



DeepLearning.AI

# Math for Machine Learning

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## Linear algebra - Week 2

Solving systems of equations

Matrix row reduction

Row operations that preserve singularity

Row-reduced echelon form

Row echelon form

Rank of a matrix



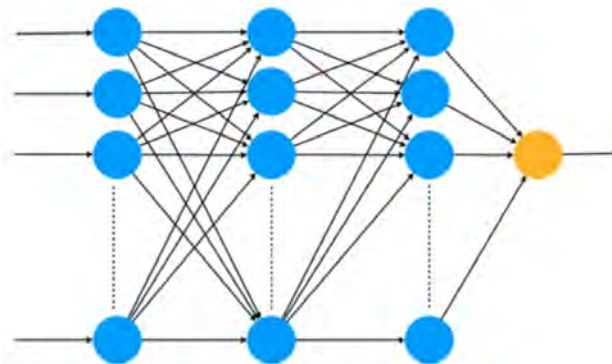
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# Solving System of Linear Equations

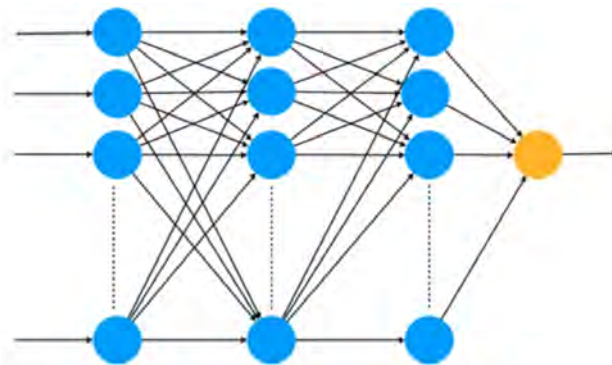
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## **Machine learning motivation**

# Neural networks - Matrix operations

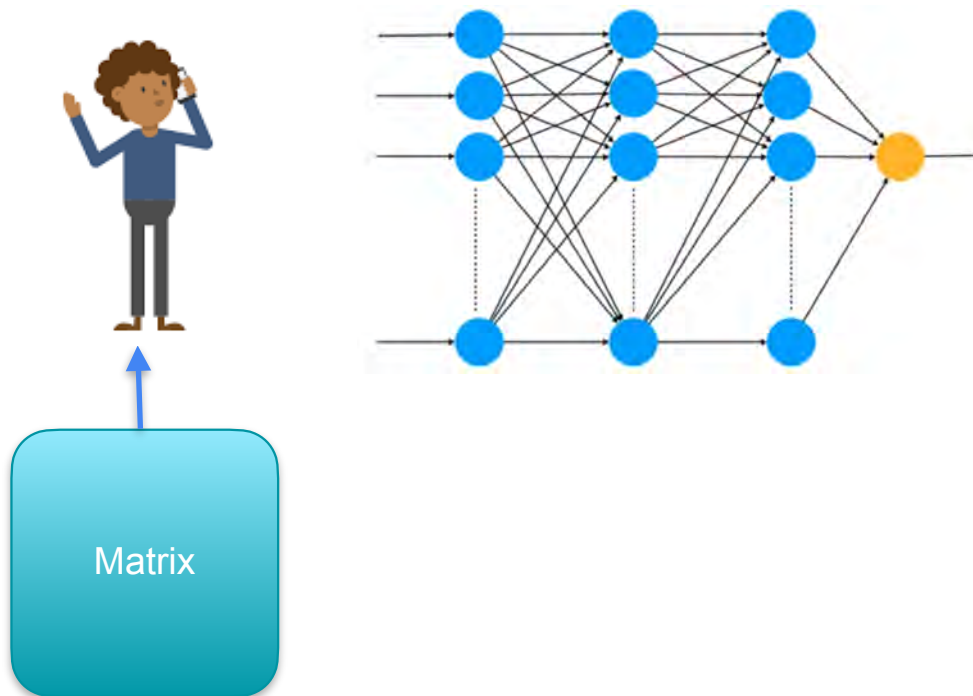


# Neural networks - Matrix operations



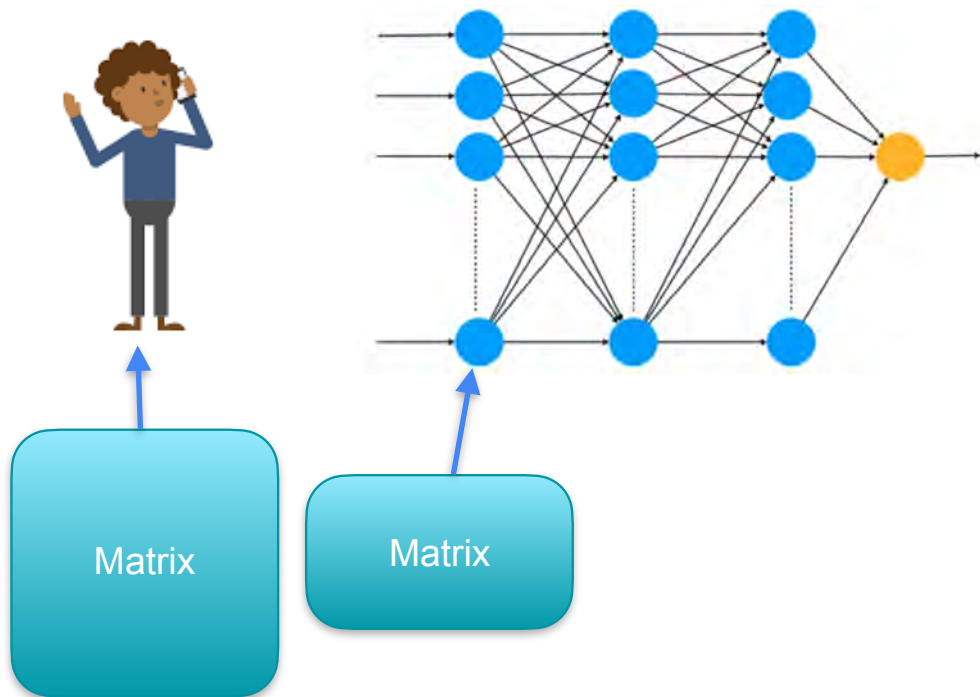
"Hello! Welcome to  
Math for Machine  
Learning!"

# Neural networks - Matrix operations



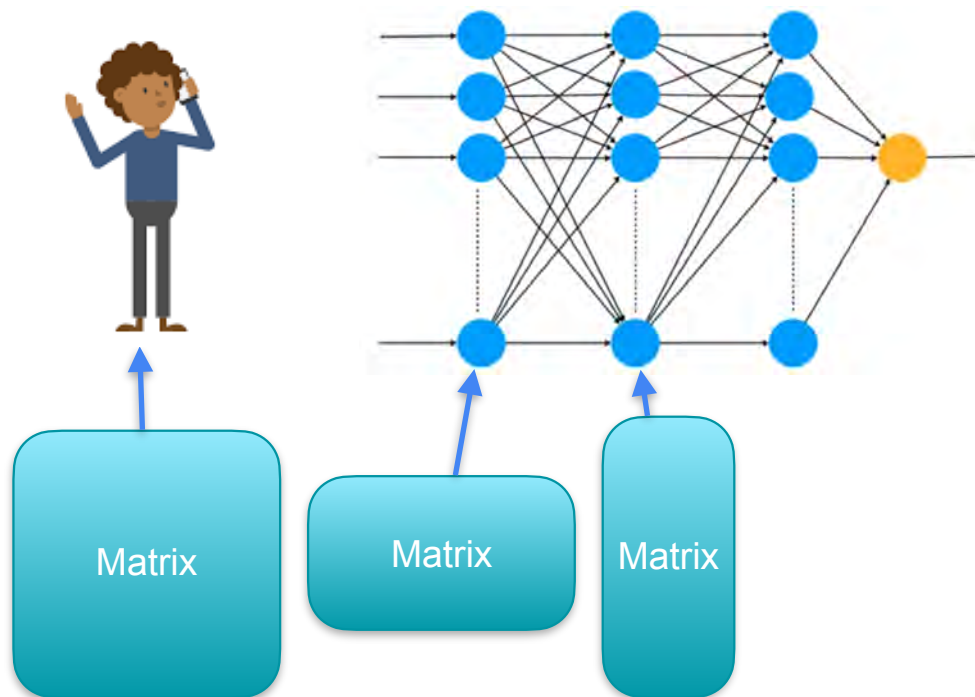
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# Neural networks - Matrix operations



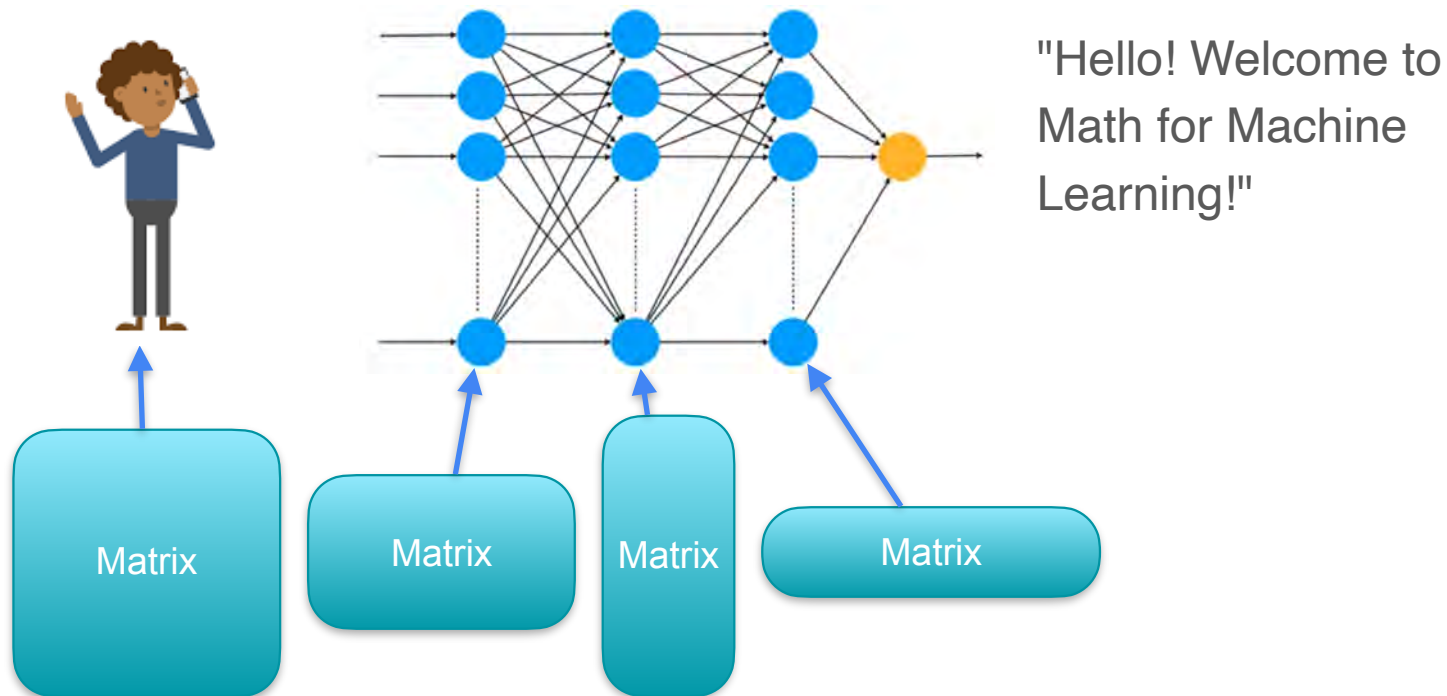
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# Neural networks - Matrix operations



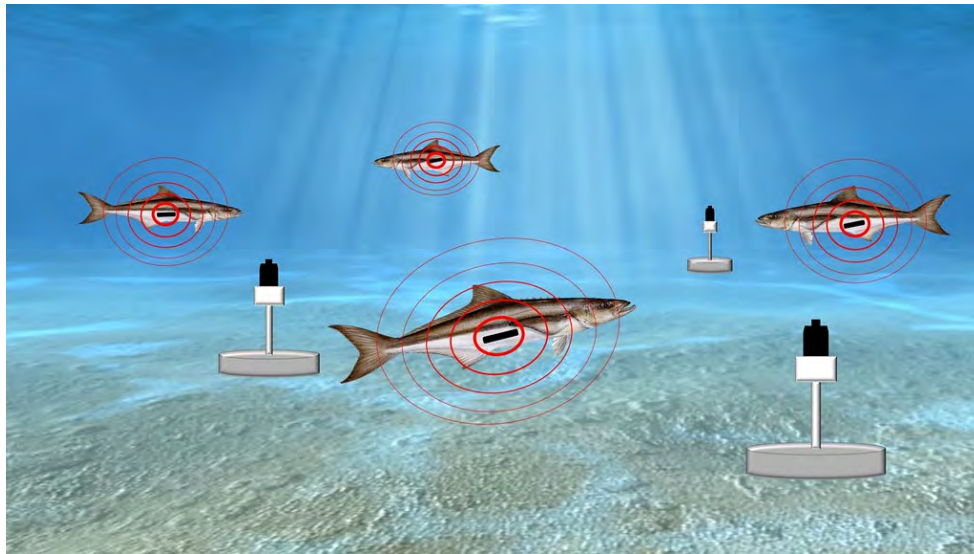
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# Neural networks - Matrix operations





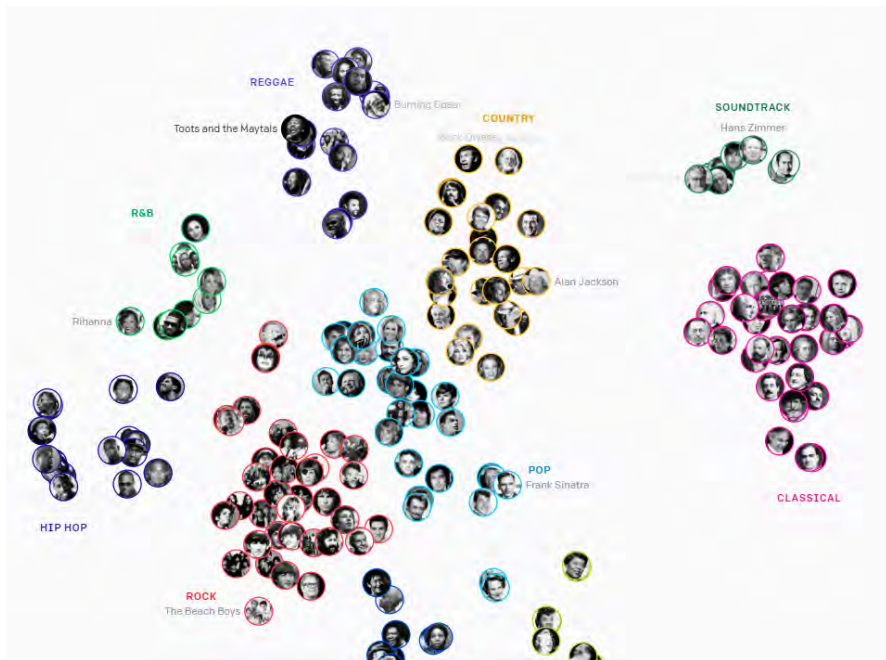
# Neural networks - Sound recognition



Acoustic monitoring: Monitoring ecosystems through sounds

- Sound recognition: tracking species through sound to preserve bio-habitats.

# Neural Networks - AI-generated music



Neural network generates music

- Automatic music generation: compressing music to discrete codes, then training the model on a specific genre to produce new music.



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# Solving System of Linear Equations



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**Solving non-singular system  
of linear equations**

# Solving systems of equations



## System



- $a + b = 10$   
 

- $a + 2b = 12$   
 

# Solving systems of equations

## System

- $a + b = 10$   
 



- $a + 2b = 12$   
  

$$\text{🍏} + \text{🍌} = \$10$$

$$\text{🍏} + \text{🍌} + \text{🍌} = \$12$$

# Solving systems of equations

## System

- $a + b = 10$   
 



- $a + 2b = 12$   
  

$$\text{apple} + \text{banana} = \$10$$

$$\text{apple} + \text{banana} + \boxed{\text{banana}} = \$12$$

# Solving systems of equations

## System

- $a + b = 10$   
 



- $a + 2b = 12$   
  

$$\text{apple} + \text{banana} = \$10$$

$$\text{apple} + \text{banana} + \boxed{\text{banana}} = \$\boxed{2}$$

# Solving systems of equations


## System

- $a + b = 10$   
 + 

- $a + 2b = 12$   
 + 

$$\text{apple} + \text{banana} = \$10$$



$$\text{apple} + \text{banana} + \boxed{\text{banana}} = \$\boxed{2}$$

 \$2











# Solving systems of equations

## System

- $a + b = 10$   
 + 



- $a + 2b = 12$   
 + 

 +  = \$10  
 \$2





 +  +  = \$  
 \$2






# Solving systems of equations

## System

- $a + b = 10$   
 

- $a + 2b = 12$   
  

 +  = \$10  
\$8   \$2

 +  +  = \$   
 \$2

# Solving systems of equations

# Solving systems of equations

## System

- $a + b = 10$







- $a + 2b = 12$





# Solving systems of equations

**System**





- $a + b = 10$   
 
- $a + 2b = 12$   
 

**Solved system**

- $a = 8$   

- $b = 2$   




# Solving systems of equations

**System**

- $a + b = 10$   
 
- $a + 2b = 12$   
 





Some process

**Solved system**

- $a = 8$   

- $b = 2$   


# Solving systems of equations



**System**

- $a + b = 10$   
 
- $a + 2b = 12$   
 

Some process





Manipulating equations

**Solved system**

- $a = 8$   

- $b = 2$   


# Solving systems of equations

**System**

- $a + b = 10$   
 
- $a + 2b = 12$   
 

**Some process**



**Manipulating equations**

**Swapping equations**

**Adding equations**

**Multiplying equations by a constant**





**Solved system**

- $a = 8$   

- $b = 2$   






# Solving systems of equations

**System**


- $a + b = 10$   
 
- $a + 2b = 12$   
 

**Solved system**

- $a = 8$   

- $b = 2$   


# Solving systems of equations


**System**


- $a + b = 10$   
 

- $a + 2b = 12$   
 

Eliminate 'a' from this equation

**Solved system**

- $a = 8$   


- $b = 2$   


# Manipulating equations

# Manipulating equations

**Multiplying by a constant**

# Manipulating equations

**Multiplying by a constant**

$$a + b = 10$$

# Manipulating equations

Multiplying by a constant

$$a + b = 10$$

$$\underline{x \qquad 7}$$

# Manipulating equations

**Multiplying by a constant**

$$a + b = 10$$

$$\begin{array}{r} x \qquad \qquad 7 \\ \hline 7a + 7b = 70 \end{array}$$

# Manipulating equations

**Multiplying by a constant**

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

**Adding two equations**



# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$a + b = 10$$

# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$a + b = 10$$

$$2a + 3b = 26$$

# Manipulating equations

## Multiplying by a constant

$$a + b = 10$$

$$\begin{array}{r} x \quad \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$a + b = 10$$

$$\begin{array}{r} + \quad 2a + 3b = 26 \\ \hline \end{array}$$

# Manipulating equations

## Multiplying by a constant

$$\begin{array}{r} a + b = 10 \\ \times \quad 7 \\ \hline 7a + 7b = 70 \end{array}$$

## Adding two equations

$$\begin{array}{r} a + b = 10 \\ + \quad 2a + 3b = 26 \\ \hline 3a + 4b = 36 \end{array}$$

Let's do a harder example

# Systems of equations

# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$

# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$

## Solved system

- $a = ?$
- $b = ?$



# Systems of equations

## System

- $5a + b = 17$
- $4a - 3b = 6$



Eliminate 'a'  
from this equation

## Solved system

- $a = ?$
- $b = ?$

# Systems of equations

## System

- $5a + b = 17$

- $4a - 3b = 6$

## Divide by coefficient of a

- $a + 0.2b = 3.4$

- $a - 0.75b = 1.5$

## Solved system

- $a = ?$

- $b = ?$

↑  
Eliminate 'a'  
from this equation

# Systems of equations

## System

- $5a + b = 17$

- $4a - 3b = 6$

## Divide by coefficient of a

- $a + 0.2b = 3.4$

- $a - 0.75b = 1.5$

## Solved system

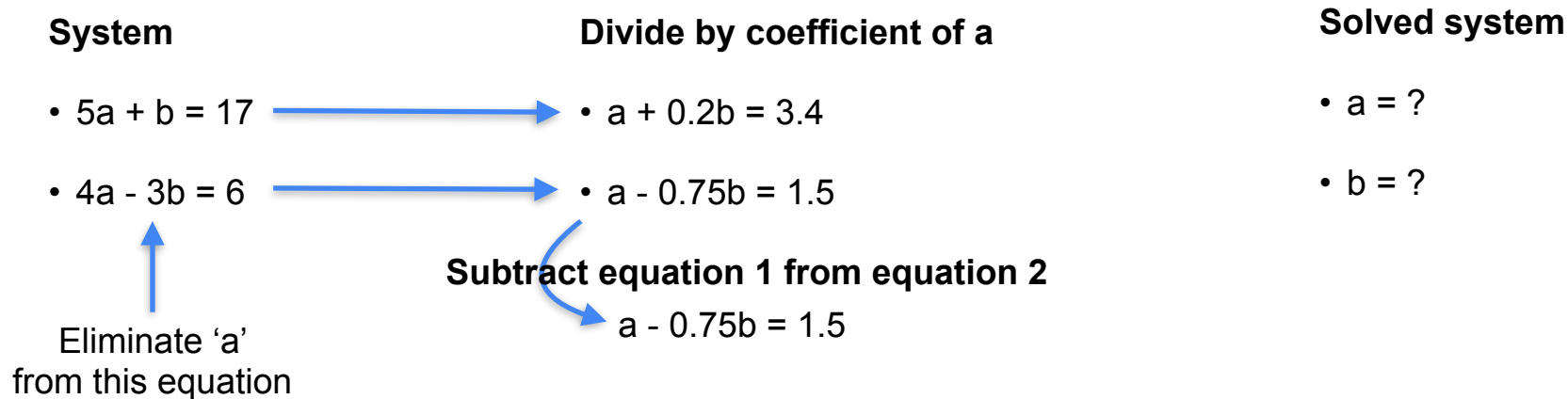
- $a = ?$

- $b = ?$

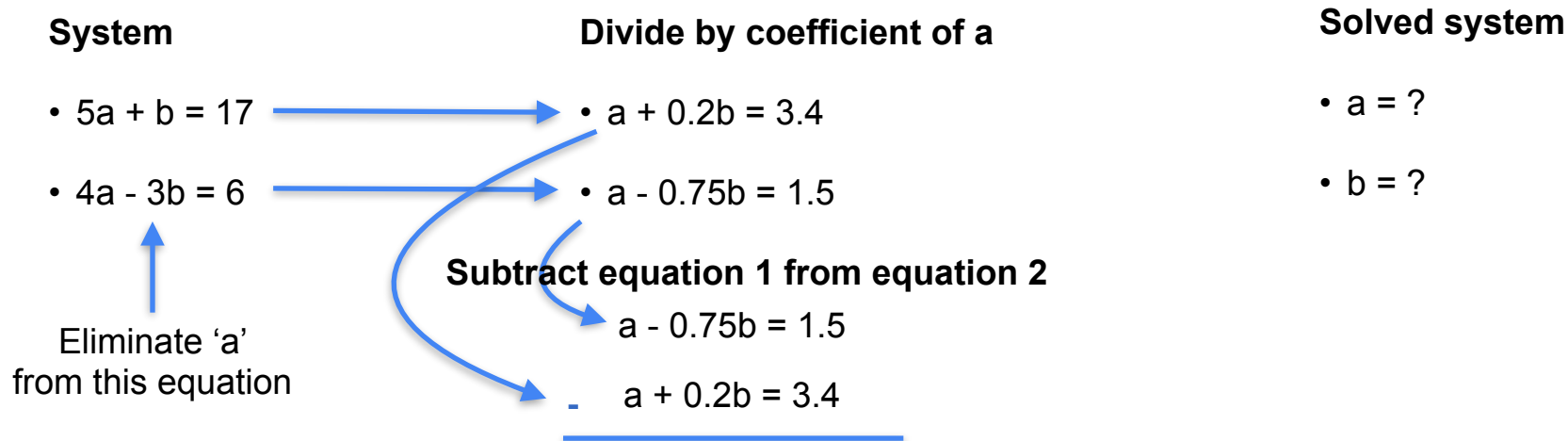
↑  
Eliminate 'a'  
from this equation

Subtract equation 1 from equation 2

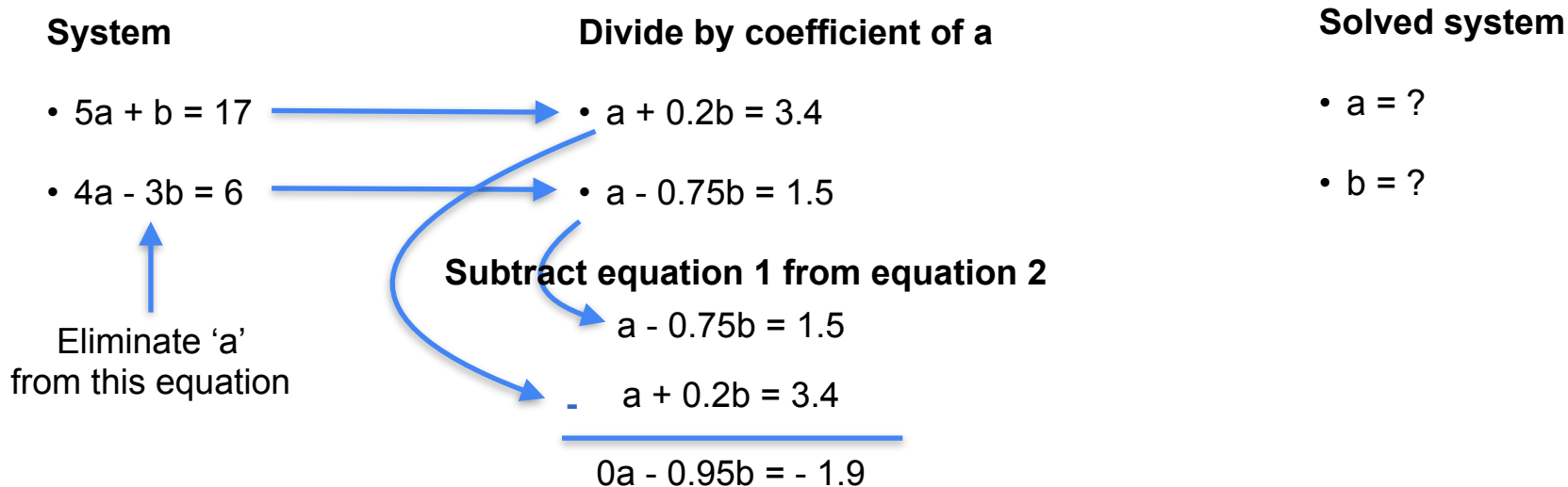
# Systems of equations



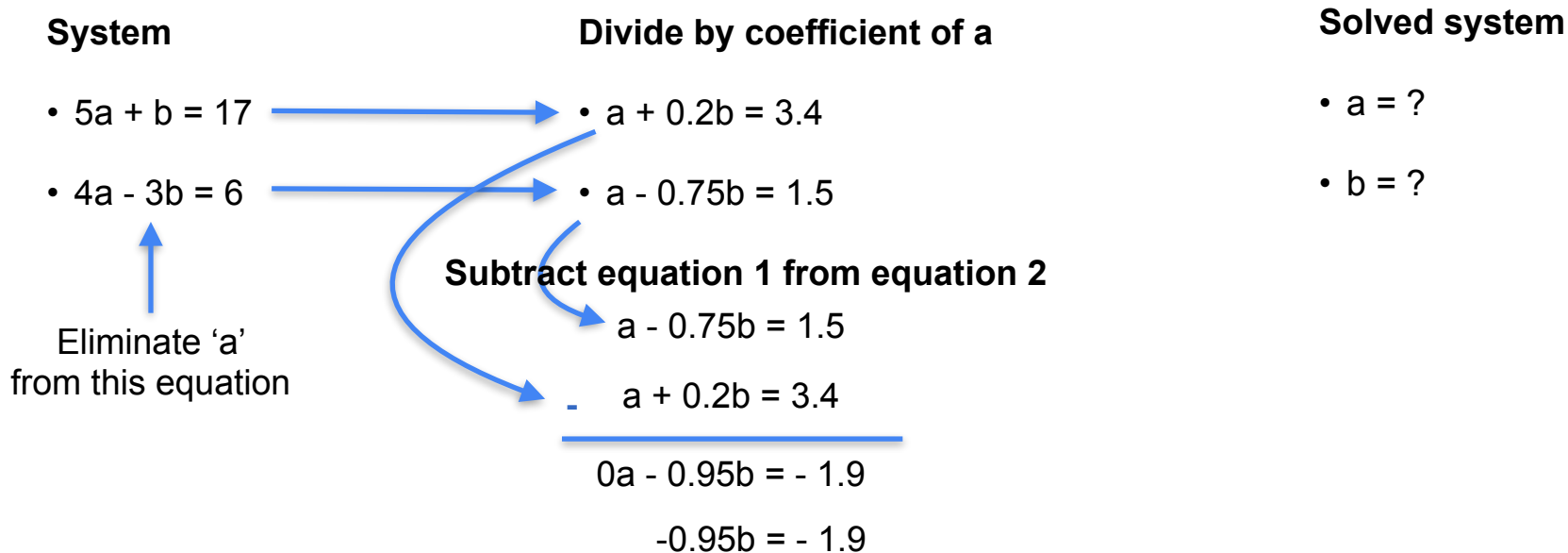
# Systems of equations



# Systems of equations



# Systems of equations

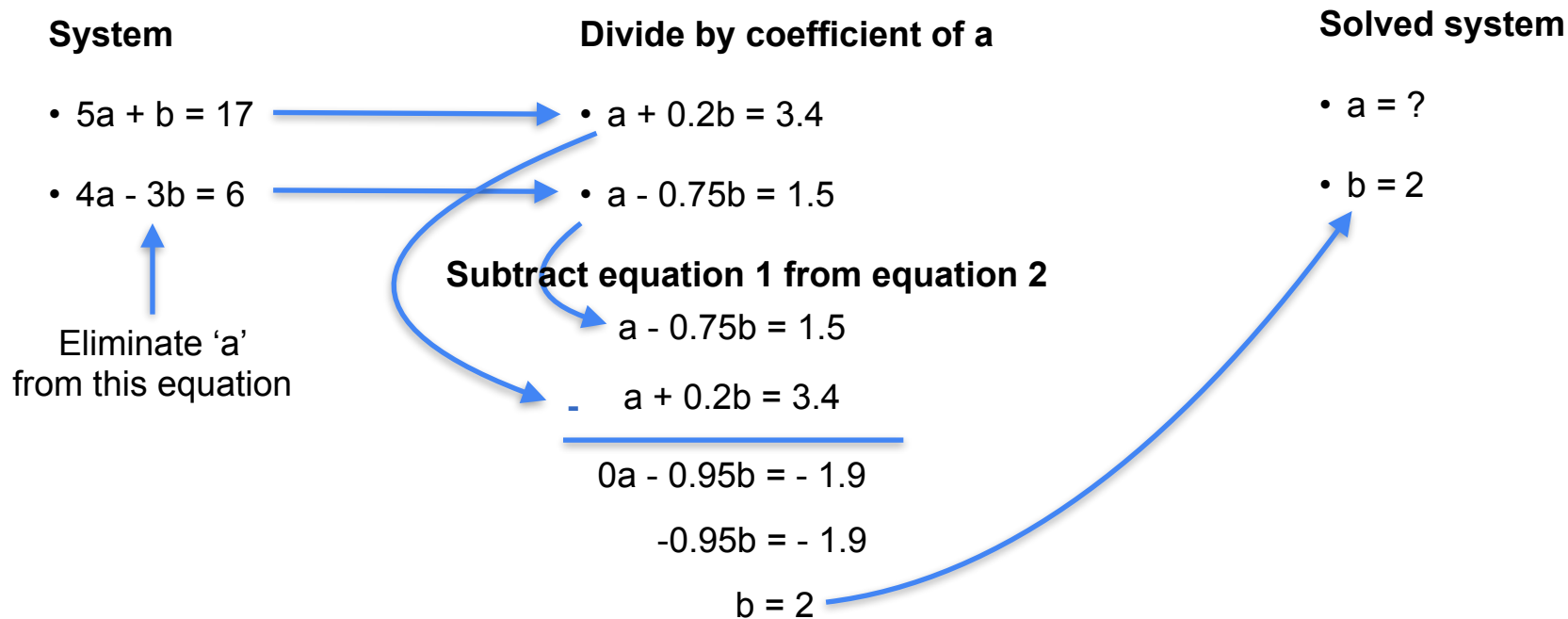


# Systems of equations

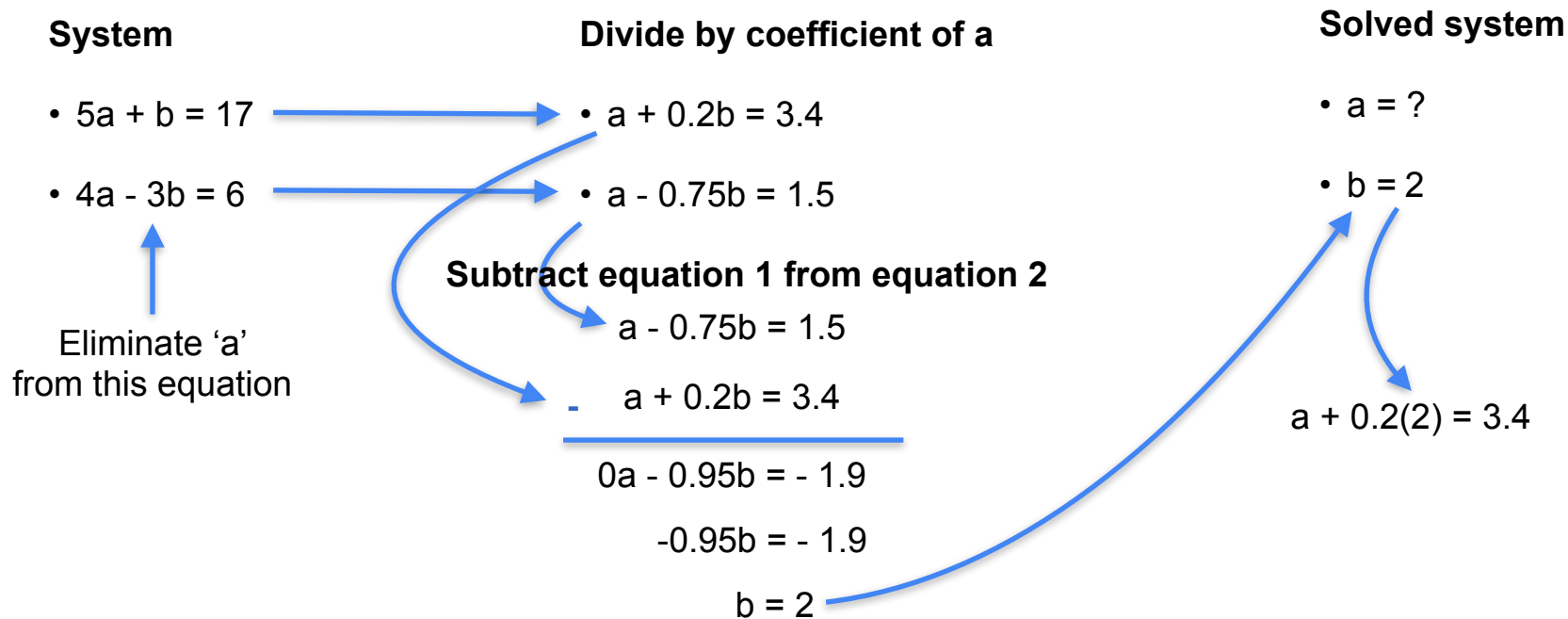
System	Divide by coefficient of a	Solved system
<ul style="list-style-type: none"><li>• <math>5a + b = 17</math></li></ul>	<ul style="list-style-type: none"><li>• <math>a + 0.2b = 3.4</math></li></ul>	<ul style="list-style-type: none"><li>• <math>a = ?</math></li></ul>
<ul style="list-style-type: none"><li>• <math>4a - 3b = 6</math></li></ul>	<ul style="list-style-type: none"><li>• <math>a - 0.75b = 1.5</math></li></ul>	<ul style="list-style-type: none"><li>• <math>b = ?</math></li></ul>
<p>↑ Eliminate 'a' from this equation</p>	<p><b>Subtract equation 1 from equation 2</b></p> $\begin{array}{r} a - 0.75b = 1.5 \\ - (a + 0.2b = 3.4) \\ \hline 0a - 0.95b = -1.9 \\ -0.95b = -1.9 \\ b = 2 \end{array}$	



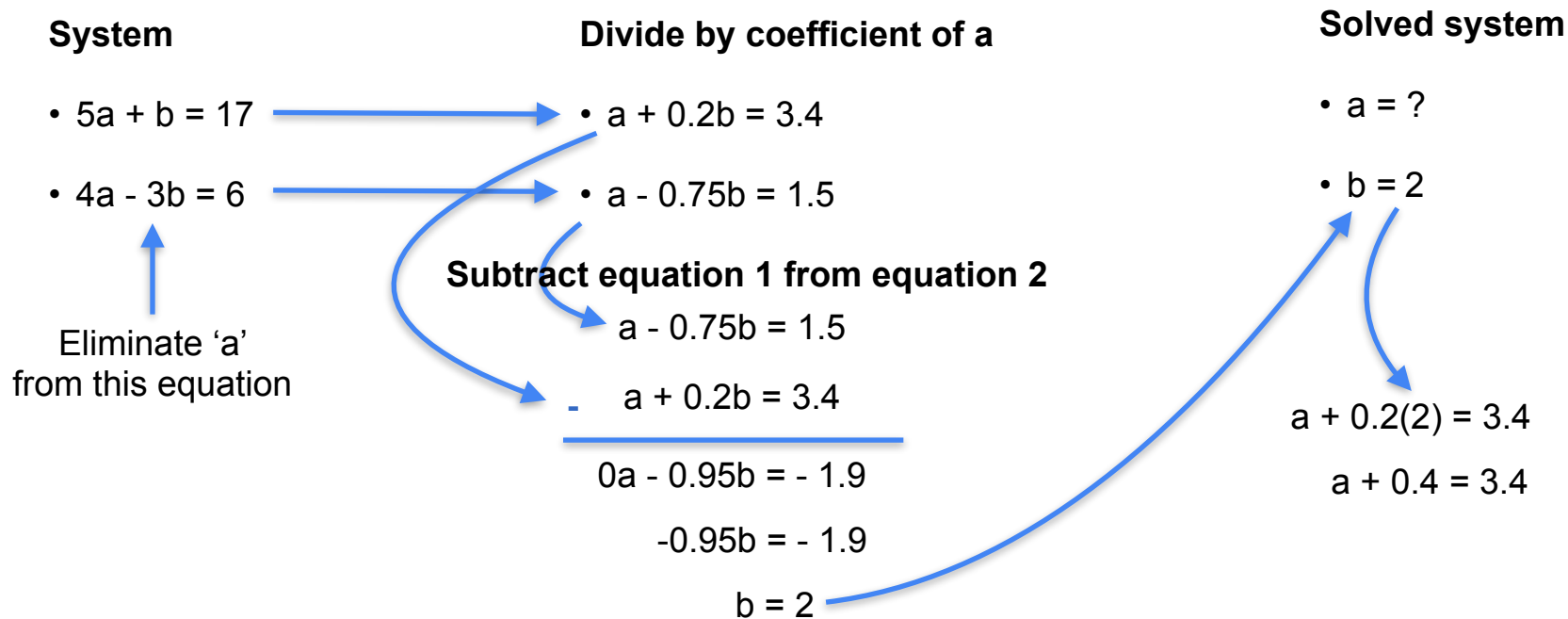
# Systems of equations



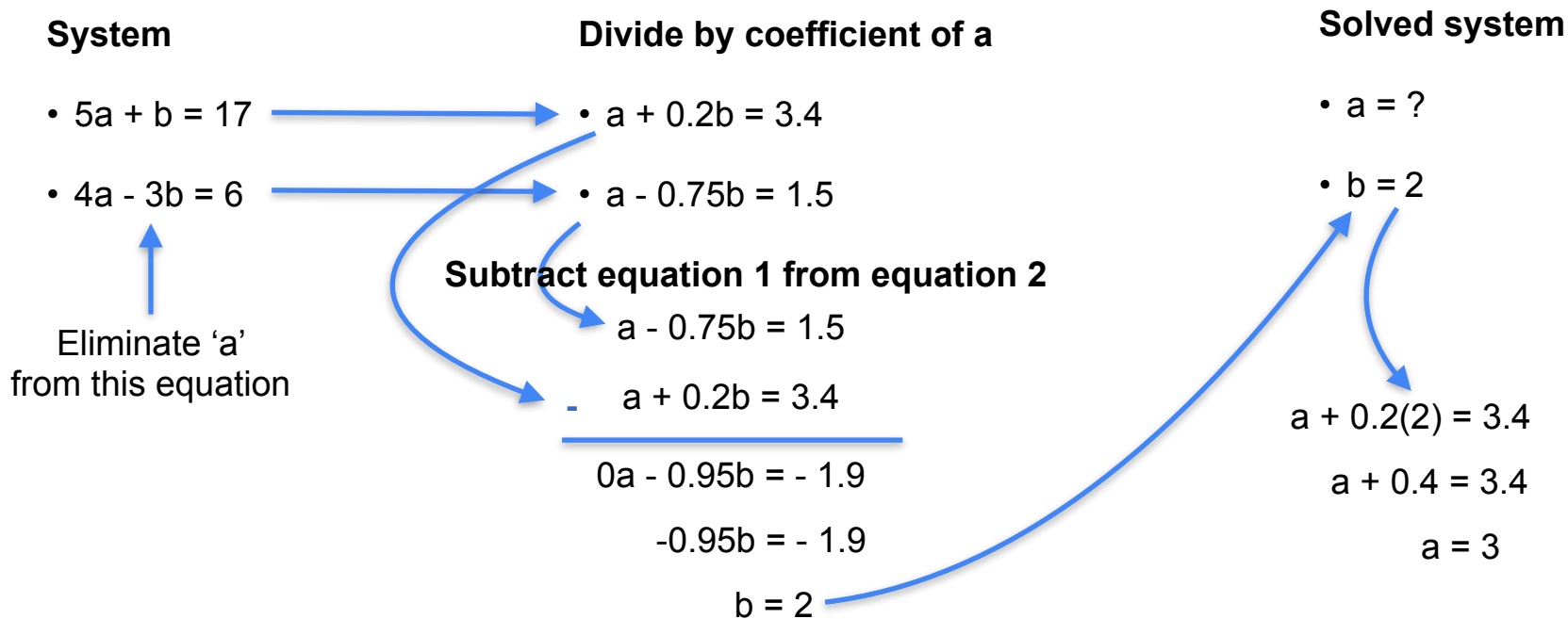
# Systems of equations



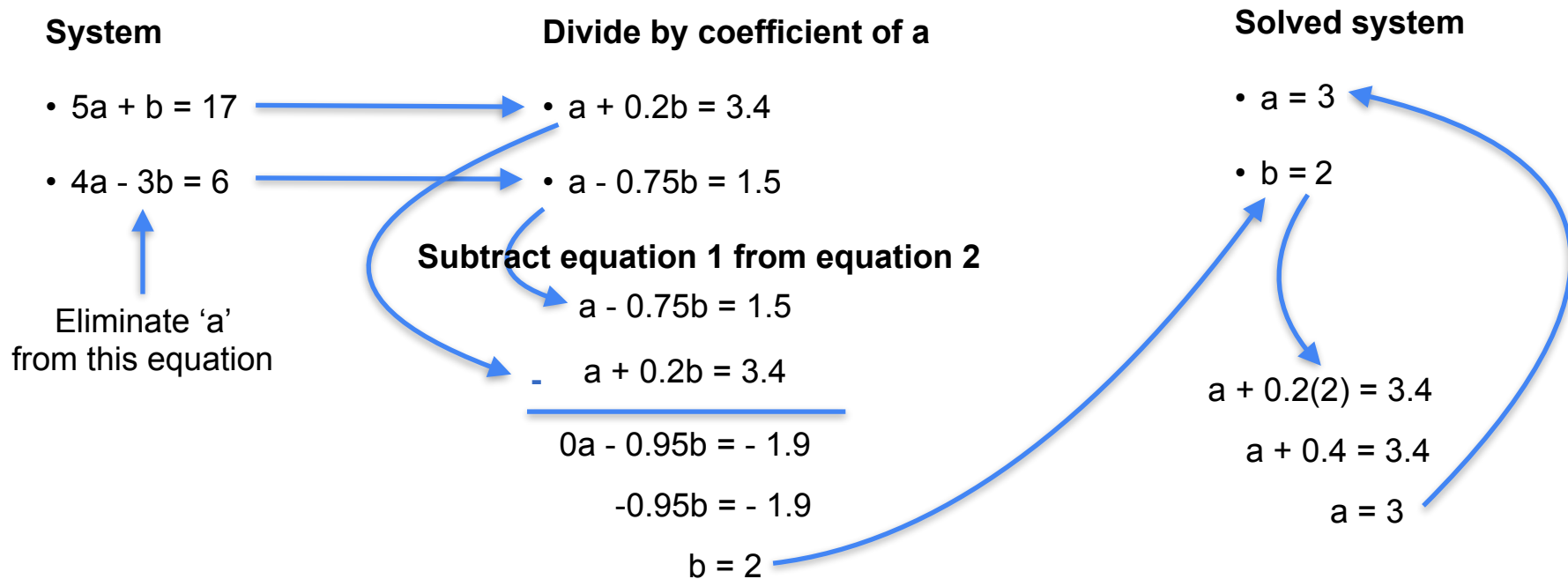
# Systems of equations



# Systems of equations



# Systems of equations



# What if one of the coefficients of a is zero?

## System

- $5a + b = 17$
- $3b = 6$

## Solved system

- $a = ?$
- $b = ?$

# What if one of the coefficients of a is zero?

## System

- $5a + b = 17$

- $3b = 6$





Eliminate 'a'  
from this equation


## Solved system

- $a = ?$

- $b = ?$

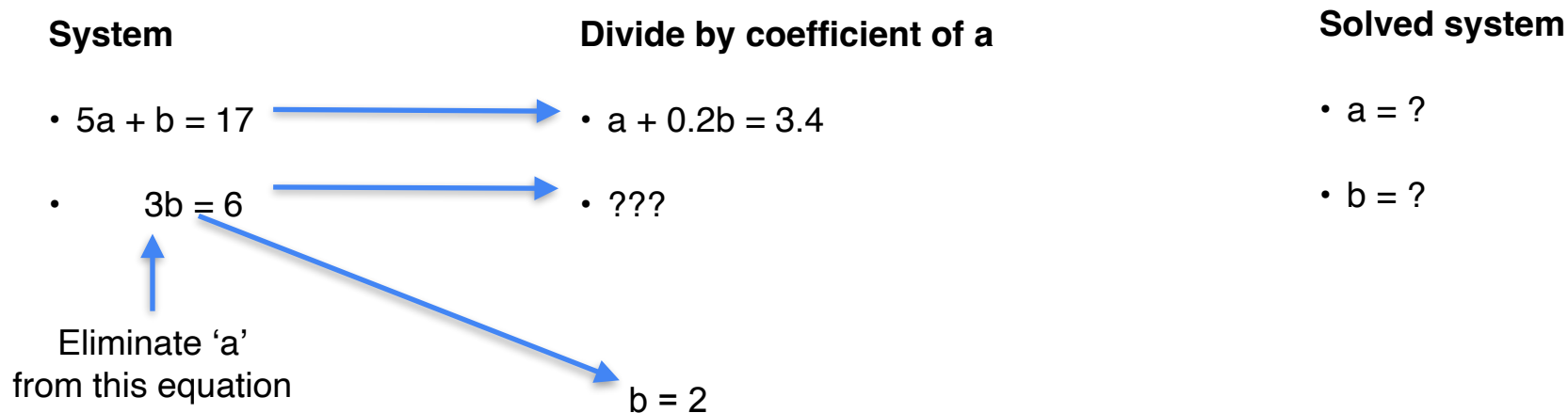
# What if one of the coefficients of a is zero?

System		Divide by coefficient of a	Solved system
• $5a + b = 17$		• $a + 0.2b = 3.4$	• $a = ?$
• $3b = 6$		• ???	• $b = ?$

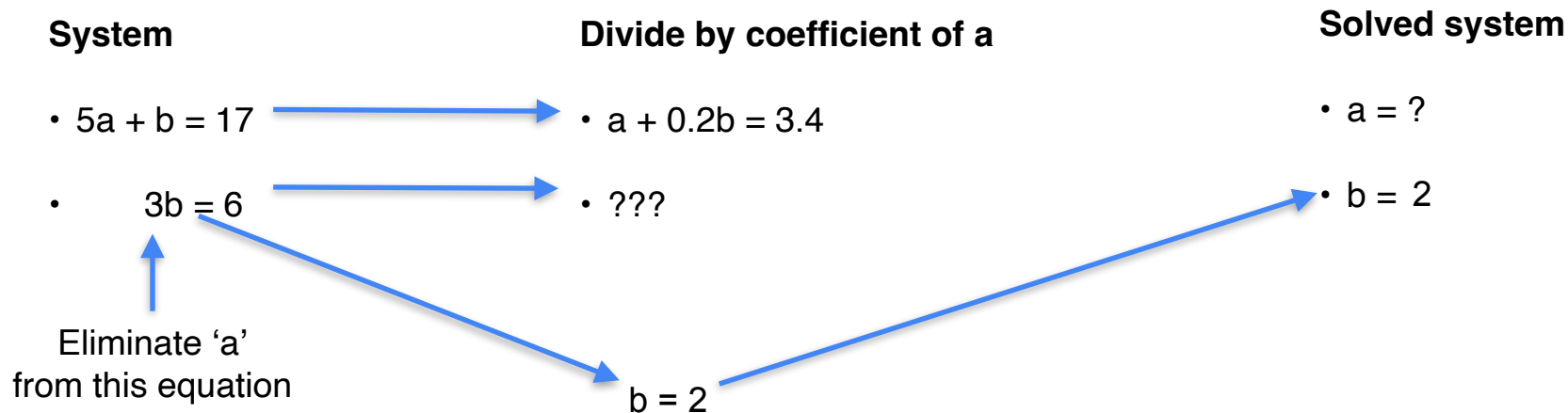
 Eliminate 'a' from this equation



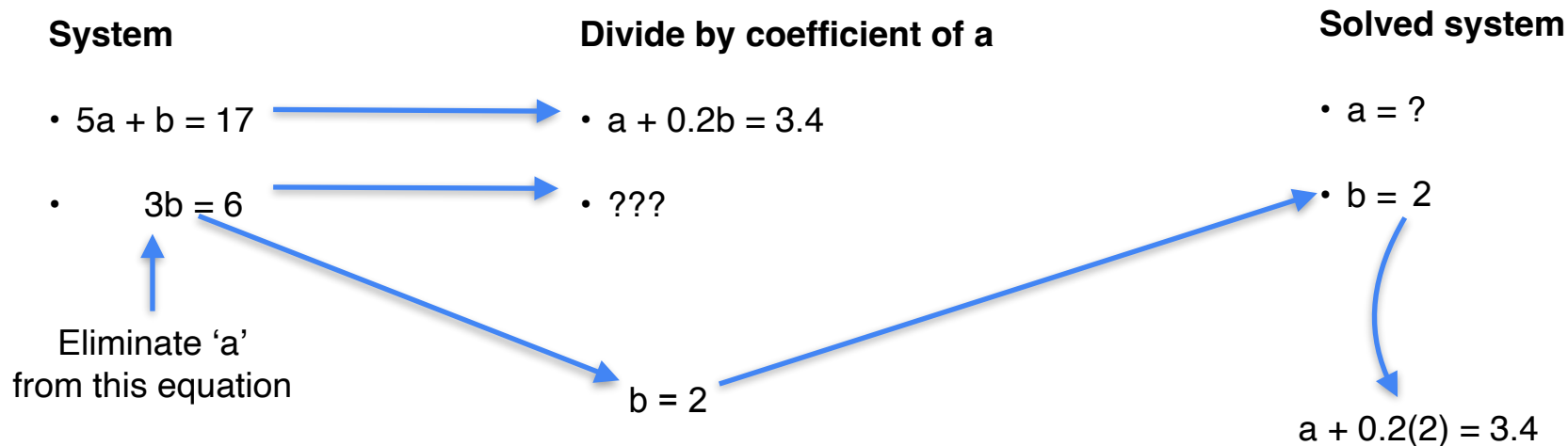
# What if one of the coefficients of a is zero?



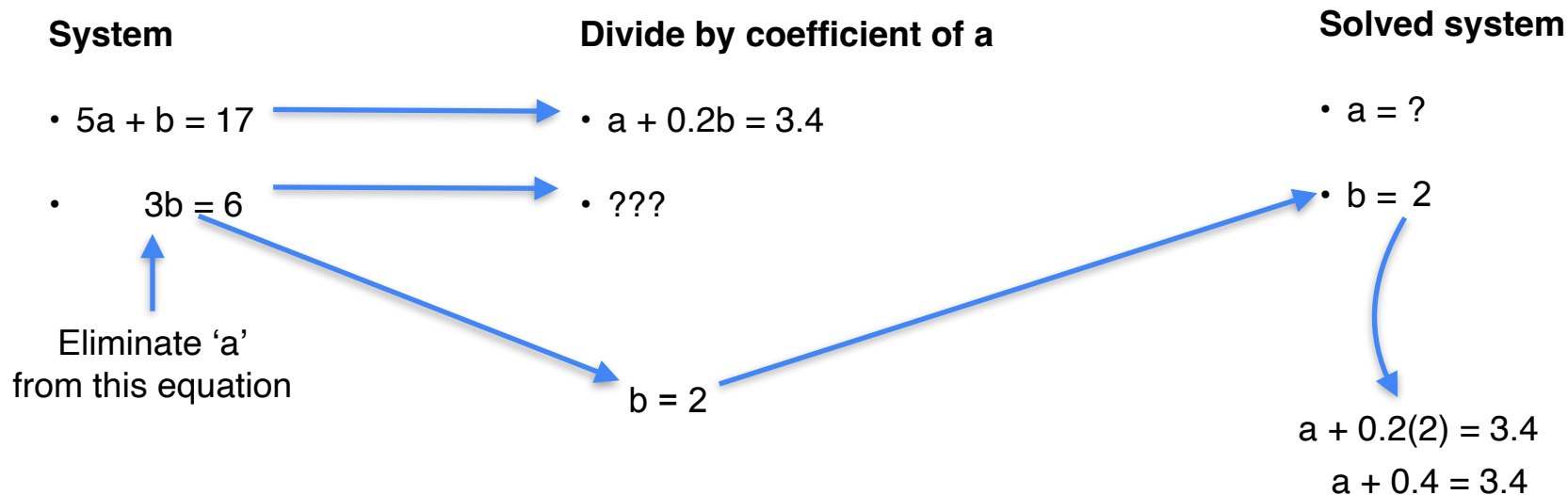
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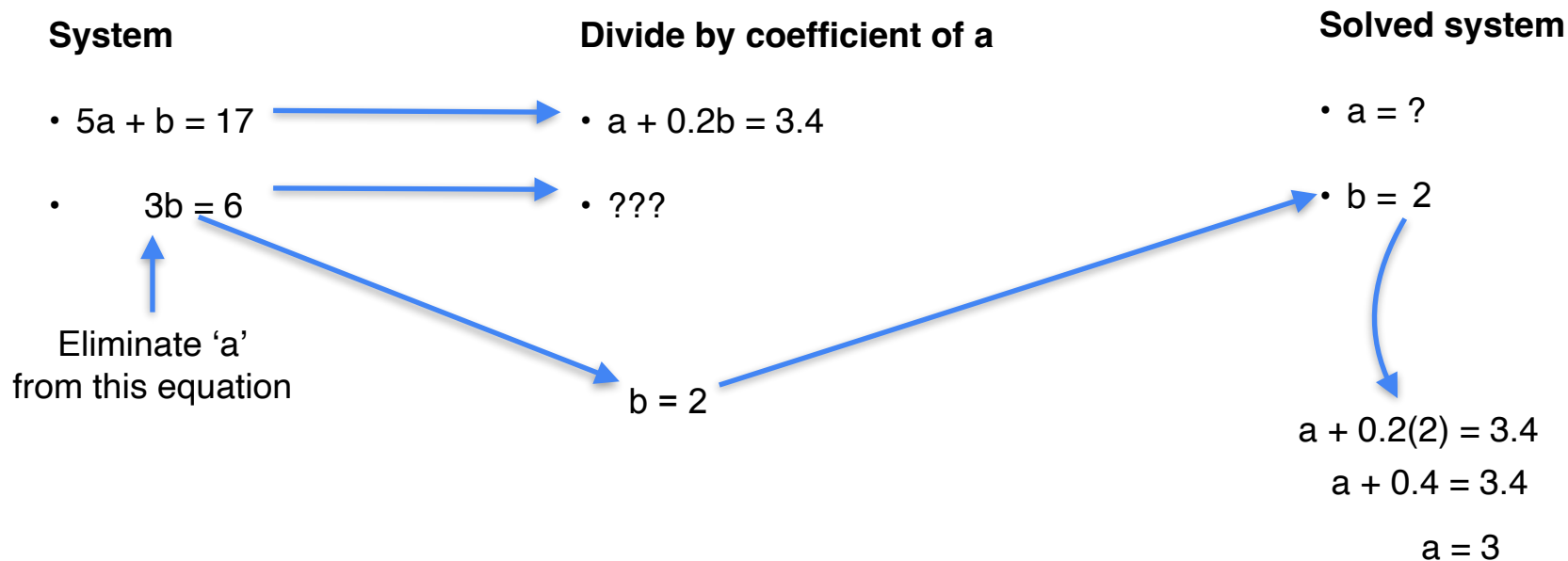
# What if one of the coefficients of a is zero?



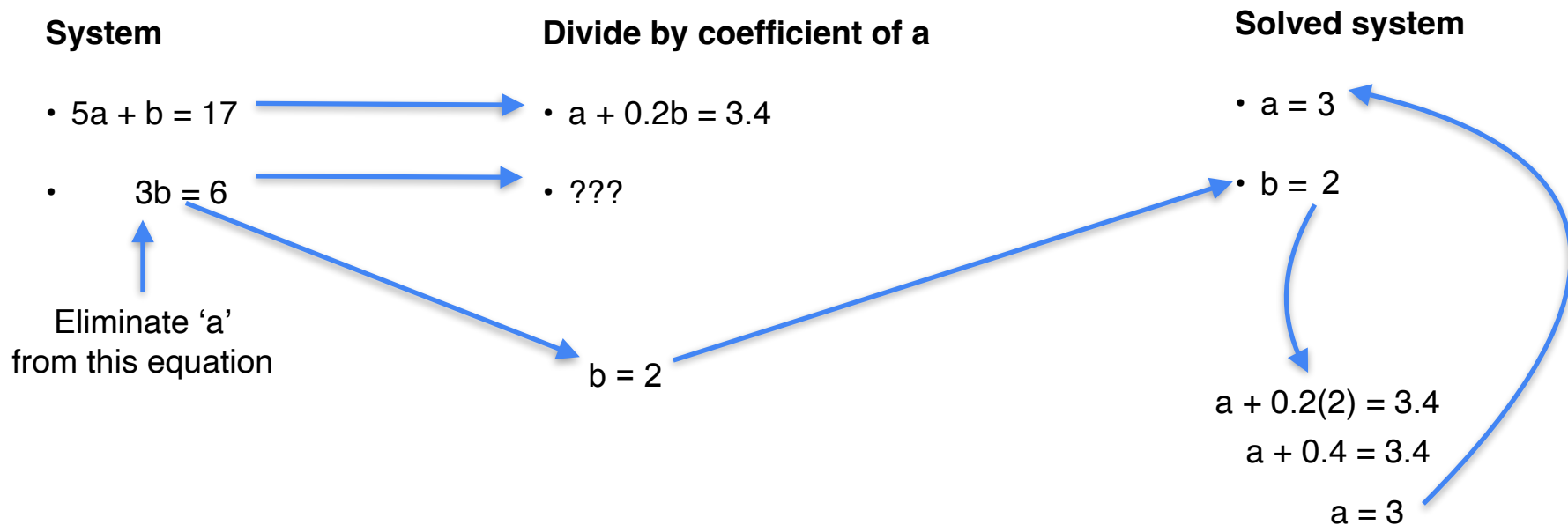
# What if one of the coefficients of a is zero?



# What if one of the coefficients of a is zero?



# What if one of the coefficients of a is zero?



# Quiz

- Solve the following system of equations

## **System**

- $2a + 5b = 46$
- $8a + b = 32$

# Solution

- Solve the following system of equations

## System

- $2a + 5b = 46$
- $8a + b = 32$

## Solution

- $a = 3$
- $b = 8$





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# Solving System of Linear Equations

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**Solving singular system of  
linear equations**

# What if the system is singular (redundant)?



## System


- $a + b = 10$
- $2a + 2b = 20$

## Solved system

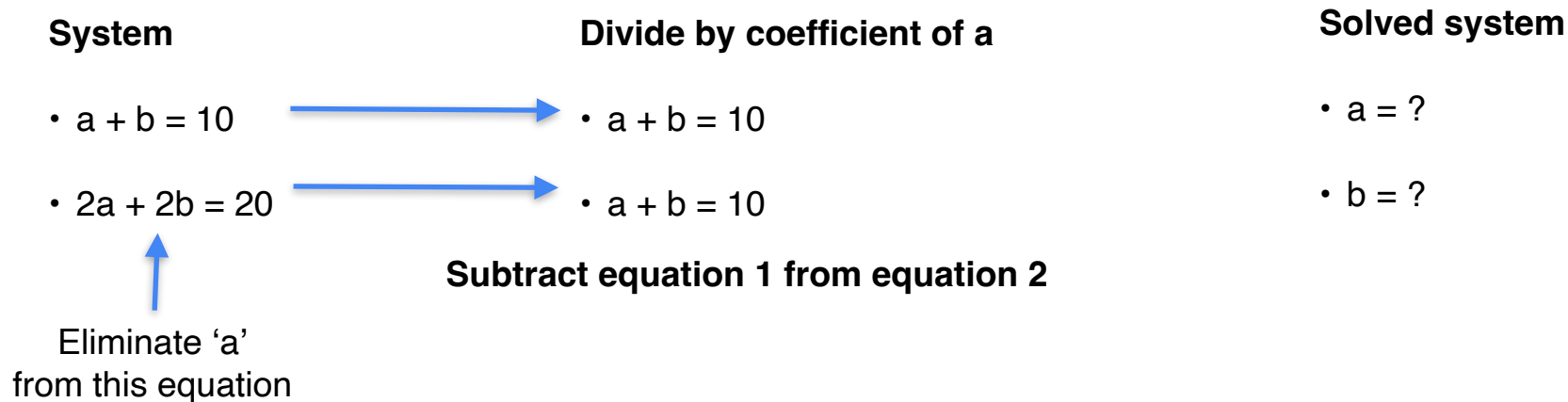
- $a = ?$
- $b = ?$

# What if the system is singular (redundant)?

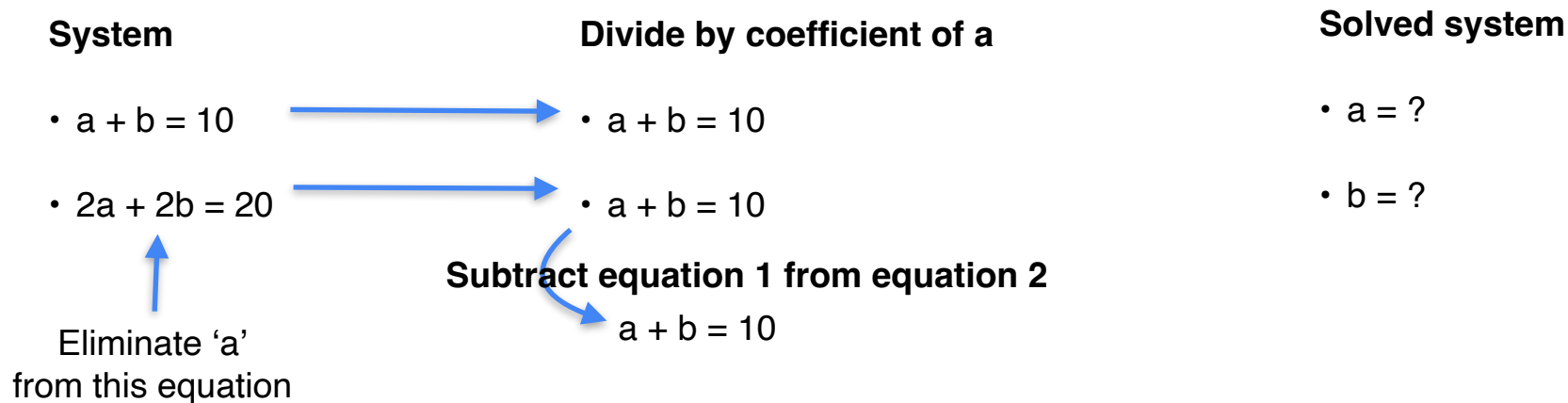
System		Divide by coefficient of a	Solved system
• $a + b = 10$		• $a + b = 10$	• $a = ?$
• $2a + 2b = 20$		• $a + b = 10$	• $b = ?$

 Eliminate 'a' from this equation

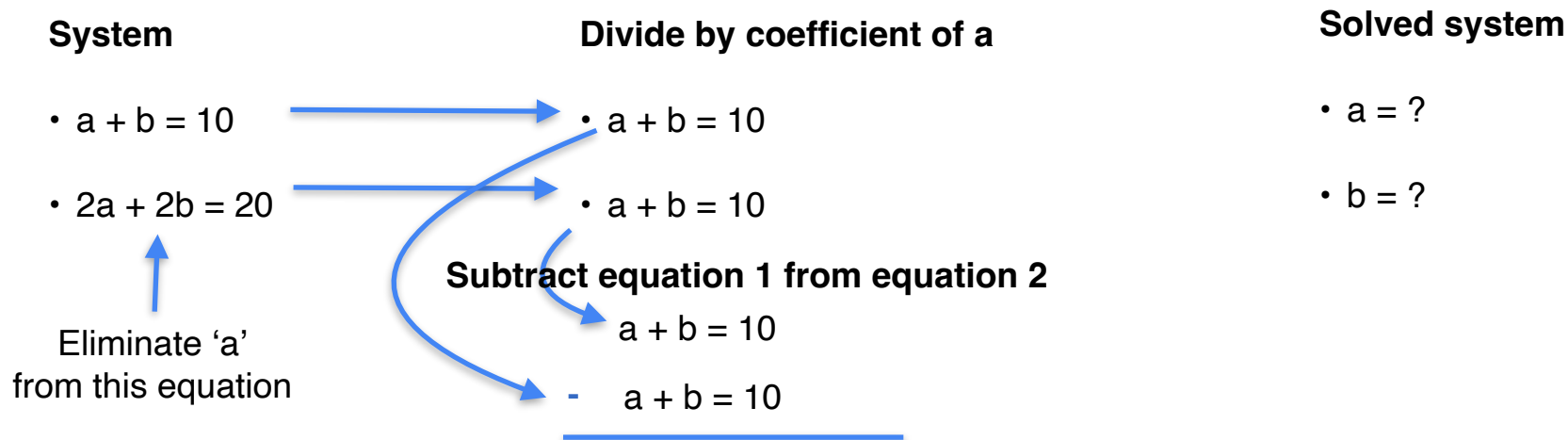
# What if the system is singular (redundant)?



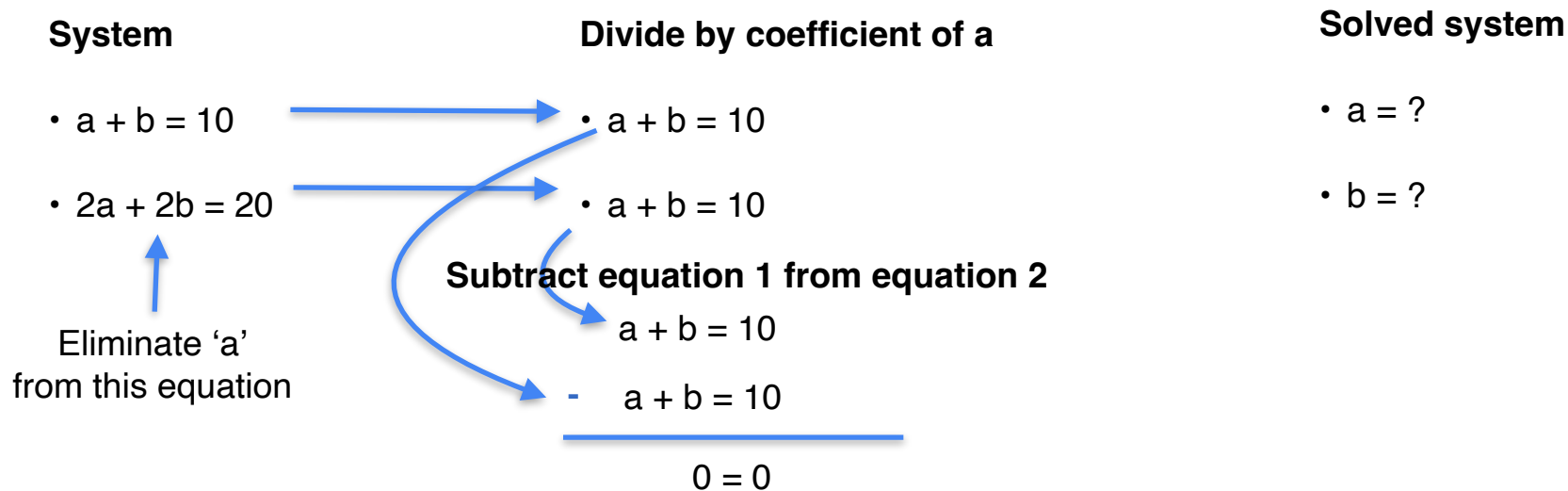
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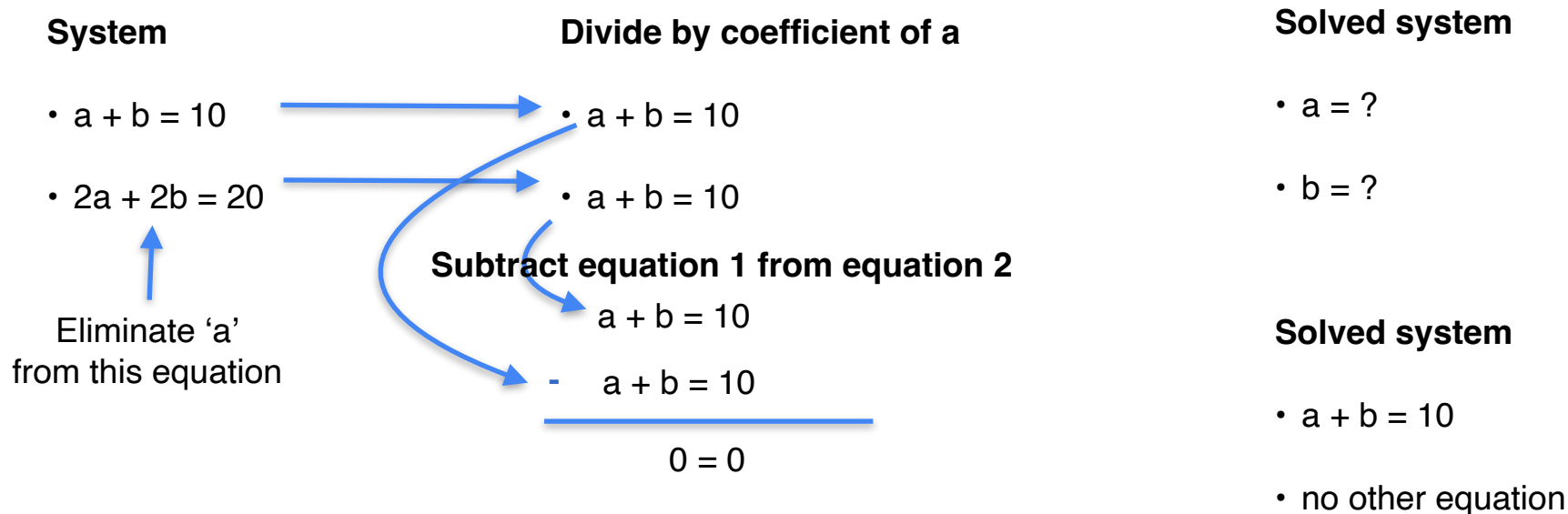
# What if the system is singular (redundant)?



# What if the system is singular (redundant)?

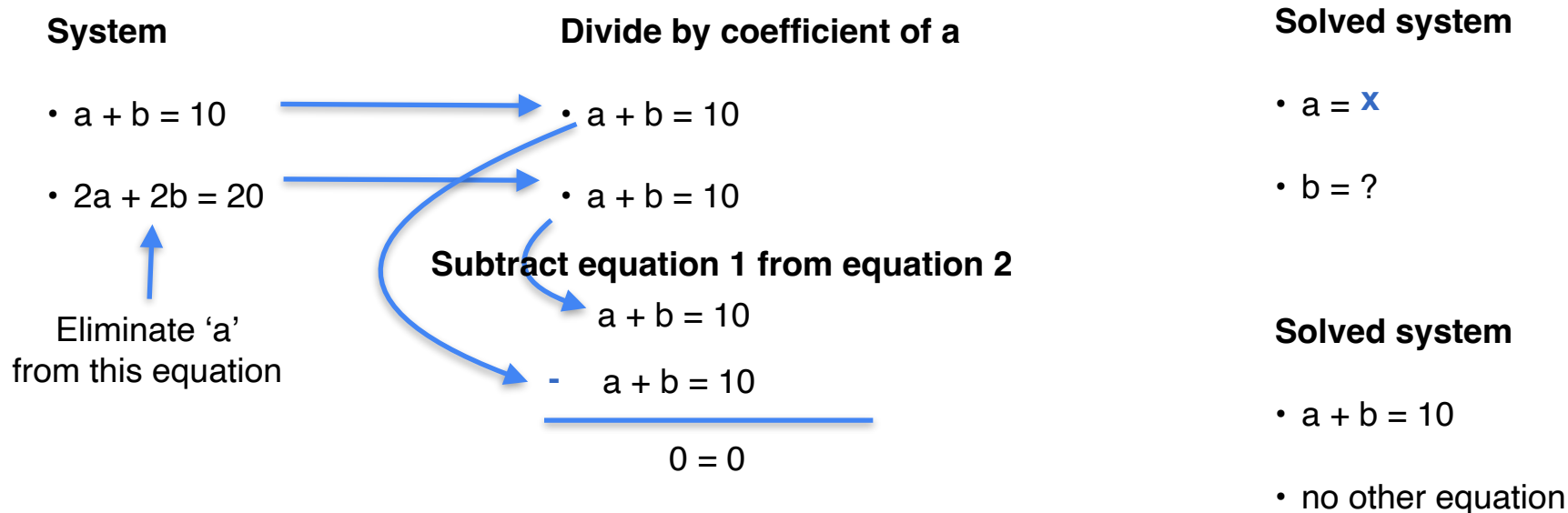


# What if the system is singular (redundant)?

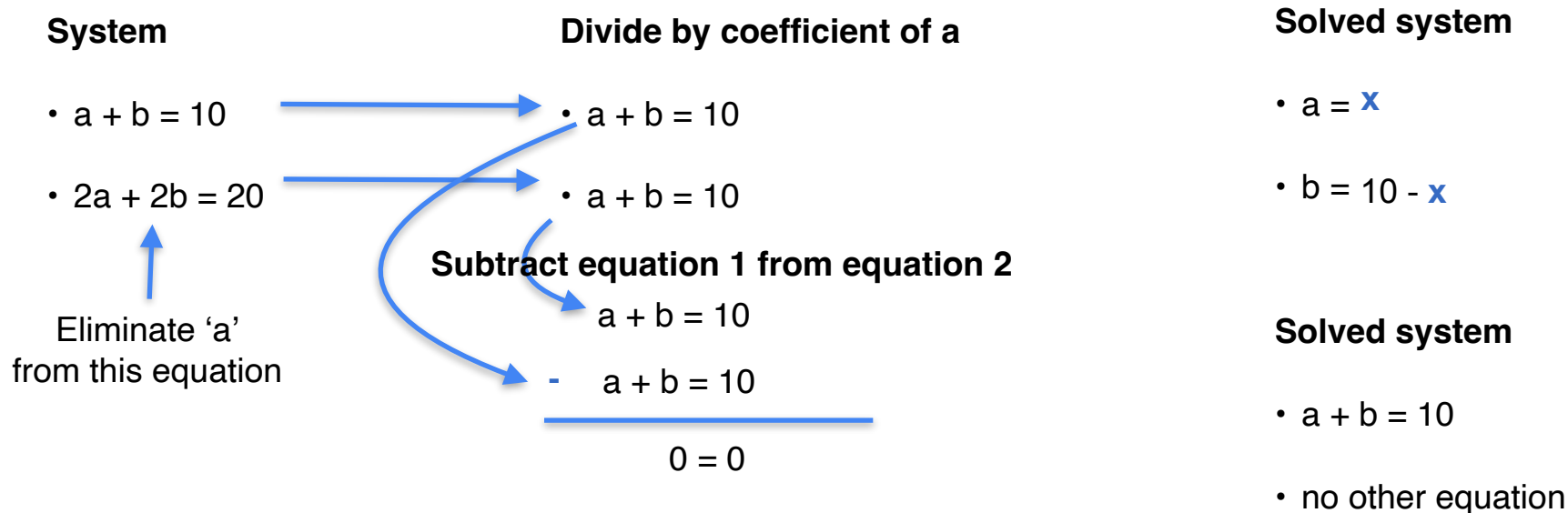




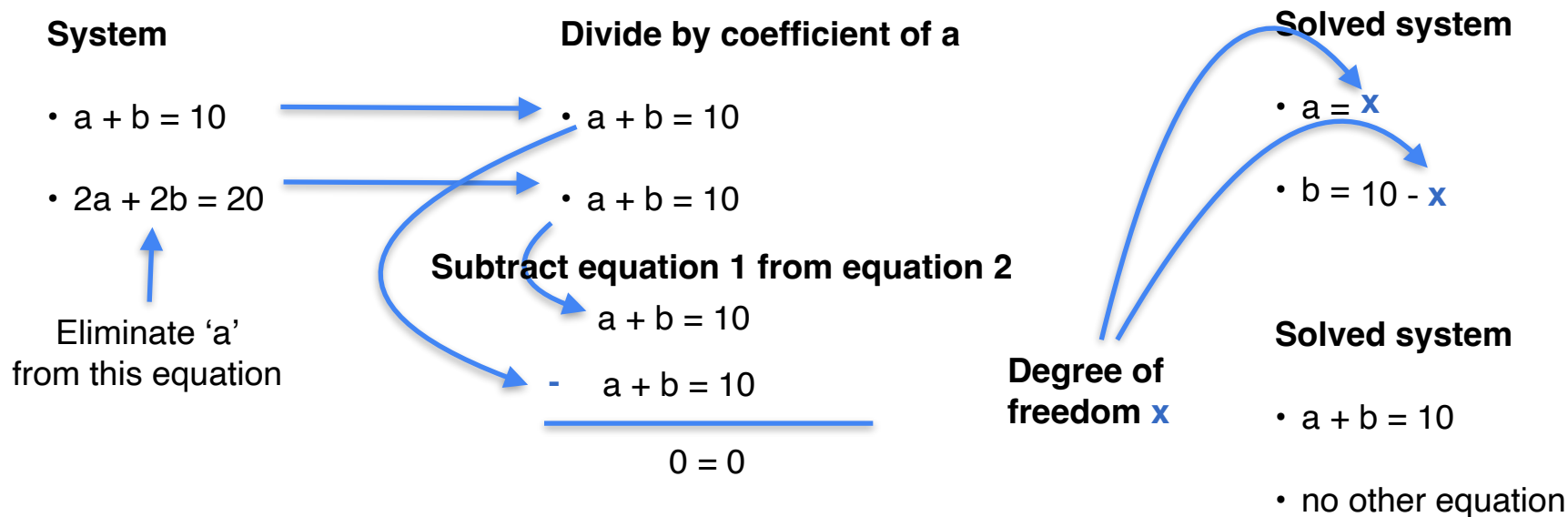
# What if the system is singular (redundant)?



# What if the system is singular (redundant)?



# What if the system is singular (redundant)?



# What if the system is singular (contradictory)?

## System

- $a + b = 10$
- $2a + 2b = 24$

## Solved system

- $a = ?$
- $b = ?$

# What if the system is singular (contradictory)?

## System

- $a + b = 10$
- $2a + 2b = 24$






Eliminate 'a'  
from this equation

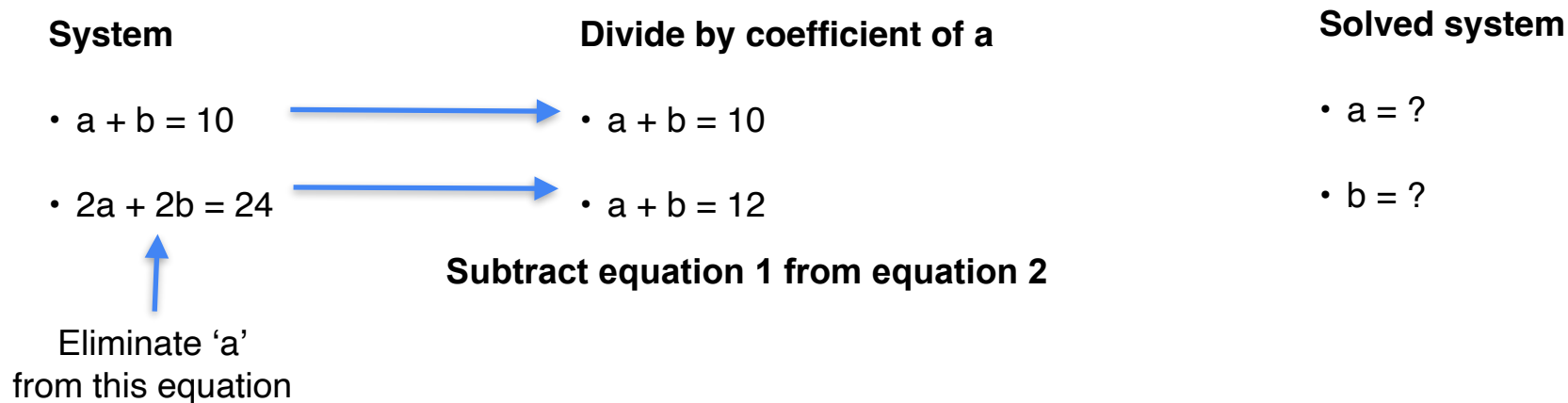
## Solved system

- $a = ?$
- $b = ?$

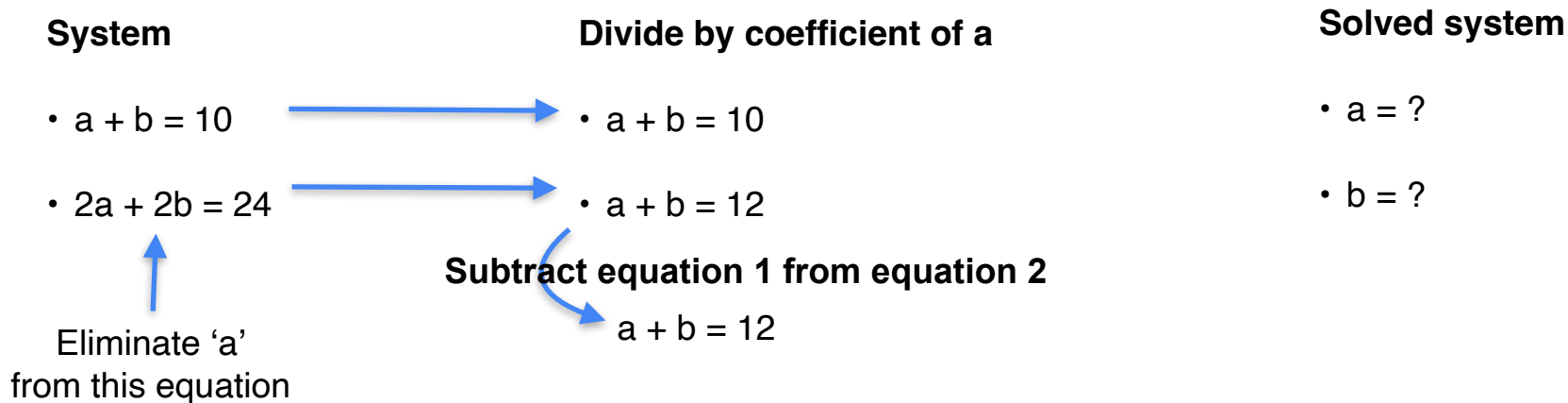
# What if the system is singular (contradictory)?

System		Divide by coefficient of a		Solved system
• $a + b = 10$		• $a + b = 10$		• $a = ?$
• $2a + 2b = 24$		• $a + b = 12$		• $b = ?$
 Eliminate 'a' from this equation				

# What if the system is singular (contradictory)?

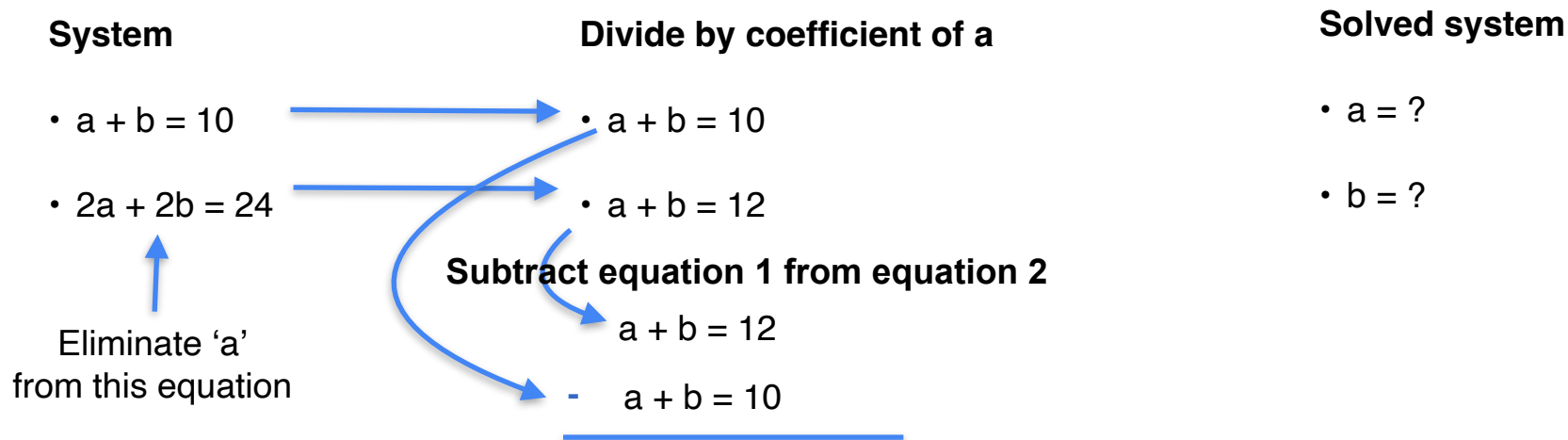


# What if the system is singular (contradictory)?

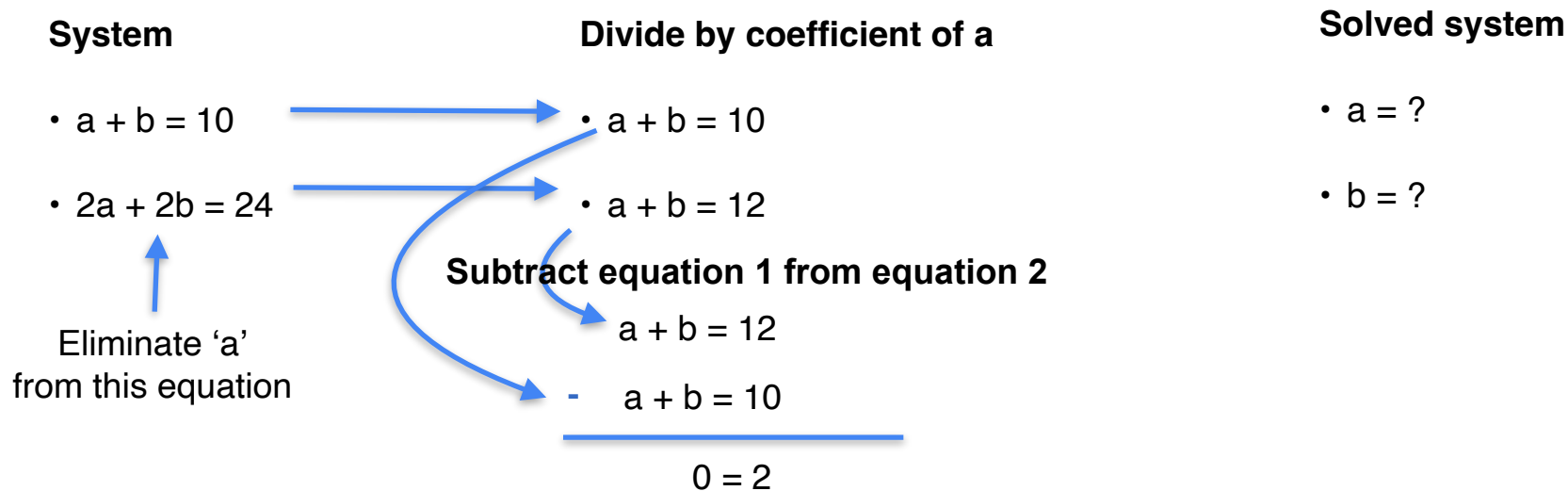




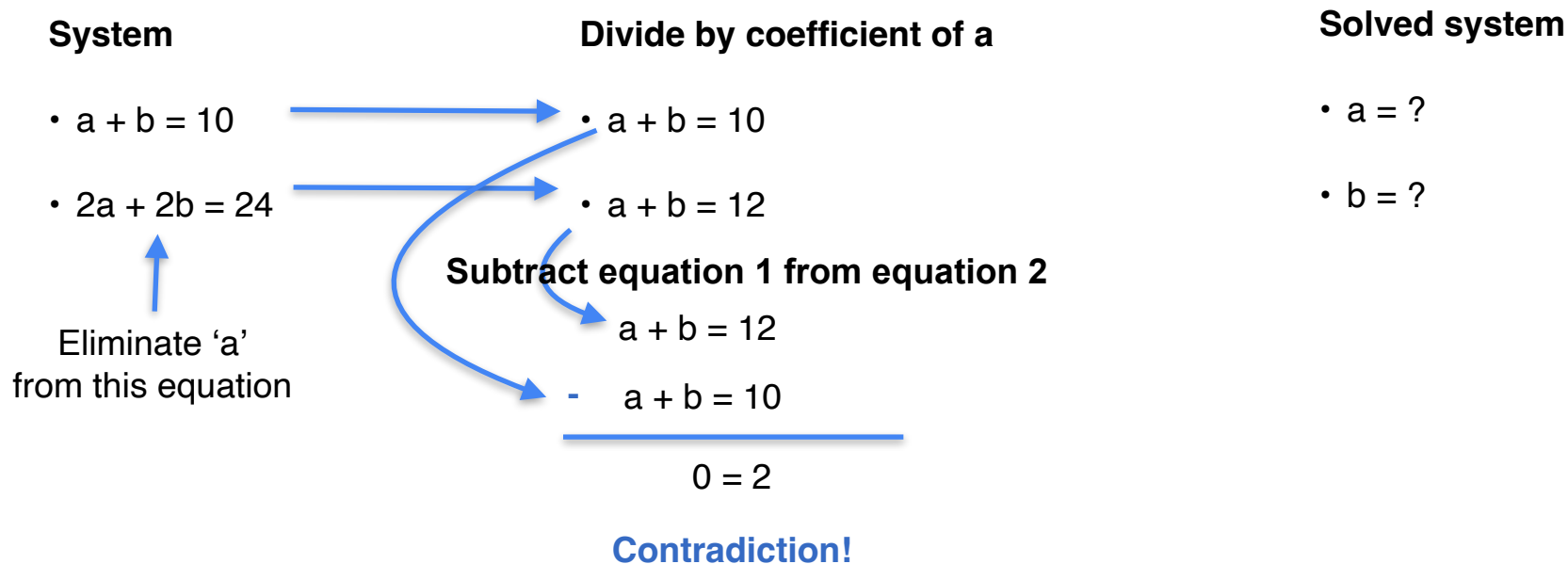
# What if the system is singular (contradictory)?



# What if the system is singular (contradictory)?



# What if the system is singular (contradictory)?



# Quiz

- Solve the following system of equations

## **System**

- $5a + b = 11$
- $10a + 2b = 22$

# Solution

- Solve the following system of equations

## System

- $5a + b = 11$
- $10a + 2b = 22$

**Solution:** If you look closely into the two equations in the system, you'll find that if equation 2 is divided by 2 you'll obtain equation 1.

Therefore, the system has infinitely many solutions.



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# Solving System of Linear Equations

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**Solving system of equations  
with more variables**

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

# Elimination method


## System

- $a + b + 2c = 12$

- $3a - 3b - c = 3$

- $2a - b + 6c = 24$

Leave 'a' by  
itself





# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each  
row by the  
coefficient of 'a'

# Elimination method

## System

- $a + b + 2c = 12$

- $3a - 3b - c = 3$

- $2a - b + 6c = 24$

- $a + b + 2c = 12$

- $a - b - 1/3 c = 1$

- $a - b/2 + 3c = 12$

Divide each  
row by the  
coefficient of 'a'

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each  
row by the  
coefficient of 'a'

$$• a + b + 2c = 12$$

- $a - b - 1/3 c = 1$
- $a - b/2 + 3c = 12$

Use the first  
equation to  
remove 'a' from  
the others

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each  
row by the  
coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - 1/3 c = 1$
- $a - b/2 + 3c = 12$

Use the first  
equation to  
remove 'a' from  
the others

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each  
row by the  
coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - 1/3 c = 1$
- $a - b/2 + 3c = 12$

Use the first  
equation to  
remove 'a' from  
the others

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Isolated 'a'

# Elimination method

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

Divide each row by the coefficient of 'a'

- $a + b + 2c = 12$
- $a - b - \frac{1}{3}c = 1$
- $a - \frac{b}{2} + 3c = 12$

Use the first equation to remove 'a' from the others

- Isolated 'a'
- $a + b + 2c = 12$
  - $-2b - \frac{7}{3}c = -11$
  - $-\frac{3}{2}b + c = 0$
- Solve this new system of 2 equations

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - \frac{7}{3}c = -11$
- $-\frac{3}{2}b + c = 0$



# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two  
rows by the  
coefficient of b

# Elimination method

## System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Divide last two  
rows by the  
coefficient of b

# Elimination method

## System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

Divide last two  
rows by the  
coefficient of b

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Use the second  
equation to  
remove 'b' from  
the third

# Elimination method

## System

- $a + b + 2c = 12$

- $-2b - 7/3 c = -11$

- $-3/2 b + c = 0$

Divide last two  
rows by the  
coefficient of b

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $b - 2/3 c = 0$

Use the second  
equation to  
remove 'b' from  
the third

- $a + b + 2c = 12$

- $b + 7/6 c = 11/2$

- $-11/6 c = -11/2$

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $-11/6 c = -11/2$

Isolated 'b'

# Elimination method

## System

- $a + b + 2c = 12$
- $-2b - 7/3 c = -11$
- $-3/2 b + c = 0$

Divide last two rows by the coefficient of b

- $a + b + 2c = 12$
- $b + 7/6 c = 11/2$
- $b - 2/3 c = 0$

Use the second equation to remove 'b' from the third

- $a + b + 2c = 12$
  - $b + 7/6 c = 11/2$
  - $-11/6 c = -11/2$
- Isolated 'b'
- $c = 3$

# Elimination method

## System

- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$
- $c = 3$

# Elimination method

## System


- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$



# Elimination method

## System

- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$    $b + \frac{7}{2} = \frac{11}{2}$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

# Elimination method

## System



- $a + b + 2c = 12$
- $b + \frac{7}{6}c = \frac{11}{2}$    $b + \frac{7}{2} = \frac{11}{2}$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

# Elimination method

## System



- $a + b + 2c = 12$    $a + 2 + 6 = 12$   
 $a = 4$
- $b + 7/6 c = 11/2$    $b + 7/2 = 11/2$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

# Elimination method

## System

- $a + b + 2c = 12$    $a + 2 + 6 = 12$   
 $a = 4$
- $b + 7/6 c = 11/2$    $b + 7/2 = 11/2$   
 $b = 2$
- $c = 3$

Replace  $c = 3$   
in the second  
equation, get  
 $b = 2$

Replace  $c = 3$   
and  $b = 2$  in the  
first equation,  
get  $a = 4$

The solution is  
 $a = 4$   
 $b = 2$   
 $c = 3$



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# Solving System of Linear Equations

---

## **Matrix row reduction**

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$



## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$



# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$

## Original matrix

5	1
4	-3

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

**Diagonal matrix**

1	0
0	1

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

**Diagonal matrix**

1	0
0	1

Row echelon form

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $1a + 0b = 3$
- $0a + 1b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

**Diagonal matrix**

1	0
0	1

Row echelon form

Reduced row echelon form

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $1a + 0b = 3$
- $0a + 1b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

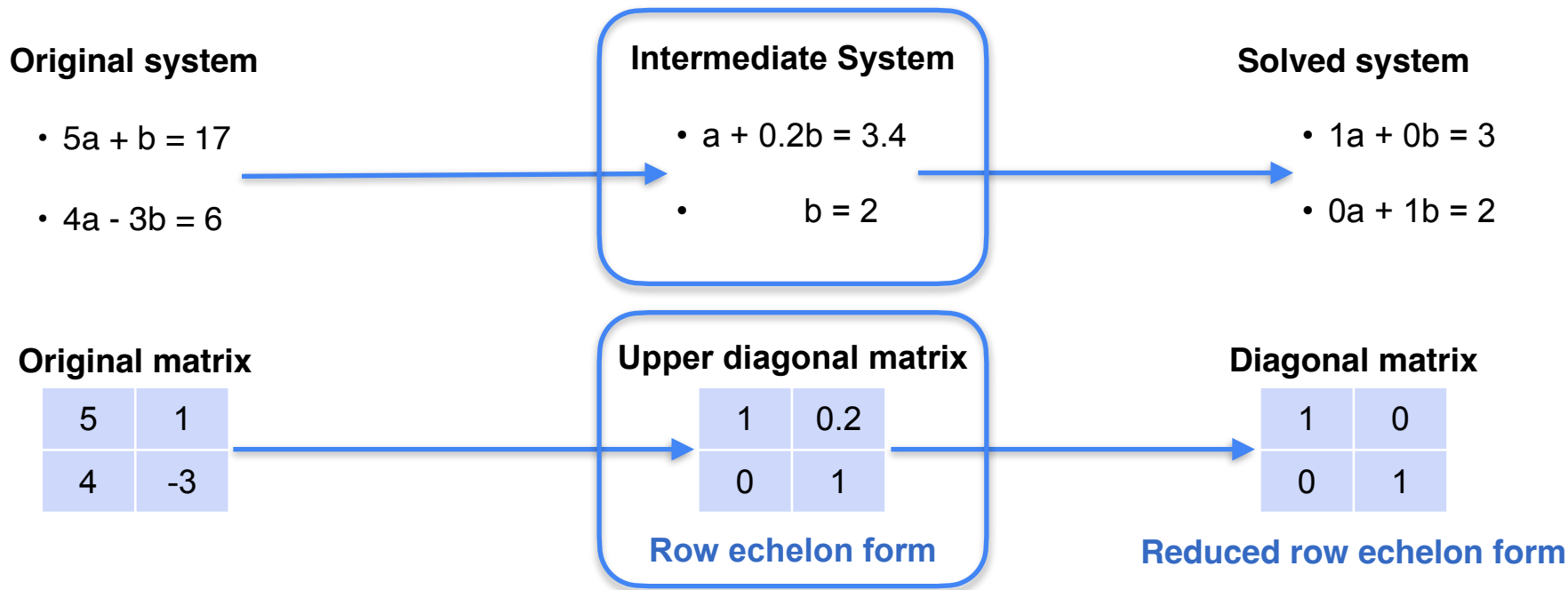
**Diagonal matrix**

1	0
0	1

Row echelon form

Reduced row echelon form

# Systems of equations to matrices



# Systems of equations to matrices

## Original system

- $a + b = 10$
- $2a + 2b = 20$



# Systems of equations to matrices

**Original system**

- $a + b = 10$
- $2a + 2b = 20$

**Intermediate System**

- $a + b = 10$
- $0a + 0b = 0$



# Systems of equations to matrices

**Original system**

- $a + b = 10$
- $2a + 2b = 20$

**Intermediate System**

- $a + b = 10$
- $0a + 0b = 0$



**Original matrix**

1	1
2	2

# Systems of equations to matrices

**Original system**

- $a + b = 10$
- $2a + 2b = 20$

**Intermediate System**

- $a + b = 10$
- $0a + 0b = 0$

**Original matrix**

1	1
2	2

**Upper diagonal matrix**

1	1
0	0

# Systems of equations to matrices

**Original system**

- $a + b = 10$
- $2a + 2b = 20$

**Intermediate System**

- $a + b = 10$
- $0a + 0b = 0$

**Original matrix**

1	1
2	2

**Upper diagonal matrix**

1	1
0	0

**Row echelon form**

# Systems of equations to matrices

**Original system**

- $a + b = 10$
- $2a + 2b = 20$

**Intermediate System**

- $a + b = 10$
- $0a + 0b = 0$

**Original matrix**

1	1
2	2

**Upper diagonal matrix**

1	1
0	0

**Row echelon form**

# Systems of equations to matrices

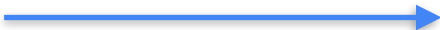
## Original system

- $5a + b = 11$
- $10a + 2b = 22$

# Systems of equations to matrices

## Original system

- $5a + b = 11$
- $10a + 2b = 22$



## Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

# Systems of equations to matrices

## Original system

- $5a + b = 11$
- $10a + 2b = 22$

## Intermediate System

- $a + 0.2b = 2.2$
- $0a + 0b = 0$



## Original matrix

5	1
10	2



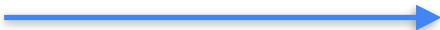
# Systems of equations to matrices

**Original system**

- $5a + b = 11$
- $10a + 2b = 22$

**Intermediate System**

- $a + 0.2b = 2.2$
- $0a + 0b = 0$



**Original matrix**

5	1
10	2

**Upper diagonal matrix**

1	0.2
0	0



# Systems of equations to matrices

**Original system**

- $5a + b = 11$
- $10a + 2b = 22$

**Intermediate System**

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

**Original matrix**

5	1
10	2

**Upper diagonal matrix**

1	0.2
0	0

**Row echelon form**

# Systems of equations to matrices

**Original system**

- $5a + b = 11$
- $10a + 2b = 22$

**Intermediate System**

- $a + 0.2b = 2.2$
- $0a + 0b = 0$

**Original matrix**

5	1
10	2

**Upper diagonal matrix**

1	0.2
0	0

**Row echelon form**

# Systems of equations to matrices

## Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

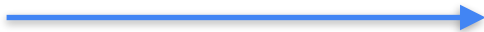
# Systems of equations to matrices

**Original system**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Intermediate System**

- $0a + 0b = 0$
- $0a + 0b = 0$



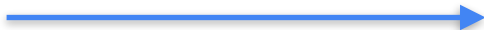
# Systems of equations to matrices

## Original system

- $0a + 0b = 0$
- $0a + 0b = 0$

## Intermediate System

- $0a + 0b = 0$
- $0a + 0b = 0$



## Original matrix

0	0
0	0

# Systems of equations to matrices

**Original system**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Intermediate System**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Original matrix**

0	0
0	0

**Upper diagonal matrix**

0	0
0	0

# Systems of equations to matrices

**Original system**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Intermediate System**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Original matrix**

0	0
0	0

**Upper diagonal matrix**

0	0
0	0

**Row echelon form**



# Systems of equations to matrices

**Original system**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Intermediate System**

- $0a + 0b = 0$
- $0a + 0b = 0$

**Original matrix**

0	0
0	0

**Upper diagonal matrix**

0	0
0	0

**Row echelon form**



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# Solving System of Linear Equations

---

**Row operations that  
preserve singularity**

# Switching rows

5	1
4	3

# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

# Switching rows

5	1
4	3

4	3
5	1

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

4	3
5	1

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$

# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

4	3
5	1

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$



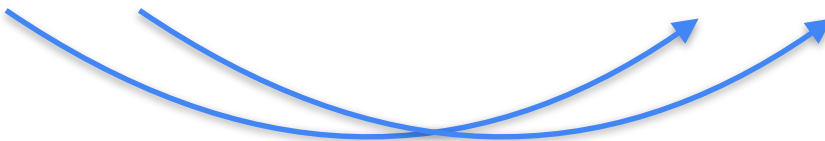
# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

4	3
5	1

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$



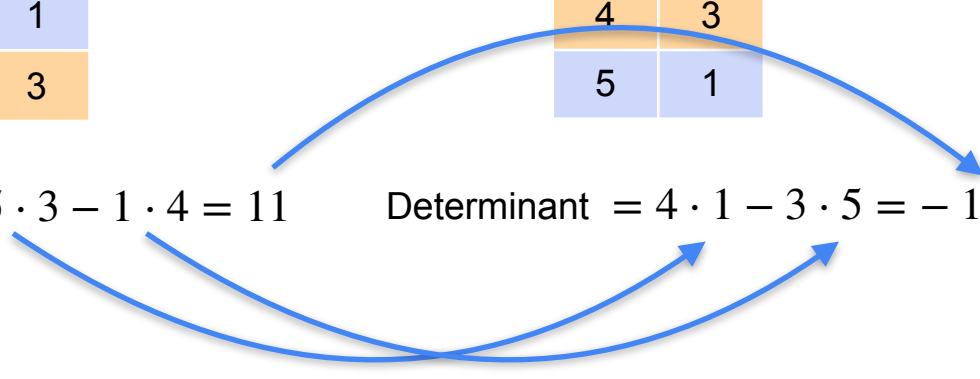


# Switching rows

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4 = 11$$

4	3
5	1

$$\text{Determinant} = 4 \cdot 1 - 3 \cdot 5 = -11$$


# Multiplying a row by a (non-zero) scalar

5	1
4	3

# Multiplying a row by a (non-zero) scalar

5	1
4	3

$$\text{Determinant} = 5 \cdot 3 - 1 \cdot 4$$

# Multiplying a row by a (non-zero) scalar

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

5	1
---	---

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

5	1
---	---

 $\times 10 =$

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

4	3
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$



# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{array}{|c|c|} \hline 5 & 1 \\ \hline \end{array} \times 10 = \begin{array}{|c|c|} \hline 50 & 10 \\ \hline \end{array}$$

50	10
4	3

$$\begin{aligned} \text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11 \end{aligned}$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

5	1
---	---

 $\times 10 =$ 

50	10
----	----

50	10
4	3

$$\text{Determinant} = 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4)$$

# Multiplying a row by a (non-zero) scalar

5	1
4	-3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

5	1
---	---

 $\times 10 =$ 

50	10
----	----

50	10
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot (10 \cdot 3) - 1 \cdot (10 \cdot 4) \\ &= 10 \cdot 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

5	1
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

+	5	1
	4	3
<hr/>		

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

+	5	1
	4	3
<hr/>		
	9	4

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$



# Adding a row to another row

5	1
4	3

+	5	1
	4	3
<hr/>		
	9	4

9	4
---	---

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

+	5	1
	4	3
<hr/>		
	9	4

9	4
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

# Adding a row to another row

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

+	<table><tr><td>5</td><td>1</td></tr><tr><td>4</td><td>3</td></tr></table>	5	1	4	3
5	1				
4	3				
	<table><tr><td>9</td><td>4</td></tr></table>	9	4		
9	4				

9	4
4	3

$$\text{Determinant} = 9 \cdot 3 - 4 \cdot 4$$

# Adding a row to another row

5	1
4	3

$$\begin{aligned}\text{Determinant} &= 5 \cdot 3 - 1 \cdot 4 \\ &= 11\end{aligned}$$

+	<table><tr><td>5</td><td>1</td></tr><tr><td>4</td><td>3</td></tr></table>	5	1	4	3
5	1				
4	3				
	<table><tr><td>9</td><td>4</td></tr></table>	9	4		
9	4				

9	4
4	3

$$\begin{aligned}\text{Determinant} &= 9 \cdot 3 - 4 \cdot 4 \\ &= 11\end{aligned}$$



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# Solving System of Linear Equations

---

## **Rank of a matrix**

# Compressing Images - Reducing rank

# Compressing Images - Reducing rank



# Compressing Images - Reducing rank

Original (Rank 200)



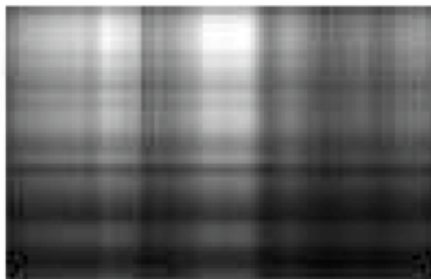


# Compressing Images - Reducing rank

Original (Rank 200)



Rank 1

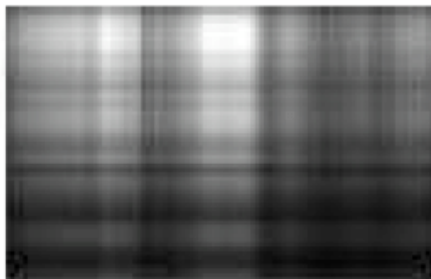


# Compressing Images - Reducing rank

Original (Rank 200)



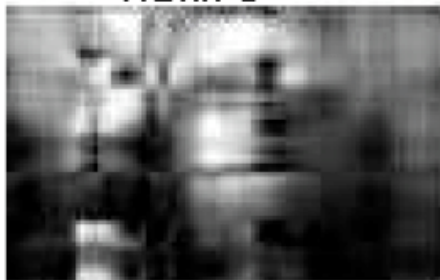
Rank 1



Rank 2



Rank 5

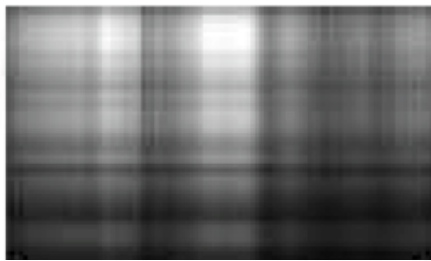


# Compressing Images - Reducing rank

Original (Rank 200)



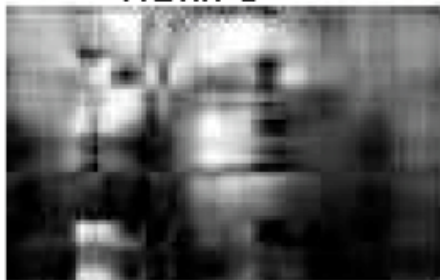
Rank 1



Rank 2



Rank 5



Rank 15

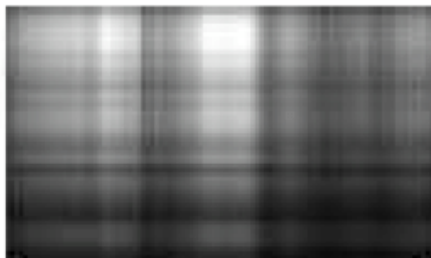


# Compressing Images - Reducing rank

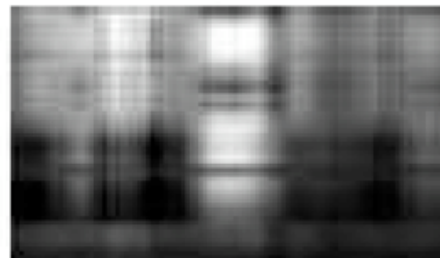
Original (Rank 200)



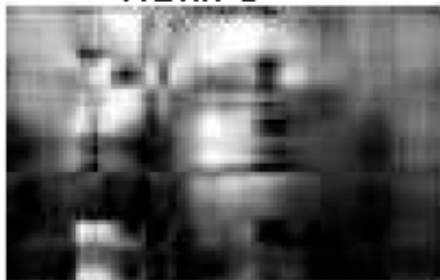
Rank 1



Rank 2



Rank 5



Rank 15





Rank 50



# Systems of information



# Systems of information

## System 1



 The dog is **black**  
 The cat is **orange**

# Systems of information

## System 1



 The dog is **black**  
 The cat is **orange**

## System 2



 The dog is **black**  
 The dog is **black**

# Systems of information

## System 1

 The dog is **black**  
 The cat is **orange**

## System 2

 The dog is **black**  
 The dog is **black**



## System 3

 The dog  
 The dog





# Systems of information

## System 1

 The dog is **black**  
 The cat is **orange**

## System 2

 The dog is **black**  
 The dog is **black**



## System 3

 The dog  
 The dog



Two sentences

# Systems of information

## System 1

 The dog is **black**  
 The cat is **orange**

## System 2

 The dog is **black**  
 The dog is **black**

## System 3

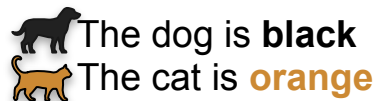
 The dog  
 The dog

**Two sentences**

**Two pieces of information**

# Systems of information

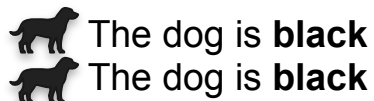
## System 1



Two sentences

Two pieces of information

## System 2



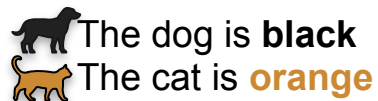
Two sentences

## System 3



# Systems of information

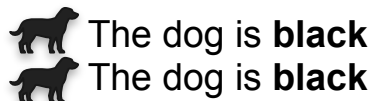
## System 1



Two sentences

Two pieces of information

## System 2



Two sentences

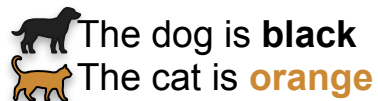
One piece of information

## System 3



# Systems of information

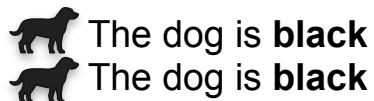
## System 1



Two sentences

Two pieces of information

## System 2



Two sentences

One piece of information

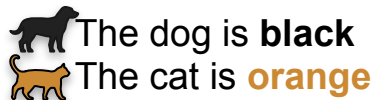
## System 3



Two sentences

# Systems of information

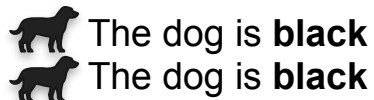
## System 1



Two sentences

Two pieces of information

## System 2



Two sentences

One piece of information

## System 3





Two sentences

Zero pieces of information

# Systems of information

## System 1



 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

## System 2

 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

## System 3



 The dog  
 The dog

Two sentences

Zero pieces of information

# Systems of information

## System 1



 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

## System 2

 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

Rank = 1

## System 3

 The dog  
 The dog



Two sentences

Zero pieces of information



# Systems of information

## System 1



 The dog is **black**  
 The cat is **orange**

Two sentences

Two pieces of information

Rank = 2

## System 2

 The dog is **black**  
 The dog is **black**

Two sentences

One piece of information

Rank = 1

## System 3

 The dog  
 The dog

Two sentences

Zero pieces of information

Rank = 0

# Systems of equations

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



# Systems of equations

## System 1

$$a + b = 0$$

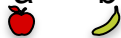


$$a + 2b = 0$$



## System 2

$$a + b = 0$$







$$2a + 2b = 0$$





# Systems of equations



## System 1

$$a + b = 0$$
 

$$a + 2b = 0$$
 

## System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 



## System 3



$$0a + 0b = 0$$

$$0a + 0b = 0$$

# Systems of equations



## System 1



$$a + b = 0$$
 

$$a + 2b = 0$$
 

Two equations

## System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 


## System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$


# Systems of equations


## System 1

$$a + b = 0$$


$$a + 2b = 0$$


## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


## System 3

$$0a + 0b = 0$$



$$0a + 0b = 0$$



Two equations

Two pieces of information



# Systems of equations



## System 1

$$a + b = 0$$
 

$$a + 2b = 0$$
 

## System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations



Two pieces of information



Rank = 2



# Systems of equations

## System 1

$$a + b = 0$$
 



$$a + 2b = 0$$
 



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$
 

$$2a + 2b = 0$$
 

Two equations


## System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

## System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

## System 3


$$0a + 0b = 0$$

$$0a + 0b = 0$$

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$


$$0a + 0b = 0$$

Two equations

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$


$$0a + 0b = 0$$


Two equations

Zero pieces of information

# Systems of equations

## System 1

$$a + b = 0$$



$$a + 2b = 0$$



Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$


$$2a + 2b = 0$$


Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

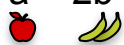
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

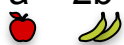
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2

## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0



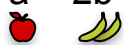
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2



## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



 	
1	1
2	2

Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

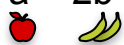
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2



## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



 	
1	1
2	2

Rank = 1

Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

$$0a + 0b = 0$$

Two equations

Zero pieces of information

Rank = 0

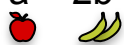
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2



## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



 	
1	1
2	2

Rank = 1



Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

 	
0	0
0	0

$$0a + 0b = 0$$

Rank = 0

Two equations

Zero pieces of information

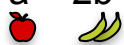
# Systems of equations



## System 1

$$a + b = 0$$



$$a + 2b = 0$$



 	
1	1
1	2

Rank = 2

Two equations

Two pieces of information

Rank = 2



## System 2

$$a + b = 0$$



$$2a + 2b = 0$$



 	
1	1
2	2

Rank = 1



Two equations

One piece of information

Rank = 1

## System 3

$$0a + 0b = 0$$

 	
0	0
0	0



Rank = 0

Two equations

Zero pieces of information



Rank = 0

# Rank and solutions to the system





1	1
1	2

**Rank = 2**



1	1
2	2



**Rank = 1**





0	0
0	0

**Rank = 0**



# Rank and solutions to the system

	
1	1
1	2

Rank = 2

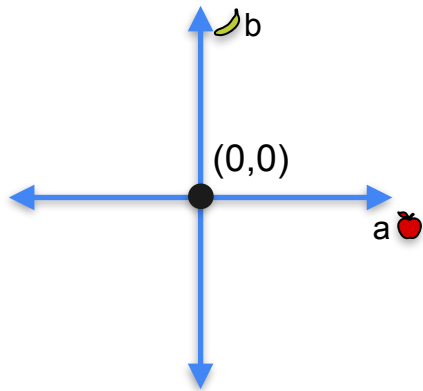
	
1	1
2	2

Rank = 1



	
0	0
0	0

Rank = 0

Dimension of solution space = 0





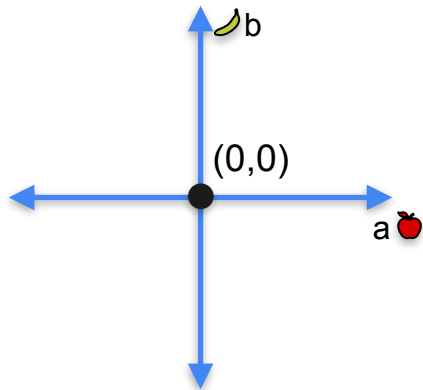
# Rank and solutions to the system



1	1
1	2

Rank = 2



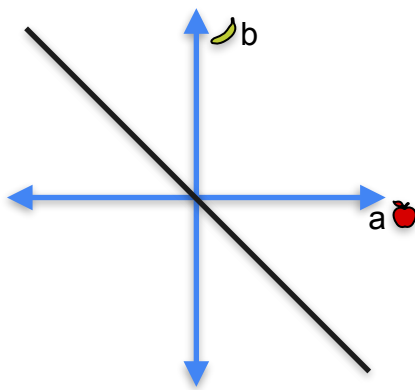
Dimension of solution space = 0



1	1
2	2

Rank = 1



Dimension of solution space = 1



0	0
0	0

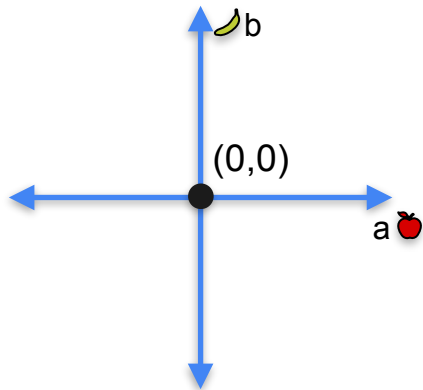
Rank = 0



# Rank and solutions to the system

	
1	1
1	2

Rank = 2

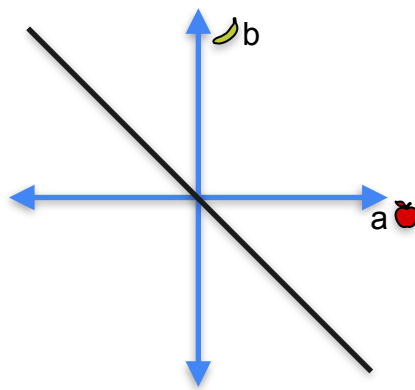
Dimension of solution space = 0





	
1	1
2	2

Rank = 1

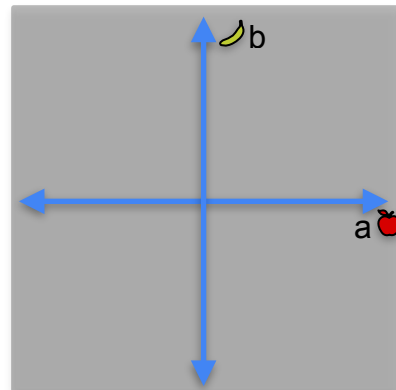
Dimension of solution space = 1



	
0	0
0	0



Rank = 0

Dimension of solution space = 2







# Rank of a matrix





1	1
1	2

**Rank = 2**



1	1
2	2

**Rank = 1**





0	0
0	0

**Rank = 0**

Dimension of solution space = 0    Dimension of solution space = 1    Dimension of solution space = 2



$$\text{Rank} = 2 - (\text{Dimension of solution space})$$

# Rank and singularity





1	1
1	2

Rank = 2



1	1
2	2



Rank = 1



0	0
0	0



Rank = 0

# Rank and singularity





1	1
1	2

Rank = 2



1	1
2	2

Rank = 1





0	0
0	0

Rank = 0

**Non-singular**



# Rank and singularity



1	1
1	2

Rank = 2



Non-singular



1	1
2	2

Rank = 1



Singular



0	0
0	0

Rank = 0



# Rank and singularity



1	1
1	2

Rank = 2



Non-singular



1	1
2	2

Rank = 1

Singular





0	0
0	0

Rank = 0

Singular



# Rank and singularity



1	1
1	2

Rank = 2



Non-singular



1	1
2	2

Rank = 1

Singular



0	0
0	0

Rank = 0

Singular

# Quiz: Rank of a matrix

Determine the rank of the following two matrices

## Matrix 1

5	1
-1	3

## Matrix 2

2	-1
-6	3

# Solutions: Rank of a matrix

Determine the rank of the following two matrices

**Matrix 1:** Since the solution space had dimension 0, the rank is **2**.

5	1
-1	3

**Matrix 2:** Since the solution space had dimension 1, the rank is **1**.

2	-1
-6	3





DeepLearning.AI

# Solving System of Linear Equations

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**Rank of a matrix:  
General case**

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$a + b + c = 0$$



$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

## System 4


$$0a + 0b + 0c = 0$$

$$0a + 0b + 0c = 0$$

$$0a + 0b + 0c = 0$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$


## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3


$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$


## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**

**3 Pieces of information**

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**  
**3 Pieces of information**

**Rank 3**

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2



# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

## Rank 3

1	1	1
1	2	1
1	1	2

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

## Rank 3

1	1	1
1	2	1
1	1	2

## Rank 2

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

## Rank 3

1	1	1
1	2	1
1	1	2

## Rank 2

1	1	1
1	1	2
1	1	3

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

## Rank 3

1	1	1
1	2	1
1	1	2

## Rank 2

1	1	1
1	1	2
1	1	3

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

## Rank 3

1	1	1
1	2	1
1	1	2

## Rank 2

1	1	1
1	1	2
1	1	3



# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

## Rank 3

1	1	1
1	2	1
1	1	2

## Rank 2

1	1	1
1	1	2
1	1	3

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



## Rank 3

1	1	1
1	2	1
1	1	2

## Rank 2

1	1	1
1	1	2
1	1	3

## Rank 1

1	1	1
2	2	2
3	3	3

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



**3 Equations**  
**0 Pieces of information**



# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



**3 Equations**  
**0 Pieces of information**

## Rank 0

# Rank for matrices

## System 1

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$



**3 Equations**  
**3 Pieces of information**

## Rank 3

1	1	1
1	2	1
1	1	2

## System 2

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$



**3 Equations**  
**2 Pieces of information**

## Rank 2

1	1	1
1	1	2
1	1	3

## System 3

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$



**3 Equations**  
**1 Piece of information**

## Rank 1

1	1	1
2	2	2
3	3	3

## System 4

$$\begin{aligned}0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0 \\0a + 0b + 0c &= 0\end{aligned}$$



**3 Equations**  
**0 Pieces of information**

## Rank 0

0	0	0
0	0	0
0	0	0

# Question

- Is there an easier way to calculate the rank?
- Answer: Yes! As before, it is the number of ones in the diagonal of the reduced row echelon form of the matrix.



DeepLearning.AI

# Solving System of Linear Equations

---

## Row echelon form

# Row echelon form of a matrix

# Row echelon form of a matrix

Original matrix

5	1
4	-3

# Row echelon form of a matrix

Original matrix

5	1
4	-3



Row echelon form

1	0.2
0	1

# Row echelon form of a matrix

Original matrix

5	1
4	-3

Row echelon form

1	0.2
0	1



5	1
10	2



# Row echelon form of a matrix

Original matrix

5	1
4	-3



Row echelon form

1	0.2
0	1

5	1
10	2



1	1
0	0

# Row echelon form of a matrix

Original matrix

5	1
4	-3



Row echelon form

1	0.2
0	1

5	1
10	2



1	1
0	0

0	0
0	0

# Row echelon form of a matrix

Original matrix

Row echelon form

5	1
4	-3



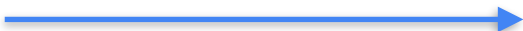
1	0.2
0	1

5	1
10	2



1	1
0	0

0	0
0	0



0	0
0	0

# Row echelon form

**Original matrix**

5	1
4	-3

# Row echelon form

**Original matrix**


5	1
4	-3

Divide each row by  
the leftmost coefficient

# Row echelon form

**Original matrix**

5	1
4	-3



Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

5	1		1	0.2
4	-3			

Divide each row by  
the leftmost coefficient

# Row echelon form

Original matrix

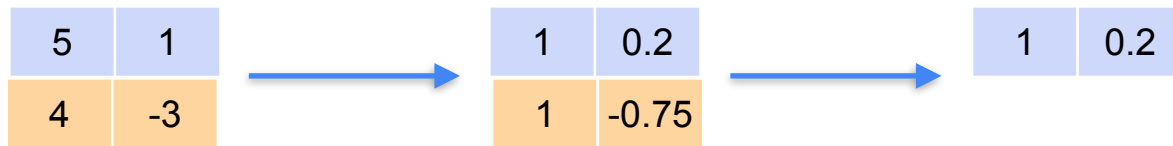
5	1		1	0.2
4	-3		1	-0.75

Divide each row by  
the leftmost coefficient



# Row echelon form

Original matrix

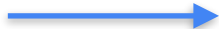


Divide each row by  
the leftmost coefficient

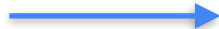
# Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



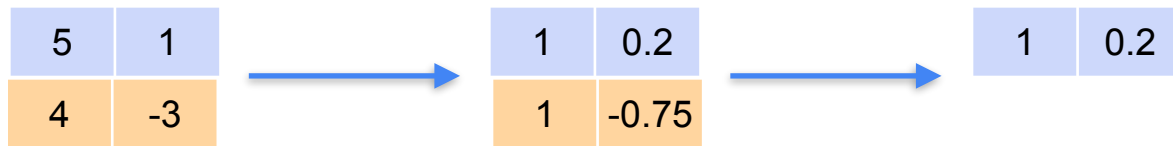
1	0.2
---	-----

Divide each row by  
the leftmost coefficient

1	-0.75
---	-------

# Row echelon form

Original matrix

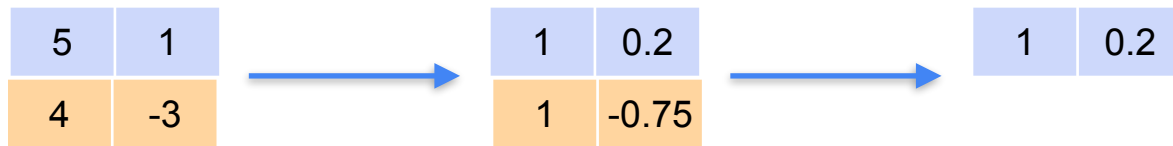


Divide each row by  
the leftmost coefficient

1	-0.75
1	0.2

# Row echelon form

Original matrix

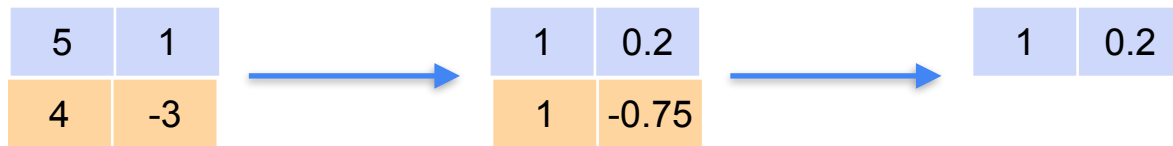


Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \end{array}$$

# Row echelon form

Original matrix

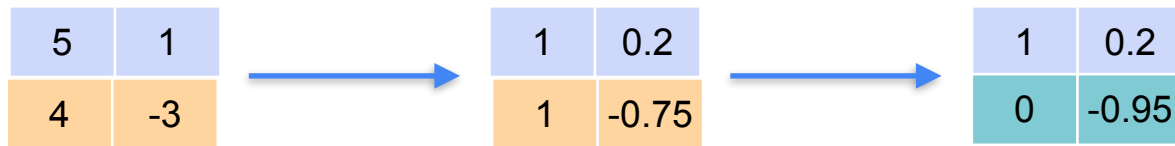


Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array} \end{array}$$

# Row echelon form

Original matrix



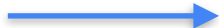
Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array} \end{array}$$

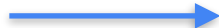
# Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95

Divide each row by  
the leftmost coefficient

	1	-0.75
-	1	0.2
<hr/>		
	0	-0.95

Divide the second row by  
the leftmost non-zero coefficient

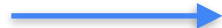
# Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



Divide each row by  
the leftmost coefficient

	1	-0.75
-	1	0.2
<hr/>		
	0	-0.95

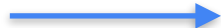
Divide the second row by  
the leftmost non-zero coefficient



# Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



1	0.2
---	-----

Divide each row by  
the leftmost coefficient

	1	-0.75
-	1	0.2
<hr/>		
	0	-0.95

Divide the second row by  
the leftmost non-zero coefficient

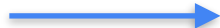
# Row echelon form

Original matrix

5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



1	0.2
0	1

Divide each row by  
the leftmost coefficient

	1	-0.75
-	1	0.2
<hr/>		
	0	-0.95

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form

Original matrix

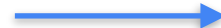
5	1
4	-3



1	0.2
1	-0.75



1	0.2
0	-0.95



Row echelon form

1	0.2
0	1

Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & -0.75 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & -0.95 \\ \hline \end{array} \end{array}$$

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

**Original matrix**

5	1
10	2

# Row echelon form for singular matrices

**Original matrix**


5	1
10	2

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2



Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2



1	0.2
---	-----

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

5	1	→	1	0.2
10	2		1	0.2

Divide each row by  
the leftmost coefficient



# Row echelon form for singular matrices

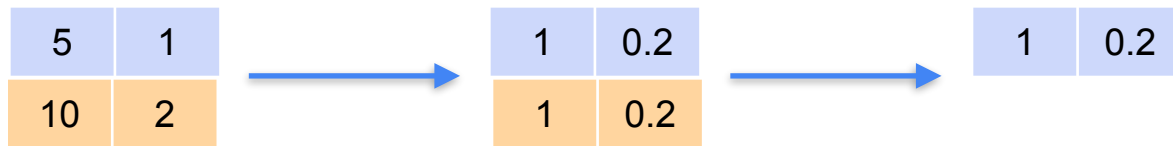
Original matrix



Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

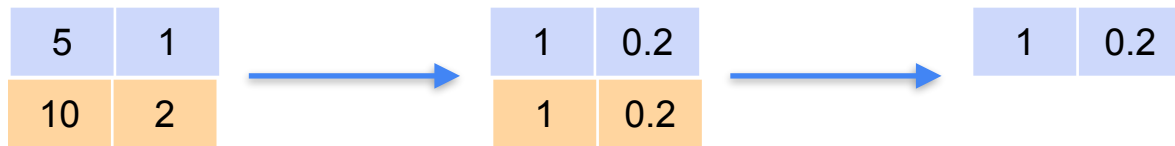


Divide each row by  
the leftmost coefficient

1	0.2
---	-----

# Row echelon form for singular matrices

Original matrix

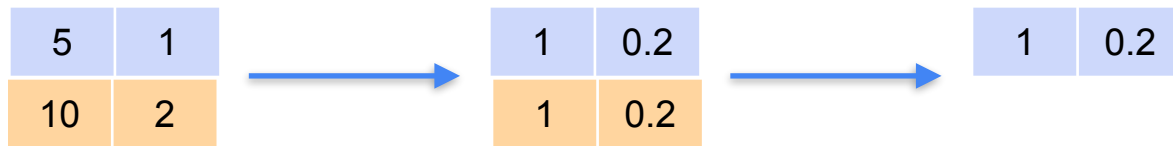


Divide each row by  
the leftmost coefficient

1	0.2
1	0.2

# Row echelon form for singular matrices

Original matrix

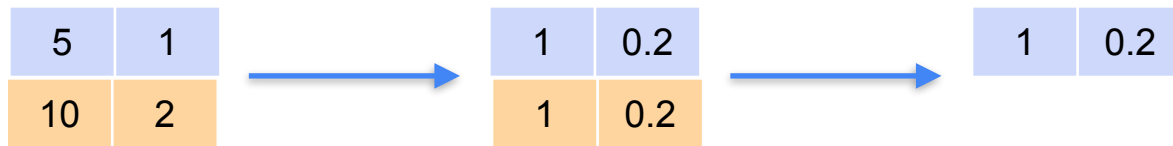


Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \end{array}$$

# Row echelon form for singular matrices

Original matrix



Divide each row by  
the leftmost coefficient

$$\begin{array}{r|cc} & 1 & 0.2 \\ - & 1 & 0.2 \\ \hline & 0 & 0 \end{array}$$

# Row echelon form for singular matrices

Original matrix



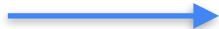
Divide each row by  
the leftmost coefficient

$$\begin{array}{r} \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ - \begin{array}{|c|c|} \hline 1 & 0.2 \\ \hline \end{array} \\ \hline \begin{array}{|c|c|} \hline 0 & 0 \\ \hline \end{array} \end{array}$$

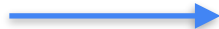
# Row echelon form for singular matrices

Original matrix

5	1
10	2



1	0.2
1	0.2



1	0.2
0	0

Divide each row by  
the leftmost coefficient

	1	0.2
-	1	0.2
<hr/>		
	0	0

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2



1	0.2
1	0.2



1	0.2
0	0



Divide each row by  
the leftmost coefficient

	1	0.2
-	1	0.2
<hr/>		
	0	0

Divide the second row by  
the leftmost non-zero coefficient



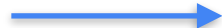
# Row echelon form for singular matrices

Original matrix

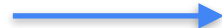
5	1
10	2



1	0.2
1	0.2



1	0.2
0	0



1	0.2
---	-----

Divide each row by  
the leftmost coefficient

	1	0.2
-	1	0.2
<hr/>		
	0	0

Divide the second row by  
the leftmost non-zero coefficient

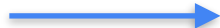
# Row echelon form for singular matrices

Original matrix

5	1
10	2



1	0.2
1	0.2



1	0.2
0	0



1	0.2
?	?

Divide each row by  
the leftmost coefficient

	1	0.2
-	1	0.2
<hr/>		
	0	0

Divide the second row by  
the leftmost non-zero coefficient

# Row echelon form for singular matrices

Original matrix

5	1
10	2

Divide each row by  
the leftmost coefficient

1	0.2
1	0.2

	1	0.2
-	1	0.2
<hr/>		
	0	0

Row echelon form

1	0.2
0	0

Divide the second row by  
the leftmost non-zero coefficient

1	0.2
?	?

# Row echelon form for singular matrices

**Original matrix**

0	0
0	0

# Row echelon form for singular matrices

**Original matrix**


0	0
0	0

Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

0	0
0	0



Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix



Divide each row by  
the leftmost coefficient

# Row echelon form for singular matrices

Original matrix

0	0	→	?	?
0	0		?	?

Divide each row by  
the leftmost coefficient



# Row echelon form for singular matrices

Row echelon form

Original matrix



Divide each row by  
the leftmost coefficient

# Row echelon form, singularity, and rank

5	1
4	-3

5	1
10	2

0	0
0	0

# Row echelon form, singularity, and rank

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

# Row echelon form, singularity, and rank

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

# Row echelon form, singularity, and rank

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 \\ 10 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

# Row echelon form, singularity, and rank

5	1	→	1	0.2
4	-3		0	1

2 ones in the diagonal

5	1	→	1	0.2
10	2		0	0

0	0	→	0	0
0	0		0	0

# Row echelon form, singularity, and rank

5	1	→	1	0.2
4	-3		0	1

**Rank 2**  
2 ones in the diagonal

5	1	→	1	0.2
10	2		0	0

0	0	→	0	0
0	0		0	0

# Row echelon form, singularity, and rank

5	1	→	1	0.2
4	-3		0	1

**Rank 2**

2 ones in the diagonal

5	1	→	1	0.2
10	2		0	0

1 one in the diagonal

0	0	→	0	0
0	0		0	0



# Row echelon form, singularity, and rank

5	1	→	1	0.2
4	-3		0	1

**Rank 2**

2 ones in the diagonal

5	1	→	1	0.2
10	2		0	0

**Rank 1**

1 one in the diagonal

0	0	→	0	0
0	0		0	0

# Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

**Rank 2**

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

**Rank 1**

1 one in the diagonal

0	0
0	0



0	0
0	0

0 ones in the diagonal

# Row echelon form, singularity, and rank

5	1
4	-3



1	0.2
0	1

**Rank 2**

2 ones in the diagonal

5	1
10	2



1	0.2
0	0

**Rank 1**

1 one in the diagonal

0	0
0	0



0	0
0	0

**Rank 0**

0 ones in the diagonal

# Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



1	0.2
0	1

**Rank 2**

**2** ones in the diagonal

5	1
10	2



1	0.2
0	0

**Rank 1**

**1** one in the diagonal

0	0
0	0



0	0
0	0

**Rank 0**

**0** ones in the diagonal

# Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



1	0.2
0	1

**Rank 2**

**2** ones in the diagonal

Singular matrix

5	1
10	2



1	0.2
0	0

**Rank 1**

**1** one in the diagonal

0	0
0	0



0	0
0	0

**Rank 0**

**0** ones in the diagonal

# Row echelon form, singularity, and rank

Non-singular matrix

5	1
4	-3



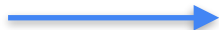
1	0.2
0	1

**Rank 2**

2 ones in the diagonal

Singular matrix

5	1
10	2



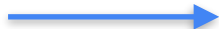
1	0.2
0	0

**Rank 1**

1 one in the diagonal

Singular matrix

0	0
0	0



0	0
0	0

**Rank 0**

0 ones in the diagonal



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# Solving System of Linear Equations

---

**Row echelon form:  
General case**

# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



## System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$



## System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

## Matrix

1	1	2
3	-3	-1
2	-1	6

# Row echelon form

## System

- $a + b + 2c = 12$
- $3a - 3b - c = 3$
- $2a - b + 6c = 24$

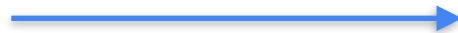


## System

- $a + b + 2c = 12$
- $-6b - 7c = -33$
- $6c = 18$

## Matrix

1	1	2
3	-3	-1
2	-1	6



## Row echelon form matrix

1	1	2
0	-6	7
0	0	6

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above

# Row echelon form

2	*	*	*	*
0	1	*	*	*
0	0	3	*	*
0	0	0	-5	*
0	0	0	0	1

**Rank 5**

3	*	*	*	*
0	0	1	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

**Rank 3**

- Zero rows at the bottom
- Each row has a pivot (leftmost non-zero entry)
- Every pivot is to the right of the pivots on the rows above
- Rank of the matrix is the number of pivots



# Another example

**Matrix**

1	1	1
1	2	1
1	1	2

# Another example

**Matrix**

1	1	1
1	2	1
1	1	2

Subtract the first row  
from the second and  
the third ones

# Another example

Matrix				Row echelon form		
1	1	1	→	1	1	1
1	2	1		0	1	0
1	1	2		0	0	1

Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3

Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3



1	1	1
0	0	1
0	0	2

Subtract the first row  
from the second and  
the third ones

# What if the matrix is singular?

**Matrix**

1	1	1
1	1	2
1	1	3

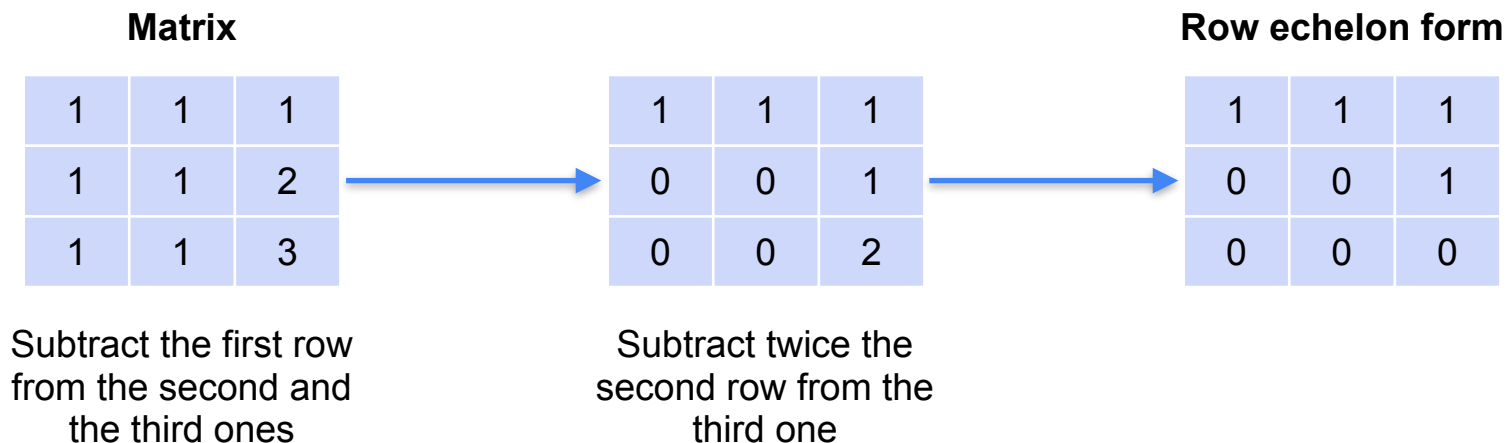
Subtract the first row  
from the second and  
the third ones



1	1	1
0	0	1
0	0	2

Subtract twice the  
second row from the  
third one

# What if the matrix is singular?





# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

Subtract twice the  
first row from the  
second row

# What if the matrix is singular?

**Matrix**

1	1	1		1	1	1
2	2	2	→	0	0	0
3	3	3		3	3	3

Subtract twice the  
first row from the  
second row

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

Subtract twice the  
first row from the  
second row



1	1	1
0	0	0
3	3	3

Subtract three times  
the first row from the  
third row

# What if the matrix is singular?

**Matrix**

1	1	1
2	2	2
3	3	3

Subtract twice the  
first row from the  
second row



1	1	1
0	0	0
3	3	3

Subtract three times  
the first row from the  
third row



**Row echelon form**

1	1	1
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1



# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

1	1	1
0	0	1
0	0	0

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

**Number of pivots = 3**

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

0	0	0
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

Number of pivots = 3

1	1	1
0	0	1
0	0	0

Number of pivots = 2

1	1	1
0	0	0
0	0	0

Number of pivots = 1

0	0	0
0	0	0
0	0	0

Number of pivots = 0

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Rank = 3**

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

**Number of pivots = 3**

1	1	1
0	0	1
0	0	0

**Number of pivots = 2**

1	1	1
0	0	0
0	0	0

**Number of pivots = 1**

0	0	0
0	0	0
0	0	0

**Number of pivots = 0**



# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Rank = 3**

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Rank = 2**

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

**Number of pivots = 3**

1	1	1
0	0	1
0	0	0

**Number of pivots = 2**

1	1	1
0	0	0
0	0	0

**Number of pivots = 1**

0	0	0
0	0	0
0	0	0

**Number of pivots = 0**

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Rank = 3**

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Rank = 2**

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Rank = 1**

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Row echelon forms**

1	1	1
0	1	0
0	0	1

**Number of pivots = 3**

1	1	1
0	0	1
0	0	0

**Number of pivots = 2**

1	1	1
0	0	0
0	0	0

**Number of pivots = 1**

0	0	0
0	0	0
0	0	0

**Number of pivots = 0**

# Rank for matrices

**Matrix 1**

1	1	1
1	2	1
1	1	2

**Rank = 3**

**Matrix 2**

1	1	1
1	1	2
1	1	3

**Rank = 2**

**Matrix 3**

1	1	1
2	2	2
3	3	3

**Rank = 1**

**Matrix 4**

0	0	0
0	0	0
0	0	0

**Rank = 0**

## Row echelon forms

1	1	1
0	1	0
0	0	1

**Number of pivots = 3**

1	1	1
0	0	1
0	0	0

**Number of pivots = 2**

1	1	1
0	0	0
0	0	0

**Number of pivots = 1**

0	0	0
0	0	0
0	0	0

**Number of pivots = 0**



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# Solving System of Linear Equations

---

## **Reduced row echelon form**

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$



**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $a = 3$
- $b = 2$

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3



# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

**Diagonal matrix**

1	0
0	1

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $a = 3$
- $b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

**Diagonal matrix**

1	0
0	1

Row echelon form

# Systems of equations to matrices

**Original system**

- $5a + b = 17$
- $4a - 3b = 6$

**Intermediate System**

- $a + 0.2b = 3.4$
- $b = 2$

**Solved system**

- $1a + 0b = 3$
- $0a + 1b = 2$

**Original matrix**

5	1
4	-3

**Upper diagonal matrix**

1	0.2
0	1

**Diagonal matrix**

1	0
0	1

Row echelon form

Reduced row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

Row echelon form

## Diagonal matrix

1	0
0	1

Reduced row echelon form

# Systems of equations to matrices

## Original system

- $5a + b = 17$
- $4a - 3b = 6$

## Intermediate System

- $a + 0.2b = 3.4$
- $b = 2$

## Solved system

- $1a + 0b = 3$
- $0a + 1b = 2$

## Original matrix

5	1
4	-3

## Upper diagonal matrix

1	0.2
0	1

Row echelon form

## Diagonal matrix

1	0
0	1

Reduced row echelon form

# Reduced row echelon form

Row echelon form

1	0.2
0	1

# Reduced row echelon form

Row echelon form





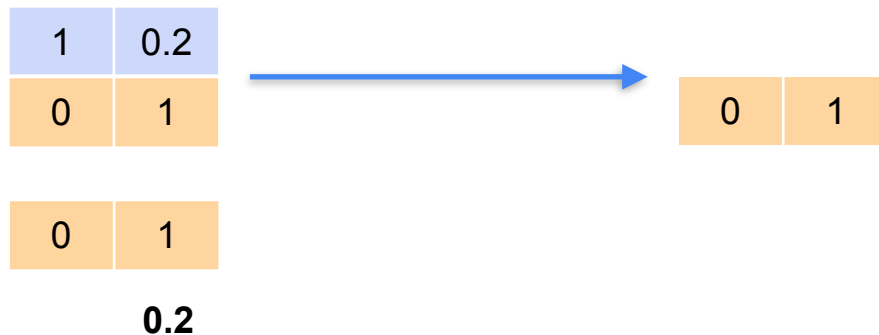
# Reduced row echelon form

Row echelon form



# Reduced row echelon form

Row echelon form



# Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

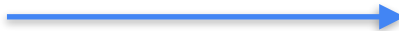
**x**      **0.2**

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# Reduced row echelon form

Row echelon form

1	0.2
0	1



0	1
---	---

0	1
---	---

x

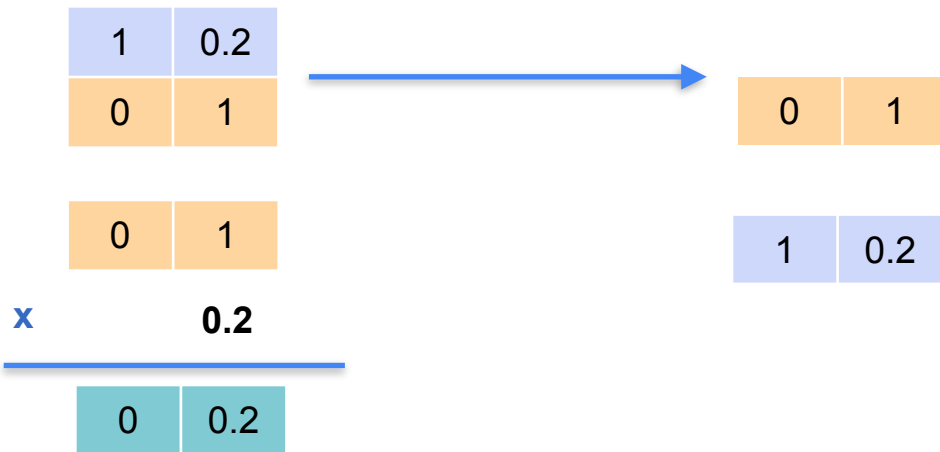
0.2

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0	0.2
---	-----

# Reduced row echelon form

Row echelon form



# Reduced row echelon form

Row echelon form

The diagram illustrates the transformation of a matrix from row echelon form to reduced row echelon form. An arrow points from the initial matrix to the final matrix.

**Initial Matrix (Row Echelon Form):**

1	0.2
0	1

**Row Operation:**  $\times 0.2$

**Intermediate Matrix:**

0	0.2
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**Final Matrix (Reduced Row Echelon Form):**

1	0.2
0	1

**Row Operation:**  $-$

**Final Matrix (Reduced Row Echelon Form):**

1	0
0	1

# Reduced row echelon form

Row echelon form

The diagram illustrates the process of converting a matrix into reduced row echelon form using row operations. It shows the initial matrix, the row operation performed, and the resulting matrix.

**Initial Matrix (Row Echelon Form):**

$$\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

**Row Operation:** Row 1 is multiplied by 0.2 (indicated by  $\times$  and 0.2).

**Intermediate Matrix:**

$$\begin{bmatrix} 0 & 0.2 \\ 0 & 1 \end{bmatrix}$$

**Row Operation:** Row 2 is subtracted from Row 1 (indicated by  $-$ ).

**Final Matrix (Reduced Row Echelon Form):**

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

# Reduced row echelon form

Row echelon form

The diagram illustrates the transformation of a matrix from row echelon form to reduced row echelon form. It shows two matrices connected by a blue arrow, with the row operation performed below each matrix.

**Left Matrix (Row Echelon Form):**

1	0.2
0	1

Row operation:  $\times$   $0.2$

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0	0.2
---	-----

**Right Matrix (Reduced Row Echelon Form):**

1	0
0	1

Row operation:  $-$

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1	0
---	---



# Reduced row echelon form

Row echelon form

1	0.2
0	1

0	1
---	---

$\times \quad 0.2$

---

0	0.2
---	-----

Reduced row echelon form

1	0
0	1

1	0.2
0	0.2

$-$

---

1	0
---	---

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

● Is in row echelon form

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form
- Each pivot is a 1

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0

# Reduced row echelon form

1	0	0	0	0
0	1	0	0	0
0	0	1	0	0
0	0	0	1	0
0	0	0	0	1

**Rank 5**

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

**Rank 3**

- Is in row echelon form
- Each pivot is a 1
- Any number above a pivot is 0
- Rank of the matrix is the number of pivots

# Reduced row echelon form

# Reduced row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0



# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by  
the value of the pivot

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

# Reduced row echelon form

Row echelon form

3	*	*	*	*
0	0	2	*	*
0	0	0	-4	*
0	0	0	0	0
0	0	0	0	0

1	*	*	*	*
0	0	1	*	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Divide each row by  
the value of the pivot

Reduced row  
echelon form

1	*	0	0	*
0	0	1	0	*
0	0	0	1	*
0	0	0	0	0
0	0	0	0	0

Turn anything above a  
pivot to 0

# Reduced row echelon form

## Row echelon form

1	2	3
0	1	4
0	0	1

# Reduced row echelon form

## Row echelon form

1	2	3
0	1	4
0	0	1

Subtract 2 times the  
second row from the  
first one

# Reduced row echelon form

## Row echelon form

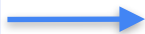
1	2	3	→	1	0	-5
0	1	4		0	1	4
0	0	1		0	0	1

Subtract 2 times the  
second row from the  
first one

# Reduced row echelon form

## Row echelon form

1	2	3
0	1	4
0	0	1



1	0	-5
0	1	4
0	0	1

Subtract 2 times the  
second row from the  
first one

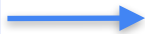
Add 5 times the third  
row to the first one

# Reduced row echelon form

## Row echelon form

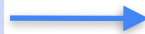
1	2	3
0	1	4
0	0	1

Subtract 2 times the  
second row from the  
first one



1	0	-5
0	1	4
0	0	1

Add 5 times the third  
row to the first one



1	0	0
0	1	4
0	0	1

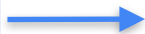


# Reduced row echelon form

## Row echelon form

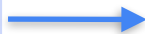
1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one



1	0	-5
0	1	4
0	0	1

Add 5 times the third row to the first one



1	0	0
0	1	4
0	0	1

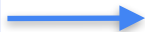
Subtract 4 times the third row from the second one

# Reduced row echelon form

## Row echelon form

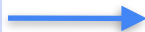
1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one



1	0	-5
0	1	4
0	0	1

Add 5 times the third row to the first one



1	0	0
0	1	4
0	0	1

Subtract 4 times the third row from the second one



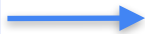
1	0	0
0	1	0
0	0	1

# Reduced row echelon form

**Row echelon form**

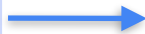
1	2	3
0	1	4
0	0	1

Subtract 2 times the second row from the first one



1	0	-5
0	1	4
0	0	1

Add 5 times the third row to the first one



1	0	0
0	1	4
0	0	1

Subtract 4 times the third row from the second one



**Reduced row echelon form**

1	0	0
0	1	0
0	0	1



DeepLearning.AI

# Solving System of Linear Equations

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## **Conclusion**