithm in [9] is a price-based water filling algorithm and is capable to be implemented in networks with distributed resource access and non-cooperative users. Another type of adapted pricing, also based on the water filling algorithm, has been offered in [10]. A different pricing approach is adopted in [11]. In [11], each user will calculate and state the price which other users should pay regarding their transmission power and the interference they make to that user. This algorithm is also possible to be implemented in distributed manner and optimizes the network in relation to Pareto optimality criterion.

In this paper we are dealing with potential games. The concept of potential games (in exact or ordinary forms) is introduced in [12]. Researchers are interested in the potential games because the convergence to Nash equilibrium in limited steps is guaranteed [13]. We first consider a cooperative CR network in which each user utilizes a pre-defined utility function that provides an exact potential game. We will then prove that using such utility function as the pricing function for another non-cooperative network, make the game to be an ordinary potential one. Specifically, the utility function presented in [5] is used as the pricing function of our non-cooperative network. Assuming that each user selects its own, personal utility function based on its preferences, the final utility function of each user will be obtained as the sum of his personal utility function and the pricing function. Despite the algorithm presented in [5], our method is thus capable of being employed in a non-cooperative network. The numerical and simulation based studies on our network, show the Pareto optimality and an appropriate level of total interference and sum-rate in all NEs of the network.

The paper has been organized as follows:

In Sect. 2, the general concepts of the game theory and its capabilities in design of a cognitive radio network have been expressed. In Sect. 3, the network structure has been studied. In Sect. 4, the network modeling using the game theory has been mentioned. In Sect. 5, simulation results and the advantages obtained due to the proposed pricing have been dealt with and finally, In part 6, the paper conclusion is offered.

2 Game Theory

The game theory is a mathematical tool which can well model interactions between some decision making individuals or groups. This theory has many applications in micro economy and has recently applied in engineering as well. It has also many applications in management, design and analysis of CR networks including the design of resource allocation algorithms and the MAC protocol [14]. The reasons for which the game theory has enjoyed significant importance in the study of CR networks can be enumerated as follows [15]:

- Ability to solve multipurpose problems (there are many tools, theorems and criteria in game theory to study and even measure the optimality of equilibrium points, in which the utility of many individuals or groups should be considered).
- The existence of the concept of non-cooperative games in the game theory which provides the possibility of studying and analyzing distributed and non-cooperative networks.

In a CR network, users usually act based on their personal preferences to fulfil their selfish objectives in question. The game theory lets us to attach a utility function to each user and have thus a model for each user's strategy adaptation. The study of the network equilibrium points (if there exists any) is among the network principal's and designer's considerations. In a CR network designed by means of game theory, optimal equilibriums in which the network enjoys appropriate communicational specifications are the best signs of efficient resource allocation.

3 Game-Theoretic Approach for Network Designing

3.1 Network Structure

Our intended network is a CR network consisting of K pairs of users able to both transmit and receive. Users have been randomly distributed in a $l^m \times l^m$ space. The distance between a user transmitter and receiver (a pair) has been assumed to be less than d_{max}. It is assumed that the moving speed of users is slow and the network topology is approximately fixed during the convergence time of network.

Data packets are transmitted in M channels. The users' transmission power lies between a maximum and minimum value. The network has also a shared control channel in which users can transmit their own signaling packets and inform others of their status. Having such a facility is essential in networks with no central controller, in order to utilize the network's resources efficiently.

The algorithm presented in [6] and pricing methods proposed in [9] and [8], are assumed to have access to such signaling data in control channel. The algorithm proposed in this paper is also feasible in terms of capability to be implemented; only needing users' local information which is reachable via the control channel.

3.2 Game Definitions

Interactions between the network users to select the channel and transmission power are here defined in the form of a game $G = (N, \mathbf{S}, \{u_i\})$ where $N = \{1, 2, \ldots, K\}$ is a limited set consisting of K players with rational behavior (a rational player make decisions to increase its utility function not to decrease that's of others). S is action space of all players and the strategy of $\mathbf{s} \in \mathbf{S}$ is defined as:

$$\mathbf{s} = \begin{bmatrix} s_{11} & \cdots & s_{1K} \\ \vdots & \ddots & \vdots \\ s_{M1} & \cdots & s_{MK} \end{bmatrix} \quad (1)$$

where M is the number of channels and s_{ij} denotes the power which the jth user transmits through the ith channel. If s_{ij} is zero, it means that no power is transmitted by the jth user through the ith channel. It is also assumed that $s_{ij} \in \{0 \cup [s_{min}, s_{max}]\}$. The ith column of matrix S is called $\mathbf{s_i}$ and represents the strategy adopted by ith player. Each player can only transmit power through one channel. In another word, in each $\mathbf{s_i}$, there is only one non-zero element which we call P_i. Also, $u_i(.) : \mathbf{S} \to \mathbb{R}$ is the ith players utility function.

Consider a game in which users' utility functions are in the form of $-P(\cdot)$ which is defined in Eq. 2

$$P(\boldsymbol{s}_i, \boldsymbol{s}_{-i}) = \sum_{\substack{j=1 \\ j \neq i}}^{N} P_j g_{ji} F(\boldsymbol{s}_i, \boldsymbol{s}_j) + \sum_{\substack{j=1 \\ j \neq i}}^{N} P_i g_{ij} F(\boldsymbol{s}_i, \boldsymbol{s}_j) \quad (2)$$

In which $F(\boldsymbol{s}_i, \boldsymbol{s}_j)$ is defined as:

$$F(\boldsymbol{s}_i, \boldsymbol{s}_j) = \begin{cases} 1 & : i, j \text{ are in the same channel} \\ 0 & : i, j \text{ are in different channels} \end{cases} \tag{3}$$

where g_{ij} is indicative of the channel gain between the ith transmitter and the jth receiver. The channel is assumed to be time invariant and the values of g_{ij} are uniquely obtained knowing the network topology. In Eq. 2, $\boldsymbol{s}_{-i}$ is the matrix of the strategies of all the users except for the ith one. $P(\boldsymbol{s}_i, \boldsymbol{s}_{-i})$ represents sum of the interference made by the ith user to the neighbouring users as well as sum of interference made by neighbouring users to it. $-P(\cdot)$ can be introduced by the network principal as an appropriate utility function to be adopted by all users of a cooperative network. A network in which all the users adopt $-P(\cdot)$ as their utility function is studied in [5]. It has been shown that in this state, the game governing the network becomes potential and quickly convergent toward the Nash equilibrium. Meanwhile, in the studied equilibria, the network users' SIR level has been optimized. In another word, $-P(\cdot)$ is an appropriate utility function which will lead to very suitable results in case of users' cooperation. In a non-cooperative environment, selfish users may select their strategy based on different utility functions because of their different personal preferences. $SIR(.) : S \to \mathbb{R}$ function, defined in Eq. 4, can be considered as an example of the network selfish users' utility function.

$$SIR_i = \frac{P_i g_{ii}}{\sum_{\substack{k=1 \\ k \neq i}}^{N} (P_k g_{ki})} \tag{4}$$

The numerator in $SIR(\cdot)$ function is the sign of the selfishness of these users in strategy selection. Based on the numerator in $SIR(\cdot)$ function, the user who selects $SIR(\cdot)$as his utility function in fact selects the highest possible transmission power, though this issue causes the making of more interference to other users.

Now, consider the game $G_1 = (N, \mathbf{S}, \{u_i\})$, in which the users with a cooperative attitude adopt $u_i(\cdot) = -P(\cdot)$as their utility function and the selfish users use their own personal utility functions. The network relevant to game G_1 is called **Net-1** hereafter. It is obvious that the advantages mentioned in [5] do not exist in **Net-1.** By adding a certain method of pricing to **Net-1**, we will consider another model and call it **Net-2**. $G_2 = (N, \mathbf{S}, \{u_{t_i}\})$ shows the game governing this network in which users are assumed to be able to freely select their personal utility functions. These functions which form a part of $u_{t_i}(.)$ will be explicitly explained in Sect. 3.4.

In order to compare two mentioned networks, the characteristics of their Nash equilibrium points will be compared. The first step for this purpose is to be assured of the convergence of the two games, or networks, toward Nash equilibrium. Evidently, in a game, which does not converge toward Nash equilibrium and in which the players are changing their strategies continuously, it is hard to provide adequate information needed to make comparisons. In the most types of potential games and under almost general circumstances the convergence of this class of games has fortunately been guaranteed.

3.3 Potential Game

Convergence of CR networks toward an equilibrium point can provide the network with adaptability. A network which quickly converges toward equilibrium can well adapt itself to changes it incurs [13]. Besides, the existence of Nash equilibrium will allow the game designer to predict the game states and apply optimization methods to promote the characteristics of equilibriums.

Broad classes of Potential game can provide us with convergence property under certain circumstances which will be discussed in the following sessions. For the first time in [16], the concept of potential games was used in the field of CR networks, followed afterwards with many applications [6, 12, 13]. Two important classes of potential games are Exact Potential and Ordinal Potentials. $PG = (N, \mathbf{S}, \{u_i\})$ is called an Exact Potential game, in which there exists function $v : \mathbf{S} \to \mathbb{R}$ so that the following condition is fulfilled:

$$\begin{aligned} &\forall i \in N, \forall \mathbf{s} \in \mathbf{S} : \\ &\quad u_i\left(\mathbf{s_i^*}, \mathbf{s_{-i}}\right) - u_i(\mathbf{s_i}, \mathbf{s_{-i}}) = v\left(\mathbf{s_i^*}, \mathbf{s_{-i}}\right) - v(\mathbf{s_i}, \mathbf{s_{-i}}) \end{aligned} \tag{5}$$

PG is named an Ordinal Potential game if:

$$\begin{aligned} &\forall i \in N, \forall \mathbf{s} \in \mathbf{S} : \\ &\quad sign\left[u_i\left(\mathbf{s_i^*}, \mathbf{s_{-i}}\right) - u_i(\mathbf{s_i}, \mathbf{s_{-i}})\right] \\ &\quad = sign\left[v\left(\mathbf{s_i^*}, \mathbf{s_{-i}}\right) - v(\mathbf{s_i}, \mathbf{s_{-i}})\right] \end{aligned} \tag{6}$$

The function $v(\cdot)$ is so called The Potential function. The Potential function for a network indicates the networks progress toward Nash Equilibrium [12].

It has been shown that all SIR-based power allocation games are Ordinal Potential [13, 17]. Interference control games, based on the utility function $-P(\cdot)$, are also proved to be Exact Potential game [13]. The potential function in this case is as follows:

$$v(\mathbf{s}) = -0.5 \sum_{i=1}^{K} P(\mathbf{s_i}, \mathbf{s_{-i}}) \tag{7}$$

note that, a game in which users separately choose one of the $-P(\cdot)$ or $(\cdot)$, as the utility functions, (like G_1) is not necessarily even an Ordinal Potential game [13].

3.4 The Proposed Pricing

Pricing can be used as a tool to persuade users to cooperate, guarantee the convergence and promote the network equilibriums optimality [8, 9]. **Net-1** and **Net-2** are non-cooperative networks. Assume that $\alpha P(\cdot)$ is applied as pricing function in **Net-2** (α is a large enough positive constant pricing factor). Function $u_{t_i}(.)$, as the ith player's utility function in G_2, will thus be as follows:

$$u_{t_i}(\mathbf{s_i}, \mathbf{s_{-i}}) = -\alpha P(\mathbf{s_i}, \mathbf{s_{-i}}) + u_{p_i}(\mathbf{s_i}, \mathbf{s_{-i}}) \tag{8}$$

where $u_{p_i}(.)$ is the personal utility function of the ith user. If he cooperatively plays, $u_{p_i}(\cdot) = 0$. Otherwise, it equals to users preferred utility function. Existence of pricing term $-\alpha P(.)$ in utility function of each user would push them to decrease the interference they cause and the interference they receive. This is the same with inducing a type of cooperation between users. We have proved in Theorem 1 that the game governing **Net-2** (i.e. G_2) is an Ordinal Potential game. Furthermore, simulations show that the network equilibrium points are considerably more appropriate than those of **Net-1**.

Theorem 1 *Using the utility function of an exact potential game G as the pricing function (with a proper multiplicative factor) of an arbitrary game G', makes G' an ordinary potential game.*

Proof Assume $G = (N, \mathbf{S}, \{u_i\})$ be an exact Potential Game with its potential function defined as $Pot(.) : \mathbf{S} \to \mathbb{R}$. Thus:

$$\begin{aligned} &\forall i \in N, \forall \mathbf{s} \in \mathbf{S} : \\ &\quad u_i\left(\mathbf{s}_\mathbf{i}^*, \mathbf{s}_{-\mathbf{i}}\right) - u_i(\mathbf{s}_\mathbf{i}, \mathbf{s}_{-\mathbf{i}}) = Pot\left(\mathbf{s}_\mathbf{i}^*, \mathbf{s}_{-\mathbf{i}}\right) - Pot(\mathbf{s}_\mathbf{i}, \mathbf{s}_{-\mathbf{i}}) \end{aligned} \tag{9}$$

Also assume $G' = \left(N, \mathbf{S}, \left\{u_{pi}\right\}\right)$ as the game with selfish users. By applying $-\alpha u_i(\cdot)$ as the pricing function in G', the total payoff function in G' can be figured as follows:

$$u_{t_i}(\mathbf{s}_\mathbf{i}, \mathbf{s}_{-\mathbf{i}}) = \alpha u_i(\mathbf{s}_\mathbf{i}, \mathbf{s}_{-\mathbf{i}}) + u_{p_i}(\mathbf{s}_\mathbf{i}, \mathbf{s}_{-\mathbf{i}}) \tag{10}$$

Regarding the general equation below

$$|a| > |b| \to Sign(a + b) = Sign(a) \tag{11}$$

And by substituting $a = \alpha u_i\left(\boldsymbol{s}_{\boldsymbol{i}}^*, \boldsymbol{s}_{-\boldsymbol{i}}\right) - \alpha u_i(\boldsymbol{s}_{\boldsymbol{i}}, \boldsymbol{s}_{-\boldsymbol{i}})$ and $b = u_{p_i}\left(\boldsymbol{s}_{\boldsymbol{i}}^*, \boldsymbol{s}_{-\boldsymbol{i}}\right) - u_{p_i}(\boldsymbol{s}_{\boldsymbol{i}}, \boldsymbol{s}_{-\boldsymbol{i}})$ and supposing a positive and sufficiently large α, for $|a| > |b|$ to be true for all i, it can be noted that

$$\begin{aligned} &\forall i \in N, \forall \mathbf{s} \in \mathbf{S} \\ &\quad sign\left(u_{t_i}\left(\boldsymbol{s}_{\boldsymbol{i}}^*, \boldsymbol{s}_{-\boldsymbol{i}}\right) - u_{t_i}(\boldsymbol{s}_{\boldsymbol{i}}, \boldsymbol{s}_{-\boldsymbol{i}})\right) \\ &\quad = sign(a + b) = sign(a) \\ &\quad = sign\left(\alpha u_i\left(\boldsymbol{s}_{\boldsymbol{i}}^*, \boldsymbol{s}_{-\boldsymbol{i}}\right) - \alpha u_i(\boldsymbol{s}_{\boldsymbol{i}}, \boldsymbol{s}_{-\boldsymbol{i}})\right) \\ &\quad = sign(\alpha) sign\left(u_i\left(\boldsymbol{s}_{\boldsymbol{i}}^*, \boldsymbol{s}_{-\boldsymbol{i}}\right) - u_i(\boldsymbol{s}_{\boldsymbol{i}}, \boldsymbol{s}_{-\boldsymbol{i}})\right) \\ &\quad = sign\left(Pot\left(\boldsymbol{s}_{\boldsymbol{i}}^*, \boldsymbol{s}_{-\boldsymbol{i}}\right) - Pot(\boldsymbol{s}_{\boldsymbol{i}}, \boldsymbol{s}_{-\boldsymbol{i}})\right) \end{aligned} \tag{12}$$

Thus G' will be an Ordinal Potential game.

Now, consider **Net-1** and the relevant game G_1 which is based on the utility function $-P(\cdot)$. G_1 is proved to be an exact potential game in [13]. Therefore according to theorem 1, by applying the proposed pricing to G_2, it will be an ordinary potential game.

Although G_2 will definitely be an Ordinal Potential game, the convergence of such a game steel needs to be proved which is presented in "Appendix".

3.5 Network Resource Access

In order to access to network resources in a distributed network a shared control channel has been considered among all the users which allows them to access information such as the index of busy channels and the power transmitting in each channel [4]. The existence of control channel data allows a cognitive radio to calculate $P(\cdot)$.

The users in the network are also considered to obey best response decision rule. Best response decision rule is obeyed in a game if:

$$\forall i \in N, \boldsymbol{s} \in \boldsymbol{S} : u_i(nxt_i(\boldsymbol{s}_{-\boldsymbol{i}}), \boldsymbol{s}_{-\boldsymbol{i}}) \geq u_i(\boldsymbol{s}_{\boldsymbol{i}}, \boldsymbol{s}_{-\boldsymbol{i}}) \tag{13}$$

where $nxt_i(\cdot)$ represents a function by which ith user determines its next strategy, knowing $\boldsymbol{s}_{-\boldsymbol{i}}$.

Users have been considered to be able to change their strategy in multiple steps. The game will continue as long as at least one player can increase its utility function. Once the game reaches Nash equilibrium, none of the players change its strategy anymore. Nash equilibrium has been shown to enjoy less optimality in the single-shot games compared with dynamic multiple-step games concerning several criteria [18].

The rule by which the times of deviation and the deviator users are determined is called the timing rule of a game. Two types of timing rules including Round-Robin and Asynchronous timing to access the control channel have been introduced in [6]. In the Round-Robin method, as a deterministic algorithm, by making a circular queue, in each step only one user is allowed to change his strategy. This method is implementable only when the network enjoys having a central controller. In the Asynchronous method, each user of the network will be allowed to change its strategy with the probability of $\frac{1}{K}$ by performing Bernoulli trial (k is the number of the network users). This method is more suitable for distributed network management [6]. Although this method slows down the convergence speed compared with the Round-Robin, it is also effective in decreasing the portability of multilateral deviation in one step and is applied in our simulations as the governing timing rule of the network.

3.6 MAC Protocol

The applied MAC protocol in this work is inspired by the one presented in [5]. In this protocol a status table, including information on the power, the channel selected by each user and the channel gains is provided and is accessible by each user via the control channel. Each transmitter performs Bernoulli trial, if the test was successful, using the status table, the transmitter calculates his own utility function for all the possible strategies and selects the one with best utility. Upon the selected strategy, the mentioned table should be updated by the winning user. Winning in the Bernoulli trial and changes in the status table will be announced to other users via appropriate exchange of the signalling packets.

3.7 Optimality

Having a game with lots of equilibrium points will pose the question that which of these equilibria outperforms others. In cognitive radio networks, criteria such as the network capacity [6], the network total interference [5], the network total SIR, sum-rate [9, 10], weighted sum-rate [19] and the network convergence speed [9] have been considered to study the network efficiency. While mathematical tools of game theory [8, 9, 14, 20] provides us with some other criteria enabling us to evaluate the optimality of the system in request. Some of those game theoretical criteria can be mentioned as: the total utility function of users [8], the players utility weighted sum [9] and Pareto optimality [8, 14] or using the concept of Stackelberg games [20]. We are steel able to utilize some criteria using both game theory tools and communicational parameters to evaluate the equilibrium point efficiency like Network Utility proposed in [21].

3.7.1 *Pareto Optimal Equilibrium*

Assuming that $u_i : \mathbf{S} \to \mathbb{R}$ is the ith player utility function ($i \in N = \{1, 2, \ldots, K\}$) and set $\mathbf{S}^* \subseteq \mathbf{S}$ is the Nash equilibrium set, equilibrium $\mathbf{s}^* \in \mathbf{S}^*$ is called a Pareto optimal equilibrium, if one cannot find another equilibrium like $\mathbf{s}$ so that $\forall i \in N : u_i(\mathbf{s}) > u_i(\mathbf{s}^*)$.

The Pareto optimal frontier in a game is an equilibrium set in which all the equilibria are Pareto optimal. In games with some non-Pareto-optimal solutions, the game designer tries to lead the game toward Pareto optimal equilibria by employing techniques such as pricing, designing a repeated game, designing a game with correlated equilibrium and etc. [14]. If all the equilibria of a game are on Pareto optimal frontier, it means that no equilibrium in this game outperform others which is indicative of efficiency of the rules governing the game.

3.7.2 Interference Minimizer Equilibrium

It is proved in [12] that given the potential game $G = (N, \boldsymbol{S}, \{u_i\})$ with $v(\cdot)$ as its potential function, the global maximizers of $v(\cdot)$ are Nash equilibria points of G. This theorem in the proposed network results in having the interference minimizer equilibrium in the equilibria set of the network. Furthermore, our network which is governed by an ordinal potential function as being introduced in (7), enjoys decrease of total interference in transient points.

4 Simulation Results

4.1 Effect of Proposed Pricing on Network Efficiency

In this part, the behaviour of two networks **Net-1** and **Net-2** is studied using numerical simulations. Due to the existence of selfish users, **Net-1** doesn't converge to NE in some cases. To be able to perform a meaningful comparison, Net-1 is compared with Net-2 when the convergence in Net-1 has been occurred. The number of users with non-cooperative behaviour is the same in both games G_1 and G_2, equalling 50 % of total users. In G_2 the $SIR(\cdot)$ function has been considered as the personal utility function of selfish users to model their non-cooperative behaviours in the network. The network structure was described in Sect. 3.1. $l = 400^m$, $M = 4$ and $K = 30$ have been assumed in simulations. d_{max} has also been considered equal 50^m. The values of s_{min} and s_{max} are 1 and 2 respectively. Users perform Bernoulli trial as discussed in Sect. 3.6 to see if they are permitted to change their strategy. Figure 1 shows the result of the **Net-2** and **Net-1** simulation under perfectly similar conditions. In this figure, each curve is indicative of the variations of the probable potential function of these two games. As Eq. (7) reveals, the game potential function size

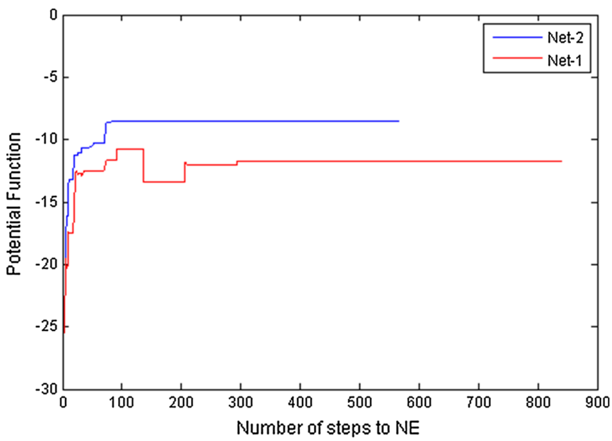


Fig. 1 The progression of v(s) in Net-2 and Net-1

in each step is proportional to the negative of the sum of the network users total interference. In a potential game, the potential function value increases continuously. As shown in Sect. 3.4, the game G_2 is an ordinary potential game which is also confirmed by Fig. 1. However, based on this figure, G_1 is not a potential game.

Furthermore, this figure shows the efficiency of **Net-2** in comparison with **Net-1** regarding total interference of network users.

Figure 2 shows the behaviour of these two networks in 200 times of simulations in which the order of Bernoulli trials winners randomly changes. Many big fluctuations are occurred in **Net-1** compared with **Net-2**, before convergence, in transient state. Besides, the potential function value of **Net-2** in most points is seen to be larger than that of **Net-1**. Also **Net-2** enjoys a faster convergence. In order to assess the effect of the proposed pricing function, another pricing function, being introduced in [8], was applied instead of our suggested pricing function in **Net-2** and its results are illustrated in Fig. 2 under the name of **Net-3** with green colour. Making comparison between results of these two pricing functions lets us to conclude that although pricing can be considered as a tool to lead the network equilibria to enjoy more efficiency, the pricing function being implemented in a network should be tailored for its taxonomy and special conditions.

All these points demonstrate the proposed method efficiency. The poor performance of **Net-1** is more emphasized when the frequency of changes in the network topology or the number of active users increases. In such conditions, network users spend a considerable time in transient states, based on which the importance of the network performance in these states will be multifold.

The diagrams of Fig. 3 have been presented to better express the information existing in Fig. 2. In Fig. 3, the distribution of the total interference of network users is displayed for four cases, including distribution of total interference in transient points of **Net-1** and **Net-2** and distribution of total interference in equilibrium points of **Net-1** and **Net-2**. Table 1 is also brought out to support and clarify this figure. Regarding Fig. 3 and Table 1, it can be seen that on average, in terms of the total interference parameter, **Net-2** has a performance 25–30 % better than **Net-1**. Fig 4 shows the distribution of sum-rate in same four cases

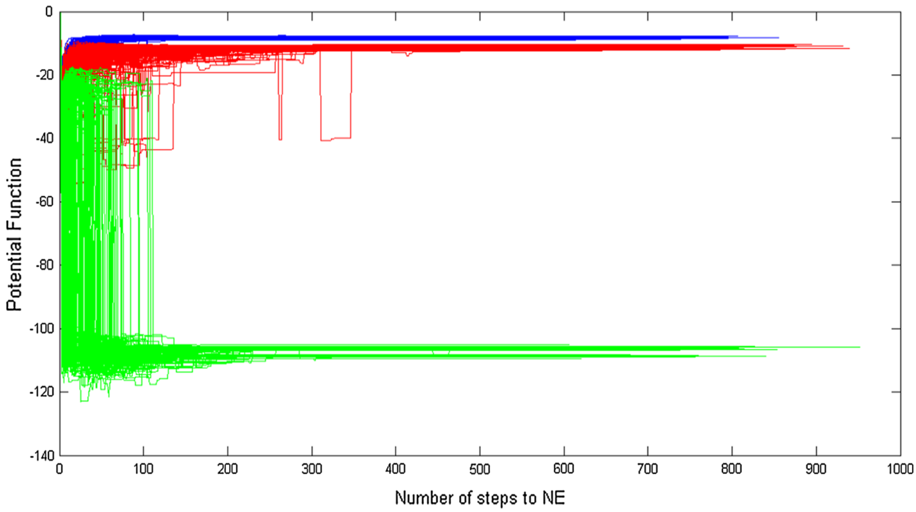


Fig. 2 Inspecting convergence and Total Interference of the three networks in 200 steps with randomly winning users in each Bernoulli trial

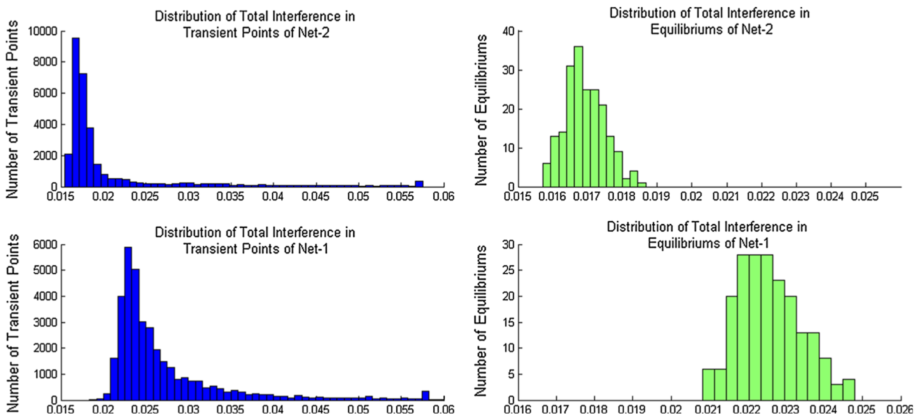


Fig. 3 Comparison between distribution of total interference in transient points and equilibriums of Net-1 and Net-2 with randomly winning users in each Bernoulli trial

Table 1 Comparison of distribution of total interference and sum-rate in transient points and equilibriums of Net-1 and Net-2 with randomly winning users in each Bernouli trial

	Net-2		Net-1	
	Transient points	Equilibrium points	Transient points	Equilibrium points
Total interference				
Mean	0.0208	0.0169	0.0271	0.0226
Median	0.0176	0.0169	0.0245	0.0225
Mode	0.0172	0.0168	0.0233	0.0221
Range	0.0421	0.0030	0.0416	0.0040
Variance	7×10^{-5}	$3\ 10^{-7}$	5×10^{-5}	6×10^{-7}
Sum-rate				
Mean	16.04	16.43	14.35	14.82
Median	16.44	16.44	14.53	14.85
Mode	16.64	16.83	15.03	14.96
Range	9.13	1.88	9.65	1.44
Variance	1.77	0.1214	0.88	0.07

inspected in Fig. 3. Sum-rate is defined using (14) and is used as a measure to study the networks efficiency [12].

$$\mathit{Sum}\text{−}\mathrm{Rate} = \sum_{i=1}^{K} \log_2 \left(\frac{P_i G_{ii}}{\sum_{\substack{k=1 \\ k \neq i}}^{N} P_k G_{ki}} \right) \tag{14}$$

As is shown in Fig. 4, considering sum-rate as an optimality criterion, both at the network equilibrium points and transient points, **Net-2** has on average 13 % better performance in comparison with **Net-1**, though the variation of this parameter is slightly higher than **Net-1**. In order to study the stability of these two networks relative to the users'

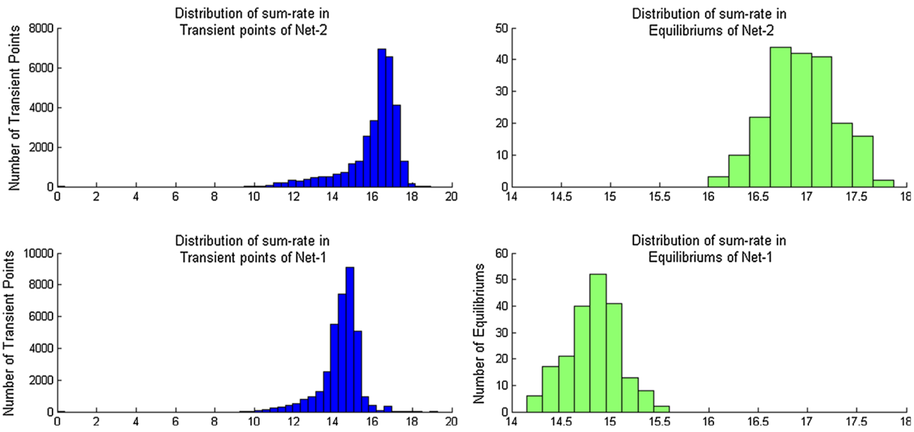
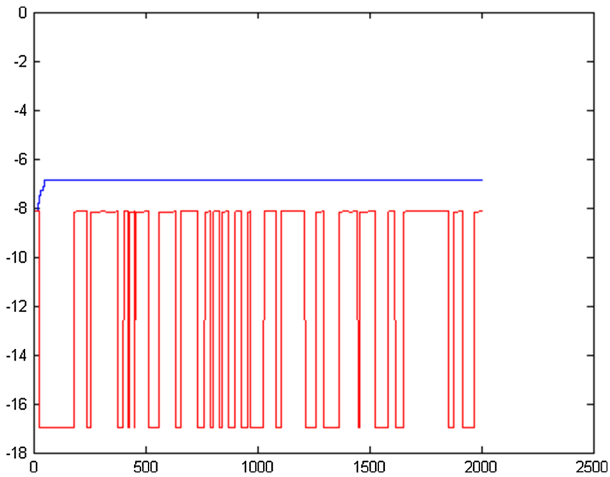


Fig. 4 Comparison between distribution of sum-rate in transient points and equilibriums of Net-1 and Net-2 with randomly winning users in each Bernoulli trial

Fig. 5 An instance topology in which Net-1 does not converge to any equilibrium whereas Net-2 does



displacement, the networks are simulated in 1000 different topologies. Simulations show that, out of 1000 topologies, **Net-1** becomes unstable in 3 of the topologies and does not reach the equilibrium point in them. The performance of **Net-2** and **Net-1** in one of these topologies has been shown in Fig. 5.

Similar to Figs. 3 and 4, Figs. 6 and 7 also are about to compare the distribution of sum-rate and distribution of total interference respectively. Figures 6 and 7 however, illustrate the distribution of these parameters in the 1000 different topology. Table 2 has been provided to present precise numerical information of Figs. 6 and 7. It is realized from this table that in transient and equilibrium points, **Net-2** has on average 20 % lower total interference than **Net-1**. In the meantime, the variance of the total interference is lower and more suitable in **Net-2** than **Net-1**. By regarding sum-rate as optimality criterion, **Net-2** express 7 $\sim$10 % better performance than **Net-1** does.

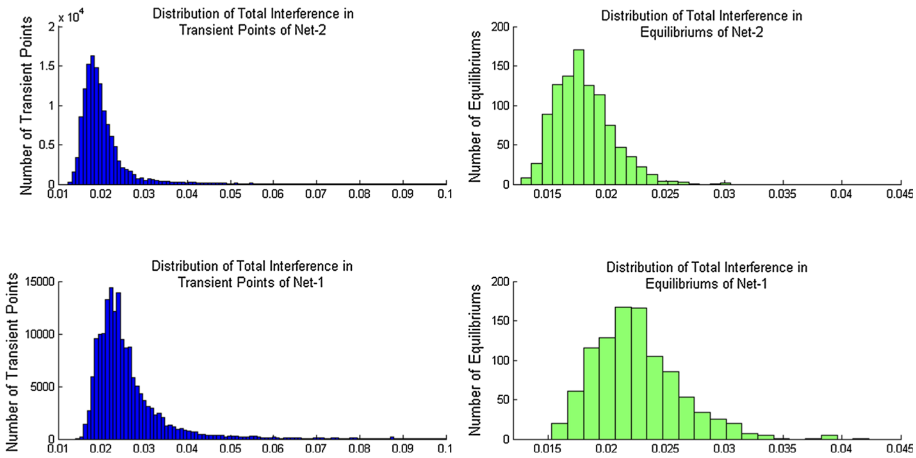


Fig. 6 Comparison between distribution of total interference in transient points and equilibriums of Net-1 and Net-2 in 1000 different topology states

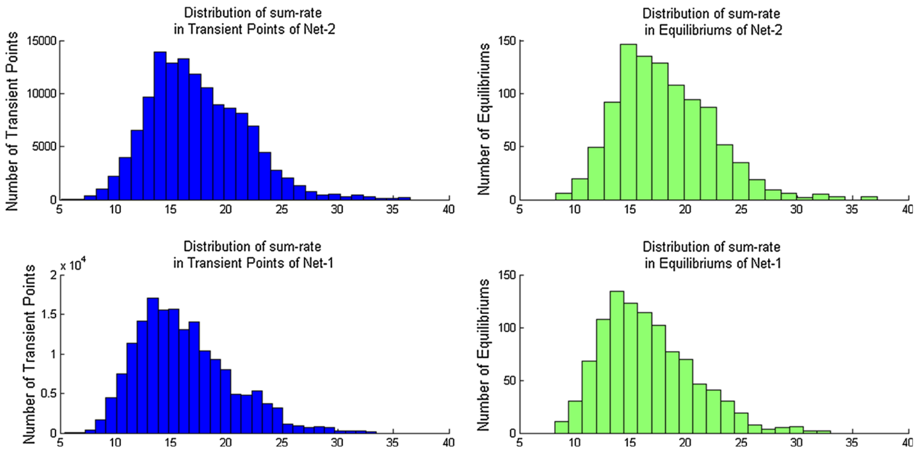


Fig. 7 Comparison between distribution of sum-rate in transient points and equilibriums of Net-1 and Net-2 in 1000 different topology states

Figure 8 is about to assess the impact of percentage of cooperation of network users upon the optimality of network equilibria. The variance of SIR of network users, which is a measure of fairness, is studied in different percentages of number of cooperative users. According to this figure, the network fairness will be reduced by the decrease in the percentage of users' cooperation. However, the figure demonstrates that being in the range of 25–75 % of cooperation, the change in network fairness is negligible. This shows the effectiveness of pricing method in order to induce cooperation among network users.

4.2 Evaluation of Optimality

A simulation-based study on existence of Pareto optimality in this paper is carried out. The simulation of **Net-2** with 8 users (8 pairs of transmitters and receivers) who were allowed

Table 2 Comparison of distribution of total interference and sum-rate in transient points and equilibriums of Net-1 and Net-2 in 1000 different topology states

	Net-2		Net-1	
	Transient points	Equilibrium points	Transient points	Equilibrium points
Total interference				
Mean	0.0204	0.0180	0.0264	0.0226
Median	0.0140	0.0177	0.0240	0.0221
Mode	0.0184	0.0175	0.0227	0.0220
Range	0.087	0.0179	0.0859	0.0270
Variance	6×10^{-5}	6×10^{-6}	9×10^{-5}	1.4×10^{-5}
Sum-rate				
Mean	17.39	18.05	16.17	16.53
Median	16.81	17.38	15.53	15.87
Mode	14.53	15.5	13.38	14.39
Range	31.38	28.92	28.08	24.84
Variance	19.33	18.87	18.87	17.45

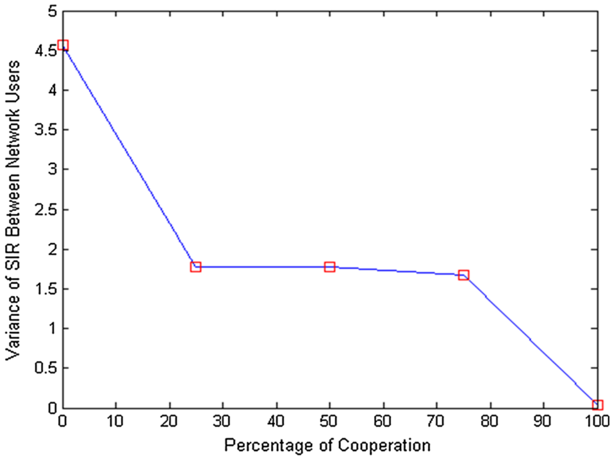


Fig. 8 The effect of percentage of cooperating users on Net-2 fairness

to use three transmission channels shows that after 1000 times of run in a specific topology, the network will become convergent to 4 Nash equilibria. In Fig. 9 the users' utility functions in these equilibrium points have been shown simultaneously to enable us to compare utility which is gained by all users in two different equilibria (each curve is related to an equilibrium).

As can be observed in the figure, no equilibrium outperforms any other one which means all of these equilibriums are positioned on Pareto optimal frontier. According to these numerical calculations, the investigated network will converge to equilibriums all of which are positioned on the optimality border, by applying $-P(\cdot)$ as the pricing function. This would issue the efficiency of the proposed methods in management of the dedicated

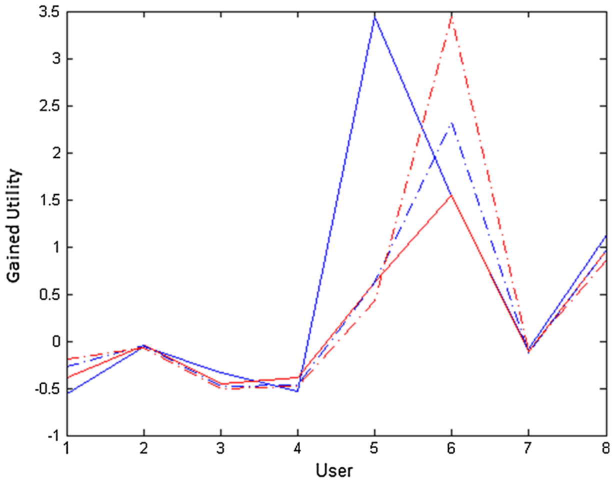


Fig. 9 The gained utility of the 8 users in 4 equilibrium points to which the Network can converge

CR networks. The employed pricing method implicitly causes the network users' to react cooperatively.

5 Conclusion

In this paper, a new pricing method is presented to be utilized in cognitive radio networks with distributed architecture of resource access. It's been shown that by utilizing the payoff function of an exact potential game, in an arbitrary game as its pricing function, the resulting game can be formulated as an ordinal potential game. This method guarantees the fast convergence of network to Nash equilibrium and controls the network users' selfish behaviors. This will result in better resource utilization and optimization of equilibrium points. The outcome of simulations performed demonstrates that the total interference of the network is well controlled and sum-rate will increase. Regarding Pareto optimality criteria, also, the equilibrium points of proposed network will be optimal.

Appendix

In order to proof the convergence of such a game, number terms and theorems need to be introduced.

Definition 1 (*path*) A path in G is a finite or infinite sequence of $\varphi = \left(\mathbf{s}^1, \mathbf{s}^2, \ldots, \mathbf{s}^l, \ldots\right)$ such that for every $l \geq 1$ there exists a unique deviator the number of which is the index of the only column differ between $\mathbf{s}^l$ and $\mathbf{s}^{l+1}$.

Based on its definition, followings can be taken as granted about a path:

- Only unilateral deviations are permitted in a path.
- Multilateral paths can be introduced as a broader class of paths in which s^k and s^{k+1} can differ in multiple columns.

Definition 2 (*improvement path*) An Improvement Path is a path in which following holds: $\forall k \geq 1, u_i(\mathbf{s}^{\mathbf{k}+1}) > u_i(\mathbf{s}^{\mathbf{k}})$ Where i is the unique deviator in kth step. Based on the above definition, players are assumed to play rational in an Improvement Path.

Definition 3 (*finite improvement property (FIP)*) Game $G = (N, \boldsymbol{S}, \{u_i\})$ is said to have the FIP if all improvement paths in it are finite.

Definition 4 (*cycle*) A cycle is a finite path of $c = (\mathbf{s}^1, \mathbf{s}^2, \ldots, \mathbf{s}^l)$ in which, $\mathbf{s}^1 = \mathbf{s}^l$. This cycle is called an improvement cycle if c is an improvement path.

Considering mentioned definitions, Theorem 2 is brought out which proves the decisive convergence of Ordinal Potential games under certain circumstances and is a step ahead to prove the convergence of our network.

Theorem 2 (ordinal potential games have FIP) As is mentioned in [12] having FIP in a finite game $G = (N, \boldsymbol{S}, \{u_i\})$ is equivalent to not having improvement cycles in it, thus below is offered the proof of Ordinal Potential games having FIP:Consider an Improvement cycle of $c = (\mathbf{s}^1, \mathbf{s}^2, \ldots, \mathbf{s}^l)$, in which:

$$\mathbf{s}^1 = \mathbf{s}^l \text{and } \forall 1 \leq k \leq l : u_i(\mathbf{s}^{\mathbf{k}+1}) > u_i(\mathbf{s}^{\mathbf{k}}) \quad \forall 1 \leq k \leq l \tag{14}$$

Where i is the unique deviator in kth step. Occurring c in an Ordinal Potential game G^* with $v(\cdot)$ as its potential function, results in:

$$\forall 1 \leq k \leq l : v(\mathbf{s}^{\mathbf{k}+1}) > v(\mathbf{s}^{\mathbf{k}}) \tag{15}$$

That results in: $v(\mathbf{s}^1) < v(\mathbf{s}^2) < \ldots < v(\mathbf{s}^l) = v(\mathbf{s}^1)$ which is not possible.Thus there exists no such c which, proving that all Ordinal Potential games have FIP.

Convergence

Being applied the Asynchronous timing rule and best response decision rule, a game having FIP is proved to enjoy convergence property [12]. For the introduced game, if the number of network users is sufficiently large, the probability of existence of two or more deviators in each step is negligible. Hence, the game is an ordinal potential one in which all the strategy sequences can be considered as improvement paths and we have proved that all such paths in an ordinal potential game are finite so the strategy sequences converge.

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