Two-way Capital Flow Management in Emerging Markets*

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Abstract

While the existing literature on capital flow management (CFM) policies mainly focuses on net capital flows, this paper studies the joint management of gross inflows and outflows. By calibrating a DSGE model to Brazil's case, we find it optimal for Brazil to shrink its external balance sheet by decreasing both inflows and outflows during periods of global financial easing. The optimal allocation can be achieved by introducing countercyclical capital flow taxes on the economy's short and long-term bond purchases. The welfare gain from such policies is equivalent to an increase of 0.2 percent in permanent consumption.

Keywords: Gross capital flows, market power, capital flow management policies

JEL Codes: F38, F41, G15

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1 Introduction

Emerging markets often face large and volatile capital flows. Many emerging market economies use capital flow management (CFM) policies to smooth the impact of external financial shocks. The main instruments of CFM policies are foreign exchange interventions and countercyclical capital controls.

Optimal capital flow management (CFM) policies have been an active research area in international macroeconomics, particularly since the Global Financial Crisis. Theoretical models have been developed to rationalize CFM policies and quantify their welfare gains. However, existing literature has been mainly focused on net capital flows, while in the real world, financial risks originate from financial transactions better captured by gross capital flows (Johnson 2009; Borio and Disyatat 2010; Obstfeld 2012). This paper thus characterizes the optimal CFM policies in a Dynamic Stochastic General Equilibrium (DSGE) model involving gross inflows and outflows.

The model features global financiers and a small open economy (SOE). Because of a wealth constraint, global financiers have a time-fluctuating demand for SOE-issued long-term bonds. This yields volatility in bond prices and gross capital flows. The model can generate capital flow events such as sudden stops and retrenchments like those observed in the data (Forbes and Warnock 2012; Broner et al. 2013). More specifically, when the wealth constraint is loose, the SOE issues long-term bonds to financiers and accumulates risk-free short-term bonds. When the constraint tightens, financiers are forced to offload their SOE bonds at a fire sale price, and the SOE decumulates its short-term bonds to buy back long-term bonds. Thus, a sudden stop and a retrenchment go hand-in-hand.

The model suggests a case for public intervention in the capital market. We characterize the decentralized equilibrium allocation and the allocation chosen by a social planner who acts on behalf of the economy in normal periods and maximizes the SOE's welfare. The social planner's allocation differs from the decentralized equilibrium allocation because she leverages the market power of the economy over its issued assets. The social planner internalizes the impact of the long-term bond price on the economy, whereas atomistic households take the bond price as given. By managing the gross positions of short and long-term bonds during normal times, the social planner can systematically affect the long-term bond price to make it more advantageous for the SOE.

In a tractable version of our model, the social planner may increase or decrease gross capital flows compared to households. Increasing gross capital flows helps stabilize long-term debt prices and lower the risk premium. By issuing more debt to finance short-term assets, the social planner increases liquidity to the long-term debt market during financial tightenings, thus supporting higher fire sale prices, even when debt rollover increases. Consequently, risk-averse foreign investors face less volatility in returns, leading to a reduced risk premium on net long-term debt issuance in easing periods. However, increasing gross capital flows also has drawbacks: it raises the fire sale price of existing long-term debt, increasing repurchase costs. The tractable model indicates that if initial debt levels are not excessively high, the social planner opts for larger, more volatile gross capital flows than households.

We resolve the above ambiguity by a quantitative analysis of the model. The model is calibrated to a typical emerging market, Brazil, via the Simulated Method of Moments to match Brazil's external liabilities and other crucial moments regarding its capital flows and asset prices. The numerical result suggests that it is optimal for Brazil to reduce its gross capital flows. We characterize the capital flow taxes that implement the social planner's allocation for Brazil. With lower inherited long-term debt, policymakers tax the long-term bond issuance and subsidize the short-term bond accumulation; with higher inherited long-term debt, they subsidize the long-term bond issuance and tax the short-term bond accumulation. On average, the optimal CFM policies increase social welfare by 0.2 percent of permanent consumption, comparable to the welfare gains in the existing literature.

The rest of this paper is organized as follows: In Section 2, we review the related literature. Section 3 details the basic model and characterizes the decentralized equilibrium and the social planner's allocations. Section 4 studies a tractable version of the DSGE model, emphasizing the importance of a quantitative study on optimal two-way capital flow management. Section 5 calibrates the DSGE model to Brazilian data and delivers our primary quantitative findings. Section 6 discusses policy tools replicating the social planner's allocation. Section 7 concludes.

2 Related Literature

This paper relates to two strands of literature: the literature on global capital flows in the global financial cycle and the literature on optimal capital flow management policies.

Our research relates to the ongoing discussions on global capital flows within the global financial cycle. Miranda-Agrippino et al. (2020) documents the pronounced co-movement in risky asset prices, capital flows, leverage, and various financial variables at a global scale, referred to as the global financial cycle. Both Davis et al. (2021) and Miranda-Agrippino and Rey (2022) identify global factors that significantly account for global capital flow variations. Numerous theories, including those by Tille and Van Wincoop (2010), Devereux and Sutherland (2011), Bruno and Shin (2015), Bräuning and Ivashina (2020a), Jiang et al. (2020), Akinci et al. (2022), and Kekre and Lenel (2021), provide insight into the dynamics of global capital flows. These works explore the roles of financial risk, production uncertainty, and preferences for safety and liquidity to shed light on the driving factors and implications of global capital flows.

Gross capital flows to emerging markets have attracted significant research attention due to their susceptibility to external financial shocks (Koepke 2019).

Several papers are motivated by the following stylized fact: even if some emerging markets heavily rely on external financing, they retain substantial safe and liquid assets, such as foreign exchange reserves. Insurance theory offers an explanation for this phenomenon. A seminal paper by Jeanne and Ranciere (2011) posits that by issuing state-contingent claims to finance safe assets, emerging markets can insure themselves against adverse future states. Many subsequent studies emphasize this insurance rationale to elucidate the observed gross capital flows in emerging markets. These risks can arise from domestic fundamentals (Alfaro and Kanczuk 2009, Bianchi et al. 2018; Alfaro and Kanczuk 2019; Bianchi and Sosa-Padilla 2020) or from global financial distress (Caballero and Simsek 2020; Jeanne and Sandri 2023). Our research aligns with the latter group, highlighting global financial risks as pivotal drivers of capital flows. We extend this line of research with a quantitative model replicating the typical capital flow and asset price dynamics observed in emerging markets.

Our study also contributes to the debate on optimal capital flow management (CFM) in emerging markets. CFM policies include foreign exchange interventions and capital controls. The global financial crisis has spawned numerous theories justifying the use of CFM policies (Jeanne and Korinek 2010; Bianchi 2011; Benigno et al. 2013; Farhi and Werning 2016; Schmitt-Grohé and Uribe 2017). Central to these theories is an externality that private agents often overlook but is of concern to a benevolent social planner. As Erten et al. (2021) underscores, key externalities include pecuniary externalities associated with financial stability and aggregate demand externality associated with unemployment. Here, we focus on pecuniary externalities, including collateral and distributive externalities, as they are more relevant to this paper.

Collateral externalities emerge when financial constraints depend on the fluctuating market value of collateral assets. Private agents often overlook their impact on these values, leading to potential financial instability and highlighting a role for public intervention (Jeanne and Korinek 2010; Bianchi 2011; Benigno et al. 2013; Schmitt-Grohé and Uribe 2017; Ottonello 2021). Bianchi (2011) explore the optimal CFM policies considering these externalities and indicate that optimal public actions can reduce the probability and the severity of financial crises for open economies. Departing from prior studies, we shift the focus of financial constraint from domestic borrowers to foreign lenders. This change resonates with the extensive research on frictional international financing (Gabaix and Maggiori 2015; Bruno and Shin 2015; Bräuning and Ivashina 2020a; Bräuning and Ivashina 2020b; Akinci et al. 2022; Morelli et al. 2022) and bolsters our model's insight into global factor-driven capital flows.

The literature discusses distributive externalities that arise when marginal rates of substitution between different dates/states differ among agents and have a zero-sum nature (Lorenzoni 2008; Dávila and Korinek 2018; Caballero and Simsek 2020). These externalities indicate that a social planner can optimize allocations by adjusting relative prices, favoring agents with higher marginal utility. Notably, Dávila and Korinek (2018) argues that while collateral externalities tend to cause overborrowing, distributive externalities might lead to underborrowing. In our model, asset prices also matter to the economy's welfare. However, what makes public intervention desirable is the market power of the economy regarding its long-term assets. The work by Jeanne and Sandri (2023) is similar to ours, wherein the model combines market power with a pure distributive externality, allowing the social planner to internalize it. This model proposes the novel policy implication that an emerging market should expand its external balance sheet by amplifying gross capital flows. In contrast to their clear-cut policy recommendation, whether the social planner should increase or decrease gross capital flows hinges on the inherited long-term debt in our model.

3 Model

This section introduces a quantitative model that describes the gross capital flows and asset price dynamics of an emerging market (henceforth EM) within the global financial cycle. During global financial easing periods, there is heightened international demand for the EM's long-term assets, leading EM households to issue such assets and accumulate short-term bonds. When the global financial condition tightens, EM households repurchase their long-term assets and decumulate their short-term bonds. We characterize both the decentralized equilibrium and the social planner's allocation, highlighting that, unlike households, the social planner proactively leverages market power in her portfolio decisions.

3.1 Households

Consider the emerging market a small open economy populated by a continuum of identical households, all consuming a single good. A representative household aims to maximize its expected lifetime utility, represented by:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to a budget constraint:

$$\bar{y} + a_t + \delta b_t = c_t + \frac{a_{t+1}}{R_f} + q_t\big(b_{t+1} - (1-\delta)b_t\big) \tag{1}$$

and a liquidity constraint:

$$a_{t+1} \geq 0 \tag{2}$$

Here, c_t represents the consumption of the good at period t. The economy can trade two assets with foreigners: a short-term bond and a long-term bond, with their initial positions denoted by a_t and b_t in period t, respectively. A long-term bond is a perpetuity contract that, for each unit issued at period t, commits to a payment of $\delta(1-\delta)^{j-1}$ units of goods as coupons at time $t+j$, where $j = 1, 2, ..., \infty$.[1] The short-term bond, on the other hand, is a one-period bond priced at $\frac{1}{R_f}$, and it assures a single unit of the consumption good in the subsequent period. Here, R_f is the constant gross international risk-free rate, and q_t is the ex-coupon price of the long-term bond. We posit that only financial center countries (not explicitly modeled here), like the United States, can be net issuers of these short-term bonds,[2] justifying the liquidity constraint specified in (2) for the emerging market.

The budget constraint (1) shows how households finance their expenditure from different income sources. At period t, a household's income comprises three elements: the constant endowment income $\bar{y} = 1$, the short-term bond return a_t, and the coupon on the long-term bond holding δb_t. Households use this to finance their consumption c_t, new acquisition of short-term bonds $\frac{a_{t+1}}{R_f}$ and new acquisition of the long-term bonds $q_t(b_{t+1} - (1-\delta)b_t)$ where $(b_{t+1} - (1-\delta)b_t)$ is the increase in long-term bond holdings. Following the convention, we represent real allocation variables in lowercase for the individual level and uppercase for the aggregate economy level.

We assume $\beta R_f < 1$. This ensures that EM households are impatient and are willing to borrow via long-term bonds to finance their consumption. Thus, b_t remains negative throughout the equilibrium path, i.e., $b_t < 0, \forall t$. The equilibrium satisfies the transversality condition (TVC):

$$\lim_{t\to\infty} \beta^t u'(c_t) b_{t+1} = 0 \tag{3}$$

[1] The benefit of specifying such long-term bonds is representing all past long-term bond purchases with a single state variable, b_t, in our model. Bianchi et al. (2018) give more details about such long-term bonds.

[2] See Maggiori (2017) for more discussion on how a country's reserve asset status is linked to its more advanced financial development level.

3.2 Foreign Financiers

We assume EM households can accumulate any positive level of short-term bonds at the international risk-free rate; however, they can only issue long-term bonds to a specialized group of traders: foreign financiers. These financiers are experts in investing in long-term bonds issued by the EM and are modeled as homogeneous atomistic agents within a unit continuum.

Financiers are short-lived, with a new generation entering the long-term bond market at the beginning of each period t. These financiers, represented by the superscript f, invest in long-term bonds during period t, taking positions given by b^f_{t+1}. In period $t+1$, these period-t born financiers collect their long-term bond coupons, δb^f_{t+1}, sell the remaining bonds $(1-\delta)b^f_{t+1}$ to EM residents and the newly entered financiers at the market price q_{t+1}, and subsequently exit the market with their returns.

Financiers born in period t possess two key attributes: an exogenous wealth level W_t and a stochastic discount factor (SDF) $M_{t,t+1}$. Each financier from the same generation shares an identical stochastic wealth amount W_t. This uniform wealth acts as a constraint, ensuring that the market value of their long-term bond holdings, expressed as $q_t b^f_{t+1}$, does not exceed W_t. This wealth constraint, in conjunction with their SDF, determines the long-term bond pricing:

$$q_t = \min\left\{ E_t\left[M_{t,t+1}\left(\delta + (1-\delta)q_{t+1}\right)\right], \frac{W_t}{b^f_{t+1}} \right\} \tag{4}$$

Equation (4) can be interpreted as follows: If the financiers' wealth is sufficiently high, ensuring the market value of their long-term bond position remains below W_t, then the SDF determines the long-term bond price. However, in scenarios where their wealth is low, such that the wealth constraint binds, the long-term bond price is determined by the ratio of financiers' wealth to their long-term bond holdings. Per standard terminology, we label periods when the wealth constraint is non-binding (binding) as financial easing (tightening) periods.

We assume that the wealth of financiers W, can take on one of two values: W_H (high) or W_L (low), with $W_H > W_L$. Specifically, W_H is assumed to be sufficiently large under which the financiers' wealth constraint is never binding. Variable W_t acts as an indicator of the global financial condition.

The transition of W across periods is governed by a Markov transition matrix:

$$Pr(W_{t+1} = W_j | W_t = W_i) = \pi_{ij} \tag{5}$$

where the transition probabilities are represented in the following matrix:

$$\pi = \begin{bmatrix} \pi_{HH} & \pi_{HL} \\ \pi_{LH} & \pi_{LL} \end{bmatrix} \tag{6}$$

in which π_{HL} (π_{LH}) is the probability of transitioning from W_H (W_L) to W_L (W_H), and π_{HH} (π_{LL}) is the probability of remaining in W_H (W_L).

Moreover, the SDF for financiers, $M_{t,t+1}$, is a function of their current and next period's financial wealth. Its formulation is given by:

$$M_{t,t+1} = \begin{cases} \dfrac{1}{R_f + \pi_{HL}(R^\kappa - 1)R_f} & \text{if } W_t = W_H \text{ and } W_{t+1} = W_H, \\ \dfrac{R^\kappa}{R_f + \pi_{HL}(R^\kappa - 1)R_f} & \text{if } W_t = W_H \text{ and } W_{t+1} = W_L, \\ \dfrac{1}{R_f + \pi_{LL}(R^\kappa - 1)R_f} & \text{if } W_t = W_L \text{ and } W_{t+1} = W_H, \\ \dfrac{R^\kappa}{R_f + \pi_{LL}(R^\kappa - 1)R_f} & \text{if } W_t = W_L \text{ and } W_{t+1} = W_L. \end{cases} \tag{7}$$

The financiers' stochastic discount factor exhibits two key features: risk aversion and alignment with the international risk-free rate. Effective risk aversion is encapsulated by R^κ; specifically, a value of R^κ exceeding 1 implies that financiers value returns more in financial tightening periods than in financial easing ones. An increased R^κ signifies an increased risk aversion of financiers. Despite this risk aversion, the risk-free rate inferred from the financiers' SDF aligns with the international risk-free rate, as evidenced by $E_t M_{t,t+1} = \frac{1}{R_f}$, $\forall t$.

Financiers can be interpreted as international financial intermediaries who intermediate funds between foreign and EM households. These intermediaries might face frictions in their intermediation role. During global easing periods, they operate with minimal financial frictions. Under such conditions, the financiers use the SDF of foreign households to price EM long-term bonds. With the small open economy assumption, foreign households' SDF can be considered exogenous to the EM. However, in times of global financial tightenings, frictional financial intermediation means the foreign households' SDF does not factor into the pricing of long-term bonds. Instead, financial intermediaries become the marginal investors in EM long-term bonds, and their available market resources can significantly influence the bond price (He and Krishnamurthy 2013).

With financiers' fluctuating wealth, our model can generate volatile capital flows and long-term bond prices. The long-term bond price stays high during easing periods when financiers value the bond payoffs via their SDF. Given the sound borrowing condition, EM households issue long-term bonds to finance their consumption and short-term bond accumulation; the latter can act as a buffer against a financial tightening. When the financial condition tightens, foreign financiers cannot accommodate the same amount of long-term bonds at a pre-financial tightening price. The bond price significantly falls to clear the market. We call such a bond price well below its fundamental value the fire sale price. EM households deplete their short-term bonds and reduce consumption to repurchase long-term bonds. Therefore, the emerging market sees positive relationships among gross inflows, gross outflows, and long-term bond prices. Notably, during tightening periods, our model generates the typical sudden stops in inflows and retrenchments in outflows as documented by Forbes and Warnock (2012) and Broner et al. (2013).

3.3 Decentralized Equilibrium Allocation

This subsection delves into the decentralized equilibrium free from public intervention. We assume a *Markov Equilibrium*. Here, decision rules and price functions depend solely on the following payoff-relevant state variables: financiers' wealth W, the EM's initial long-term bond position B, and the EM's initial short-term bond position A. We denote the state vector by $s \equiv (W, A, B)$. Thus, the decentralized equilibrium can be defined by:

1. Portfolio rules $A'(s)$ and $B'(s)$

2. A consumption function $c(s)$

3. A bond price function $q(s)$

such that

(a) $A'(s)$, $B'(s)$, $c(s)$, and $q(s)$ satisfy the budget constraint (1), the liquidity constraint (2), and the TVC (3).

(b) Given $q(s)$, $A'(s)$, $B'(s)$ and $c(s)$ must solve individual households' optimization conditions:

$$\frac{u'(c(s))}{R_f} = \beta E_{W'|W} u'(c(s')) + \mu(s),$$
$$u'(c(s))q(s) = \beta E_{W'|W} \left[u'(c(s')) \left(\delta + (1-\delta)q(s')\right)\right]$$

where $s' \equiv (W', A'(s), B'(s))$ and $\mu(s)$ denotes the Lagrange multiplier corresponding to the liquidity constraint.

(c) Given $A'(s)$ and $B'(s)$, $q(s)$ must satisfy:

$$q(s) = \min \left\{ E_{W'|W} \left[M_{W,W'} \left(\delta + (1-\delta) q\left(W', A'(s), B'(s)\right)\right)\right], -\frac{W}{B'(s)} \right\}.$$

The equilibrium condition (c) incorporates the market clearing condition of the long-term bonds $b_{t+1} + b^f_{t+1} = 0$. When expressing financiers' SDF, $M_{W,W'}$ represents the SDF between the current and following periods' financial conditions, indicated by W and W', respectively.

3.4 Social Planner's Allocation

This subsection characterizes the allocation chosen by a social planner. The planner encounters the same constraints as private households: the budget constraint (1) and the liquidity constraint (2). Foreign financiers price the long-term bonds still by (4). Moreover, this planner is discretionary and thus cannot commit to future actions. The planner is also prudential as its interventions occur only during financial easing periods when the financiers' wealth is W_H. We assume the prudential nature of the social planner for three reasons: first, it aligns with the IMF's 2022 emphasis on prudential or preemptive capital flow measures; second, this assumption offers more tractability to the model as shown in Section 4; third, under our benchmark calibration, the outcomes and welfare gains from an unrestricted planner that intervenes under any financial condition are very close to those of a prudential planner discussed here.

In the social planner's allocation, decision rules and the value function are denoted with a subscript "SP", and the bond price function carries a superscript "SP." The social planner's allocation is defined by:

1. Portfolio rules $A'_{SP}(s)$ and $B'_{SP}(s)$,
2. A consumption function $C_{SP}(s)$
3. A value function $V_{SP}(s)$,
4. A long-term bond price function $q^{SP}(s)$.

such that when $W = W_H$:

(a) Given $q^{SP}(s)$, $A'_{SP}(s)$, $B'_{SP}(s)$, and $V_{SP}(s)$ solve the following Bellman equation:

$$V_{SP}(W_H, A, B) \equiv \max_{\substack{A'_{SP} \geq 0, \\ B'_{SP}}} \{u(C) + \beta E_{W'|W_H} V(W', A'_{SP}, B'_{SP})\}$$

s.t.

$$C = \bar{Y} + A + \delta B - Q\big(W_H, A'_{SP}, B'_{SP}\big)\big(B'_{SP} - (1-\delta)B\big) - \frac{A'_{SP}}{R_f}$$

and

$$Q(W_H, A'_{SP}, B'_{SP}) \equiv E_{W'|W_H}\big[M_{W_H,W'}\big(\delta + (1-\delta)q^{SP}(W', A'_{SP}, B'_{SP})\big)\big]$$

where $Q(W_H, A'_{SP}, B'_{SP})$ is the current bond price given an arbitrary portfolio choice (A'_{SP}, B'_{SP}).

(b) Given $A'_{SP}(s)$ and $B'_{SP}(s)$, $q^{SP}(s)$ must satisfy:

$$q^{SP}(W_H, A, B) = Q\big(W_H, A'_{SP}(W_H, A, B), B'_{SP}(W_H, A, B)\big).$$

when $W = W_L$:

(a) $A'_{SP}(s)$, $B'_{SP}(s)$, $C_{SP}(s)$ and $q^{SP}(s)$ satisfy the budget constraint (1), the liquidity constraint (2), and the TVC (3).

(b) Given $q^{SP}(s)$, $A'_{SP}(s)$, $B'_{SP}(s)$ and $C_{SP}(s)$ must solve:

$$\frac{u'(C_{SP}(s))}{R_f} = \beta E_{W'|W} u'(C_{SP}(s')) + \mu_{SP}(s) \tag{8}$$

$$u'(C_{SP}(s))q^{SP}(s) = \beta E_{W'|W}\left[u'(C_{SP}(s'))\big(\delta + (1-\delta)q^{SP}(s')\big)\right] \tag{9}$$

where $s' \equiv (W', A'_{SP}(s), B'_{SP}(s))$ and $\mu_{SP}(s)$ denotes the Lagrange multiplier corresponding to the liquidity constraint.

(c) Given $A'_{SP}(s)$ and $B'_{SP}(s)$, $q^{SP}(s)$ must satisfy:

$$q^{SP}(s) = \min\left\{E_{W'|W}\left[M_{W,W'}\left(\delta + (1-\delta)q^{SP}\left(W', A'_{SP}(s), B'_{SP}(s)\right)\right)\right], -\frac{W}{B'_{SP}(s)}\right\}.$$

The equilibrium conditions for the social planner's allocation resemble those of the decentralized equilibrium when financial conditions are tight, owing to the lack of the planner's intervention. Nevertheless, differences in the bond price functions $q^{SP}(s)$ and $q(s)$ mean that the equilibrium outcomes still differ between the two allocations regardless of financial conditions.

3.5 A Comparison between the Two Allocations

A critical difference between the social planner's and the decentralized equilibrium allocations is that the social planner internalizes the effect of gross international positions (A', B') on long-term bond prices. In other words, the social planner leverages her market power to influence the long-term bond price while private households take the bond price as given. To better understand this, we examine the social planner's first-order condition (FOC) concerning A'_{SP}:

$$\frac{u'(C_{SP}(s))}{R_f} + \underbrace{\beta\pi_{HL}u'\big(C_{SP}(W_L, A', B')\big)\frac{\partial q^{SP}(W_L, A', B')}{\partial A'}(B'' - (1-\delta)B')}_{\text{additional marginal loss by increasing } A'} =$$
$$\beta E_{W'|W_H}u'(C_{SP}(s')) + \mu_{SP}(s) \underbrace{-u'(C_{SP}(s))\frac{\partial Q(W_H, A', B')}{\partial A'}(B' - (1-\delta)B)}_{\text{additional marginal gain by increasing } A'} \tag{10}$$

and B'_{SP}:

$$u'(C_{SP}(s))q^{SP}(s) + \underbrace{\beta\pi_{HL}u'\big(C_{SP}(W_L, A', B')\big)\frac{\partial q(W_L, A', B')}{\partial B'}(B'' - (1-\delta)B')}_{\text{additional marginal loss by increasing } B'} =$$

$$\beta E_{W'|W_H}[u'(C_{SP}(s')) * (\delta + (1-\delta)q^{SP}(s'))]$$
$$\underbrace{-u'(C_{SP}(s))\frac{\partial Q(W_H, A', B')}{\partial B'}(B' - (1-\delta)B)}_{\text{additional marginal gain by increasing } B'} \quad (11)$$

The above FOCs have more terms than their counterparts in the decentralized equilibrium. Specifically, the left-hand side (LHS) of (10) and (11) represents the marginal loss from purchasing an additional unit of the short-term and long-term bonds, respectively, and the right-hand side (RHS) represents the marginal gain. When juxtaposed with their counterparts in the decentralized equilibrium—(8) and (9)—it is evident they both incorporate an additional marginal gain and an additional marginal loss; these additional terms are highlighted (underbraced) in the social planner's FOCs. Next, we will focus on these additional terms and the economic concerns they capture.

Let's start with the social planner's choice of long-term bonds as captured by (11). The first underbraced term in (11) emphasizes the social planner's evaluation of how B' impacts long-term bonds' future fire sale price. A lower long-term debt level (an increased B') increases the fire sale price, as it reduces the long-term debt that needs to be rolled over during financial tightenings. More specifically, an increase in B' increases the fire sale price by $\frac{\partial q(W_L,A',B')}{\partial B'}$. This price increase makes the economy pay more money to buy back the long-term bonds, where the increased spending should be equal to the product of $\frac{\partial q(W_L,A',B')}{\partial B'}$, the marginal increase in bond price, and $(B'' - (1-\delta)B')$, the units of net bond purchase. The utility effect is the extra spending times the contemporaneous marginal utility $u'(C_{SP}(W_L, A', B'))$ after being adjusted by the subjective discount factor β and the probability of upcoming financial tightenings π_{HL}. This is a welfare loss to the economy as long as the economy repurchases the long-term bonds during a financial tightening.

The second underbraced term in (11) captures the social planner's concern about the impact of B' on the current bond price. As mentioned before, reducing long-term debt increases the fire sale bond price. An increased fire sale price then improves the current bond price. Notably, a rise in B' directly improves the current long-term bond price by $\frac{\partial Q(W_H, A', B')}{\partial B'}$. This effect is first modulated by $-(B' - (1-\delta)B)$, the net bond issuance in the same period, given that all these issued bonds carry an identical price. After that, the result is magnified by $u'(C_{SP}(s))$, reflecting the change in households' utility. As long as the EM is a net issuer of the long-term bond, this utility change is a welfare gain.

We now consider the social planner's portfolio decision regarding short-term bonds as characterized by (10). The portfolio selection of short-term bonds can affect the long-term bond price in the current and subsequent periods. This is because short-term bonds can provide liquidity to buy back long-term bonds in a financial tightening period, which strengthens the ex-post-fire sale price and prices in easing periods ex-ante through financiers' expectations. Like in (11), the social planner values the short-term bond choice differently from households, as captured by the two additional terms in (10): when the economy accumulates more short-term assets, on the one hand, it boosts the current long-term bond price and improves the economy's borrowing condition, thus a welfare gain; on the other hand, it also increases the fire sale long-term bond price and worsens the economy's potential redemption condition in the next period, thus a welfare loss.

A combined examination of these additional welfare gains and losses reveals the social planner's dilemma in adjusting the long-term bond position. The planner's general objective is to enhance the EM's welfare. For easing periods, this is achieved by decreasing borrowing (by moving B' closer to zero), thereby elevating the price at which the EM offloads its debt. By contrast, for tightening periods, the planner might seek benefits for the EM by increasing its borrowing during easing times, consequently lowering the debt's fire sale price in potential tightening episodes. Given the positive correlation between the current bond price and the future fire sale price, the planner faces two conflicting economic forces: augmenting the fire sale bond price for more favorable current borrowing conditions or suppressing it to ensure better debt redemption. The social planner has to determine whether to increase or decrease the fire sale bond price relative to its decentralized allocation level.

The social planner's problem becomes even more complicated if we consider the adjustment of A' and B' together. Unlike adjusting only B' or A', which necessarily leads to a change in net inflows and consumption, the social planner can maintain a fixed net inflow and consumption by adjusting gross capital flows. In other words, the social planner can finance short-term assets by issuing long-term debt without affecting the current consumption level. This case needs careful examination as it can substantially affect the long-term bond price in the social planner's allocation and policy implications regarding gross capital flow management, as illustrated by Jeanne and Sandri (2023). The following subsection details how the social planner manages such debt-financed assets.

3.6 Management of Debt-Financed Assets

This subsection illustrates how the social planner manages the gross external balance sheet while keeping current consumption unchanged. We assume the social planner wishes to maintain a fixed consumption level at $\bar{C}$. Consequently, the budget constraint is given by:

$$\bar{Y} + \delta B + A - \frac{\tilde{A}'}{R_f} - Q\left(W_H, \tilde{A}', B'\right)(B' - (1-\delta)B) - \bar{C} = 0 \tag{12}$$

Here, B', the long-term bond choice serves as the social planner's control variable. Meanwhile, $\tilde{A}'$, representing the choice of short-term bonds, adjusts automatically in response to changes in B' to keep current consumption stable. The selection of B' is inherently bound by constraints ensuring non-negative consumption and adherence to the liquidity constraint (2). Using the implicit function theorem, we deduce that

$$\frac{\partial \tilde{A}'}{\partial(-B')} = \frac{Q + \frac{\partial Q}{\partial B'}(B' - (1-\delta)B)}{\frac{1}{R_f} + \frac{\partial Q}{\partial \tilde{A}'}(B' - (1-\delta)B)}$$

The social planner's optimization problem is as follows:

$$\max_{B'} u(\bar{C}) + \beta E_{W'|W_H} V_{SP}\left(W', \tilde{A}', B'\right)$$
$$\text{s.t.} \quad (2)\ \&\ (12)$$

In the scenario where B' ensures a non-negative $\tilde{A}'$, the marginal utility of the social planner concerning debt that is issued to finance assets, $-B'$, is expressed by

$$E_{W'|W}\big[u'(C')\big(\frac{\partial \tilde{A}'}{\partial(-B')} - \delta - (1-\delta)q^{SP}(W', \tilde{A}', B')\big)\big]$$
$$-\pi_{HL} u'(C'_L)(B'' - (1-\delta)B')\left(\frac{\partial q(W_L, \tilde{A}', B')}{\partial \tilde{A}'}\frac{\partial \tilde{A}'}{\partial(-B')} - \frac{\partial q(W_L, \tilde{A}', B')}{\partial B'}\right) \quad (13)$$

The above marginal utility has two components. The first is the "insurance" effect of debt-financed assets, as described by Bianchi et al. (2018). This effect maintains unchanged current consumption while safeguarding future consumption in adverse states W_L. Specifically, an increase in long-term debt by one unit allows the social planner to accumulate $\frac{\partial \tilde{A}'}{\partial(-B')}$ units of short-term bonds. Consequently, in the following period, the debt obligation's market value is $\delta + (1-\delta)q'$, whereas the short-term bonds provide $\frac{\partial \tilde{A}'}{\partial(-B')}$ units of liquidity. In high consumption future states (W_H), the payoff to debt-financed assets, $\frac{\partial \tilde{A}'}{\partial(-B')} - \delta - (1-\delta)q'$, usually becomes negative due to q''s high value. Conversely, the payoff tends to be positive in low consumption states (W_L) with a fire sale price of q'. This is precisely what insurance means.

The second component arises from the effect on fire sale prices caused by the adjustment in debt-financed assets. An increase of one unit in debt-financed assets impacts the future fire sale price through two channels. The first is the direct effect of this increase on the gross debt position, $\frac{\partial q(W_L,\tilde{A}',B')}{\partial(-B')}$, and the second is the indirect effect due to the corresponding adjustment in short-term assets, $\frac{\partial q(W_L,\tilde{A}',B')}{\partial \tilde{A}'} \times \frac{\partial \tilde{A}'}{\partial(-B')}$. The related welfare impact is the negation of this net change in the fire sale price, multiplied by the debt redemption value $B'' - (1-\delta)B'$, the probability of the fire sale, and the marginal utility during the fire sale, $u'(C'_L)$. A positive net change in the fire sale price implies a welfare loss under a positive debt repurchase $B'' - (1-\delta)B' > 0$.

If the fire sale risk π_{HL} is a small number[3], we can approximate $\frac{\partial \tilde{A}'}{\partial(-B')}$ by $QR_f + (\frac{\partial Q}{\partial B'} - QR_f\frac{\partial Q}{\partial \tilde{A}'})\big(B' - (1-\delta)B\big)R_f$. With this approximated expression for $\frac{\partial \tilde{A}'}{\partial(-B')}$, (13) becomes

$$
\begin{aligned}
&E_{W'|W}\big[u'(C')\big(Q(W_H,\tilde{A}',B')R_f - \delta - (1-\delta)q^{SP}(W',\tilde{A}',B')\big)\big)\big] \\
&\underbrace{-E_{W'|W}\big[u'(C')(-\frac{\partial Q}{\partial B'} + QR_f\frac{\partial Q}{\partial \tilde{A}'})(B' - (1-\delta)B)R_f\big]}_{\text{additional marginal gain by increasing } -B'} \\
&\underbrace{-\pi_{HL}u'(C'_L)(B'' - (1-\delta)B')\left(\frac{\partial q(W_L,\tilde{A}',B')}{\partial \tilde{A}'}\frac{\partial \tilde{A}'}{\partial(-B')} - \frac{\partial q(W_L,\tilde{A}',B')}{\partial B'}\right)}_{\text{additional marginal loss by increasing } -B'} \qquad (14)
\end{aligned}
$$

To compare the social planner's marginal valuation of debt-financed assets with that of households, let's imagine another scenario where a representative household makes its portfolio choice for only one period, and for all subsequent periods,

[3]This is because in the special case of $\pi_{HL} = 0$, $q_t = \bar{q}$ for all t.

the economy is taken over by a discretionary prudential social planner[4], the representative household's marginal utility of debt-financed-assets at $(\tilde{A}', B')$ is given by

$$E_{W'|W}\big[u'(C')\big(Q(W_H, \tilde{A}', B')R_f - \delta - (1-\delta)q^{SP}(W', \tilde{A}', B')\big)\big] \tag{15}$$

which can be derived by a linear combination of (8) and (9). (15) only includes the insurance part because the representative household takes long-term bond prices as given.

Compared to (15), (14) introduces two additional terms. The first term indicates a potential welfare gain when debt-financed assets are increased. This gain occurs under certain conditions: first, if the current net long-term debt issuance$-(B' - (1-\delta)B)$ is positive; and second, if there is a resulting positive valuation gain in the current long-term bond price, i.e., $-\frac{\partial Q}{\partial B'} + QR_f\frac{\partial Q}{\partial \tilde{A}'} > 0$. Under these conditions, the money spared by issuing one more unit of debt-financed assets during easing periods equals the product of the net debt issuance and the net change in the long-term bond price. The social planner can then allocate the spared money to short-term bonds, yielding a liquidity value in the next period of $-(B' - (1-\delta)B) * (-\frac{\partial Q}{\partial B'} + QR_f\frac{\partial Q}{\partial \tilde{A}'}) * R_f$, regardless of the prevailing financial conditions.

We label the second additional component a welfare loss when increasing debt-financed assets. Let's temporarily assume the marginal increase in such debt-financed assets raises the fire sale price, i.e., $\frac{\partial q(W_L,\tilde{A}',B')}{\partial \tilde{A}'}\frac{\partial \tilde{A}'}{\partial(-B')} - \frac{\partial q(W_L,\tilde{A}',B')}{\partial B'} > 0$. Furthermore, assume the economy is a net buyer of long-term bonds during a future fire sale with $B'' - (1-\delta)B' > 0$. Under these conditions, the total loss in consumption goods equates to the product of this net change in the fire sale price and the amount of net debt redemption.

[4]This assumption ensures that both private households and the social planner face the same pricing schedule of long-term bonds $Q(W_H, A', B')$.

Analyzing whether the social planner values debt-financed assets more than households is challenging. The ambiguity of whether the sum of two additional terms in equation (14) is positive or negative complicates this analysis. Under the assumptions that capital inflows are positive during financial easing, negative during tightening and that expanding the external balance sheet boosts bond prices, one might hypothesize that a social planner is more likely to expand the external balance sheet with more sizeable net long-term bond issuance relative to net redemption. In addition, we need to check the validity of these ad-hoc assumptions. The following section delves into a tractable case of the full model to shed light on when the social planner accumulates more debt-financed assets than households.

4 A Tractable Case

This section examines a tractable version of the DSGE model to distill its intuition. The model begins at period 0, with inherited gross positions (a_0, b_0) for the open economy and abundant foreign financiers' wealth W_H. In period 1, the financial condition tightens with probability π. From period 2 onward, the model transitions into a frictionless steady-state, ensuring $q_t = \bar{q}$ for all $t \geq 2$.

To simplify our analysis, we make the following assumptions: households have linear utility functions represented by $u(\cdot) = \cdot$, and $\beta R_f = 1$. These assumptions imply that households choose not to consume if any asset yields a return exceeding R_f. We employ a backward induction approach to analyze this streamlined model.

4.1 Equilibrium at Period 1

In period 1, financiers' wealth can be either W_H or W_L. Under W_H, since long-term and short-term bonds yield identical returns, without sacrificing any insights, we set the long-term bond choice at the period's end, b_2^H, to

$$\bar{y} + a_1 + \delta b_1 = \bar{q}\left(b_2^H - (1-\delta)b_1\right) \tag{16}$$

with the subscript H designating the "high" financiers' wealth. $\bar{q} \equiv \frac{\delta}{\delta + R_f}$ is the frictionless long-term bond price, equal to the present value of all future coupons discounted by the risk-free rate R_f.

On the other hand, under a tightened financial condition, the equilibrium portfolio choice becomes singular. In this case, households choose zero consumption and run out of short-term bonds to buy back long-term ones. This situation can be described by

$$\bar{y} + a_1 + \delta b_1 = q_1^L\left(b_2^L - (1-\delta)b_1\right), \tag{17}$$

with the subscript L symbolizing the "low" financiers' wealth. At the same time, foreign financiers price the long-term bond by

$$q_1^L = -\frac{W_L}{b_2^L}. \tag{18}$$

4.2 Equilibrium at Period 0

In this subsection, we begin by showing how, at period 0, enlarging the external balance sheet invariably raises the long-term bond price. We then analyze the decentralized equilibrium and the social planner's allocations.

4.2.1 Balance Sheet Size and Long-term Bond Price

Regardless of the equilibrium type, the following conditions always hold at period 0.

$$m_0 = \beta a_1 + q_0 \left(b_1 - (1-\delta)b_0\right) \tag{19}$$

$$q_0 = \beta\delta + (1-\pi)(1-\delta)M^H\bar{q} + \pi(1-\delta)M^L q_1^L \tag{20}$$

Equation (19) is the budget constraint of households in period 0; equation (20) is the pricing rule of financiers regarding the long-term bond, where M^H is the financiers' SDF between period 0 and period 1's easing state, and M^L is the SDF between period 0 and period 1's tightening state. Financiers' risk-aversion implies $M^L > M^H$. As period 1 corresponds to the easing state, the price is pinned down by the stochastic discount factor instead of the financiers' wealth.

If we combine equations (17) to (20), we can derive the fire sale price q_1^L as follows

$$q_1^L = \bar{q} - \frac{1}{1-\delta} * \frac{\beta(W_L + \bar{y}) + n_0}{(1-\delta)\pi M^L b_0 + (\beta - \pi M^L)b_1} \tag{21}$$

where $n_0 \equiv \bar{y} + a_0 + \bar{q}R_f b_0$ is the country's net foreign assets in period 0. Let us assume $\beta(W_L + \bar{y}) + n_0 < 0$. Then it follows from equation (21) that q_1^L is decreasing with b_1, that is, the fire sale price of a long-term bond is increasing with the number of long-term bonds issued in period 0. Like in Jeanne and Sandri (2023) this is because the additional long-term bonds finance more short-term bonds that can be used to buy back the long-term bonds in period 1. It follows from equation (20) that q_0 is also decreasing in b_1, thus increasing in the external balance sheet size chosen in period 0. The facts that q_1 and q_0 increases in $-b_1$ corresponds to $\frac{\partial q(W_L,\tilde{A}',B')}{\partial \tilde{A}'}\frac{\partial \tilde{A}'}{\partial(-B')} - \frac{\partial q(W_L,\tilde{A}',B')}{\partial B'} > 0$ and $-\frac{\partial Q}{\partial B'} + QR_f\frac{\partial Q}{\partial \tilde{A}'} > 0$ in Section 3.6, respectively. Therefore, these two ad-hoc assumptions hold for this simplified case.

Since the household does not consume in period 0 or in period 1, period 0-welfare is equal to the expected discounted value of period-2 welfare, $U_0 = \beta^2 E_0 U_2$. Given $a_2 = 0$, period 2-welfare is equal to the PDV of income plus the payment on the long-term bonds held by households, $U_2 = \bar{y}/(1-\beta) + \bar{q} R_f b_2$. Hence maximizing U_0 is equivalent to maximizing $E_0 b_2$. The difference between the decentralized equilibrium allocation and the social planner's allocation is that in the decentralized equilibrium, households take the prices q_0 and q_1^L as given; in contrast, the social planner takes into account that these prices are endogenous to the economy's external balance sheet. The rest of this subsection compares the two allocations.

4.2.2 Decentralized Equilibrium

The period-1 budget constraint implies $b_2 = \frac{y+a_1}{q_1} + b_1 \left(\frac{\delta}{q_1} + 1 - \delta \right)$. Using the period-0 budget constraint (19) to substitute out b_1, and ommitting irrelevant terms gives

$$E_0 b_2 = a_1 E_0 \left[\frac{1}{q_1} - \frac{\beta}{q_0} \left(1 - \delta + \frac{\delta}{q_1} \right) \right] + ...$$

In an equilibrium where households hold a non-zero finite level of short-term bonds a_1, the factor of a_1 in the equation above must be equal to zero, which implies

$$q_0 = \beta \left[\delta + \frac{1-\delta}{E_0 \left(1/q_1 \right)} \right].$$

Using (20) to substitute out q_0 gives a fixed-point equation for q_1^L. As $q_1^L < \bar{q}$, we solve for q_1^L as follows

$$q_1^L = \frac{M^H}{M^L} \bar{q} \tag{22}$$

In the face of financial tightening, the long-term bond price declines below its intrinsic value $\bar{q}$. This allows us to derive

$$q_0 = \beta\delta + (1-\delta)M^L\bar{q}$$

Next, we can plug (22) back to (21) to obtain b_1, then solve for a_1 using (19) with the obtained b_1.

4.2.3 Social Planner's Allocation

Now we examine the social planner's allocation. The social planner in period 0 determines A_1 and B_1 with the intent of maximizing E_0B_2. This leads to:

$$\begin{aligned}
E_0B_2 &= (1-\pi)B_2^H + \pi B_2^L, \\
&= \frac{1-\pi}{\bar{q}}(A_1 + \bar{q}R_f B_1) - \pi\frac{W_L}{q_1^L} + \cdots, \\
&= \frac{1-\pi}{\bar{q}}\left(\frac{q_0B_0(1-\delta)}{(1-\pi)M^H}\right) - \pi\frac{W_L}{q_1^L} + \cdots, \\
&= -\pi\left[(-B_0)\frac{(1-\delta)^2q^L}{\bar{q}M^H/M^L} + \frac{W_L}{q_1^L}\right] + \cdots,
\end{aligned}$$

where the ellipsis $(\cdots)$ signifies constant terms, which we can conveniently overlook due to their independence from q_1^L. The above derivations are obtained by combining equations (16)-(19).

Therefore, the social planner aims to minimize $(-B_0)\frac{(1-\delta)^2q^L}{\bar{q}M^H/M^L} + \frac{W_L}{q_1^L}$, leading to the formulation:

$$q_1^L = \frac{1}{1-\delta}\sqrt{\left(\frac{M^H}{M^L}\bar{q}\right)\left(\frac{W_L}{-B_0}\right)}. \tag{23}$$

When contrasted with the decentralized equilibrium, as depicted by equation (22), it is ambiguous whether the social planner chooses to raise or reduce q_1^L. This ambiguity extends to whether the social planner increases or decreases the external balance sheets compared to the decentralized outcome.

The social planner will opt to augment the external balance sheet and raise q_1^L

relative to uncoordinated private households if and only if:

$$B_0 > -\frac{1}{(1-\delta)^2}\frac{W_L}{\bar{q}}\frac{M^L}{M^H},$$

That is, when the economy does not carry excessive long-term debt at period 0. Intuitively, the social planner can expand the external balance sheet so as to accumulate short-term bonds and buy back the long-term debt in a financial tightening, or it can use the short-term bonds that it already has to buy back the long-term debt in period 0. The social planner follows the first strategy when inherited long-term debt is low.

4.3 A Comparison between the Two Allocations

To understand why the optimal CFM policies depend on the inherited long-term debt level, we need to know that the emerging market is both a seller (during the financial easing in period 0) and a buyer (during the financial tightening in period 1) of the long-term bonds. The social planner's decision may differ from private households' because she actively uses market power over long-term bonds. The social planner wants to sell bonds at higher prices and buy them back at lower prices than households. However, a tradeoff arises because the social planner cannot increase the period-0 bond price and simultaneously decrease the fire sale price. Equation (20) reveals that these two prices must be positively correlated.

Given the tradeoff above, the social planner faces a decision: Should the fire sale price be set above or below the level found in decentralized equilibrium? Suppose the social planner opts for a fire sale bond price higher than the decentralized equilibrium. In that case, it indicates a greater valuation of the benefits from an enhanced bond price in period 0. Conversely, a lower fire sale bond price would mean prioritizing a reduced price in period 1. The question is what determines the relative dominance between increasing the fire sale price for a better borrowing condition and lowering the fire sale price for a better redemption condition.

Our analysis identifies the inherited debt level at period 0 as a crucial determining factor. The bonds in the fire sale consist of two components: the residual portion of the net bond issuance $(1-\delta)|B_1-(1-\delta)B_0|$ and the remaining inherited long-term debt $(1-\delta)^2|B_0|$. For the net bond issuance, the social planner invariably aims to elevate the fire sale price towards the frictionless level, $\bar{q}$. Like in Jeanne and Sandri (2023), from an ex-ante perspective, a risk-neutral social planner seeks to minimize the risk premium associated with net borrowing. Any actions that depress the fire sale price compared to the decentralized equilibrium would amplify the risk premium, ex-ante, given that rational and risk-averse international financiers would factor this in when setting long-term bond prices[5]. In contrast, for the inherited debt—where the economy is only a purchaser in period 1—the optimal move is to push the fire sale price towards zero for social welfare. Hence, when both net bond issuance and inherited debt are considered, the importance of the inherited debt level at period 0 as a determinant for the social planner's optimal choices becomes evident.

Finally, let's see how results in this tractable case help us understand the social planner's evaluation of an increase in debt-financed assets in the full model as captured in (14). (14) indicates that whether the social planner has a higher or lower evaluation than households depends on the relative size between the marginal welfare gain and marginal welfare loss. This relative size, in turn, depends on the ratio of net debt issuance to net debt redemption. The tractable model suggests that the inherited long-term debt position is critical in determining this ratio. The larger the inherited long-term debt level, the smaller this ratio, and the more inclined the social planner is to lower the holding of debt-financed assets.

[5] This can be understood if we set the inherited long-term bonds at 0. In this case, $E_0 B_2 = -\pi \frac{W_L}{q_1^L} + \ldots$, which increases in the fire sale price q_1^L.

5 Quantitative Analysis

This section describes the calibration of the model and quantitatively studies the prudential social planner's intervention and its welfare implications. We numerically solve for the decentralized equilibrium and the social planner's allocations using global nonlinear methods (described in detail in the appendix).

5.1 Calibration

Table 1 lists the values assigned to all parameters in the model. A period in the model represents a quarter. The baseline calibration uses data from Brazil, a typical emerging market with substantial financial openness, and the best available external balance sheet data dating back to 2002 among all emerging markets. The appendix gives a detailed description of the variables we use in our estimation and their data sources.

We calibrate three parameters based on existing literature or by matching specific data moments. We set the risk aversion at $\sigma = 4$, in line with the literature's typical range of 2 to 10. The international risk-free rate, R_f, is determined to be 1.0017, reflecting the average real return on Brazil's external assets. We configure the coupon-decaying rate, δ, at 0.0174 to correspond with the average ratio of current capital inflows to liabilities from the previous period.[6]

We estimate the remaining parameters using the simulated moments method (SMM). Our estimation process intentionally over-identifies, choosing more empirical moments related to gross capital flows, gross international positions, and asset prices than the parameters estimated via SMM. The subjective discount factor, β, is estimated at 0.9859, while the effective risk-aversion of financiers, R^{κ}, stands at 1.3627. The financiers' wealth during financial tightening is set at 1.1101. Transi-

[6] The model's counterpart ratio is denoted as $\frac{-q_t(B_{t+1}-(1-\delta)B_t)}{q_{t-1}B_t}$. The calibration of δ rests on the idea that capital inflows should compensate for the decaying portion of liabilities in the long run. Simulations indicate a model-implied ratio of $\frac{\text{Capital inflow}_t}{\text{Liabilities}_{t-1}}$ of approximately 0.0180, quite close to the calibrated value of 0.0174.

tion probabilities are 0.9664 for switching from financial easing to tightening and 0.5336 for the reverse.

Table 1: Parameter Estimation Results

Parameter	Value	Determination	Description
R_f	1.0017	Calibration	International risk-free rate
δ	0.0174	Calibration	Depreciation rate of long-term bonds
σ	4.0	Calibration	Relative risk-aversion
β	0.9859	Estimation	Subjective discount factor for households
R^κ	1.3627	Estimation	Risk-aversion measure for financiers
W_L	1.1101	Estimation	Lower bound of financiers' wealth
π_{HH}	0.9664	Estimation	Continuation probability in a financial easing
π_{LL}	0.5366	Estimation	Continuation probability in a financial tightening

5.2 Model Evaluation

Table 2 shows the model's fit with empirical data. Our model accurately captures the observed volatility in inflows, outflows, and excess returns (the external liability return minus the external asset return). While slight discrepancies exist in these volatility terms, the deviations are within acceptable tolerances. Regarding correlations, our model captures the positive relationship between inflows and outflows and their negative correlations with net foreign asset positions (not targetted). The model overestimates the correlation between inflows and excess returns and underestimates that between outflows and excess returns but with correct signs.

Our model's estimation of means, especially the liability-to-GDP ratio, aligns with empirical data. The mean of the inflow-to-GDP ratio closely matches its empirical counterpart, which is primarily attributed to our calibration of δ. However, the calibrated model struggles to accurately reflect the external asset-to-GDP ratio and the official reserve-to-GDP ratio — approximately 40 percent of the external asset-to-GDP ratio — observed in the real world.

Overall, our model generates a good fit for the data, especially regarding the volatility of capital flows and excess returns. The model does not perform well when fitting external asset aggregates. This limitation might arise because, in our model, the external asset only offers insurance. While we recognize that emerging markets accumulate short-term assets for other reasons, this paper restricts the attention to the insurance role of debt-financed assets.

Table 2: Comparison between data moments and model moments

Moments	Theoretical Moments	Empirical Moments	Targeted
std(inflows)	0.0263	0.0235	Yes
std(outflows)	0.0170	0.0191	Yes
std(excess return)	0.0730	0.0798	Yes
std(NFA)	0.1395	0.2194	No
ρ(inflows, outflows)	0.8278	0.7740	Yes
ρ(inflows, excess return)	0.4148	0.1939	Yes
ρ(outflows, excess return)	0.2554	0.4191	Yes
ρ(inflows, NFA)	−0.6393	−0.6806	No
ρ(outflows, NFA)	−0.2709	−0.2918	No
E(excess return)	0.0151	0.0107	Yes
E(liability-GDP-ratio)	1.5785	1.5585	Yes
E(inflow-GDP-ratio)	0.0289	0.0277	No
E(asset-GDP-ratio)	0.0340	0.7984	No

Note: E[X], std[X], and ρ[X, Y] denote the mean of variable X, the standard deviation of variable X, and the correlation between variables X and Y, respectively.

5.3 Numerical Results

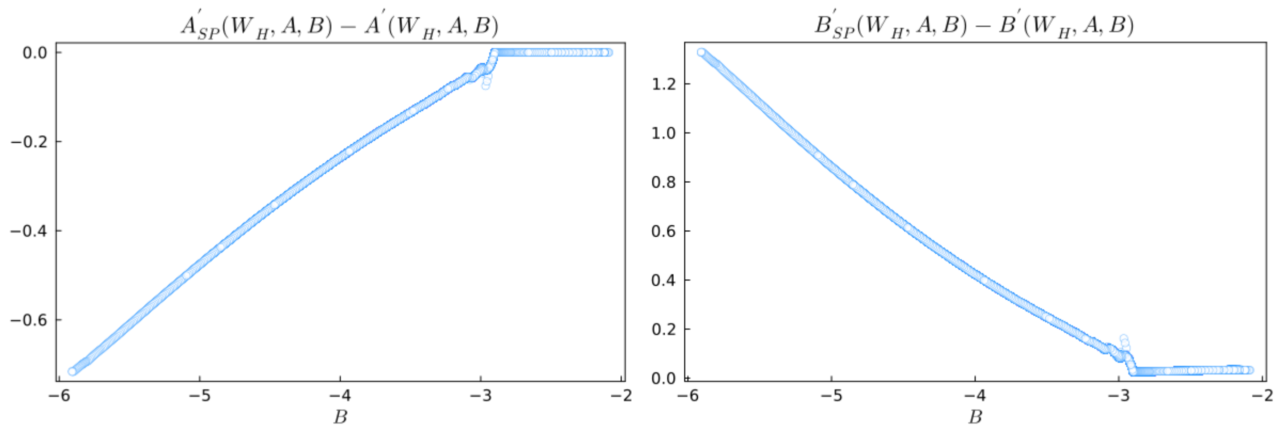
This subsection presents the quantitative results of our DSGE model, comparing the decentralized equilibrium and the social planner's allocations. The results show that the prudential social planner shrinks the external balance sheet relative to the decentralized equilibrium. The welfare gain from the public intervention is equivalent to a 0.2 percent increase in permanent consumption.

5.3.1 Portfolio Decisions

We first show how the portfolio decisions of the social planner differ from those of private households and how that difference affects the long-term bond pricing, then simulate the model to analyze the welfare gain from public interventions.

Figure 1 illustrates how portfolio decision rules differ between the decentralized equilibrium and the social planner's allocations. To facilitate comparison, we define difference measures $\Delta A'(W_H, A, B) = A'_{SP}(W_H, A, B) - A'(W_H, A, B)$ and $\Delta B'(W_H, A, B) = B'_{SP}(W_H, A, B) - B'(W_H, A, B)$. Here, "$SP$" denotes the social planner's decision rules, while unlabelled rules pertain to private households. These measures highlight short-run allocation disparities, given that the social planner and private households originate from the same initial gross positions (A, B). Within the entire state space of (A, B), we narrow our focus to a subset where (A, B) possesses a non-zero probability of occurrence in the decentralized equilibrium, essentially the ergodic path of (A, B). Each point in Figure 1 corresponds to an ergodic point, emphasizing the initial long-term bond position B.

Figure 1: Portfolio Differentials

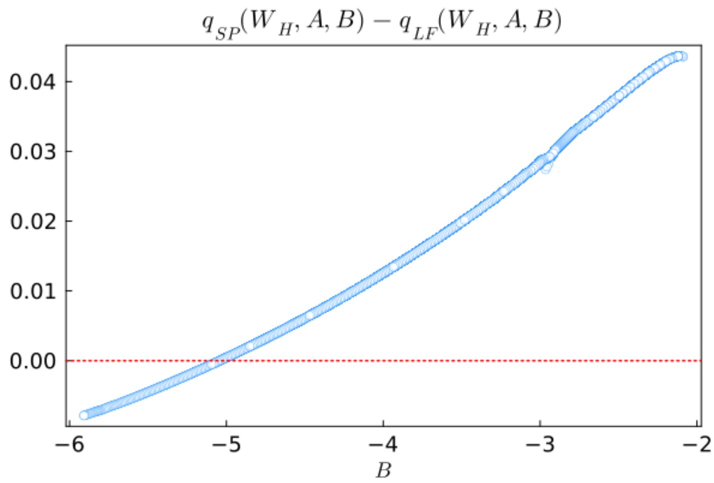


A primary observation from Figure 1 is that the social planner shrinks the external balance sheet relative to private households. Across all states on the ergodic path, the planner opts to diminish long-term bond issuance (evident from a positive $\Delta B'(W_H, A, B)$) and curtail short-term bond accumulation (indicated by a non-positive $\Delta A'(W_H, A, B)$). The mean values of $\Delta A'$ and $\Delta B'$ stand at -0.024 and 0.067, respectively. Furthermore, $\Delta A'(W_H, A, B)$ displays a strong positive correlation of 0.93 with the inherited long-term bond position, B, while $\Delta B'(W_H, A, B)$ exhibits a correlation of -0.91 with B. These findings are consistent with the insight from the tractable model that a prudential social planner is inclined to shrink the external balance sheet as inherited long-term debt increases.

5.3.2 Asset Prices

After examining the social planner's portfolio decisions, we focus on how public intervention affects the long-term bond price, a key aspect of the global financial cycle. In Figure 2, we introduce the price differential term $\Delta q(W_H, A, B) = q^{SP}(W_H, A, B) - q(W_H, A, B)$, analogous to the previously defined portfolio differential terms. The graph illustrates that the social planner's intervention typically boosts the long-term bond price, as evidenced by a positive $\Delta q(W_H, A, B)$. Notably, when initial debt levels are high, the planner's intervention lowers the bond price compared to the decentralized equilibrium. However, the likelihood of the economy venturing into such a high-indebtedness state is less than 5×10^{-4} in equilibrium.

Figure 2: Bond Price Differentials



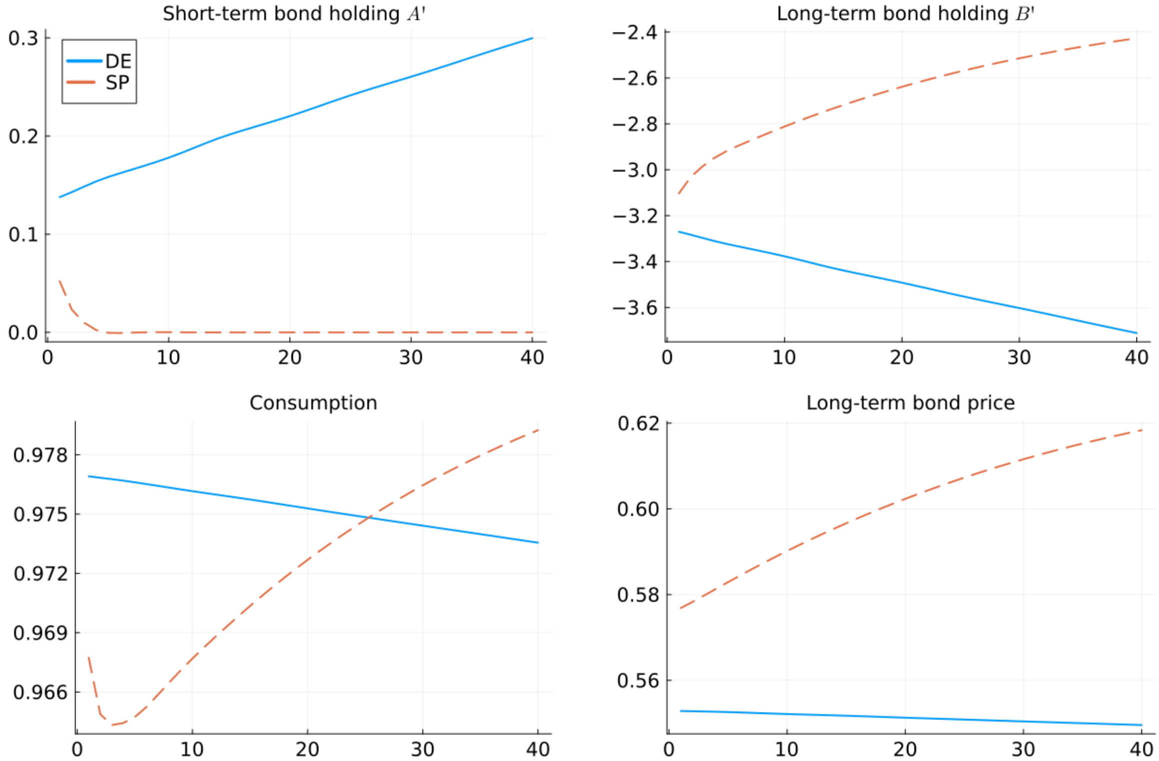
5.3.3 Transition Path Analysis

Our prior analysis indicates that the social planner reduces the external balance sheet and raises long-term bond prices. However, the earlier tractable model cannot explain this outcome, as it always suggests a positive correlation between the size of the external balance sheet and long-term bond prices. The DSGE model allows for time-variant consumption, opening up more possibilities for the relationship between the bond price and external balance sheet size. Specifically, suppose the social planner desires a higher current long-term bond price. Expanding the external balance sheet while maintaining the current consumption unchanged at the decentralized equilibrium level could be a choice. However, she could also opt to decrease the current consumption to boost the bond price, which leads to less long-term debt and, thus, a shrunk external balance sheet. In this subsection, we conduct a transition path analysis to understand the differences between the decentralized equilibrium and the social planner's allocations.

To conduct the transition path analysis, we simulate the model under the decentralized equilibrium and the social planner's allocations separately, starting from a common initial state (W_H, A_0, B_0) in period 0. This simulation spans 40 periods, during which no financial tightenings are introduced, i.e., $W_t = W_H$ for $t = 0, 1, ..., 40$. Subsequently, we collect and compare the trajectories of pertinent macro variables across both allocations. The transition paths are plotted in Figure 3.

The upper two panels restate our earlier observation that the social planner curtails the external balance sheet size compared to private households. The bottom left panel shows that while diminishing the aggregate external balance sheet, the social planner concurrently curtails consumption, thus, net capital inflows. This reduced consumption enhances the long-term bond price, attributed to a reduced net debt burden during periods of financial tightening. Although the economy can also strengthen its net foreign asset position by expanding the external balance sheet—increasing assets more than liabilities—this is deemed unnecessary because, at period 0, the social planner's regulated economy manifests as a net buyer of long-term bonds rather than a net seller. Therefore, expanding the external balance sheet to increase the current bond price does not appeal to the social planner. This scenario is evidenced by the negative capital inflows at period 0 upon the social planner taking over the economy (not depicted).

Figure 3: A Comparison between Two Alternative Allocations' Dynamics



Note: The figures juxtapose the decentralized equilibrium allocation ("DE") with the social planner's allocation ("SP") over time. Both start from identical initial gross positions $(A_0, B_0) = (0.13, -3.26)$. Here, A_0 and B_0 denote the ergodic means in the decentralized equilibrium when $A > 0$. The foreign financiers' wealth consistently remains at W_H for periods 0 to 40.

The social planner's allocation of negative capital inflows in our model presents a notable divergence from the existing literature. Typically, previous studies show that economies borrow using short-term debt. In these scenarios, the social planner incurs positive debt but less than what is seen in the decentralized equilibrium. This prudential debt accumulation approach is often referred to as "leaning against the wind." In contrast, due to low coupon payments (a small δ) and a reserve of short-term bonds, our model allows the social planner to act as a net purchaser of long-term debt without significantly reducing consumption. This strategy in our model is more aptly described as "reversing the wind," indicating a proactive approach to managing long-term debt.

5.3.4 Welfare Implications

While we have shown that the social planner tends to limit the external balance sheet, an essential question remains: Does this intervention genuinely enhance welfare? Our study revolves around a discretionary social planner; inherently, only a fully committed social planner guarantees welfare improvement over a decentralized equilibrium. To address this, we delve into a welfare analysis in this subsection.

We define the metric $\Delta(W_H, A, B)$ to assess the welfare benefits resulting from the social planner's intervention. This metric contrasts the lifetime utilities of households under the two allocations, converting the differences into equivalent consumption terms. The formulation is:

$$\Delta(W_H, A, B) = 100 * \left[\left(\frac{V_{SP}(W_H, A, B)}{V(W_H, A, B)} \right)^{\frac{1}{1-\sigma}} - 1 \right]$$

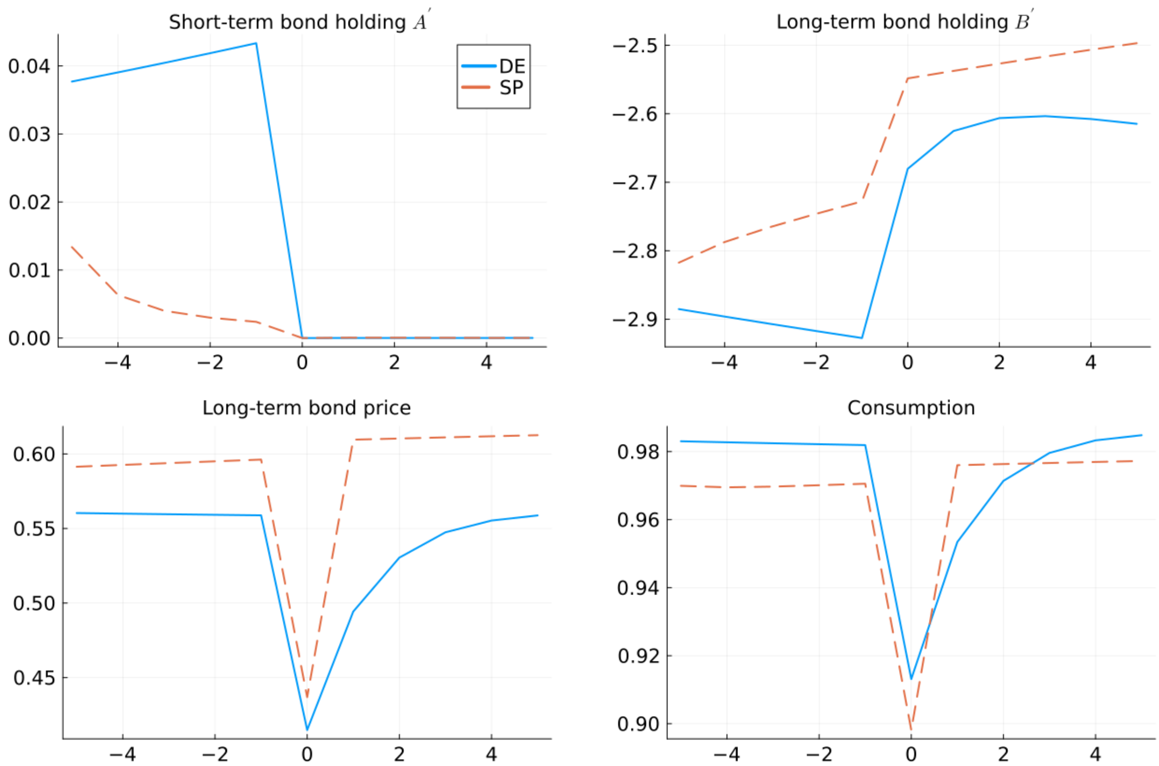
Here, $V_{SP}(W_H, A, B)$ represents the lifetime welfare with the social planner's intervention, and $V(W_H, A, B)$ denotes the welfare without such intervention.

Numerically determining the values for (W_H, A, B) across all model states reveals that the welfare gain is consistently positive (the relevant graph is not presented here). This finding indicates that the intervention by a discretionary social planner benefits the economy. The positive welfare gain is not an obvious result but a quantitative finding, as the social planner is discretionary in our context. To provide a clearer picture of the average benefit of public intervention, we calculate the expected welfare gain, represented by $E\Delta(W_H, A, B)$ where the expectation is taken over the ergodic distribution of (W_H, A, B) in decentralized equilibrium.

The unconditional welfare gain $E\Delta(W_H, A, B)$ is estimated to be 0.20 percent of permanent consumption. This value is comparable to the welfare improvement in the existing literature, e.g., 0.14 percent. Welfare gain here should be viewed as a conservative estimate. More significant gains might be realized when accounting for inefficient resource use during financial tightenings, risks specific to individual firms, or potential long-term effects on economic growth.

We analyze the dynamics surrounding typical financial tightening episodes to elucidate the welfare gains from intervention. We conducted simulations in a decentralized economy for one million periods, identifying 27382 unique episodes, each covering eleven periods, with a financial tightening occurring midway. In each case, we posit that the social planner begins with the same gross positions at period -5. Our analysis focuses on the social planner's allocation during these episodes, assuming a regulated economy with an identical sequence of financier wealth W_t. The findings are illustrated in Figure 4.

Figure 4: Dynamics around A Typical Financial Tightening



Note: The figures plot dynamics of various macro variables around a typical financial tightening at period 0 under the two allocations. "DE" and "SP" represent the decentralized equilibrium allocation and the social planner's allocation, respectively. Both scenarios commence from identical initial gross positions (A_{-5}, B_{-5}).

Figure 4 demonstrates that before a financial tightening, the social planner reduces the external balance sheet and consumption expenditure compared to private households. As a result of this intervention, consumption recovers more swiftly after the tightening. In a decentralized equilibrium, consumption takes over four quarters to return to its pre-tightening level; however, with the social planner's allocation, it rebounds in just one quarter.

6 Policy Instruments

This subsection discusses the policy instruments that can replicate the social planner's allocation in the short run when the ergodic distribution of initial gross positions follows that of decentralized equilibrium. We focus on capital flow taxes, a prevalent tool for capital controls. The social planner can impose ad-valorem taxes such that when EM residents purchase long-term or short-term bonds, they must pay $q(W_H, A, B) + \tau_B$ or $\frac{1}{R_f} + \tau_A$ units of goods, respectively. The collected tax revenue is then returned to the households in a lump sum.

By analyzing the social planner's portfolio rules in (10) and (11) , we can get the taxation rule for short-term bond purchases

$$\tau_A(W_H, A, B) = \frac{\partial Q(W_H, A'_{SP}, B'_{SP})}{\partial A'}(B'_{SP} - (1-\delta)B)$$
$$+\beta(1-\pi_{HH})\frac{u'(C_{SP}(W_L, A'_{SP}, B'_{SP}))}{u'(C_{SP}(W_H, A, B))}\frac{\partial q^{SP}(W_L, A'_{SP}, B'_{SP})}{\partial A'}(B''_{SP} - (1-\delta)B'_{SP})$$

and that for long-term bond purchases

$$\tau_B(W_H, A, B) = \frac{\partial Q(W_H, A'_{SP}, B'_{SP})}{\partial B'}(B'_{SP} - (1-\delta)B)$$
$$+\beta(1-\pi_{HH})\frac{u'(C_{SP}(W_L, A'_{SP}, B'_{SP}))}{u'(C_{SP}(W_H, A, B))}\frac{\partial q^{SP}(W_L, A'_{SP}, B'_{SP})}{\partial B'}(B''_{SP} - (1-\delta)B'_{SP})$$

where:

$$Q(W_H, A', B') \equiv E_{W'|W_H}\left[M_{W_H,W'}\left(\delta + (1-\delta)q^{SP}(W', A', B')\right)\right]$$

Recall that A'_{SP} and B'_{SP} represent the prudential social planner's portfolio choice in the current state (W_H, A, B), and B''_{SP} pertains to the equilibrium long-term bond choice in the state (W_L, A'_{SP}, B'_{SP}). The signs of these taxes, positive or negative, depend on the capital flow dynamics of both the present and the subsequent periods. On the ergodic path of decentralized equilibrium, the metric

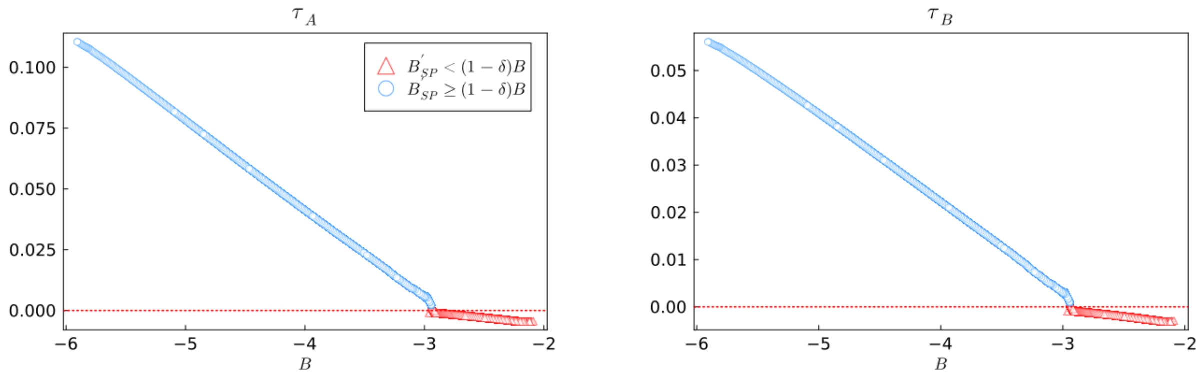
$B''_{SP} - (1-\delta)B'_{SP}$ consistently remains positive, reflecting the open economy's tendency for long-term bond redemptions during financial tightenings. However, the measure $B'_{SP} - (1-\delta)B$ varies between positive and negative values, depending on the current state.

Figure 5 plots capital flow taxes τ_A and τ_B against the ergodic long-term bond position in the decentralized equilibrium, B. Bubble points are shaded red when the social planner's choice for long-term bonds, B'_{SP}, is less than $(1-\delta)B$, indicating a positive capital inflow in the social planner's allocation. This suggests that the social planner is a net issuer of long-term bonds only when inherited long-term debt is low.

In Figure 5's red region where the social planner is a net long-term debt issuer, both τ_A and τ_B are negative. Within this context, a negative τ_B indicates a tax on long-term bond issuance. In contrast, a negative τ_A is a subsidy for short-term bond accumulation. These taxes are modest in terms of magnitude: τ_B averages at -0.13 percent and τ_A at -0.17 percent; when the long-term bond tax is represented as the tax-price ratio $\frac{\tau_B}{q}$, it has an average value of -0.22 percent.

On the other hand, in the blue region, where the social planner acts as a long-term bond buyer, the tax dynamics flip. Here, the economy benefits from subsidized long-term borrowing while short-term savings incur a tax. The tax magnitudes are significant: τ_B averages 0.008 and τ_A is around 0.015. When this long-term bond tax is expressed as the tax-price ratio, $\frac{\tau_B}{q}$, it averages a notable 1.5 percent.

Figure 5: Ad-valorem Capital Flow Taxes



Note: The figures display capital flow taxes for short-term bond purchases, τ_A (left panel), and long-term bond purchases, τ_B (right panel), plotted against the ergodic initial long-term bond position B (x-axis) in the decentralized equilibrium.

7 Conclusion

This paper investigates the optimal gross capital flow management by a prudential social planner in an open economy. During financial easing periods, private households in the economy issue long-term debt and accumulate short-term bonds; during financial tightening periods, they reduce their short-term bond holdings and repurchase long-term debt. Despite using two financial instruments to share risks, households fail to exploit their collective market power over their issued long-term debt. The prudential social planner rebalances portfolios between short-term and long-term bonds to improve the long-term bond pricing in the economy's favor.

Our paper makes a novel normative contribution by highlighting how optimal capital flow management depends on the initial long-term debt position of the economy. For economies with low initial long-term debt, the optimal CFM encourages an expansion of the external balance sheet, thereby increasing gross capital flows and improving long-term bond prices during fire sales. Because of the theoretical ambiguity regarding the inherited debt, we seek to view the optimal CFM in a specific numerical case of Brazil. The quantitative results suggest that for Brazil, an optimal CFM involves contracting the external balance sheet through reduced gross capital flows, yielding a welfare gain equivalent to approximately 0.2 percent of permanent consumption.

From a policy perspective, the paper argues against one-size-fits-all CFM policies for emerging markets. Appropriate CFM policies need to consider the specific indebtedness of an emerging market. Our analysis identifies capital flow taxes on both inflows and outflows as pertinent CFM policy instruments, the rates of which should be adjusted according to the economy's inherited long-term debt levels. In substantially low indebtedness cases, the model advises a policy that subsidizes short-term bond purchases while levying taxes on long-term debt creation—the recommended policy shifts towards the opposite as the economy's debt increases.

8 Appendix

8.1 Derivation of The Social Planner's Decision Rule

Given the social planner's Bellman equation in Section 3.4, we can derive the first-order conditions concerning A' and B' as follows:

$$-u'(C_{SP}(s))\Big(\frac{1}{R_f} + \frac{\partial Q(W_H, A', B')}{\partial A'}(B' - (1-\delta)B)\Big) + \beta E_{W'|W}\frac{\partial V_{SP}(W', A', B')}{\partial A'} + \mu^A_{SP} = 0 \tag{24}$$

$$-u'(C_{SP}(s))\Big(Q(W_H, A', B') + \frac{\partial Q(W_H, A', B')}{\partial B'}(B' - (1-\delta)B)\Big) + \beta E_{W'|W}\frac{\partial V_{SP}(W', A', B')}{\partial B'} = 0 \tag{25}$$

where μ^A_{SP} is the Lagrangian multiplier concerning the liquidity constraint.

We aim to determine the values of $\frac{\partial V_{SP}(W',A',B')}{\partial A'}$ and $\frac{\partial V_{SP}(W',A',B')}{\partial B'}$. The form of these partial derivatives is contingent upon W, the financiers' wealth, stemming from our model's premise that the social planner is absent during financial tightenings. However, in periods of financial easing, the envelope theorem implies:

$$\frac{\partial V_{SP}(W_H, A, B)}{\partial A} = u'(C)$$
$$\frac{\partial V_{SP}(W_H, A, B)}{\partial B} = u'(C)(\delta + (1-\delta)Q(W_H, A, B))$$

In contrast, during financial tightening periods, the social planner's problem is formulated as

$$V_{SP}(W_L, A, B) \equiv \max_{A' \geq 0, B'} u(\bar{y} + A + \delta B - q^{SP}(W_L, A, B)(B' - (1-\delta)B) - \frac{A'}{R_f})$$
$$+\beta E_{W'|W_L} V_{SP}(W', A', B')$$

the envelope theorem then implies

$$\frac{\partial V_{SP}(W_L, A, B)}{\partial A} = u'(C)\Big(1 - \frac{\partial q^{SP}(W_L, A, B)}{\partial A}\big(B' - (1-\delta)B\big)\Big)$$

$$\frac{\partial V_{SP}(W_L, A, B)}{\partial B} = u'(C)\Big(\delta + (1-\delta)q(W_L, A, B) - \frac{\partial q^{SP}(W_L, A, B)}{\partial B}\big(B' - (1-\delta)B\big)\Big)$$

Therefore, we have the following equations regarding $\frac{\partial V_{SP}(W', A', B')}{\partial A'}$ and $\frac{\partial V_{SP}(W', A', B')}{\partial B'}$:

$$\frac{\partial V_{SP}(W'_H, A', B')}{\partial A'} = u'(C(W_H, A', B')) \tag{26}$$

$$\frac{\partial V_{SP}(W_H, A', B')}{\partial B'} = u'(C(W_H, A', B'))(\delta + (1-\delta)Q(W_H, A'', B'')) \tag{27}$$

$$\frac{\partial V_{SP}(W_L, A', B')}{\partial A'} = u'(C(W_L, A', B'))\Big(1 - \frac{\partial q^{SP}(W_L, A', B')}{\partial A'}(B'' - (1-\delta)B')\Big) \tag{28}$$

$$\frac{\partial V_{SP}(W_L, A', B')}{\partial B'} = u'(C(W_L, A', B'))\Big(\delta + (1-\delta)q(W_L, A', B') - \frac{\partial q^{SP}(W_L, A', B')}{\partial B'}(B'' - (1-\delta)B')\Big) \tag{29}$$

Plugging (26) to (29) back into (24) and (25) and utilizing the relationship that $Q(W_H, A'', B'') = q^{SP}(W_H, A', B')$, we can get the Euler equations for the social planners shown in Section 3.4.

8.2 Data Source and Transformation

Table 3: Data Sources and Descriptions (2001q4 - 2022q4)

Variable	Description	Source
Baseline Variables		
Gross Inflows	Sum of FDI, portfolio, and other inflows (current \$)	BOP-IIP
Gross Outflows	Sum of FDI, portfolio, reserve, other inflows (current \$)	BOP-IIP
External Liabilities	Liabilities (current \$)	BOP-IIP
External Assets	Assets (current \$)	BOP-IIP
GDP	PPP-adjusted Brazil's GDP (current \$)	OECD Stats
GDP Deflator	U.S. GDP deflator	Fred

Following Caballero and Simsek (2020), we categorize gross inflows into three main components: Foreign Direct Investment (FDI), portfolio investment, and other investment inflows. Similarly, gross outflows consist of four components:

FDI, portfolio investment, other investment, and official reserve outflows. External liabilities and assets are documented as their end-of-period market values. All capital flows and gross positions are expressed in current dollars. Therefore, we transform them into real terms using the U.S. GDP deflator. Regarding Brazil's GDP, we first obtain Brazil's PPP-adjusted GDP in current dollars and again convert it into its real counterpart using the U.S. GDP deflator.

To align the data with our model, we normalize both the real capital flows and real gross positions using the concurrent real GDP trend derived from an HP filter. This process effectively strips away trends in capital flows and domestic income, making the capital flow data stationary and more apt for our analysis.

In addition to measuring capital flows, evaluating the excess returns on Brazil's external balance sheet is crucial. This involves assessing returns on both external liabilities and assets separately. Using the conventional register method, we characterize the gross return on liabilities as the sum of investment payments and valuation changes divided by the liabilities from the preceding period. The following equation elucidates the unobserved valuation change:

$$\text{valuation change}_t = \text{liabilities}_t - \text{liabilities}_{t-1} - \text{gross capital inflows}_t$$

We can compute the real returns on Brazil's external liabilities using the valuation change and investment payment as follows:

$$\text{external liability return}_{t-1,t} = \frac{\text{investment payment}_t + \text{valuation change}_t}{\text{liabilities}_{t-1}}$$

Similarly, we determine the gross returns on Brazil's external assets. The excess return is then derived by subtracting the return on external liabilities from the return on external assets.

8.3 Numerical Method

We create an equation solver-free method to solve the decentralized equilibrium and the social planner's allocations. To elucidate the procedure, we detail the steps for the decentralized equilibrium. First, we create grids for state variables (A, B): $\mathbf{A}$ with 41 points spanning from $\underline{A} = 0$ to $\bar{A} = 2$ and $\mathbf{B}$ with 41 points between $\underline{B} = -6$ and $\bar{B} = -2$. Next, we initialize decision rules and the bond price function using $A'_1(s)$, $B'_1(s)$, and $q'_1(s)$, where subscripts denote the iteration round. Upon initialization, we commence the iterative process. Specifically, throughout iterations $i = 1, 2, ..., n$, we execute the following steps:

- Step 1: Construct the consumption $c_i(s)$ for grid $s \equiv (W, A, B)$ by setting

  $$C_i(s) = \bar{Y} + \delta B + A - \frac{A'_i(s)}{R_f} - q_i(s)(B'_i(s) - (1-\delta)B)$$

  and then interpolate $C_i(s)$ over grids of s linearly to get the consumption function $C_i(\cdot)$.

- Step 2: Solve for C_i^b by applying the fixed-point iteration method to the Euler equation for the long-term bond with

  $$C_i^b(s) = \left(\beta E_{W'|W} \frac{C_i\big(W', A'_i(s), B'_i(s)\big)^{-\sigma}\big(\delta + (1-\delta)q_i(W', A'_i(s), B'_i(s))\big)}{q_i(s)}\right)^{-\frac{1}{\sigma}}$$

  As $A'_i(s), B'_i(s)$ generally do not match node grids in $\mathbf{A}$ and $\mathbf{B}$, respectively, $C_i(\cdot)$ and $q_i(\cdot)$ are evaluated at $\big(W', A'_i(s), B'_i(s)\big)$ to determine $C_i(W', A'_i(s), B'_i(s))$ and $q_i(W', A'_i(s), B'_i(s))$ using cubic spline interpolations.

- Step 3: Solve for $B'_{i+1}(s)$ using the budget constraint

  $$B'_{i+1}(s) = \frac{\bar{y} + \delta B + A - C_i^b(s) - \frac{A'_i(s)}{R_f}}{q_i(s)} + (1-\delta)B$$

  If $B'_{i+1}(s)$ is between $\underline{B}$ and $\bar{B}$, keep it; If $B'_{i+1}(s)$ is lower than $\underline{B}$, change it to $\underline{B}$; If $B'_{i+1}(s)$ is higher than $\bar{B}$, change it to $\bar{B}$.

- Step 4: Solve for C_i^a by applying the fixed-point iteration method to the Euler equation for the one-period bond with

$$C_i^a(s) = \left[\beta R_f E_{W'|W} c_i(W', A'_i(s), B'_i(s))^{-\sigma}\right]^{-\frac{1}{\sigma}}$$

- Step 5: Solve for $A'_{i+1}(s)$ using the budget constraint

$$A'_{i+1}(s) = R_f\big(\bar{y} + \delta B + A - q_i(s)(B'_{i+1}(s) - (1-\delta)B) - C_i^a(s)\big)$$

  If $A'_{i+1}(s)$ is between $\underline{A}$ and $\bar{A}$, keep it; If $A'_{i+1}(s)$ is lower than $\underline{A}$, change it to $\underline{A}$; If $A'_{i+1}(s)$ is higher than $\bar{A}$, change it to $\bar{A}$.

- Step 6: Update the long-term bond price.

$$q_{i+1}(W, A, B) = \min\left\{E_t\left[M_{W'|W}\left(\delta + (1-\delta)q_i(W', A'_{i+1}(W, A, B), B'_{i+1}(W, A, B))\right)\right], -\frac{W}{B'_{i+1}(W, A, B)}\right\}$$

  Like in Step 2, $q_i(\cdot)$ needs to be evaluated at $(W', A'_{i+1}(W, A, B), B'_{i+1}(W, A, B))$ using the cubic spline interpolation.

- Step 7: Check the convergence. Compute $||A_{i+1}(s) - A_i(s)||$, $||B_{i+1}(s) - B_i(s)||$, and $||q_{i+1}(s) - q_i(s)||$ to see whether they are smaller than the numerical tolerance, which is 10^{-6}. If they are, stop the algorithm and collect $A_i(s)$, $B_i(s)$, and $q_i(s)$ as the decision rules and the price function. If not, return to Step 1 with the updated decision rules and price function $A_{i+1}(s)$, $B_{i+1}(s)$, and $q_{i+1}(s)$.

We employ the same method used for the decentralized equilibrium to determine the social planner's allocation. The only distinction is the need to compute the partial derivatives related to bond price functions. For these derivatives, we utilize Julia's "ForwardDiff" package.

We also utilize the exogenous grid method for solving both the decentralized and social planner's allocations in our model. This conventional method uses an equation solver to solve the two Euler equations, identifying updated portfolio decision rules as the roots. Although this approach is more time-consuming due to its heavy reliance on the equation solver, it contrasts with the quicker, solver-free method we previously employed. Notably, the exogenous grid method corroborates our fast method, as it yields consistent solutions for both equilibria.

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