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Joint spectrum load balancing and handoff management in cognitive radio networks: a non-cooperative game approach

Mina Fahimi[1] · **Abdorasoul Ghasemi**[1]

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Abstract We propose a non-cooperative game theory based algorithm for spectrum management problem in cognitive radio networks taking into account the spectrum handoff effects. The objective is to minimize the spectrum access time of Secondary Users (SUs) which are competing for spectrum opportunities in heterogeneous environment. In this paper, the preemptive resume priority (PRP) M/G/1 queuing model is used to characterize the multiple handoff and data delivery time of SUs. Also an explicit solution for channels selection probabilities of each SU is extracted for PRP M/M/1 model specifically. The effect of handoffs is considered as the interrupted packets which return to the SUs' low priority queue when the high priority Primary User's packets are arrived to take service. The queuing delay of SUs' and the effect of these returned packets are considered in order to balance the load of SUs on channels so that the minimum spectrum access time is sensed by each SU. The non-cooperative spectrum load balancing with handoff management game is proposed to find a distributed solution for each SU. It is shown that this game has a unique Nash equilibrium point which can be achieved by SUs as decision makers. At this equilibrium, each SU incurs the minimum delay on all channels while the free spectrum holes of channels are utilized efficiently. Simulation results are provided to evaluate the performance of the proposed scheme in terms of spectrum access delay, fairness, and channels spectrum holes utilization.

✉ Abdorasoul Ghasemi
arghasemi@kntu.ac.ir

[1] Faculty of Computer Engineering, K. N. Toosi University of Technology, Tehran, Iran

Keywords Cognitive radio networks · Spectrum handoff · Load balancing · Queuing theory · Game theory

1 Introduction

The explosive growth of demands for wireless spectrum access and inefficiency of traditional licensed spectrum management have made cognitive radio (CR) technology more interesting in recent years. CR methods refer to dynamic spectrum access (DSA) in which unlicensed users at a particular time and specific location can intelligently utilize the idle spectrums [1]. That is, CR user or Secondary User (SU) is a context aware intelligent user who aims to opportunistically utilize the spectrum holes of licensed or Primary Users (PUs) [2, 3]. In the hierarchical access model [3], the SUs are allowed to utilize these holes provided that the incurred performance degradation of PUs is tolerable. In this model, two approaches for spectrum sharing between SUs and PUs are proposed. In the spectrum underlay approach, the transmissions scheduling and powers of SUs are adjusted such that the resulted interference on primary receivers is tolerable. In the spectrum overlay, which is adopted in this paper, SUs should explore the PUs' channels to sense and exploit the spectrum holes opportunistically. That is, in the exploration phase, the SU deploys a sensing module, e.g., an energy detector, to decide about the existence of PUs on a given channel. The SU can exploit that channel if it is idle.

More specifically, to have a reliable communication in this approach, four important functionalities are required [4]. The first one is spectrum sensing, i.e., exploring channels to find the spectrum holes. The second one is spectrum management which refers to choosing the best available channel. The third is spectrum sharing, i.e., sharing the spectrum holes among multiple SUs in order to provide fairness between them and to achieve the maximum system utilization. The last one is spectrum mobility for switching to another available channel when it is necessary. There are several researches on each of these functionalities [4].

This paper focuses on a solution for spectrum management considering the handoff effects. More specifically, to share the spectrum holes efficiently and fairly, and to avoid collisions between competing SUs, the channel allocation scheme should balance the SUs load on the channels' spectrum holes. Regarding this problem, centralized and distributed solutions can be used. In a centralized solution, a central controller allocates the spectrum to SUs while in a distributed scheme, each SU accesses the spectrum based on its strategy [4].

On the other hand, spectrum mobility refers to the fact that SU must leave the channel when a PU starts to take service on this channel. Spectrum handoff helps the SU for returning the channel to the PU and selects another channel to find spectrum opportunity or wait until the transmission of PU is completed on the current channel [4]. Clearly, multiple interruption during the transmission of a SU increases the data delivery time of the SU. An efficient spectrum management scheme should avoid unnecessary waiting time or channel switching that may exceed a sustainable threshold for delay sensitive applications.

In this paper, the spectrum management problem with spectrum handoff effect is considered to find a spectrum management scheme among SUs that leads to minimum delay for each SU. The objective is to minimize the spectrum access delay or Overall System Time (OST) [5] of each SU taking into account the spectrum handoff effects. The probability based spectrum management is adopted for analysis in which SUs select channels according to predetermined probability and adjust them to achieve the spectrum management aims. By assuming the spectrum handoff effect, an algorithm is developed to minimize the OST of each SU.

The main contributions of this work are:

- By adopting the PRP M/G/1 queuing model for each channel, the effect of multiple handoffs on OST is considered in the cost function of each SU.
- The rate of handoff occurrence is computed based on the arrival rate of low priority SU's queue and the probability of handoff occurrence. Then, the effect of these returned packets are considered in computing the delay of each SU.
- For the special case of PRP M/M/1 queueing model, we derive an explicit solution for spectrum management and load balancing problem. A non-cooperative game is designed to adjust the channels selection probabilities as the best response solution for each SU. It is shown that this game has a unique Nash equilibrium which leads to the load balanced spectrum management.
- A simple algorithm based on nonlinear optimization is proposed for each SU to minimize the OST to reach the Nash equilibrium.

To evaluate the performance of the proposed scheme, an event based simulation environment is developed using C++ programming language to simulate the behavior of SUs and PUs at the MAC layer. The packet delay of SUs is computed using this simulation environment and compared with analytical results for different load balancing schemes.

The rest of this paper is organized as follows. In Sect. 2 some recent related works on spectrum management and spectrum handoff are reviewed. The system model and problem statement are presented in Sect. 3. Assuming a queuing model for each channel, the cost function is estimated in Sect. 4. In Sect. 5, a centralized solution for spectrum management with spectrum handoff effects is presented and then a non-cooperative game is analyzed to derive a distributed solution to find the best strategy of each SU for channel selection. The developed simulation environment, simulation results, performance evaluations and discussions are provided in Sect. 6 before concluding the paper in Sect. 7.

2 Related work

There are different studies on spectrum load balancing and spectrum handoff management that are reviewed in the two following subsections.

2.1 Spectrum load balancing

In a class of spectrum management schemes, SUs select the best available channel taking into account the channels' traffic loads or their expected throughput on each channel and then compete with each other to exploit this channel [6–8]. These methods lead to collision on this channel and do not exploit less available channels efficiently. Spectrum load balancing schemes are another class of solutions for spectrum sharing. Prior researches such as [9] suggests an algorithm to smooth the traffic load of each SU on channels periodically which is not fair. In [10], Spectrum Load Balancing based on Dynamic feedback theory and Hash Table (SLBDH) is proposed for load balancing in CR networks. The load of SUs is divided into small portions or cells then each cell is allocated to a free space according to hash table information. Game theoretic based solutions are suggested in [11–14]. In [11], the number of SUs on each channel is balanced. Specifically, a cost function based on the number of SUs on each channel is introduced and two algorithms are developed in order to minimize this cost for each SU. Specifically, SUs select the channel that has the minimum number of competing SUs and minimum cost in each step of the algorithm. In [12], a randomized protocol based on local information for each SU is proposed. The authors have considered a cost function based on delay and throughput. This algorithm reaches a steady state in which SUs sustain a certain threshold of cost function. The SUs decide to change the channel by comparing this cost function with a given threshold. In [13], the priority of PUs to SUs is considered. The M/G/1 queuing system is used to compute the delay and throughput of each SU as utility function. Then, dynamic strategy learning (DSL) algorithm as a better response algorithm is proposed to derive the channel selection probability. The effect of handoff is considered as a function of the difference between the selected probability value and its previous value. The convergence of DSL algorithm is not guaranteed in delay sensitive applications and for these applications DSL does not achieve a steady state. In [14], spectrum load balancing (SLB) algorithm is proposed to find the optimal spectrum management probability. Using the M/M/1 queuing delay as cost function for each SU, a game is designed to minimize this cost for each SU. This algorithm that is executed by each SU selfishly, is a best reply scheme which converges to the Nash equilibrium. The SLB algorithm has good results in terms of spectrum load balancing and convergence time.

Although the main objective of spectrum load balancing in SLB and other similar schemes is to minimize the spectrum access delay for SUs, they have not considered the effect of multiple handoffs. Spectrum handoff delay incurs an additional delay for SUs which should be considered jointly in spectrum load balancing schemes.

2.2 Spectrum handoff management

Spectrum handoff occurs when a PU returns to take service on a channel which is currently exploited by SUs. This channel must be released by SUs and they should explore other channels or wait until the PU's transmissions on this channel is finished. Therefore, most of the studies and analysis on handoff management is based on the effect of multiple handoffs on data transmission time of SUs. Spectrum handoff strategies can be categorized to proactive-sensing and reactive-sensing [15]. In proactive-sensing spectrum handoff, the SUs make decision about the target channel in a probability-based scheme taking into account the channel long term observations. That is, the SUs predict the likely time instant where a handoff will be required and trigger the handoff before that to avoid interference to the PU. While in reactive-sensing spectrum handoff, SUs determine the target channel in an on demand manner using the instantaneous outcomes from wideband sensing. Therefore, the SUs should sense all of the candidate channels to select the target channel. The advantage of proactive-sensing is reducing the handoff delay because the time consuming wideband sensing is not required. However, SUs should periodically explore all channels to obtain the required channels statistics for prediction. The level of PUs protection depends on the accuracy of the prediction. On the other hand, the reactive sensing scheme avoid such precomputation while incurs more delay when a handoff is required. In [16], a Markov transition model for PUs' traffic and the PRP M/G/1 model for SUs channel access are used to characterize the reactive-sensing handoff delay. Also, the admission region for delay and transmission probability for different channels' are computed based on the SUs' and PUs' arrival rates. There are some researches on modeling spectrum handoff probability in CR networks in order to study the effect of handoff occurrence in transmission delay of SUs. In [17], the reactive-sensing spectrum handoff in space and time domains is analyzed to minimize the hidden terminal problem in CR ad hoc networks. Assuming a two states Markov chain for PU's traffic model, the competition between SUs is investigated. Channels are assumed to have the same and fixed busy-idle model and all channels become unavailable to SUs when one or more PUs are active. In [18], the proactive-sensing spectrum handoff and M/G/1 queuing model are assumed and the total service time is evaluated. The SU makes decision about stay or change the channel. In stay situation, each SU waits on the current channel until the transmission of PU is finished. On the other hand, in change state, SU selects another channel. In addition, a greedy algorithm is proposed to select the best channel in each situation. However, the greedy solution leads to inefficiency in channel utilization. In [19, 20], proactive-sensing spectrum handoff protocols are proposed where SU changes the channel before the PU starts to take service on this channel. In [19], SUs predict the probabilities of idle periods on PUs' channels. In [20], SUs analyze channel history to predict the probability of future spectrum availability. Assuming that the idle periods are fixed and the busy periods are exponentially distributed, the SU estimates the probabilities of idle periods. The efficient usage of idle spaces and balancing issue are not considered in [19, 20] and the SUs select the channel with highest idle probability simultaneously. Modeling the spectrum holes by Poisson distribution, the spectrum handoff probability is also discussed in [21]. In [22, 23] the QoS performance of spectrum management is discussed and an analytical framework based on PRP M/G/1 is studied to characterize the multiple handoffs effect on the channel selection scheme. The probability of handoff occurrence and different rates of returned packets to SUs' queues are investigated in [23] and the delay model for each stay and change situation is analyzed based on PUs' interruptions on the channels and handoff rate.

In [24], a proactive-sensing spectrum scheme with different priority queue is considered for SU's packets. That is, the packets which are interrupted by PUs' arrivals and returned to SU's queue have higher priority compared to the non backlogged packets. Also, a delay model based on the rate of the returned packets is analyzed and a scheduler which compute the channel selection probability for a single SU scenario is designed.

In this paper, we adopt a more accurate model for the interaction of PUs and SUs while jointly considering the spectrum utilization and load balancing issues.

2.3 Spectrum load balancing with handoff management

The objective of this paper is characterizing the SUs spectrum load balancing taking into account the PUs' interruptions. We use PRP M/G/1 queuing model to analyze the priority of the PUs to the SUs. We compute the rate of interruptions during the transmission of SUs and the arrival rate of interrupted packets which are returned to the SUs' queue. We consider a scheduler to compute the channels selection probabilities for each SU which make the minimum delay for different SUs on different channels. Also, the closed form solution for the handoff and queueing delays using the PRP M/M/1 model is derived. Then, a non-cooperative game which considers both queuing and handoff delays, is used to find a distributed solution for channels selection probabilities of SUs. The proposed spectrum load balancing and handoff management (SLBHM) game is analyzed and the convergence of SUs' decisions to the Nash equilibrium of SLBHM is discussed. Based on this game, an algorithm is presented in order to balance the load of SUs on primary channels.

We also solve the load balancing and handoff management problem as an optimization problem numerically in a centralized manner to discuss about the efficiency of the SLBHM game equilibrium. On the other hand, we compare SLBHM solution with SLB scheme [14] as a distributed solution to show the gains of joint spectrum load balancing and handoff management in CR networks. In fact, SLB can be considered as a special case of SLBHM while the QoS of SUs in CR networks is better satisfied by SLBHM scheme. The result of each scheme is evaluated using a developed event based simulator which accurately simulate the behavior of SUs and PUs at the MAC layer.

3 System model and problem statement

We consider a wireless network environment with M heterogeneous frequency channels which are exploited by licensed PUs. The set of channels and PUs are denoted by $\mathcal{F} = \{F_1, F_2, \ldots, F_M\}$ and $\mathcal{PU} = \{PU_1, PU_2, \ldots, PU_M\}$, respectively. Channel F_i is dedicated to PU_i and is modeled by an PRP M/G/1 queuing system. That is, the packet arrival process of PU_i on F_i is Poisson process which is denoted by $A_i^{(PU)}(t)$ with average $\lambda_i^{(PU)}$ (pkt/s) and the average packet service time of PU on channel F_i is $E\left[X_i^{(PU)}\right]$.

There are N SUs which share the available spectrum holes of primary network and their set is denoted by $\mathcal{SU} = \{SU_1, SU_2, \ldots, SU_N\}$. It is assumed that the packet arrival process of SU_j is also Poisson process which is denoted by $A_j^{(SU)}(t)$ with average arrival rate $\lambda_j^{(SU)}$ (pkt/s). The system is assumed to be time slotted and each SU is equipped with a sensor to explore the primary channels for possible exploitation of primaries protection. That is, the SUs sense the channels at the beginning of each time slot and are allowed to exploit a channel white spaces if find it idle [20]. The sensing is assumed to be perfect. The transmission of SUs are interrupted by the PU if it returns to take service on this channel. The interrupted SU has to wait until the PU's transmission is finished.

Let s_{ji} be the load fraction of SU_j on channel F_i. That is, SU_j selects channel F_i with probability s_{ji}. Hence, the packet arrival process of SU_j on channel F_i is a Poisson process which is denoted by stochastic process $A_{ji}^{(SU)}(t)$ with average $\lambda_{ji}^{(SU)} = s_{ji}\lambda_j^{(SU)}$. Also, the average service time of SU_j on channel F_i is $E\left[X_{ji}^{(SU)}\right]$.

The SU will attempt to transmit the interrupted or backlogged packets when the channel becomes idle again. In fact, there is another packet arrival to the low priority SU's queue which is denoted by stochastic process $A_{ji}^{(f)}$ with average rate $\lambda_{ji}^{(f)}(t)$ for SU_j on channel F_i due to these returned packets. The average service time of these packets is $E[X_{ji}^{(f)}]$. Table 1 summarizes the definition of the main system model parameters. The objective is to balance the SUs' loads on M channels by designing an appropriate scheduler. The scheduler should adjust the channels selection probabilities of each SU and lead to the minimum incurred SU's transmission delay. The queuing system model is depicted in Fig. 1.

In order to balance the load on the channels, SU_j should adjust its channel selection probability profile which is denoted by $\mathbf{s}_j = [s_{j1}, s_{j2}, \ldots, s_{jM}]$ where $0 \le s_{ji} \le 1$ is the probability of selecting channel F_i, and $\sum_{i=1}^{M} s_{ji} = 1$. Let

Table 1 System parameters

Symbols	Meaning
$A_i^{(PU)}(t)$	Stochastic process of PU_i's packet arrival
$\lambda_i^{(PU)}$	Average arrival rate of PU_i's packets
$E\left[X_i^{(PU)}\right]$	Average service time for PU_i
$A_j^{(SU)}(t)$	Stochastic process of SU_j's packet arrival
$\lambda_j^{(SU)}$	Average arrival rate of SU_j's packets
s_{ji}	Load fraction of SU_j on channel F_i
$A_{ji}^{(SU)}(t)$	Stochastic process of SU_j's packet arrival on channel F_i
$\lambda_{ji}^{(SU)} = s_{ji}\lambda_j^{(SU)}$	Average arrival rate of SU_j's packets on channel F_i
$E\left[X_{ji}^{(SU)}\right]$	Average service time for SU_j on channel F_i
$A_{ji}^{(f)}(t)$	Stochastic process of interrupted packets arrival of SU_j'on channel F_i
$\lambda_{ji}^{(f)}$	Average arrival rate of interrupted packets of SU_j on channel F_i
$E[X_{ji}^{(f)}]$	Average service time for interrupted packets of SU_j on channel F_i
λ_i	Average total packets arrival rate on channel F_i

$\mathbf{S} = [\mathbf{s}_1^T, \mathbf{s}_2^T, \ldots, \mathbf{s}_N^T]^T$ denotes the SUs' channel selection probabilities. That is, $\mathbf{S}$ is a two dimensional matrix which its rows and columns corresponds to the SUs and channels, respectively. When SU_j chooses a channel, it perceives queuing and handoff delays on that channel which affects its QoS. Since these delays depend on the load on the selected channel, the QoS of SU_j is influenced by the strategies of other SUs in channel selection. The sum of perceived delays or OST of SU_j is shown by cost function $c_j(\mathbf{s}_j, \mathbf{s}_{-j})$; where $\mathbf{s}_j$ is the probability profile of SU_j and $\mathbf{s}_{-j}$ is the probability profiles of other SUs in channel selection. Any decrease in c_j leads to an increase in the QoS of SU_j. In a probability based model, this cost function is given by (1).

$$c_j(\mathbf{s}_j, \mathbf{s}_{-j}) = \sum_{i=1}^{M} s_{ji} E_{(\mathbf{s}_j,\mathbf{s}_{-j})}[OST_{ji}] \tag{1}$$

where $E_{(\mathbf{s}_j,\mathbf{s}_{-j})}[OST_{ji}]$ is the expected OST based on channel selection probability profiles $(\mathbf{s}_j, \mathbf{s}_{-j})$ that is incurred by SU_j on channel F_i. The objective of SU_j is to selfishly minimize c_j by adjusting $\mathbf{s}_j$ taking into account other SUs' profiles, i.e., $\mathbf{s}_{-j}$.

In Fig. 2, the details of the OST of a SU which takes service on a channel is depicted. Following the arrival of SU, it waits until the channel becomes idle. The SU starts to exploit the first spectrum hole when it may be interrupted several times by the PU which exploits this channel. Each handoff incur a delay on the cost function of SU. According to Fig. 2, the OST of SU_j on channel F_i is given by:

$$E[OST_{ji}] = E\left[W_{ji}\right] + E\left[SD_{ji}\right] \tag{2}$$

where $E[W_{ji}]$ and $E[SD_{ji}]$ are the expected waiting time and service duration of SU_j on channel F_i, respectively. The service duration starts from the time that SU_j takes service on channel F_i until its departure and includes the average service time and handoff delay as given by (3) [5].

$$E[SD_{ji}] = E\left[X_{ji}^{(SU)}\right] + E[N_{ji}]E[HD_{ji}] \tag{3}$$

where $E[N_{ji}]$ is the average number of spectrum handoffs that occur during the service time of SU_j on channel F_i and $E[HD_{ji}]$ is the average delay per each handoff. Therefore, the cost function in (1) can be written as:

$$c_j(\mathbf{s}_j, \mathbf{s}_{-j}) = \sum_{i=1}^{M} s_{ji} \Big(E_{(\mathbf{s}_j,\mathbf{s}_{-j})}[W_{ji}] + E_{(\mathbf{s}_j,\mathbf{s}_{-j})}\left[X_{ji}^{(SU)}\right] + E_{(\mathbf{s}_j,\mathbf{s}_{-j})}[N_{ji}] E_{(\mathbf{s}_j,\mathbf{s}_{-j})}[HD_{ji}] \Big) \tag{4}$$

where $E_{(\mathbf{s}_j,\mathbf{s}_{-j})}\left[W_{ji}\right]$, $E_{(\mathbf{s}_j,\mathbf{s}_{-j})}\left[X_{ji}^{(SU)}\right]$, $E_{(\mathbf{s}_j,\mathbf{s}_{-j})}\left[N_{ji}\right]$, $E_{(\mathbf{s}_j,\mathbf{s}_{-j})}\left[HD_{ji}\right]$ are, respectively, the average waiting time, service time, number of interruption, and handoff delay of SU_j on channel F_i according to channel selection probabilities $\mathbf{S} = (\mathbf{s}_j, \mathbf{s}_{-j})$. In the next section, we estimate this cost function for future analysis.

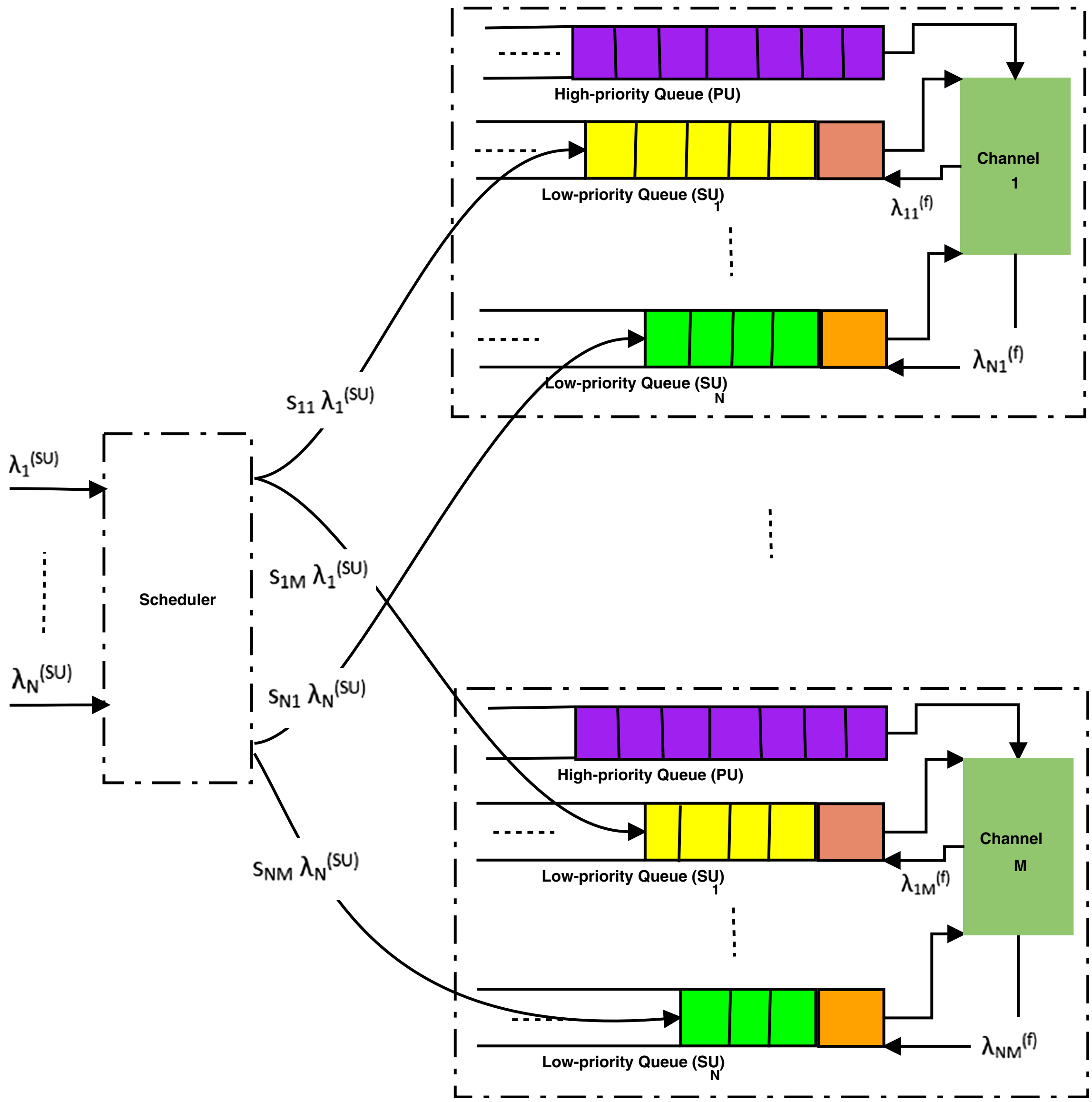

Fig. 1 Queuing model of PUs and SUs. For each channel a high priority PUs' packets and a low priority SUs' packets are considered. Packets of SUs are assigned with different probabilities which are computed by SUs' scheduler to channels. Interrupted packets of SU come back to SU's queue in order to retransmit when SU starts to take service on the channel

4 Spectrum access delay analysis based on queuing model

In this section, queuing analysis is used to estimate the cost function $c_j(\mathbf{s}_j, \mathbf{s}_{-j})$ for SU_j. First by adopting PRP M/G/1 queuing model, we follow the spectrum access delay of each low priority SU analytically and characterize the effect of high priority PU interruption on the spectrum usage of SU [25, 26]. Based on this model, the data delivery time of each SU is derived.

Considering Fig. 1, in order to compute the rate of packets which are returned back to the SU's queue, we can use the *PASTA*, i.e., Poisson Arrivals See Time Average, property of the Poisson process. Let the probability of spectrum handoff for SU_j on channel F_i is denoted by P^h_{ji}. Therefore, the probability that packets of SU_j will see this channel on handoff state is P^h_{ji}. On the other hand, packets of SU_j arrive to channel F_i with average rate $\lambda^{(SU)}_{ji}$. Therefore, the rate of handoff occurrence on channel F_i for SU_j which is equal to the arrival rate of interrupted packets to SU's queue is given by:

$$\lambda^{(f)}_{ji} = P^h_{ji}\lambda^{(SU)}_{ji} = P^h_{ji}s_{ji}\lambda^{(SU)}_j \tag{5}$$

Based on [23], $P^h_{ji} = \frac{\lambda^{(PU)}_i}{\lambda^{(PU)}_i + \mu^s_{ji}}$ where μ^s_{ji} is the average service rate of SU_j on channel F_i.

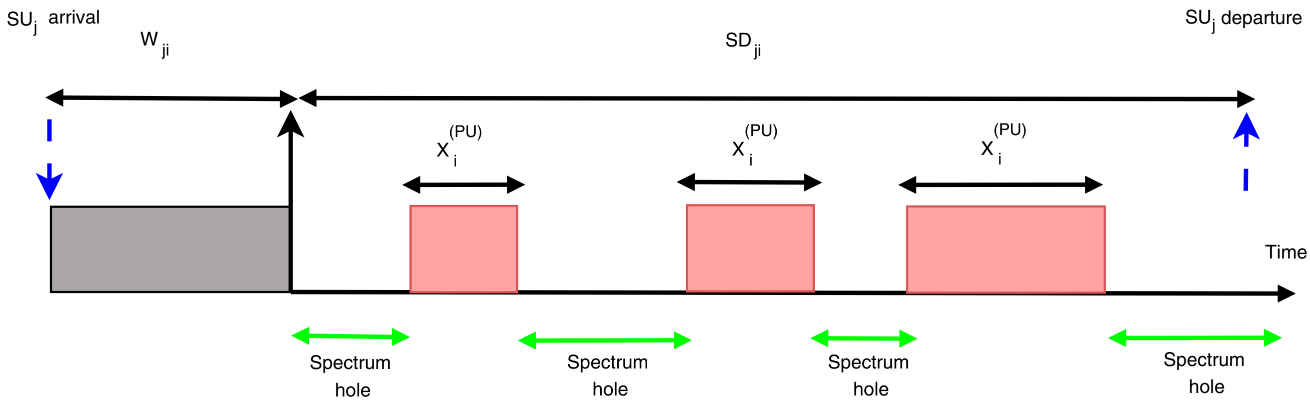

Fig. 2 Overall system time of SU_j on channel F_i

While the stochastic process $A_{ji}^{(f)}(t)$ is a Poisson process, it should be noted that the Poisson process $A_{ji}^{(f)}(t)$ is weakly dependent on two Poisson processes $A_{ji}^{(SU)}(t)$ and $A_i^{(PU)}(t)$. However, as it will be justified by simulation, when $\lambda_j^{(SU)}$ is small enough, there is a weak dependency between stochastic processes $A_{ji}^{(f)}(t)$ and $A_{ji}^{(SU)}(t)$. On the other hand, the dependency of two Poisson processes $A_i^{(PU)}(t)$ and $A_{ji}^{(f)}(t)$, relates to the amount of available opportunities on channel F_i. The independent assumption is reasonable if the channel is not heavily utilized by the PU as it is shown in simulation results. Therefore, to be able to follow the problem analytically we assume that stochastic processes $A_{ji}^{(SU)}(t)$ and $A_{ji}^{(f)}(t)$, as well as $A_{ji}^{(f)}(t)$ and $A_i^{(PU)}(t)$ are independent. Using this assumption and noting that the merging of two independent Poisson processes is a Poisson process, the total arrival rate of the packets on channel F_i is a Poisson process with rate λ_i which is given by:

$$\lambda_i = \lambda_i^{(PU)} + \sum_{j=1}^{N} s_{ji}\left(\lambda_j^{(SU)} + P_{ji}^h \lambda_j^{(SU)}\right) \tag{6}$$

and the traffic load of channel F_i is:

$$\rho_i = \rho_i^{(PU)} + \sum_{j=1}^{N} \rho_{ji}^{(SU)} \tag{7}$$

where $\rho_i^{(PU)} = \lambda_i^{(PU)} E\left[X_i^{(PU)}\right]$ and $\rho_{ji}^{(SU)} = s_{ji}\lambda_j^{(SU)} E\left[X_{ji}^{(SU)}\right] + P_{ji}^h s_{ji}\lambda_j^{(SU)} E\left[X_{ji}^{(f)}\right]$.

Using the PRP M/G/1 queuing analysis [25, 26], the average waiting time is computed by (8). Since the $A_i^{(PU)}(t)$ is a Poisson process, the number of interruption incurred by SU_j on this channel is $E[N_{ji}] = \lambda_i^{(PU)} E[X_{ji}^{(SU)}]$. Also, the delay after each interruption is equal to the queuing delay of PUs' packets which are arrived to take service on channel F_i. Therefore, the handoff delay without changing channel after each interruption is $E[HD_{ji}] = \frac{E\left[X_i^{(PU)}\right]}{1-\lambda_i^{(PU)} E\left[X_i^{(PU)}\right]}$. The final cost function in (4) can then be written as (10).

$$E_{(\mathbf{s}_j,\mathbf{s}_{-j})}[W_{ji}] = \frac{E[R_i]}{\left(1 - E\left[X_i^{(PU)}\right]\lambda_i^{(PU)}\right)\left(1 - E\left[X_i^{(PU)}\right]\lambda_i^{(PU)} - \sum_{j=1}^{N} s_{ji}\lambda_j^{(SU)}\left(E\left[X_{ji}^{(f)}\right]P_{ji}^h + E\left[X_{ji}^{(SU)}\right]\right)\right)} \tag{8}$$

where $E[R_i]$ is the average remaining service time on channel F_i:

$$E[R_i] = \frac{1}{2}\left(E\left[\left(X_i^{(PU)}\right)^2\right]\lambda_i^{(PU)} + \sum_{j=1}^{N}\left(E\left[\left(X_{ji}^{(f)}\right)^2\right]P_{ji}^h s_{ji}\lambda_j^{(SU)} + E\left[\left(X_{ji}^{(SU)}\right)^2\right]s_{ji}\lambda_j^{(SU)}\right)\right) \tag{9}$$

and

$$c_j(\mathbf{s}_j, \mathbf{s}_{-j}) = \sum_{i=1}^{M} s_{ji}\left(E_{(\mathbf{s}_j,\mathbf{s}_{-j})}[W_{ji}] + E\left[X_{ji}^{(SU)}\right] + \lambda_i^{(PU)} E\left[X_{ji}^{(SU)}\right]\frac{E\left[X_i^{(PU)}\right]}{1 - \lambda_i^{(PU)} E\left[X_i^{(PU)}\right]}\right) \tag{10}$$

In the next section, we propose a game theoretic scheme to find the best solution for spectrum load balancing which minimizes the spectrum access delay of each SU.

5 SLBHM: centralized and distributed solutions

In order to solve the spectrum load balancing and handoff management problem we follow two approaches and compare the results. In the first approach, the problem is formulated as an optimization problem and the optimum channels selection probabilities are computed in a centralized manner. This solution is useful if there is a central controller that balances the load of SUs on primary channels. In addition, it can be used to compare the efficiency of distributed schemes as a reference. The second solution approach is a distributed one based on non-cooperative game theory in which each user decides about its load distribution selfishly. In this scheme, the SUs interchange their strategy profiles by messages passing. Each SU then minimizes its cost function taking into account the received messages.

5.1 SLBHM as an optimization problem

The Global Optimum Scheme (GoS) for load balancing is introduced in [27] where the objective is to minimize the weighted sum of job completion time in a computer system. We adopt this static load balancing scheme as a centralized solution when the objective is to minimize the spectrum access time of all SUs which are processed in the system. The probability profiles of users are obtained by solving the nonlinear optimization problem in (11–14).

$$\min_{\mathbf{s}} \frac{1}{\sum_{j=1}^{N} \lambda_j^{(SU)}} \sum_{j=1}^{N} \lambda_j^{(SU)} c_j(\mathbf{s}_j, \mathbf{s}_{-j}) \tag{11}$$

s.t.

$$0 \le s_{ji} \le 1 \quad i = 1, \ldots, M \quad \text{and} \quad j = 1, \ldots, N \tag{12}$$

$$\sum_{i=1}^{M} s_{ji} = 1 \quad j = 1, \ldots, N \tag{13}$$

$$E\left[X_i^{(PU)}\right]\lambda_i^{(PU)} + \sum_{j=1}^{N} s_{ji}\lambda_j^{(SU)}\left(E\left[X_{ji}^{(f)}\right]P_{ji}^{h} + E\left[X_{ji}^{(SU)}\right]\right) < 1 \quad i = 1, \ldots, M \tag{14}$$

The objective function in (11) is the weighted sum of all SUs spectrum access delay. Regarding the fairness criterion between SUs, the weight of each SU is considered as the ratio of its load to the sum of all SUs' loads. Constraint (14) indicates the stability requirement on channel F_i, i.e., the sum of PU's and SU's rates on each channel must be less than the channel service rate.

The solution of this problem can be used by a central controller for joint spectrum load balancing and handoff management regrading the channels residual capacities and SUs' loads. Furthermore, it can be used as a reference to compare the results of distributed schemes. We use MATLAB optimization toolbox to solve this problem in the evaluation section.

5.2 SLBHM as a non-cooperative game problem

In this section, we consider a distributed solution in which each SU aims to selfishly minimize its cost function using the non-cooperative SLBHM game. A non-cooperative strategic game has three main components: a finite set of N decision makers or players, the strategy space S for each player, and a utility function for players per each strategy, which are shown by the triple $\mathcal{G} = <N, S, U>$. Each individual player makes decisions independently to maximize its utility or minimize its cost function selfishly [28, 29].

In SLBHM game, players are the SUs which make decisions on distributing their traffic load on the primary system channels. The objective of each SU is to minimize its spectrum access delay selfishly. Therefore, the strategy space of SU_j is its probability profile for channel selection $\mathbf{s}_j = [s_{j1}, s_{j2}, \ldots, s_{jM}]$. The cost function of SU_j, $c_j(\mathbf{s}_j, \mathbf{s}_{-j})$, reflects its perceived delay as a function of its strategy, $\mathbf{s}_j$, and the other SUs' strategies, $\mathbf{s}_{-j}$. The SLBHM game is formally given by (15–18).

$$\textit{SLBHM Game}: \min_{\mathbf{s}_j} c_j(\mathbf{s}_j, \mathbf{s}_{-j}) \tag{15}$$

s.t.

$$0 \le s_{ji} \le 1 \quad i = 1, \ldots, M \tag{16}$$

$$\sum_{i=1}^{M} s_{ji} = 1 \tag{17}$$

$$E\left[X_i^{(PU)}\right]\lambda_i^{(PU)} + \sum_{j=1}^{N} s_{ji}\lambda_j^{(SU)}\left(E\left[X_{ji}^{(f)}\right]P_{ji}^{h} + E\left[X_{ji}^{(SU)}\right]\right) < 1 \quad i = 1, \ldots, M \tag{18}$$

where (16–18) are the local constraints of SU_j in its decision making.

In the following we argue that the best response of each SU to solve the problem in (15–18) converges to a unique Nash equilibrium point. The best response of SU_j in SLBHM game is a strategy profile $\mathbf{s}_j^*$ for which we have:

$$\mathbf{s}_j^* = \arg\min_{\mathbf{s}_j} c_j(\mathbf{s}_j, \mathbf{s}_{-j}) \tag{19}$$

Using the constraints in (16–18) and the cost function in (10), SU_j makes its decision by solving the optimization problem in (20).

$$\begin{cases} \mathbf{s}_j^* = \arg\min_{\mathbf{s}_j} \sum_{i=1}^{M} s_{ji} \left(E_{(\mathbf{s}_j,\mathbf{s}_{-j})}[W_{ji}] + E\left[X_{ji}^{(SU)}\right] + \lambda_i^{(PU)} E\left[X_{ji}^{(SU)}\right] \dfrac{E\left[X_i^{(PU)}\right]}{1-\lambda_i^{(PU)} E\left[X_i^{(PU)}\right]} \right) \\ subject\ to \begin{cases} 0 \le s_{ji} \le 1 \quad \text{for} \quad i = 1,\ldots,M \\ \sum_{i=1}^{M} s_{ji} = 1 \\ E\left[X_i^{(PU)}\right]\lambda_i^{(PU)} + \sum_{j=1}^{N} s_{ji}\lambda_j^{(SU)}\left(E\left[X_{ji}^{(f)}\right]P_{ji}^h + E\left[X_{ji}^{(SU)}\right]\right) < 1 \;\; i = 1,\ldots,M \end{cases} \end{cases} \tag{20}$$

Proposition 1 *The SLBHM non-cooperative game with cost function* (10) *and constraints* (16–18) *for each player has a unique Nash equilibrium solution.*

Proof Please see "Appendix 1".

The proposed game is applicable for general M/G/1 queueing system if we can derive $E\left[X_i^{(PU)}\right]$, $E\left[X_{ji}^{(SU)}\right]$, $E\left[X_{ji}^{(f)}\right]$, $E\left[\left(X_i^{(PU)}\right)^2\right]$, $E\left[(X_{ji}^{(SU)})^2\right]$, and $E\left[(X_{ji}^{(f)})^2\right]$. In the next section, the explicit solution of the SLBHM game when the service time of SU's packets are exponential and deterministic are derived, i.e., for M/M/1 and M/D/1 queueing systems. In addition, we use M/G/1 model assuming hyperexponential service time to verify the analytical and simulation results for M/G/1 queuing system.

6 SLBHM game analysis in special cases

6.1 SLBHM Game for M/M/1

As a special case when the average service time of the packets in each class of priority is exponentially distributed, i.e., for M/M/1 queueing model, closed form solution can be derived for spectrum access delay of SUs. Let the average service time on channel F_i is equal for high and low priority queues and is given by $E\left[X_i^{(PU)}\right] = E\left[X_{ji}^{(SU)}\right] = \frac{1}{\mu_i}$ and $E\left[(X_i^{(PU)})^2\right] = E\left[\left(X_{ji}^{(SU)}\right)^2\right] = \frac{2}{\mu_i{}^2}$. Due to memoryless property of exponential distribution the service time of interrupted packets is also exponential. Therefore, $E\left[X_{ji}^{(f)}\right] = \frac{1}{\mu_i}$ and $E\left[\left(X_{ji}^{(f)}\right)^2\right] = \frac{2}{\mu_i{}^2}$. Based on these assumptions the cost function in (10) can be written as in (21) where c_j^m denotes the cost function for M/M/1 model and the optimization problem (20) can be written as (25).

$$c_j^m(\mathbf{s}_j,\mathbf{s}_{-j}) = \sum_{i=1}^{M} s_{ji}\left(\frac{s_{ji}\lambda_j^{(SU)}\left(1+P_{ji}^h\right)+R_{ji}}{\left(\mu_i-\lambda_i^{(PU)}\right)\left(\mu_{ji}-s_{ji}\lambda_j^{(SU)}\left(1+P_{ji}^h\right)\right)} + \frac{1}{\mu_i} + \frac{\lambda_i^{(PU)}}{\mu_i}\left(\frac{1}{\mu_i-\lambda_i^{(PU)}}\right)\right) \tag{21}$$

where, P_{ji}^h for SU_j on channel F_i is:

$$P_{ji}^h = \frac{\lambda_i^{(PU)}}{\lambda_i^{(PU)}+\mu_i} \tag{22}$$

and, R_{ji} and μ_{ji} are:

$$R_{ji} = \lambda_i^{(PU)} + \sum_{k=1,k\neq j}^{N} s_{ki}\lambda_k^{(SU)}\left(1+P_{ki}^h\right) \tag{23}$$

$$\mu_{ji} = \mu_i - R_{ji} \tag{24}$$

$$\begin{cases} \mathbf{s}_j^* = \arg\min_{\mathbf{s}_j} \sum_{i=1}^{M} s_{ji} \left(\frac{s_{ji}\lambda_j^{(SU)}\left(1+P_{ji}^h\right)+R_{ji}}{\left(\mu_i-\lambda_i^{(PU)}\right)\left(\mu_{ji}-s_{ji}\lambda_j^{(SU)}\left(1+P_{ji}^h\right)\right)} + \frac{1}{\mu_i} + \frac{\lambda_i^{(PU)}}{\mu_i}\left(\frac{1}{\mu_i-\lambda_i^{(PU)}}\right)\right) \\ \quad subject\ to \begin{cases} 0 \le s_{ji} \le 1 \quad i=1,\ldots,M \\ \sum_{i=1}^{M} s_{ji} = 1 \\ \lambda_i^{(PU)} + \sum_{j=1}^{N} s_{ji}\lambda_j^{(SU)}\left(1+P_{ji}^h\right) < \mu_i \quad i=1,\ldots,M \end{cases} \end{cases} \tag{25}$$

Proposition 2 *The best strategy of SU_j using the SLBHM game in (25) is given by*:

$$s_{ji}^* = \frac{\mu_{ji}(1+Q)+\sqrt{\mu_{ji}^2(1+Q)^2-(1+Q)\left(Q\mu_{ji}-R_{ji}\mu_{ji}\right)}}{1+Q} \quad for\ i = 1\ldots M \tag{26}$$

where $Q = \left(-\alpha_j - \frac{1}{\mu_i} - \frac{\lambda_i^{(PU)}}{\mu_i}\left(\frac{1}{\mu_i-\lambda_i^{(PU)}}\right)\right)\left(\mu_i-\lambda_i^{(PU)}\right)$ and α_j *is the Lagrange multiplier for constraint* (17).

Proof Please see "Appendix 2".

In (26), each SU requires other SUs' strategies to make its decision. In a practical scenario, each SU can broadcast its strategy to other SUs. Also, an iterative solution is required to gradually improve each SU decision upon receiving new messages of other SU until its convergence to the Nash equilibrium. Therefore, after receiving a new message from other SU, SU_j will updates its channels selection probabilities by:

$$s_{ji}^{l+1} = s_{ji}^l + \kappa\left(\frac{-G_{ji}}{H_{ji}} - \frac{\alpha_j}{H_{ji}}\right) \tag{27}$$

where l is the step of Newton method, $\kappa < 1$ is an appropriate step size which ensures the convergence, G_{ji} and H_{ji} are derivation and hessian of cost function (21) respectively.

Algorithm 1 summarizes the main steps that each SU should do in order to reach its best response decision by solving (25). Since G_{ji} and H_{ji} for SU_j on channel F_i are existed and H_{ji} is positive, this algorithm converges to a unique Nash equilibrium [30]. In each step, each SU solves the optimization problem (25) by updating its strategy based on (27) until $\sum_{j=1}^{N} \Delta c_j^m(\mathbf{s}_j, \mathbf{s}_{-j})$, which is the difference between utility function in current step and previous step is less than a termination criterion. In each step, SU_j uses the value of $\mathbf{s_{-j}}$ from the previous step.

Algorithm 1 The iterative algorithm to find the best response of SLBHM game

```
Initialization Step:
for all j ⊆ N do
    Choose a feasible starting point s_j^(0) for SU_j.
    Broadcasts the initial strategy profile to other SUs.
end for
Iteration Phase:
loop
    if s^(l) satisfies a suitable termination criterion, Σ_{j=1}^N Δc_j^m(s_j, s_{-j}) < ε then
        STOP.
    end if
    l ← l + 1
    for all j ⊆ N do
        SU_j receives a message of other SUs' strategies profiles.
        SU_j updates its strategy at this stage, s_j^(l), using (27).
        SU_j broadcasts the updated strategy profile to other SUs.
    end for
end loop
```

6.2 SLBHM game for M/D/1 queuing model

As another special case, we investigate the M/D/1 queuing model, when the service time of the SUs and PUs are deterministic and is equal to D where D is a constant. In this case, we consider $X_i^{(PU)} = D$ and $X_{ji}^{(SU)} = D$. We should compute the service time of interrupted packets, $E\left[X_{ji}^{(f)}\right]$.

A returned SU's packet on a channel means that a PU packet arrival is happened in [0, D]. That is a Poisson event is happened during this interval. It is known that the distribution of the time at which this event occurred is uniform over $[0, D]$ [33]. When an event of Poisson process $A_i^{(PU)}(t)$ occurs during service time $[0, D]$ of a SU's packet on channel F_i, $X_{ji}^{(f)}$ is the remaining time of this interrupted packet and we have $E\left[X_{ji}^{(f)}\right] = \frac{D}{2}$. Also, $E\left[\left(X_i^{(PU)}\right)^2\right] = D^2$, $E\left[\left(X_{ji}^{(SU)}\right)^2\right] = D^2$, and $E\left[\left(X_{ji}^{(f)}\right)^2\right] = \frac{D^2}{3}$. Using these values we can derive the cost function in (10) for M/D/1 queueing system. Hence the SLBHM game can be analyzed in a similar manner using this cost function as in the previous subsection.

6.3 SLBHM game for M/G/1 queuing model

In order to investigate the M/G/1 model, we use a hyper-exponential distribution for service time of packets. In this model, it is assumed that the service time of 20, 30 and 50 % of arriving packets of SU_j on channel F_i follow different exponential distributions with means $\frac{1}{\mu_{ji}^{(1)}}$, $\frac{1}{\mu_{ji}^{(2)}}$, and $\frac{1}{\mu_{ji}^{(3)}}$ respectively. It is also assumed that PUs' packets have exponential service time with average $E\left[X_i^{(PU)}\right] = \frac{1}{\mu_i}$ and $E\left[(X_i^{(PU)})^2\right] = \frac{2}{(\mu_i)^2}$. By conditioning, the average service time of SUs is given by $E\left[X_{ji}^{(SU)}\right] = 0.2\,\frac{1}{\mu_{ji}^{(1)}} + 0.3\,\frac{1}{\mu_{ji}^{(2)}} + 0.5\,\frac{1}{\mu_{ji}^{(3)}}$ and $E\left[(X_{ji}^{(SU)})^2\right] = 0.2\,\frac{2}{(\mu_{ji}^{(1)})^2} + 0.3\,\frac{2}{(\mu_{ji}^{(2)})^2} + 0.5\,\frac{2}{(\mu_{ji}^{(3)})^2}$. Due to memoryless property of exponential distribution the service time of interrupted packets is also $E\left[X_{ji}^{(f)}\right] = 0.2\,\frac{1}{\mu_{ji}^{(1)}} + 0.3\,\frac{1}{\mu_{ji}^{(2)}} + 0.5\,\frac{1}{\mu_{ji}^{(3)}}$ and $E\left[(X_{ji}^{(f)})^2\right] = 0.2\,\frac{2}{\left(\mu_{ji}^{(1)}\right)^2} + 0.3\,\frac{2}{\left(\mu_{ji}^{(2)}\right)^2} + 0.5\,\frac{2}{(\mu_{ji}^{(3)})^2}$. Using these values we can derive the cost function in (10) for this queueing system and SLBHM game can be analyzed for this cost function.

7 Simulation environment and results

In order to simulate the CR networks we develop an advanced event based simulator using programming language C++. This simulator has been developed taking advantage of extensible modular structure and class diagram for improved scalability. The benefit of our simulator is its flexibility in choosing the parameters of interest, including number of SUs, number of channels, different traffic models and arrival rates for SUs and PUs, different service time model on each channel and different schedulers for channel selection probability of each SU. In this simulation environment, we use event-driven programming where different events such as arrival events, departure events, and handoff events on all channels are processed in a time slotted manner. Each event is processed by the event handler based on the event occurrence time. Also, each component in this simulator is modeled as a class with different processing functions which are able to evaluate various parameters of the system. For example, each channel is modeled by a class in C++ environment with different properties such as service rate, idle period, and busy period. Also, there are two C++ classes for two types of priorities, i.e., SUs which are low-priority users and PUs which are high-priority users. Each of these classes has different properties such as traffic model, arrival rate, number of packets, waiting time, service duration and different methods in order to set or compute these properties. For SUs, the number of handoff, interrupted packets, and the rate of interruptions are considered too. When a high priority packet arrives, the handoff event occurs and is processed. Then, the interrupted SU's packet returns to the low priority queue of the current channel and the PU's packet starts to take service. When the PU's transmission is finished, the channel state becomes idle. Then, the low priority interrupted packet starts to take service on this channel. After each transmission of packets, the departure event is generated and channels state becomes idle. Finally, sorted results are saved into a file that is delivered to a MATLAB parser for the purposes of visualization. In Fig. 3, the simplified structure of the simulator is captured.

In the next subsections, the proposed delay model and SLBHM game is evaluated and compared with other schemes using the developed simulator.

7.1 Overall system time evaluation

The simulation results in this section are provided to show the gain of joint load balancing and handoff management compared to the centralized solution and SLB [14] which just takes into account the load balancing using the algorithm which is introduced in [32], regardless of the priority

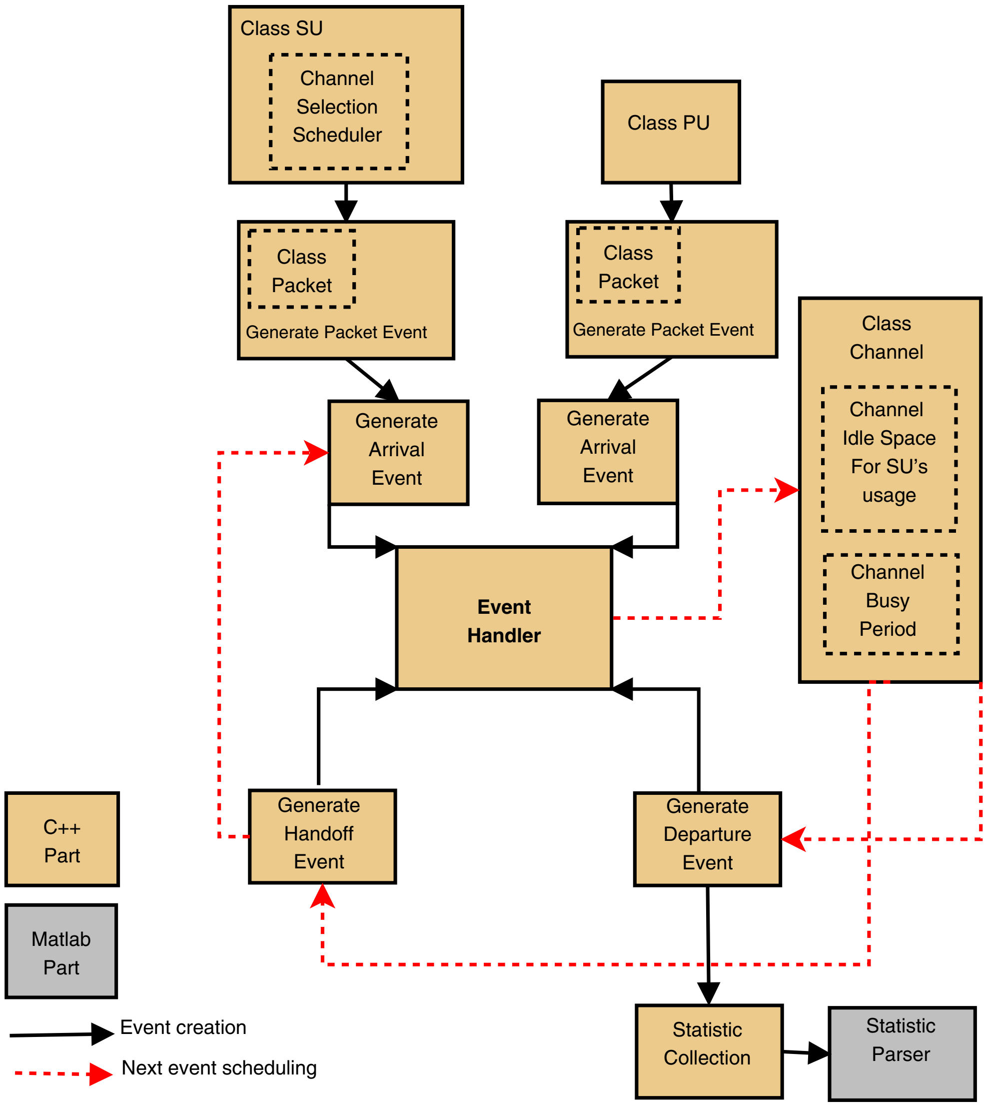

Fig. 3 Simplified simulator structure

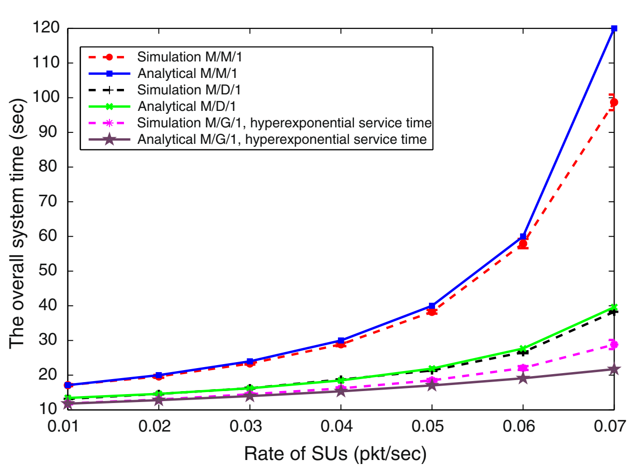

Fig. 4 Validation of proposed analytical delay model with simulation result by changing SU's packets arrival rate

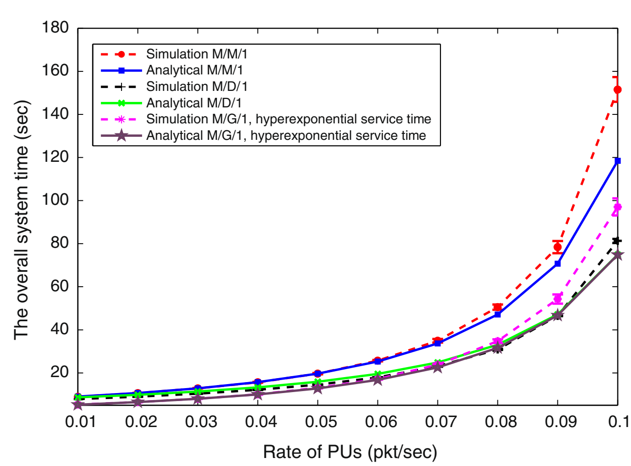

Fig. 5 Validation of proposed analytical delay model with simulation result by changing PU's packets arrival rate

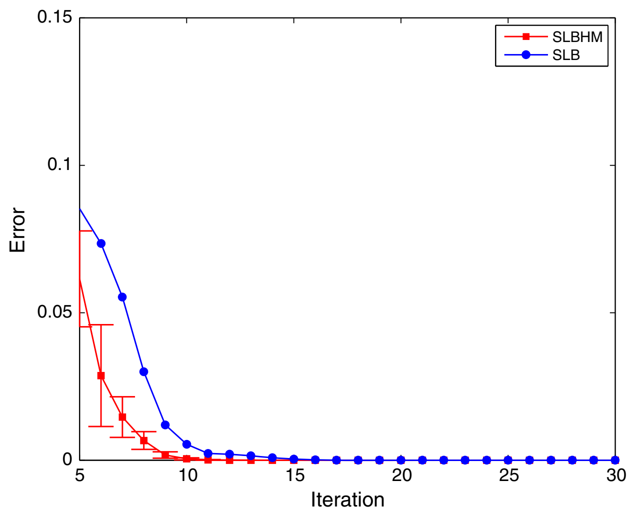

Fig. 6 The convergence behavior of SLB and SLBHM

of PUs to SUs and handoff effect. In [14] all of SUs and PUs have the same the priority and simple M/M/1 queueing model is used to balance the load of SUs on the channels. In addition the efficiency of the distributed solution is discussed by comparing the results with the optimal scheme. In all simulations, the reported results are the average of 50 times run where in each run we use 10,000 packets per SU and 50,000 packets per PU. In addition, to the average of each reported result the corresponding 90 % confidence intervals are depicted on figures to show their reliabilities.

First, we justify the cost function in (10) for the special cases of M/M/1, M/D/1, and M/G/1 queueing systems using the developed simulator. For this simulation we consider a simple scenario in which one SU is going to deploy the spectrum opportunities of one PU channel. The results of simulations and the analytical results are shown

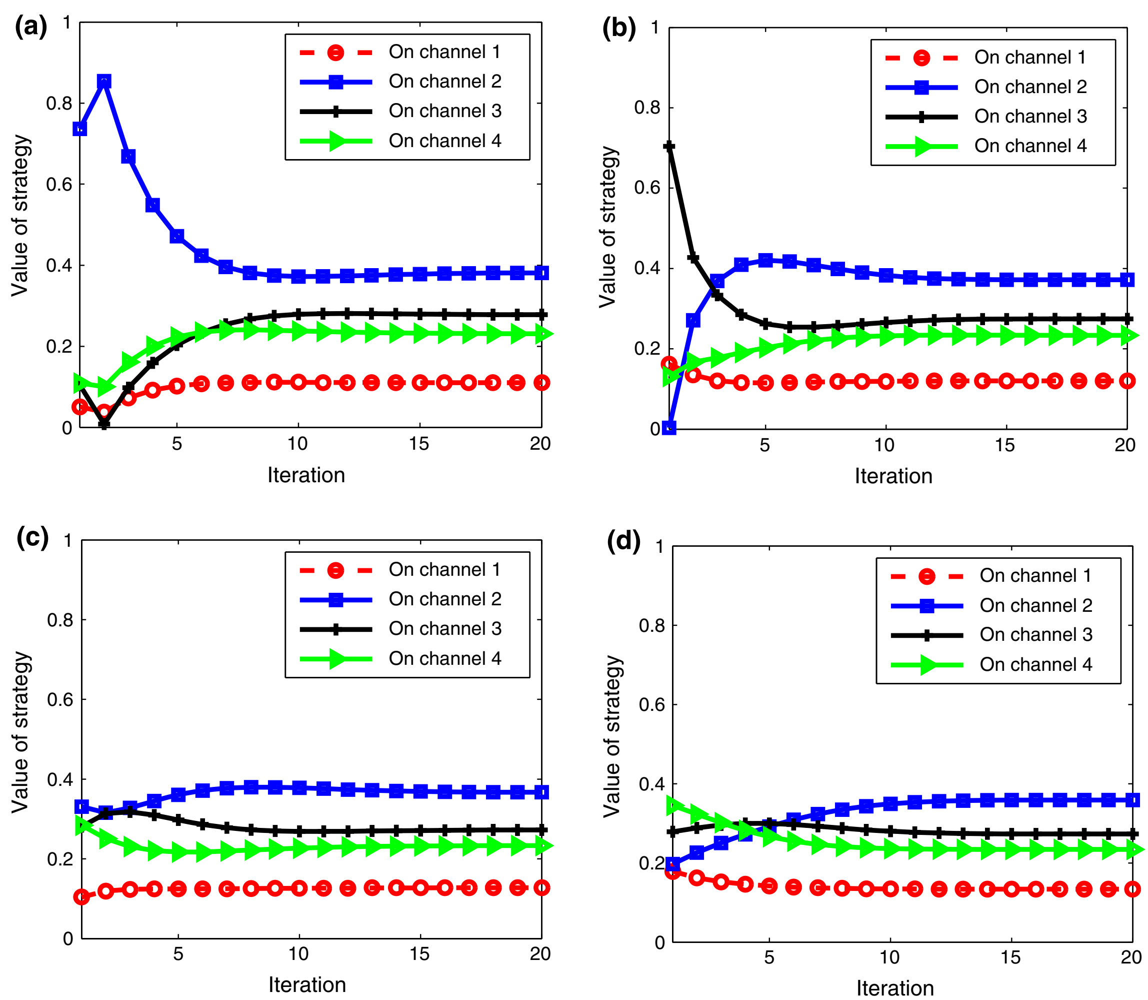

Fig. 7 **a** Convergence of strategies for SU1, **b** convergence of strategies for SU2, **c** convergence of strategies of strategies for SU3, **d** convergence of strategies for SU4

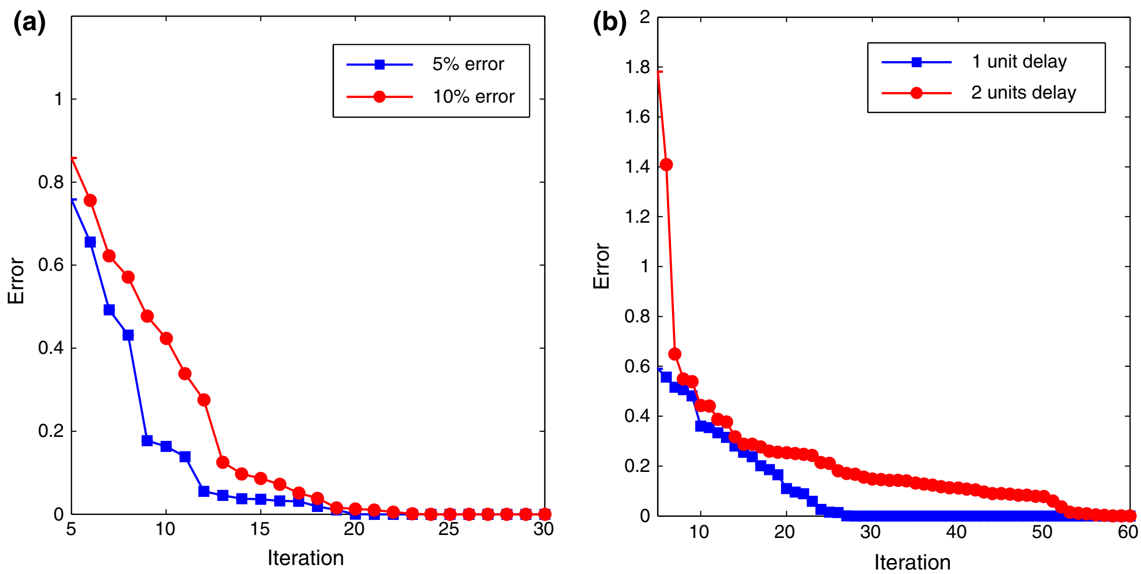

Fig. 8 Convergence behavior of SLBHM when information is received, **a** with different error, **b** with different delay units

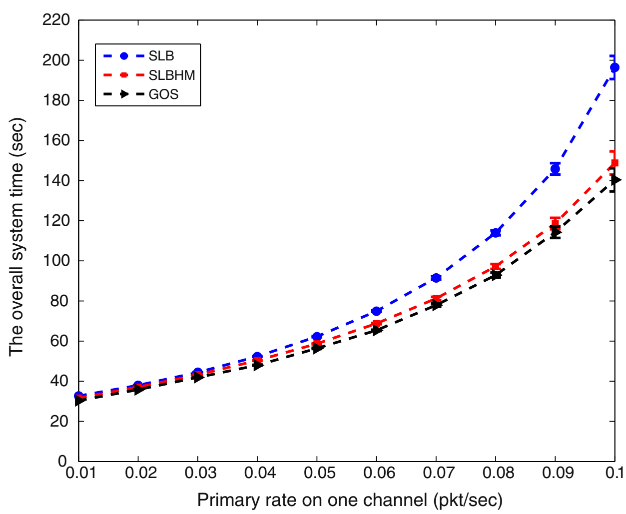

Fig. 9 The overall system time of GOS, SLBHM, and SLB

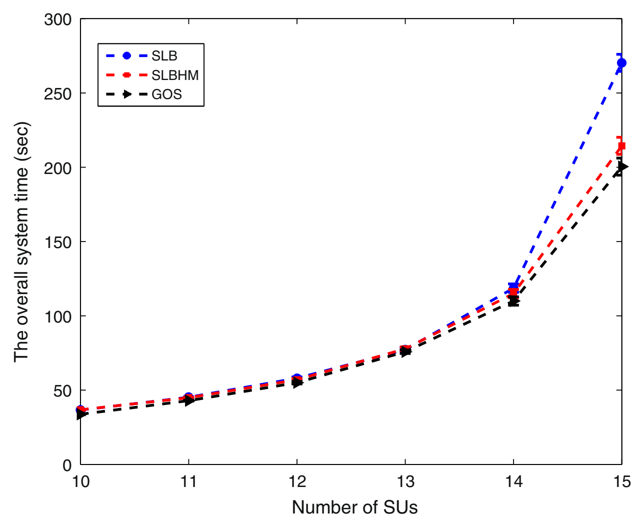

Fig. 10 The number of SUs impact on the overall system time

in Figs. 4 and 5. In Fig. 4 the SU's rate, $\lambda^{(SU)}$, is increased from 0.01 to 0.07 in the system and the PU's rate is $\lambda^{(PU)} = 0.05$ and in Fig. 5 the PU's rate, $\lambda^{(PU)}$ is increased from 0.01 to 0.1 in the system and the SU's rate is $\lambda^{(SU)} = 0.02$. In M/M/1 and M/D/1 model the average service time for PUs and SUs packets is considered $\frac{1}{\mu_i} = \frac{1}{\mu_{ji}} = 0.15$ and for M/G/1 model for SUs we suppose 20 30 and 50 % of packets follow different exponential distributions with means $\frac{1}{\mu_{ji}^{(1)}} = \frac{1}{0.15}$, $\frac{1}{\mu_{ji}^{(2)}} = \frac{1}{0.25}$, and $\frac{1}{\mu_{ji}^{(3)}} = \frac{1}{0.35}$ respectively. It is also assumed that PUs' packets have exponential service time with average $\frac{1}{\mu_i} = \frac{1}{0.15}$. As it is shown in Figs. 4 and 5, analytical results predict the system behavior well.

In the next simulation, we investigate the convergence behavior of SLBHM game for the M/M/1 queueing system and compare the results with SLB game [14]. In order to investigate the convergence of the algorithm, the difference between the sum of all SUs utility functions in current iteration and previous one is computed as the error. In this simulation, it is assumed that $\epsilon = 0.0001$, and when $\sum_{j=1}^{N} \Delta c_j^m(\mathbf{s}_j, \mathbf{s}_{-j}) < 0.0001$ the suitable termination criterion is satisfied. This error for SLBHM in consecutive iterations is shown in Fig. 6. In this scenario, we consider a network with $N = 4$ SUs and $M = 4$ primary channels. Also, the arrival rates of PUs and SUs are: $\lambda_1^{(PU)} = 0.1, \lambda_2^{(PU)} = 0.02, \lambda_3^{(PU)} = 0.04, \lambda_4^{(PU)} = 0.05$ and the arrival rates of SUs are: $\lambda_1^{(SU)} = 0.05, \lambda_2^{(SU)} = 0.06, \lambda_3^{(SU)} = 0.07, \lambda_4^{(SU)} = 0.08$. The service rates of all channels are the same, i.e., $\mu_i = 0.15, i = 1, 2, 3, 4$. In this figure, the average of error and the corresponding 95 % confidence interval for 50 runs with different random initial values of strategies is shown for SLBHM. Also the initial starting point of SLB is set to zero according to [32]. From this figure, we find that both algorithms converge after about 10 iterations. In Algorithm 1, we use logarithmic barrier method for each SU to solve (23) which is one of the fast method in solving non-linear optimization problems [31].

The convergence of the SUs' decisions to the Nash equilibrium is also shown in Fig. 7 in which the channels selection probabilities of each SU for a random initial feasible solution is depicted.

In the simulation we compare the results of SLBHM game with GOS and SLB schemes in terms of OST and channel utilization. The final strategy $\mathbf{S} = \left[\mathbf{s}_1^T, \mathbf{s}_2^T, \ldots, \mathbf{s}_N^T\right]^T$ is consistent with (26) for all SUs.

Figure 8 shows the convergence behavior of SLBHM algorithm in different scenarios when the other SU's strategies, $\mathbf{s}_{-j}$ receives at SU_j with error or delay. In Fig. 8(a) it is assumed that the strategies of other SU, $\mathbf{s}_{-j}$, are received with error. In this scenario, the strategies of SU_{-j} are affected by a random error which is selected according to a uniform distribution in the range of $\pm 0.05\mathbf{s}_{-j}$ or $\pm 0.1\mathbf{s}_{-j}$. That is when the received strategies of other SUs in SU_j have 5 or 10 % error. In another scenario, the strategies of other SU, $\mathbf{s}_{-j}$, are received with different time units of delay and results are shown in Fig. 8(b). Note that the step size of the iterative optimization algorithm should be decreased to guarantee the convergence in these scenarios.

Figure 9 shows the OST of the SLHB, SLB, and GOS when the load of the PUs is increased. The same simulation setup with $N = 4$ SUs and $M = 4$ is used as in the previous simulation. The arrival rate of PU_3 is increased from 0.05 to 0.1 and the spectrum access delay is shown in Fig. 9. This figure shows that SLBHM result outperforms SLB specially when the traffic load of PUs are heavy and also is close to the global optimal scheme.

The OST for SLBHM is compared with GOS and SLB as the number of SUs is increased in Fig. 10. We consider a network with $N = 10 \sim 15$ SUs and $M = 4$ primary channels. In order to investigate the effect of number of SUs, we consider that a new SU_j which comes to take service has the rate $\lambda_j^{(SU)} = 0.02$. This figure shows that the OST of SUs in SLBHM is decreased compared to SLB when the number of SUs is increased which means the load of the system is increased. Therefore, the SLBHM is more efficient in heavy loaded conditions. In addition, the results of SLBHM as a distributed solution are close to GOS scheme especially when the number of SUs is increased.

In order to compare the fairness of the proposed scheme, the Jain's fairness index on the spectrum access delay of the SUs is computed using (28) [34] for different schemes. This index for SLB and SLBH is 0.99 and for GOS is 0.89 which shows that the results of SLBHM is fair enough. This result is also reasonable since the GOS considers the fairness roughly for channel selection.

$$I(\mathbf{C}) = \frac{\left(\sum_{\mathbf{j=1}}^{\mathbf{N}} \mathbf{c_j}\left(\mathbf{s_j}, \mathbf{s_{-j}}\right)\right)^{\mathbf{2}}}{\mathbf{N}\sum_{\mathbf{j=1}}^{\mathbf{N}} \mathbf{c_j}\left(\mathbf{s_j}, \mathbf{s_{-j}}\right)^{\mathbf{2}}} \quad (28)$$

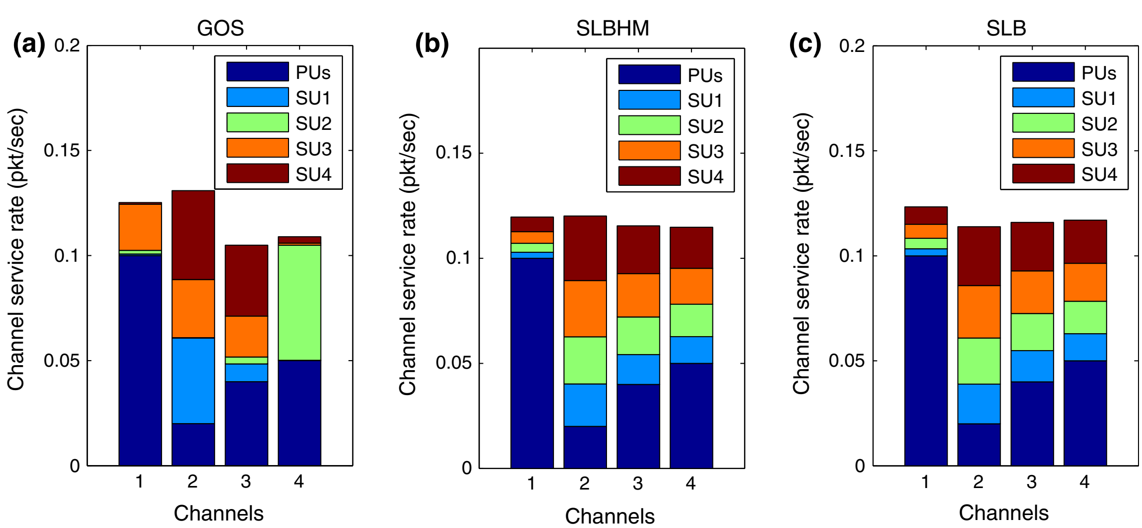

Fig. 11 The usage of channels, **a** GOS, **b** SLBHM, **c** SLB

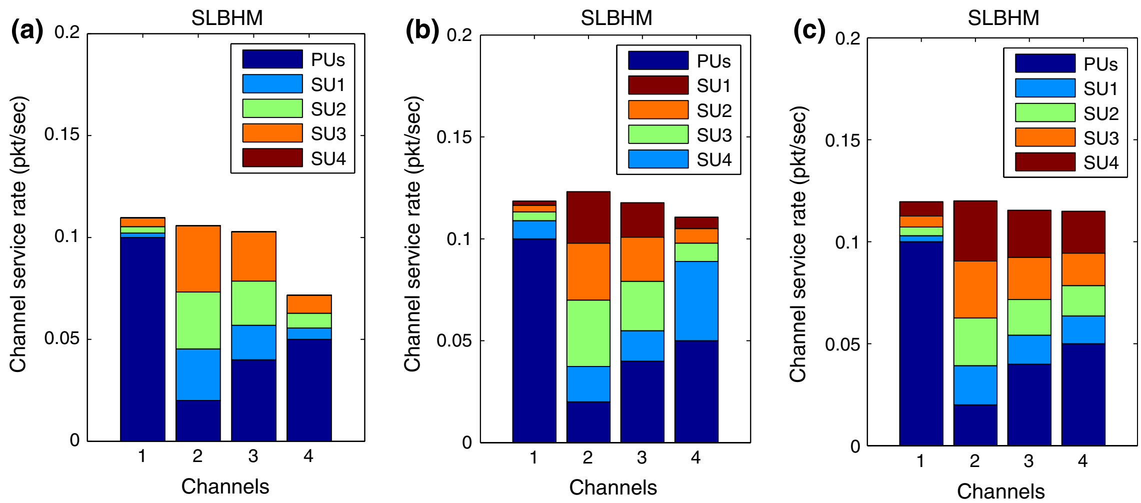

Fig. 12 The system at different iterations. **a** The system at iteration 20, **b** at iteration 21, SU_4 arrives to take service, **c** the system with four SUs in iteration 30 when it is converged

7.2 Exploitation of channels

The simulations in this section investigate how the discussed schemes exploit the idle spaces of channels. That is we look at load balancing from the primary system point of view. This may be important if the primary network channels statistics are varying in time.

In Fig. 11, using the same simulation setup as in the previous simulation with $N = 4$ SUs and $M = 4$, the ratio of total arrival rate to the total service rate of the system is shown. The results of this figure are at the system utilization 78 %.

Note that the available spectrum opportunities on channel 2 is higher than other channels, from Fig. 11 we find that this channel is mostly exploited by GOS which does not consider fairness criterion strictly. Also, this channel is more exploited by SLBHM compared to SLB. The consistent results are also derived for channel 1 which is less available for SUs. These results are also reasonable because the least and most number of handoffs and hence incurred delays will be on channel 2 and 1 respectively. This means that from the primary network point of view, GOS and SLBHM exploit the idle spaces of channels more efficiently compared to SLB but GOS does not balance the SUs' loads on primary channels as good as SLB and SLBHM.

In the last simulation we investigate how SLBHM behaves when a new SU join or remove from the secondary system. We consider the system when the first three SUs are active and their load is balanced with each algorithm, see Fig. 12(a). Then SU_4 join the system with an initial load distribution on channels as shown in Fig. 12(b). In Fig. 12(c) the channels load is shown when the load of SU_4 is balanced is shown. The results are the same as Fig. 12(b) as expected.

8 Conclusion

In this paper, we characterize data delivery time of SUs in CR networks by developing PRP M/G/1 queuing model and considering the handoff effects. We propose an algorithm for spectrum load balancing with handoff management in cognitive radio networks based on exponential service time. The problem is formulated as a non-cooperative game in which each SU decides on the channel selection probabilities taking into account the received messages from other SUs. The uniqueness of the Nash equilibrium point of the proposed game is analyzed and its efficiency is discussed by comparing with the optimal centralized solution. We develop an event based simulator to investigate the CR networks behavior and simulation results are provided to show the performance of the proposed scheme in terms of spectrum access delay, load balancing, and exploitation of channels' idle spaces.

Appendix 1: Proof of proposition 1

If the strategy set of players is compact then a nonempty set of Nash equilibrium solution will exist [30]. In addition, the sufficient condition for the uniqueness of Nash equilibrium is that the cost function of each player is a strongly convex function in the strategy space. The domain of the problem which is given by (16) and (17) is a convex set. Using (28) and (29) we find that the derivation of the cost function in (10), exists and taking into account (18) its hessian is positive definite. Therefore, the cost function is strongly convex. In (29) and (30), we have:

$$I_{ji} = E\left[\left(X_{ji}^{(f)}\right)^2\right] P_{ji}^h \lambda_j^{(SU)} + E\left[\left(X_{ji}^{(SU)}\right)^2\right] \lambda_j^{(SU)}$$

$$B_{ji} = E\left[\left(X_i^{(PU)}\right)^2\right] \lambda_i^{(PU)} + \sum_{k=1,k\neq j}^{N} \left(E\left[\left(X_{ki}^{(f)}\right)^2\right] P_{ki}^h s_{ki} \lambda_k^{(SU)} + E\left[\left(X_{ki}^{(SU)}\right)^2\right] s_{ki} \lambda_k^{(SU)} \right)$$

$$C_{ji} = E\left[X_{ji}^{(f)}\right] P_{ji}^h \lambda_j^{(SU)} + E\left[X_{ji}^{(SU)}\right] \lambda_j^{(SU)}$$

$$D_{ji} = E\left[X_i^{(PU)}\right] \lambda_i^{(PU)} + \sum_{k=1,k\neq j}^{N} \left(E\left[X_{ki}^{(f)}\right] P_{ki}^h s_{ki} \lambda_k^{(SU)} + E\left[X_{ki}^{(SU)}\right] s_{ki} \lambda_k^{(SU)} \right)$$

$$\frac{\partial c_j}{\partial s_{ji}} = \frac{1}{2\left(1 - E\left[X_i^{(PU)}\right]\lambda_i^{(PU)}\right)} \left(\frac{I_{ji}s_{ji} + B_{ji}}{1 - C_{ji}s_{ji} - D_{ji}} + \frac{\left(I_{ji}\left(1 - D_{ji}\right) + C_{ji}B_{ji}\right)s_{ji}}{\left(1 - C_{ji}s_{ji} - D_{ji}\right)^2} \right) + E\left[X_{ji}^{(SU)}\right] + \lambda_i^{(PU)} E\left[X_{ji}^{(SU)}\right] \frac{E\left[X_i^{(PU)}\right]}{1 - \lambda_i^{(PU)} E\left[X_i^{(PU)}\right]} \quad (29)$$

$$\mathsf{H} = \begin{pmatrix} \frac{\partial^2 c_j}{\partial s_{j1}^2} & 0 & \dots & 0 \\ 0 & \frac{\partial^2 c_j}{\partial s_{j2}^2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{\partial^2 c_j}{\partial s_{jM}^2} \end{pmatrix}$$
$$= \begin{pmatrix} \frac{1}{1 - E\left[X_1^{(PU)}\right]\lambda_1^{(PU)}} \frac{\left(I_{j1} - I_{j1}D_{j1} + C_{j1}B_{j1}\right)\left(1 - D_{j1}\right)}{\left(1 - C_{j1}s_{j1} - D_{j1}\right)^3} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{1}{1 - E\left[X_M^{(PU)}\right]\lambda_M^{(PU)}} \frac{\left(I_{jM} - I_{jM}D_{jM} + C_{jM}B_{jM}\right)\left(1 - D_{jM}\right)}{\left(1 - C_{jM}s_{jM} - D_{jM}\right)^3} \end{pmatrix} \quad (30)$$

Appendix 2: Proof of proposition 2

In order to compute an explicit solution for (25), we use the logarithmic barrier method by applying KKT conditions [30, 31]. In optimization problem (25), there is one equality condition and three inequality conditions. Since $\sum_{i=1}^{M} s_{ji} = 1$ we can ignore the condition $s_{ji} \leq 1$. Let α_j, β_{ji}, and η_{ji} denote the Lagrange multipliers for constraints $\sum_{i=1}^{M} s_{ji} = 1$, $s_{ji} \geq 0$, and the last constraint in (25). The Lagrangian of (25) is given by:

$$L_j = c_j^m(\mathbf{s}_j, \mathbf{s}_{-j}) + \alpha_j \sum_{i=1}^{M}(s_{ji} - 1) + \sum_{i=1}^{M} \beta_{ji}(-s_{ji}) + \sum_{i=1}^{M} \eta_{ji}\left(s_{ji}\lambda_j^{(SU)}\left(1 + P_{ji}^h\right) + \sum_{k=1,k\neq j}^{N}\left(s_{ki}\lambda_k^{(SU)}\left(1 + P_{ki}^h\right)\right) + \lambda_i^{(PU)} - \mu_i\right) \tag{31}$$

In (37), $\beta_{ji}(-s_{ji}) = 0$ and $\eta_{ji}\left(s_{ji}\lambda_j^{(SU)}(1+P_{ji}^h) + \sum_{k=1,k\neq j}^{N}(s_{ki}\lambda_k^{(SU)}(1+P_{ki}^h)) + \lambda_i^{(PU)} - \mu_i\right) = 0, i = 1,\ldots,M$.In order to find the optimum solution of this Lagrangian, we use Logarithmic barrier method which approximates the cost function without inequality conditions. In Logarithmic barrier method the cost function $c_j^m(\mathbf{s}_j, \mathbf{s}_{-j})$ can be written as:

$$c_j^m(\mathbf{s}_j, \mathbf{s}_{-j}) \approx c_j^m(\mathbf{s}_j, \mathbf{s}_{-j}) + \sum_{i=1}^{M}\left(-\frac{1}{r}\right)\log(s_{ji}) + \sum_{i=1}^{M}\left(-\frac{1}{r}\right)\log\left(\mu_i - \left(\lambda_i^{(PU)} + s_{ji}\lambda_j^{(SU)}\left(1 + P_{ji}^h\right) + \sum_{k=1,k\neq j}^{N}\left(s_{ki}\lambda_k^{(SU)}\left(1 + P_{ki}^h\right)\right)\right)\right) \tag{32}$$

and the Lagrangian is:

$$L_j \approx c_j^m(\mathbf{s}_j, \mathbf{s}_{-j}) + \sum_{i=1}^{M}\left(-\frac{1}{r}\right)\log(s_{ji}) + \sum_{i=1}^{M}\left(-\frac{1}{r}\right)\log\left(\mu_i - \left(\lambda_i^{(PU)} + s_{ji}\lambda_j^{(SU)}(1 + P_{ji}^h) + \sum_{k=1,k\neq j}^{N}\left(s_{ki}\lambda_k^{(SU)}(1 + P_{ki}^h)\right)\right)\right) + \alpha_j \sum_{i=1}^{M}(s_{ji} - 1) \tag{33}$$

If r is large enough, $\beta_{ji}(-s_{ji}) = \frac{1}{r}$ and $\eta_{ji}\left(s_{ji}\lambda_j^{(SU)}(1 + P_{ji}^h) + \sum_{k=1,k\neq j}^{N}(s_{ki}\lambda_k^{(SU)}(1 + P_{ki}^h)) + \lambda_i^{(PU)} - \mu_i\right) = \frac{1}{r}$. The derivation and hessian of this Lagrangian can be computed by (33) and (34):

$$G_{ji} = \frac{\partial c_j^m(\mathbf{s}_j, \mathbf{s}_{-j})}{\partial s_{ji}} - \frac{1}{r\, s_{ji}} - \frac{1}{r(\mu_i - (\lambda_i^{(PU)} + s_{ji}\lambda_j^{(SU)}(1 + P_{ji}^h) + \sum_{k=1,k\neq j}^{N}(s_{ki}\lambda_k^{(SU)}(1 + P_{ki}^h))))} \tag{34}$$

$$H_{ji} = \frac{\partial^2 c_j^m(\mathbf{s}_j, \mathbf{s}_{-j})}{\partial s_{ji}^2} + \frac{1}{r\,(s_{ji})^2} + \frac{(\lambda_j^{(SU)}(1 + P_{ji}^h))^2}{r(\mu_i - (\lambda_i^{(PU)} + s_{ji}\lambda_j^{(SU)}(1 + P_{ji}^h) + \sum_{k=1,k\neq j}^{N}(s_{ki}\lambda_k^{(SU)}(1 + P_{ki}^h))))^2} \tag{35}$$

By applying Newton step in logarithmic barrier method, we can compute the explicit solution for α_j and s_{ji}. The Newton step in logarithmic barrier method is given by [31]:

$$rH_{ji}\Delta s_{ji} + \alpha_j = -rG_{ji} \tag{36}$$

By considering the condition (17), $\sum_{i=1}^{M} \Delta s_{ji} = 0$ and by this condition the value of α_j in each step can be computed by:

$$\sum_{i=1}^{M} \Delta s_{ji} + \sum_{i=1}^{M} \frac{\alpha_j}{rH_{ji}} = \sum_{i=1}^{M} \frac{-rG_{ji}}{H_{ji}} \tag{37}$$

$$\alpha_j = r\frac{\sum_{i=1}^{M} \frac{-G_{ji}}{H_{ji}}}{\sum_{i=1}^{M} \frac{1}{H_{ji}}} \tag{38}$$

By computing the derivation, hessian and Lagrange multiplier we can reach to optimum solution of (25) iteratively by:

$$s_{ji}^{l+1} = s_{ji}^{l} + \kappa\left(\frac{-G_{ji}}{H_{ji}} - \frac{\alpha_j}{H_{ji}}\right) \tag{39}$$

In optimum solution, which is denoted by s_{ji}^*, we have $-\alpha_j = G_{ji}$. We can compute the explicit solution for s_{ji}^* in M/M/1 queuing model. For M/M/1, the value of G_{ji} is:

$$G_{ji} = \frac{2s_{ji}\mu_{ji}\lambda_j^{(SU)}(1+P_{ji}^h) - \left(s_{ji}\lambda_j^{(SU)}(1+P_{ji}^h)\right)^2 + R_{ji}\mu_{ji}}{\left(\mu_i - \lambda_i^{(PU)}\right)(\mu_{ji} - s_{ji}\lambda_j^{(SU)}(1+P_{ji}^h))^2} + \frac{1}{\mu_i} + \frac{\lambda_i^{(PU)}}{\mu_i}\left(\frac{1}{\mu_i - \lambda_i^{(PU)}}\right) \tag{40}$$

By solving the $-\alpha_j = G_{ji}$, we have:

$$\left(s_{ji}\lambda_j^{(SU)}\left(1+P_{ji}^h\right)\right)^2(1+Q) - 2\left(s_{ji}\mu_{ji}\lambda_j^{(SU)}\left(1+P_{ji}^h\right)\right)(1+Q) + \left(Q\mu_{ji}^2 - R_{ji}\mu_{ji}\right) = 0 \tag{41}$$

where $Q = \left(-\alpha_j - \frac{1}{\mu_i} - \frac{\lambda_i^{(PU)}}{\mu_i}\left(\frac{1}{\mu_i - \lambda_i^{(PU)}}\right)\right)\left(\mu_i - \lambda_i^{(PU)}\right)$.

Therefore if there exists a feasible solution for the system the best strategy of SU_j for selecting channel F_i at the Nash equilibrium, s_{ji}^*, is given by:

$$s_{ji}^* = \frac{\mu_{ji}(1+Q) + \sqrt{\mu_{ji}^2(1+Q)^2 - (1+Q)\left(Q\mu_{ji} - R_{ji}\mu_{ji}\right)}}{1+Q} \tag{42}$$

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Mina Fahimi received her B.S in Computer Software Engineering from Al-Zahra University, Tehran, Iran and her M.Sc. degree from K. N. Toosi University of Technology, Tehran, Iran, in Artificial Intelligence, Computer Engineering. Her research interests include machine learning, game and optimization theories, distributed algorithms, and wireless networks.

Abdorasoul Ghasemi received his B.S. degree (with honors) from Isfahan University of Technology, Isfahan, Iran and his M.Sc. and Ph.D. degrees from Amirkabir University of Technology (Tehran Polytechnique), Tehran, Iran all in Electrical Engineering in 2001, 2003 and 2008, respectively. He is currently an Assistant Professor with the Faculty of Computer Engineering of K. N. Toosi University of Technology, Tehran, Iran. His research interests include communication networks, network protocols, resource management in wireless networks, and applications of optimization and game theories in networking.