# A 3-D Scan Matching using Improved 3-D Normal Distributions Transform for Mobile Robotic Mapping

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Abstract—A 3D scan matching is an important component for sensor based localization and mapping by a mobile robot in natural environment. In this paper, the present authors propose a way to extend 2D Normal Distributions Transform(NDT) scan matching method to 3D scan matching, and its improvement for faster processing time. This scan matching method divides scan into voxels, and approximates scan points in each cell into normal distribution. That matching time is  $\mathcal{O}(N)$  with N of the number of input scan points. The authors describe in this paper, NDT for 3D scan points, its acceleration using the dual resolutions of NDT, and experiments of map building in large scale environments.

#### I. INTRODUCTION

In this paper, the present authors propose a scan matching method for 3-D space and its improvement, that registers two 3-D scan data-sets by a Laser Range Finder (LRF) fast.

Nowadays, many research issues of automatic 3-D mapping for large unknown environment are actively reported. Such mapping methods build an environments map using 3-D LRF by a mobile robot moving on rough terrain. Generally, it is very difficult for a mobile robot on rough terrain to get its own position by internal sensors. Therefore, the mapping requires techniques for concurrent mapping and localization in 3-D space using internal and external sensors. Such techniques are well known as CML or SLAM.

This research objective is to realize fast scan matching in 3-D space, and contributes to 3-D SLAM. In other words, the proposed method builds an environment map by scanned 3-D shape from mechanically nodding LRF on a mobile robot, and solves its position by scan matching in real-time.

Hereafter, the term "input scan" denotes newly obtained 3-D data from the LRF, and the term "reference scan" denotes a map which has been built by the past input scans. Let N and M be the number of points in the input scan and in the reference scan respectively. One of the issues for real-time 3-D scan matching is a computational load with respect to the number of scan points. The number of 3-D scan points is larger than that of 2-D scan points. The number of reference scan points is increasing as the map is extended under scan matching process. The proposed method achieves a fast 3-D scan matching, where Peter Biber's 2-D Normal Distributions Transform (NDT)[1] is incorporated. The computation time

order of the proposed method achieves  $\mathcal{O}(N)$ . Also, 3-D NDT scan matching and its improvement of convergence range are proposed. Experimental results of 3-D NDT scan matching in realistic environments are demonstrated.

#### II. RELATED WORKS

Research issues of 2-D SLAM using sensor data on horizontal plane are reported by many researchers. Especially, scan matching using high accurate LRF realizes high accurate localization and mapping in real-time[1][2]. Recently, many companies develops real-time 3-D range sensing devices and enriches that performance. These devices are promissing for real-time 3-D SLAM and position tracking in 3-D map[3]. The most popular methods usually follow the Iterative Closest Point(ICP) algorithm[4]. They are based on iterative process where they first compute the correspondences between reference scan and transformed input scan points by searching a closest point each other, and then they minimize the least square distance using newton's algorithm. The ICP algorithm can be applied to various shapes of the objects because it uses raw data of scanned points. Calculation load of ICP arises from making correspondences of the scan points. This load takes  $\mathcal{O}(MN)$  if brute force search algorithm is used, and  $\mathcal{O}(Nloq(M))$  if k-DTree space quantization algorithm is used[5]. For other improvement of ICP, Benjemaa [6] uses zbuffer to search corresponding scan points. This method does not necessarily obtain the closest points, and minimize the distance between every point of the input scan and every point of reference scan which is rendered at the view point of the input scan. This method is hopeful to accelerate computation time by incorporating GPU. However, the computational time becomes slower as the map is extended because of whole rendering of the reference scan. On the other hand, Neugebauer [7] proposed acceleration for ICP that uses linearized transformation matrix under small rotation angle hypothesis. This method calculates transformation matrix by least square method. Peter's NDT[1] divides euclid space into lattice cells, and converts reference scan points in a cell into normal distribution. This method can achieve calculation load of  $\mathcal{O}(N)$ . Therefore, NDT is suitable even for 3-D SLAM and

position tracking using a 3-D range camera. However, Peter et al. only implement 2-D NDT and they do not discuss the relationship between the matching performance and cell size.

#### III. 3-D SCAN MATCHING USING 3-D NDT

In this section, 3-D Normal Distributions Transform (NDT) which is applied to reference scan, and 3-D scan matching using 3-D NDT are illustrated.

## A. NDT scan matching algorithm

The NDT scan matching process takes the following steps;

- 1) Apply NDT to reference scan.
- Read an input scan and set initial tentative position by preview or odometry.
- Transform the input scan by 3-D coordinate transformation.
- 4) Select a corresponding ND voxel (see III-B) of each input scan point.
- 5) Update the matching parameter by newton's method.
- 6) If matching parameter is converged, then go to 7, else
- 7) Add transformed input scan to reference scan.
- 8) Go to 2.

Hereafter, details of the process flow are illustrated.

# B. Normal Distributions Transform (NDT)

NDT is a process to transform point data in a voxel into normal distribution. The voxel is a cubic lattice in a 3-D space, where the whole space is divided into voxels. In this paper, a normal distribution in a voxel is named "ND voxel".

NDT process is as follows (Fig.1(a)); First, reference scan point vectors  $\boldsymbol{x}_i = (x_i, y_i, z_i)^t (i = 0 \dots M - 1)$  are measured, then they are voted in the appropriate voxels by rounding the coordinate value of  $\boldsymbol{x}_i$ . Let  $M_k$  be the number of points in a ND voxel k, and  $\boldsymbol{x}_{ki}$  ( $i = 0 \dots M_k - 1$ ) is a coordinate vector of the voted point in a ND voxel k. An average  $\boldsymbol{p}_k$  and covariance matrix  $\Sigma_k$  of the ND voxel k are calculated as follows;

$$p_k = \frac{1}{M_k} \sum_{i=0}^{M_k - 1} x_{ki}, \tag{1}$$

$$\Sigma_{k} = \frac{1}{M_{k}} \sum_{i=0}^{M_{k}-1} (\boldsymbol{x}_{ki} - \boldsymbol{p}_{k}) (\boldsymbol{x}_{ki} - \boldsymbol{p}_{k})^{t}, \qquad (2)$$

and estimation value e(x) of ND voxel k is defined as,

$$e(\mathbf{x}) \sim exp(-\frac{(\mathbf{x} - \mathbf{p_k})^t \mathbf{\Sigma_k^{-1}}(\mathbf{x} - \mathbf{p_k})}{2}).$$
 (3)

In case of applying NDT to reference scan, it is necessary for us to pay careful treatment on the following two problems. One of them is the effect of quantization of the space into voxels. Peter's 2-D NDT [1] improves it by overlapping the ND cells half by half. Therefore, one point falls into four ND cells (Fig.1(b)). The other one is degeneration of the covariance matrix. If the voted point vectors in a ND voxel are aligned in a line or the vector is unique in the voxel, then the covariance matrix loses holomorphy and the inverse matrix of the covariance matrix can not exist. A measure of this problem

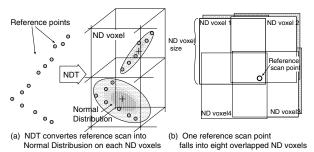


Fig. 1. NDT and Overlapped ND voxels

in [1] is to modify the minimum eigen value of the covariance matrix with 0.001 times as much as the maximum eigen value, if the minimum one is smaller than 0.001 times as much as the maximum one. The authors take same approaches. However, the number of overlapping ND voxel becomes eight in 3-D space.

## C. Incremental update of ND voxel

The number of reference scan points  $M_k$  in the ND voxel k is increased as the environment map is expanded. Therefore, the computational load to obtain mean vectors  $\boldsymbol{p}_k(k=1,...,M_k)$  and covariance matrix  $\boldsymbol{\Sigma}_k(k=1,...,M_k)$  will increase according to equations (1), and (2).

To decrease this load, the authors take the following incremental update equations to apply NDT to reference scan in ND voxel. When ND voxel k gets new reference points  $x_{ki}$ , then mean vector  $p_k$  and covariance matrix  $\Sigma_k$  are updated by following equations;

$$\boldsymbol{m}_k = \boldsymbol{m}_{k_{old}} + \boldsymbol{x}_{ki}, \quad \boldsymbol{S}_k = \boldsymbol{S}_{k_{old}} + \boldsymbol{x}_{ki} \boldsymbol{x}_{ki}^t,$$
 (4)

$$\boldsymbol{p}_k = \frac{\boldsymbol{m}_k}{M_k}, \ \boldsymbol{\Sigma}_k = \frac{\boldsymbol{S}_k - \boldsymbol{p}_k \boldsymbol{m}_k^t}{M_k}.$$
 (5)

In the implementation,  $m_k$  and  $S_k$  will be maintained in each ND voxel k. When new input scan is associated with the existing reference scan,  $m_k$  and  $S_k$  are updated by only Equation (4). A mean vector  $p_k$  and covariance matrix  $\Sigma_k$  are updated by Equation (5) only when  $m_k$  and  $S_k$  have been updated at the matching operation.

This incremental update makes it possible to calculate the update with  $\mathcal{O}(N_{k-1})$  that is in accordance with the number of the input scan points  $N_{k-1}$ , and is independent from the number of reference scan points  $M_k$ .

# D. Coordinate transform of input scan

3-D coordinate transform equations for input scan points  $X = x_i (i = 0...N - 1)$  are given as follows;

$$x_i' = Rx_i + t', (6)$$

where R is rotation matrix of euler angle  $\alpha, \beta, \gamma$  to rotate z, y, z axis. R is calculated as follows:

$$\mathbf{R} = \begin{pmatrix} \cos \alpha - \sin \alpha & 0 \\ \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \cos \beta & 0 & \sin \beta \\ 0 & 1 & 0 \\ -\sin \beta & 0 & \cos \beta \end{pmatrix} \\ \times \begin{pmatrix} \cos \gamma & -\sin \gamma & 0 \\ \sin \gamma & \cos \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
(7)

Here, t' is a translation vector;

$$\mathbf{t'} = \begin{pmatrix} t_x & t_y & t_z \end{pmatrix}^t. \tag{8}$$

In this paper, the coordinate transform parameter vector  $\mathbf{t} = (t_x, t_y, t_z, \alpha, \beta, \gamma)^t$  is named matching parameter.

# E. 3-D NDT scan matching

Scan matching is a parameter search problem that finds the best coordinate transform parameter vector  $\boldsymbol{t}$  for the input scan to match the existing reference scan. For this propose, a measure is provided for evaluation of fitness between the existing reference scan and the input scan, and such a coordinate transform vector  $\boldsymbol{t}$  that maximize the measure is obtained.

The measure function of NDT is defined as follows;

$$E(\boldsymbol{X}, \boldsymbol{t}) = \sum_{i}^{N-1} exp \frac{-(\boldsymbol{x}_{i}' - \boldsymbol{p}_{i})^{t} \Sigma_{i}^{-1} (\boldsymbol{x}_{i}' - \boldsymbol{p}_{i})}{2}, \quad (9)$$

where  $p_i$  and  $\Sigma_i$  are mean vector  $p_{k'}$  and covariance matrix  $\Sigma_{k'}$  of ND voxel k' corresponding to the vector  $x_i'$  respectively. If the value for the measure function E is high, then input scan and reference scan are well-corresponding. Newton's nonlinear function optimizer is utilized to find a parameter vector t such that E(X,t) is maximized on the NDT scan matching. Because Newton's algorithm is basically a nonlinear function minimization algorithm, an optimizing function f(t) for NDT scan matching must be;

$$f(t) = -E(X, t).$$

Parameter vector t is updated by following equation;

$$t_{new} = t - H^{-1}g, \tag{10}$$

where, g and H are partial differential and second order partial differential of optimizing function. They are ;

$$g = \sum_{i=0}^{N-1} \tilde{g}, \quad H = \sum_{i=0}^{N-1} \tilde{H}, \quad q = x_i' - p_i,$$
 (11)

$$\tilde{g}_n = \frac{\partial f}{\partial t_n} = \mathbf{q}^t \Sigma_i^{-1} \frac{\partial \mathbf{q}}{\partial t_n} \exp \frac{-\mathbf{q}^t \Sigma_i^{-1} \mathbf{q}}{2},$$
 (12)

$$\tilde{H}_{nm} = \frac{\partial^{2} f}{\partial t_{n} \partial t_{m}} = -\exp \frac{-\boldsymbol{q}^{t} \Sigma_{i}^{-1} \boldsymbol{q}}{2} \\
 ((-\boldsymbol{q}^{t} \Sigma_{i}^{-1} \frac{\partial \boldsymbol{q}}{\partial t_{n}}) (\boldsymbol{q}^{t} \Sigma_{i}^{-1} \frac{\partial \boldsymbol{q}}{\partial t_{m}}) - \\
 (\boldsymbol{q}^{t} \Sigma_{i}^{-1} \frac{\partial^{2} \boldsymbol{q}}{\partial t_{n} \partial t_{m}}) - (\frac{\partial \boldsymbol{q}}{\partial t_{m}} \boldsymbol{q}^{t} \Sigma_{i}^{-1} \frac{\partial \boldsymbol{q}}{\partial t_{n}})),$$
(13)

where,  $\frac{\partial \boldsymbol{q}}{\partial t_i}$  and  $\frac{\partial^2 \boldsymbol{q}}{\partial t_i \partial t_j}$  are partial differential and second partial differential of  $\boldsymbol{q}$  by  $\boldsymbol{t}=(t_x,t_y,t_z,\alpha,\beta,\gamma)^t=(t_1,t_2,t_3,t_4,t_5,t_6)^t$ .  $\frac{\partial \boldsymbol{q}}{\partial t_i}$  is derived from Equation (14) and the second order differential can be derived by approximation such as:

$$\frac{\partial^2 \mathbf{q}}{\partial t_n \partial t_m} \simeq \frac{\frac{\partial \mathbf{q}(t_1, \dots, t_m + \Delta t_m, \dots, t_6)}{\partial (t_n)} - \frac{\partial \mathbf{q}(\mathbf{t})}{\partial t_n}}{\Delta t_m}.$$
 (15)

# IV. NDT CHARACTERISTICS AND ITS IMPROVEMENT

In this section, NDT scan matching characteristics and its improvements are illustrated.

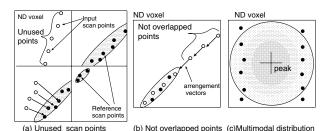


Fig. 2. Causes of matching error

#### A. ND voxel size and matching characteristics

In NDT method, the reference points are voted in the corresponding voxels. Mean vector and covariance matrix are calculated from the positions of the voted reference points in each voxel. Then the voxel is characterized by the mean vector and the covariance matrix.

In case that ND voxel size is too small, some input scan points may not correspond to a ND voxel which is characterized with the reference scan points (Fig.2(a)). Such points will not contribute to position correction, therefore, convergence time and extremal value are to increase. On the other hand, if such points were outliers or truly newly captured, Fig.2(a) can be recognized as a proper case. The voxel size is smaller, the mismatch case becomes less.

The other case that ND voxel size is large, will provide the fact that many input scan points will correspond to the characterized ND voxel. Its converging area becomes large and converging speed is fast. However, some ND voxel has non overlapping points between reference scan points and input scan points. This fact causes matching error (Fig.2(b)). In addition, if ND voxel size is larger than shape detail of environment, then distribution in a ND voxel isn't unimodal (Fig.2(c)). That is not appropriate approximation and causes matching error.

# B. Improve matching speed of NDT

As is mentioned in Section IV-A, convergence behavior will vary according to the voxel size. If it is large, convergence will be fast but it will take high possibility of miss matching. If it is small, vice versa. To improve this problem, the authors propose an approach to take different size of ND voxel size.

- 1) Converging stage: In the beginning of iterative process of the scan matching, a large ND voxel whose size is n times as much as the size of the normal voxel will be applied to any of input scan points which are further than the threshold distance from the scanner position (Fig.3(a)) (n is 4 in this paper).
- 2) Adjusting stage: When the value of the measure function E becomes stable during the matching iteration, put all the ND voxel size back to normal. Then continue the iteration until E converges (Fig. 3(b)).

This idea arise from the fact that even small angular deviation of the scanner posture causes large displacement of the scanned point if it is far from the scanner position. Therefore, this algorithm takes an advantage of large ND voxel at far points and small ND voxel at near points at the beginning

Reference scan points

Scanner

Far scan points

Near scan points

Adjustment vector

Large ND voxels

(a) Converging state

Reference scan points

Scanner

Fig. 3. NDvoxel size selection by distance of a input scan

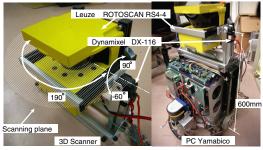


Fig. 4. 3-D Scanner with Leuze RS4-4 and mobile robot

of the matching. Finally, matching is converged then taking accuracy of matching using small ND voxel only.

#### V. EXPERIMENTS

In this Section, experimental results of proposed ND voxel selecting scan matching algorithm are illustrated.

The 3-D range sensor and a mobile robot for this experiments are shown in Fig. 4. The range sensor is Leuze ROTOSCAN RS-4 laser scanner. The authors equipped the sensor with tilting mechanism, which makes it possible to scan in 3-D space. Sensors view angle is -60 to 90 degrees in tilt direction and  $\pm 95$  degrees in horizontal direction of LRF. Its resolution is 0.36 degrees, accuracy in depth is  $\pm 81mm$ . The mobile robot "PC Yamabico" is a mobile base for robotic mapping in large environments. This mobile robot can measure own position by odometry. NDT matching program are implemented in C language which runs on Pentium4 3.4GHz PC under Linux OS.

#### A. Matching performance

The authors took scan data by using the laser scanner at the several positions and postures in the experimental environment. The environment is a laboratory room of the

TABLE I
POSITION ESTIMATION AND TIME

Actual sensor	Estimated	Error of	Time,
displacement	displacement	estimated	iterations
(relative)	by scanmatching	displacement	
$400mm, 0^{\circ}$	391.2 <i>mm</i> ,0.28°	$8.8mm, -0.28^{\circ}$	0.23s (47)
$800mm, 0^{\circ}$	$802.5mm, 0.19^{\circ}$	$2.5mm, -0.19^{\circ}$	0.46s (167)
$0mm, 30^{\circ}$	17.4 <i>mm</i> , 30.1°	$-17.4mm, -0.1^{\circ}$	0.38s (155)
$0mm, -30^{\circ}$	1.1 <i>mm</i> , -29.93°	$-1.1mm, -0.1^{\circ}$	0.67s (252)
$800mm, -30^{\circ}$	793.3 <i>mm</i> ,-29.7°	$6.7mm, -0.3^{\circ}$	0.51s (209)

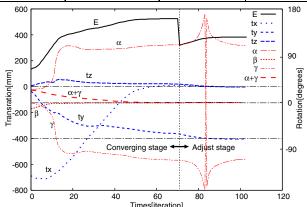


Fig. 5. Transition of matching parameter

authors (Fig.8). The number of every input scan points is about 10000 points, and actually a tenth of the input scan points are used for the scan matching program.

Table 1 shows the results of the position estimation after the proposed scan matching process. The reference scan was obtained at the origin of the local coordinate and its posture is set to the reference. Then, input scans are obtained at the relative displacements and postures which are presented on the first column of Table I. The 2nd column of Table I presents estimated displacements after the scan matching which begins with the zero initial displacement and posture.

These posture angles of matching results are sum of  $\alpha$  and  $\gamma$ , because  $\beta$  of y axis rotation value are very small over this experiments.

In this Table, matching error between actual position and matching position shows small. Especially, rotation angles are very small. In addition, matching time is very fast that is faster than 1 second.

Transition of the measure function value E and matching parameters at 400mm in displacement and 0 degree in angle is illustrated in Fig. 5. In initial state of the matching, the matching parameters  $t_x, t_y, \alpha$  and  $\beta$  are not on the target value.

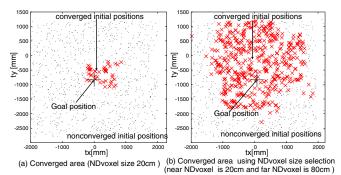


Fig. 6. Convergence area

In the converging stage, evaluation function E increases and matching parameters converges target value. In addition, at 70 times of matching process, evaluation function E decreases. That is changing from converging stage to adjusting stage. In adjusting stage, matching parameters becomes more closer in target value.

# B. Converging area

NDT scan matching converging area will change according to the ND voxel size. Converging area tests of NDT scan matching are shown in Fig.6. This experimental setting is 800mm in displacement between input scan and reference scan. These initial matching parameters  $t_x$  and  $t_y$  are given by random sampling, and  $t_z$ ,  $\alpha$ ,  $\beta$  and  $\gamma$  are set to 0. In this Figure, a symbol X is converged initial position and dot is else. In Fig.6, (a) is result of constant voxel size 20cm and (b) is result of proposed voxel size selection algorithm where voxel size is 20cm if the scanned point is nearer than 3m and else voxel size is 80cm in converging stage , and 20cm all the cases in adjusting stage.

In this experiments, converging area of constant ND voxel size as Fig.6(a) are about  $\pm 20cm$ . On the other hand, converging area of proposed ND voxel size selection as Fig.6(a) are extended to  $\pm 1m$ . These results show that proposed ND voxel size selection algorithm is effective to extend convergence area.

#### C. Computational load

A ND voxel which corresponds to each point of the input scan is uniquely found from the coordinate values of the point. The computational load of calculation for mean and covariance matrix of the voted points in a ND voxel has upper bound according to the formula in section III-C. Therefore, the proposed NDT scan matching method achieves computational load at  $\mathcal{O}(N)$  with the number of input scan points N independent from the reference scan points number M.

Figure 7 presents the performance of the calculation time with respect to the number of input scan points. In Fig. 7, two performances for the number of reference scan of M=24404 and M=337956 are illustrated.

In this Figure, the two computation times are linearly increased by the number of input scan points N and these are independent from the number of reference scan points number M. Therefore, computational load of NDT scan matching proves  $\mathcal{O}(N)$  experimentally .

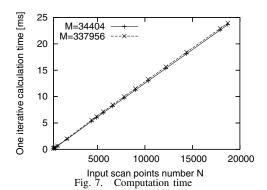


Fig. 8. Experimental environment

# D. Large scale environments mapping experiments

A 3-D mapping in large scale environment using mobile robot is experimented. Experimental environment is the author's laboratory (Fig.8) which is rectangular shape of 7m in width and 15m in length.

Matching results of the environment are shown in Fig.9 (bird view) and Fig.10(top view). In Fig.10, each mark of L are origin of the scanner, dotted line are odometry of the mobile robot, continuous line are trajectory of the mobile robot. Camera position icon in Fig. 10 is view point for Fig.8 and 9 in the laboratory room. In Figure 10, scanned points of floor and ceiling planes are not displayed. In Figure 9, color of the points changes according to the height from the floor.

In this experiment, the number of reference scan is 38, and the number of total scanned points is 410351. Matching time in total is 3.3 second (average 0.09 second per one input scan). Initial matching parameter is to add previous scan position and odometry. This NDT matching program uses  $20cm \times 128 \times 80 \times 40$  ND voxels, and  $80cm \times 32 \times 20 \times 10$  ND voxels. Required amount of the memory for all the voxels is 172MB. These voxel sizes are decided by experiments.

In Fig.10, odometry path is strained, on the other hand, shape of the room from matching result is not strained and sensing position are corrected by the matching.

## VI. CONCLUSION

In this paper, 3-D scan matching using NDT, and convergence performance improvement using ND voxel size

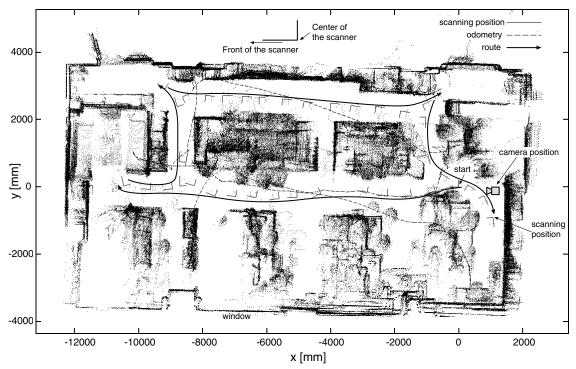


Fig. 10. Matching result (top view)

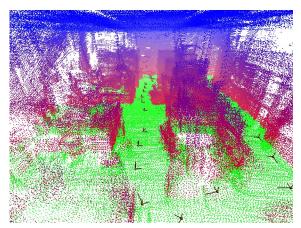


Fig. 9. Matching result(bird's view)

selection are described. In addition, convergence area and computation load of the proposed algorithm, and experiments in large scale environments which use odometry as initial matching parameter are demonstrated. Computation load of the proposed methods is independent from the number of reference scan points, therefore, it could be effective for real-time localization and mapping.

If a matching process could not get good initial matching parameter or high frequency scanning rate, then an improved convergence area of NDT scan matching is required. In addition, improvement of convergence area in this paper is proposed. However, convergence area of this method changes according to the shape and complexity of the environment. Therefore, good size of ND voxel must be determined. Moreover, this method need large amount of memory for keeping ND voxel data. These improvement are future tasks to realize

robust and large environment robotic mapping. An another issue of SLAM, a loop closing is not discussed in this paper. In future work, the authors will extend the proposed method and apply it to mobile robot for rough terrain or real-time 3-D SLAM using real-time 3-D range sensor.

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