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pyPopFit

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ABSTRACT
Abstract.

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1 THEORETICAL FRAMEWORK

$$\mathcal{L}(t^i, A_V^i \mid \boldsymbol{D}^i) \equiv p(\boldsymbol{D}^i \mid t^i, A_V^i) = (1 - P_{\rm bin}) \times p(\boldsymbol{D}^i \mid t^i, A_V^i, \textbf{sin}) + P_{\rm bin} \times p(\boldsymbol{D}^i \mid t^i, A_V^i, \textbf{bin}). \tag{1}$$

The single-star part:

$$p(\boldsymbol{D}^i \mid t^i, A_V^i, \textbf{sin}) = \int_{M^*_{\rm min}}^{M^*_{\rm max}} dM^i \left[p(\boldsymbol{D}^i \mid M^i, t^i, A_V^i, \textbf{sin}) p(M^i \mid t^i, \textbf{sin}) \right]; \tag{2}$$

$$p(M^i \mid t^i, \textbf{sin}) = \begin{cases} \dfrac{(M^i)^\alpha}{\int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM^i \left[(M^i)^\alpha \right]} \equiv \dfrac{(M^i)^\alpha}{A(t^i)} & \text{if } M^*_{\rm min} < M^i < M_{\rm max}(t^i), \\ 0 & \text{otherwise;} \end{cases} \tag{3}$$

$$p(\boldsymbol{D}^i \mid t^i, A_V^i, \textbf{sin}) = \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM^i \left[\mathcal{L}_{\rm sin}(M^i, t^i, A_V^i \mid \boldsymbol{D}^i) \frac{(M^i)^\alpha}{A(t^i)} \right]; \tag{4}$$

The binary-star part:

$$\begin{aligned} p(\boldsymbol{D}^i \mid t^i, A_V^i, \textbf{bin}) &= \int_{M^*_{\rm min}}^{M^*_{\rm max}} dM_1^i \int_{M^*_{\rm min}}^{M^*_{\rm max}} dM_2^i \left[p(\boldsymbol{D}^i \mid M_1^i, M_2^i, t^i, A_V^i, \textbf{bin}) p(M_1^i, M_2^i \mid t^i, \textbf{bin}) \right] \\ &= \int_{M^*_{\rm min}}^{M^*_{\rm max}} dM_1^i \left\{ p(M_1^i) \int_{M^*_{\rm min}}^{M^*_{\rm max}} dM_2^i \left[p(\boldsymbol{D}^i \mid M_1^i, M_2^i, t^i, A_V^i, \textbf{bin}) p(M_2^i \mid M_1^i, t^i, \textbf{bin}) \right] \right\} \end{aligned} \tag{5}$$

$$p(M_1^i) = \frac{(M_1^i)^\alpha}{\int_{M^*_{\rm min}}^{M^*_{\rm max}} dM^i \left[(M^i)^\alpha \right]} \equiv \frac{(M_1^i)^\alpha}{C} \quad \text{for } M^*_{\rm min} < M_1^i < M^*_{\rm max} \tag{6}$$

$$p(M_2^i \mid M_1^i, t^i, \textbf{bin}) = \begin{cases} \dfrac{1}{\min\left[M_1^i, M_{\rm max}(t^i) \right] - M^*_{\rm min}} & \text{if } M_2^i < M_1^i \text{ and } M_2^i < M_{\rm max}(t^i), \\ 0 & \text{otherwise;} \end{cases} \tag{7}$$

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Divide the binary-star function into two parts

$$p(\boldsymbol{D}^i \mid t^i, A_V^i, \mathbf{bin}) = \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_1^i \left\{ p(M_1^i) \int_{M^*_{\rm min}}^{M^*_{\rm max}} dM_2^i \left[p(\boldsymbol{D}^i \mid M_1^i, M_2^i, t^i, A_V^i, \mathbf{bin}) p(M_2^i \mid M_1^i, t^i, \mathbf{bin}) \right] \right\}$$
$$+ \int_{M_{\rm max}(t^i)}^{M^*_{\rm max}} dM_1^i \left\{ p(M_1^i) \int_{M^*_{\rm min}}^{M^*_{\rm max}} dM_2^i \left[p(\boldsymbol{D}^i \mid M_1^i, M_2^i, t^i, A_V^i, \mathbf{bin}) p(M_2^i \mid M_1^i, t^i, \mathbf{bin}) \right] \right\}$$
$$= \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_1^i \left\{ \frac{(M_1^i)^\alpha}{C} \int_{M^*_{\rm min}}^{M_1^i} dM_2^i \left[\frac{\mathcal{L}_{\rm bin}(M_1^i, M_2^i, t^i, A_V^i \mid \boldsymbol{D}^i)}{M_1^i - M^*_{\rm min}} \right] \right\}$$
$$+ \int_{M_{\rm max}(t^i)}^{M^*_{\rm max}} dM_1^i \left\{ \frac{(M_1^i)^\alpha}{C} \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_2^i \left[\frac{\mathcal{L}_{\rm sin}(M_2^i, t^i, A_V^i \mid \boldsymbol{D}^i)}{M_{\rm max}(t^i) - M^*_{\rm min}} \right] \right\} \quad (8)$$

The second term is equal to

$$\int_{M_{\rm max}(t^i)}^{M^*_{\rm max}} dM_1^i \left[\frac{(M_1^i)^\alpha}{C(M_{\rm max}(t^i) - M^*_{\rm min})} \right] \times \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_2^i \left[\mathcal{L}_{\rm sin}(M_2^i, t^i, A_V^i \mid \boldsymbol{D}^i) \right]$$
$$\equiv \frac{B(t^i)}{C(M_{\rm max}(t^i) - M^*_{\rm min})} \times \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_2^i \left[\mathcal{L}_{\rm sin}(M_2^i, t^i, A_V^i \mid \boldsymbol{D}^i) \right] \quad (9)$$

The total likelihood is

$$p(\boldsymbol{D}^i \mid t^i, A_V^i) = (1 - P_{\rm bin}) \times \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM^i \left[\mathcal{L}_{\rm sin}(M^i, t^i, A_V^i \mid \boldsymbol{D}^i) \frac{(M^i)^\alpha}{A(t^i)} \right]$$
$$+ P_{\rm bin} \times \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_1^i \left\{ \frac{(M_1^i)^\alpha}{C} \int_{M^*_{\rm min}}^{M_1^i} dM_2^i \left[\frac{\mathcal{L}_{\rm bin}(M_1^i, M_2^i, t^i, A_V^i \mid \boldsymbol{D}^i)}{M_1^i - M^*_{\rm min}} \right] \right\}$$
$$+ P_{\rm bin} \times \frac{B(t^i)}{C(M_{\rm max}(t^i) - M^*_{\rm min})} \times \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_2^i \left[\mathcal{L}_{\rm sin}(M_2^i, t^i, A_V^i \mid \boldsymbol{D}^i) \right] \quad (10)$$

Changing the label of the integral in the first and third terms do not change their values

$$p(\boldsymbol{D}^i \mid t^i, A_V^i) = \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_1^i \left\{ \left[\frac{(1 - P_{\rm bin})(M_1^i)^\alpha}{A(t^i)} + \frac{P_{\rm bin} B(t^i)}{C(M_{\rm max}(t^i) - M^*_{\rm min})} \right] \mathcal{L}_{\rm sin}(M_1^i, t^i, A_V^i \mid \boldsymbol{D}^i) \right\}$$
$$+ \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_1^i \left\{ \frac{P_{\rm bin}(M_1^i)^\alpha}{C(M_1^i - M^*_{\rm min})} \int_{M^*_{\rm min}}^{M_1^i} dM_2^i \left[\mathcal{L}_{\rm bin}(M_1^i, M_2^i, t^i, A_V^i \mid \boldsymbol{D}^i) \right] \right\}$$
$$\equiv \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_1^i \left[\mathcal{S}(M_1^i \mid t^i) \mathcal{L}_{\rm sin}(M_1^i, t^i, A_V^i \mid \boldsymbol{D}^i) \right] + \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_1^i \left\{ \mathcal{T}(M_1^i) \int_{M^*_{\rm min}}^{M_1^i} dM_2^i \left[\mathcal{L}_{\rm bin}(M_1^i, M_2^i, t^i, A_V^i \mid \boldsymbol{D}^i) \right] \right\}$$
$$\equiv \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM_1^i \left\{ \mathcal{S}(M_1^i \mid t^i) \mathcal{L}_{\rm sin}(M_1^i, t^i, A_V^i \mid \boldsymbol{D}^i) + \mathcal{T}(M_1^i) \int_{M^*_{\rm min}}^{M_1^i} dM_2^i \left[\mathcal{L}_{\rm bin}(M_1^i, M_2^i, t^i, A_V^i \mid \boldsymbol{D}^i) \right] \right\} \quad (11)$$

The user may specify a value of $M_{\rm min}(t^i)$ below which a star would become too faint to match the observations. Below this value, the likelihood is zero. So the likelihood can be calculated more efficiently with

$$\mathcal{L}(t^i, A_V^i \mid \boldsymbol{D}^i) \equiv p(\boldsymbol{D}^i \mid t^i, A_V^i) =$$
$$\int_{M_{\rm min}(t^i)}^{M_{\rm max}(t^i)} dM_1^i \left\{ \mathcal{S}(M_1^i \mid t^i) \mathcal{L}_{\rm sin}(M_1^i, t^i, A_V^i \mid \boldsymbol{D}^i) + \mathcal{T}(M_1^i) \int_{M_{\rm min}(t^i)}^{M_1^i} dM_2^i \left[\mathcal{L}_{\rm bin}(M_1^i, M_2^i, t^i, A_V^i \mid \boldsymbol{D}^i) \right] \right\}; \quad (12)$$

where

$$\mathcal{S}(M_1^i \mid t^i) = \frac{(1 - P_{\rm bin})(M_1^i)^\alpha}{A(t^i)} + \frac{P_{\rm bin} B(t^i)}{C(M_{\rm max}(t^i) - M^*_{\rm min})}; \mathcal{T}(M_1^i) = \frac{P_{\rm bin}(M_1^i)^\alpha}{C(M_1^i - M^*_{\rm min})};$$
$$A(t^i) = \int_{M^*_{\rm min}}^{M_{\rm max}(t^i)} dM^i \left[(M^i)^\alpha \right]; B(t^i) = \int_{M_{\rm max}(t^i)}^{M^*_{\rm max}} dM^i \left[(M^i)^\alpha \right]; C = \int_{M^*_{\rm min}}^{M^*_{\rm max}} dM^i \left[(M^i)^\alpha \right] \equiv A(t^i) + B(t^i). \quad (13)$$

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