IN3050/IN4050 Mandatory Assignment 2, 2023: Supervised Learning

Rules

Before you begin the exercise, review the rules at this website:

- https://www.uio.no/english/studies/examinations/compulsory-activities/mn-ifi-mandatory.html

in particular the paragraph on cooperation. This is an individual assignment. You are not allowed to deliver together or copy/share source-code/answers with others. Read also the "Routines for handling suspicion of cheating and attempted cheating at the University of Oslo":

- https://www.uio.no/english/studies/examinations/cheating/index.html

By submitting this assignment, you confirm that you are familiar with the rules and the consequences of breaking them.

Delivery

Deadline: Friday, March 24, 2023, 23:59

Your submission should be delivered in Devilry. You may redeliver in Devilry before the deadline, but include all files in the last delivery, as only the last delivery will be read. You are recommended to upload preliminary versions hours (or days) before the final deadline.

What to deliver?

You are recommended to solve the exercise in a Jupyter notebook, but you might solve it in a Python program if you prefer.

Alternative 1

If you choose Jupyter, you should deliver the notebook. You should answer all questions and explain what you are doing in Markdown. Still, the code should be properly commented. The notebook should contain results of your runs. In addition, you should make a pdf of your solution which shows the results of the runs. (If you can't export: notebook -> latex -> pdf on your own machine, you may do this on the IFI linux machines.)

Alternative 2

If you prefer not to use notebooks, you should deliver the code, your run results, and a pdf-report where you answer all the questions and explain your work.

Here is a list of *absolutely necessary* (but not sufficient) conditions to get the assignment marked as passed:

- You must deliver your code (python file or notebook) you used to solve the assignment.
- The code used for making the output and plots must be included in the assignment.
- You must include example runs that clearly shows how to run all implemented functions and methods.
- All the code (in notebook cells or python main-blocks) must run. If you have unfinished code that crashes, please comment it out and document what you think causes it to crash.
- You must also deliver a pdf of the code, outputs, comments and plots as explained above.

Your report/notebook should contain your name and username.

Deliver one single zipped folder (.zip, .tgz or .tar.gz) which contains your complete solution.

Important: if you weren’t able to finish the assignment, use the PDF report/Markdown to elaborate on what you’ve tried and what problems you encountered. Students who have made an effort and attempted all parts of the assignment will get a second chance even if they fail initially. This exercise will be graded PASS/FAIL.

Goals of the assignment

The goal of this assignment is to get a better understanding of supervised learning with gradient descent. It will, in particular, consider the similarities and differences between linear classifiers and multi-layer feed forward networks (multi-layer perceptron, MLP) and the differences and similarities between binary and multi-class classification. A main part will be dedicated to implementing and understanding the backpropagation algorithm.

Tools

The aim of the exercises is to give you a look inside the learning algorithms. You may freely use code from the weekly exercises and the published solutions. You should not use ML libraries like scikit-learn or tensorflow.

You may use tools like NumPy and Pandas, which are not specific ML-tools.

The given precode uses NumPy. You are recommended to use NumPy since it results in more compact code, but feel free to use pure python if you prefer.

Beware

There might occur typos or ambiguities. This is a revised assignment compared to earlier years, and there might be new typos. If anything is unclear, do not hesitate to ask. Also, if you think some assumptions are missing, make your own and explain them!

Initialization

In [ ]:

```
import numpy as np
import matplotlib.pyplot as plt
import sklearn #for datasets
```

Datasets

We start by making a synthetic dataset of 2000 datapoints and five classes, with 400 individuals in each class. (See https://scikit-learn.org/stable/modules/generated/sklearn.datasets.make_blobs.html regarding how the data are generated.) We choose to use a synthetic dataset---and not a set of natural occuring data---because we are mostly interested in properties of the various learning algorithms, in particular the differences between linear classifiers and multi-layer neural networks together with the difference between binary and multi-class data.

When we are doing experiments in supervised learning, and the data are not already split into training and test sets, we should start by splitting the data. Sometimes there are natural ways to split the data, say training on data from one year and testing on data from a later year, but if that is not the case, we should shuffle the data randomly before splitting. (OK, that is not necessary with this particular synthetic data set, since it is already shuffled by default by scikit, but that will not be the case with real-world data.) We should split the data so that we keep the alignment between X and t, which may be achieved by shuffling the indices. We split into 50% for training, 25% for validation, and 25% for final testing. The set for final testing *must not be used* till the end of the assignment in part 3.

We fix the seed both for data set generation and for shuffling, so that we work on the same datasets when we rerun the experiments. This is done by the `random_state` argument and the `rng = np.random.RandomState(2022)`.

In [ ]:
```
from sklearn.datasets import make_blobs
X, t_multi = make_blobs(n_samples=[400,400,400, 400, 400],
                        centers=[[0,1],[4,2],[8,1],[2,0],[6,0]],
                        cluster_std=[1.0, 2.0, 1.0, 0.5, 0.5],
                        n_features=2, random_state=2022)
```

In [ ]:
```
indices = np.arange(X.shape[0])
rng = np.random.RandomState(2022)
rng.shuffle(indices)
indices[:10]
```

Out[ ]:
```
array([1018, 1295,  643, 1842, 1669,   86,  164, 1653, 1174,  747])
```

In [ ]:
```
X_train = X[indices[:1000],:]
X_val = X[indices[1000:1500],:]
X_test = X[indices[1500:],:]
t_multi_train = t_multi[indices[:1000]]
t_multi_val = t_multi[indices[1000:1500]]
t_multi_test = t_multi[indices[1500:]]
```

Next, we will make a second dataset by merging classes in (X,t) into two classes and call the new set (X, t2). This will be a binary set. We now have two datasets:

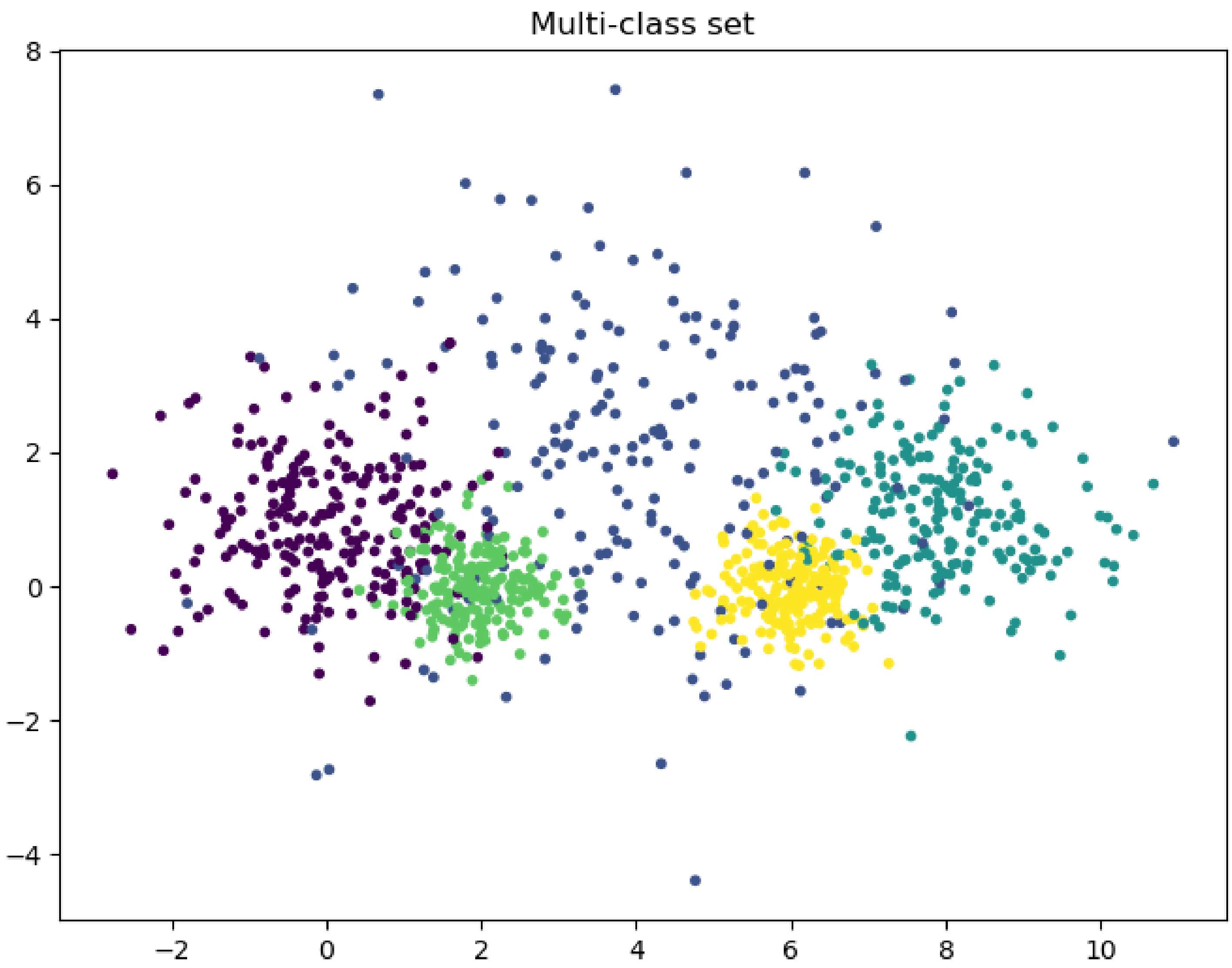
- Binary set: `(X, t2)`
- Multi-class set: `(X, t_multi)`

In [ ]:
```
t2_train = t_multi_train >= 3
t2_train = t2_train.astype('int')
t2_val = (t_multi_val >= 3).astype('int')
t2_test = (t_multi_test >= 3).astype('int')
```

We can plot the two traning sets.

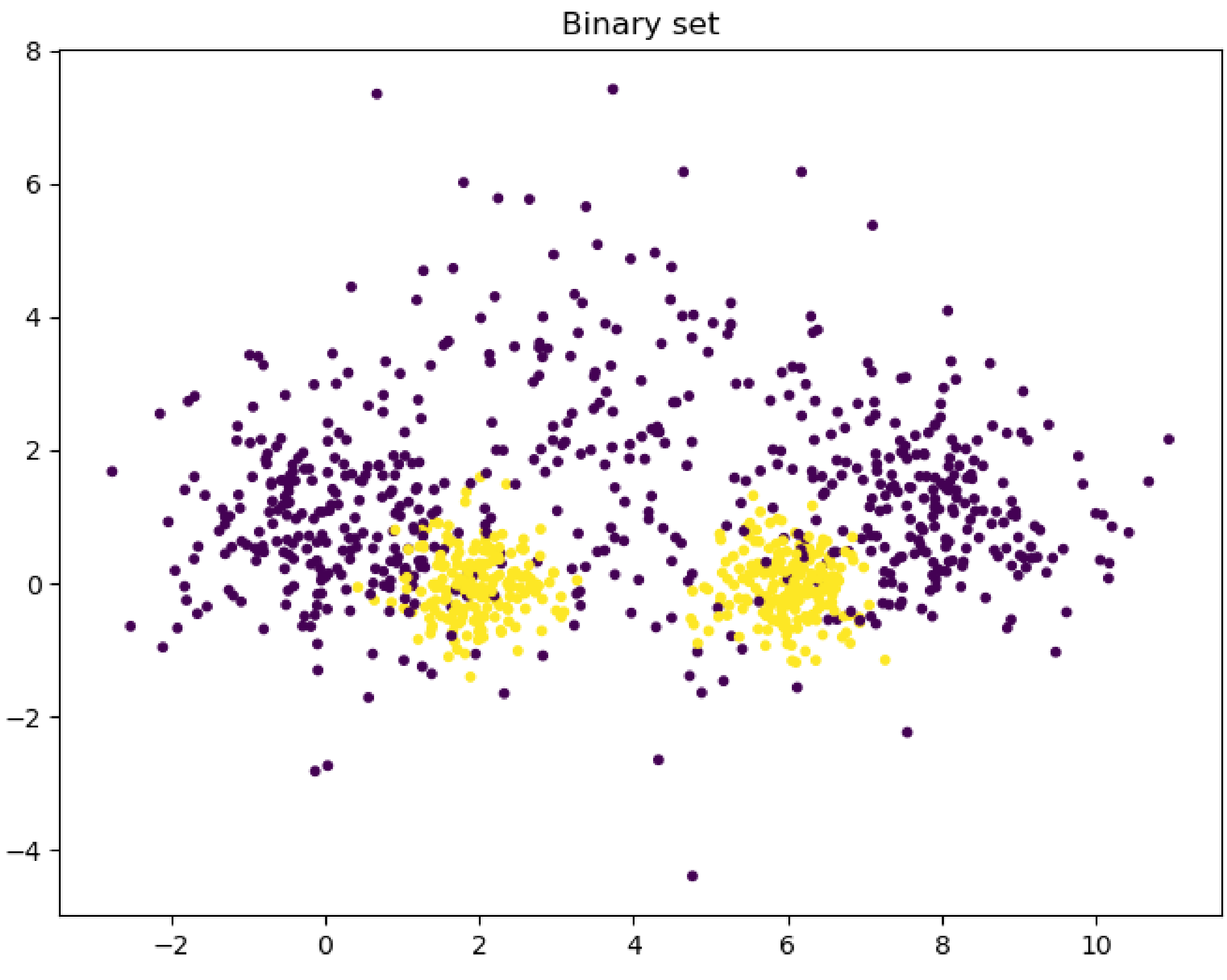
In [ ]:
```
plt.figure(figsize=(8,6)) # You may adjust the size
plt.scatter(X_train[:, 0], X_train[:, 1], c=t_multi_train, s=10.0)
plt.title('Multi-class set')
```

Out[ ]:
```
Text(0.5, 1.0, 'Multi-class set')
```



In [ ]:
```
plt.figure(figsize=(8,6))
plt.scatter(X_train[:, 0], X_train[:, 1], c=t2_train, s=10.0)
plt.title('Binary set')
```

Out[ ]:
```
Text(0.5, 1.0, 'Binary set')
```



Part I: Linear classifiers

Linear regression

We see that that set (X, t2) is far from linearly separable, and we will explore how various classifiers are able to handle this. We start with linear regression. You may make your own implementation from scratch or start with the solution to the weekly exercise set 7. We include it here with a little added flexibility.

In [ ]:
```
def add_bias(X, bias):
    """X is a Nxm matrix: N datapoints, m features
    bias is a bias term, -1 or 1. Use 0 for no bias
    Return a Nx(m+1) matrix with added bias in position zero
    """
    N = X.shape[0]
    biases = np.ones((N, 1))*bias # Make a N*1 matrix of bias-s
    # Concatenate the column of biases in front of the columns of X.
    return np.concatenate((biases, X), axis  = 1)
```

In [ ]:
```
class NumpyClassifier():
    """Common methods to all numpy classifiers --- if any"""

    def MSE(self, x, y):
        return sum((x - y)**2)/x.shape[0]

    # Logistic (Used in Logistic Regression)
    def logistic(self, x):
        return 1/(1+np.exp(-x))

    # Binary cross-entropy loss for logistic regression
    def BCE(self, t_train, y_pred):
        loss = - t_train * np.log(y_pred) - (1 - t_train) * np.log(1 - y_pred)
        return np.mean(loss)

    # # Binary cross-entropy loss for logistic regression
    # def BCE(self, x, t_train, y_pred):
    #     N = len(y_pred)
    #     sum = 0
    #     for j in range(N):
    #         for i in range(len(x[0])):
    #             sum -= (t_train[j] - y_pred[j])*x[j][i]
    #     return sum/N
```

```
    def accuracy(self, predicted, gold):
        return np.mean(predicted == gold)
```

In [ ]:

```
class NumpyLinRegClass(NumpyClassifier): # Gradient descent batch training?

    def __init__(self, bias=-1):
        self.bias=bias

    def fit(self, X_train, t_train, X_test, t_test, eta = 0.1, epochs=10):
        # eta - learning rate
        """X_train is a Nxm matrix, N data points, m features
        t_train is a vector of length N,
        the targets values for the training data"""

        # X_train_no_bias = np.copy(X_train)

        (N, m) = X_train.shape

        if self.bias:
            X_train = add_bias(X_train, self.bias)

        self.weights = weights = np.zeros(m + 1)
        # Added for Task:Loss {
        self.loss = loss = []
        self.accuracy_f = accuracy_f = []
        #}

        for e in range(epochs):
            weights -= eta / N * X_train.T @ (X_train @ weights - t_train)
            # Task: loss {
            loss.append(self.MSE(t_train, X_train @ weights))
            # loss.append(self.MSE(t_train, self.predict(X_train_no_bias)))
            accuracy_f.append(self.accuracy(self.predict(X_test), t_test))
            # accuracy_f.append(np.mean(self.predict(X_val) == t_val))
            # accuracy_f.append(np.mean(self.predict(X_train_no_bias) == t_val))
            # }

    # Task:Loss {
    def mse_loss(self):
```

```python
        return self.loss

    def accuracy_func(self):
        return self.accuracy_f
    # }

    def predict(self, X, threshold=0.5):
        """X is a Kxm matrix for some K>=1
        predict the value for each point in X"""
        if self.bias:
            X = add_bias(X, self.bias)
        ys = X @ self.weights
        return ys > threshold
```

We can train and test a first classifier.

In [ ]:
```python
def accuracy_(predicted, gold):
    return np.mean(predicted == gold)
```

In [ ]:
```python
cl = NumpyLinRegClass()
cl.fit(X_train, t2_train, X_val, t2_val)
print(accuracy_(cl.predict(X_val), t2_val))
print(cl.accuracy(cl.predict(X_val), t2_val))
```

```
0.522
0.522
```

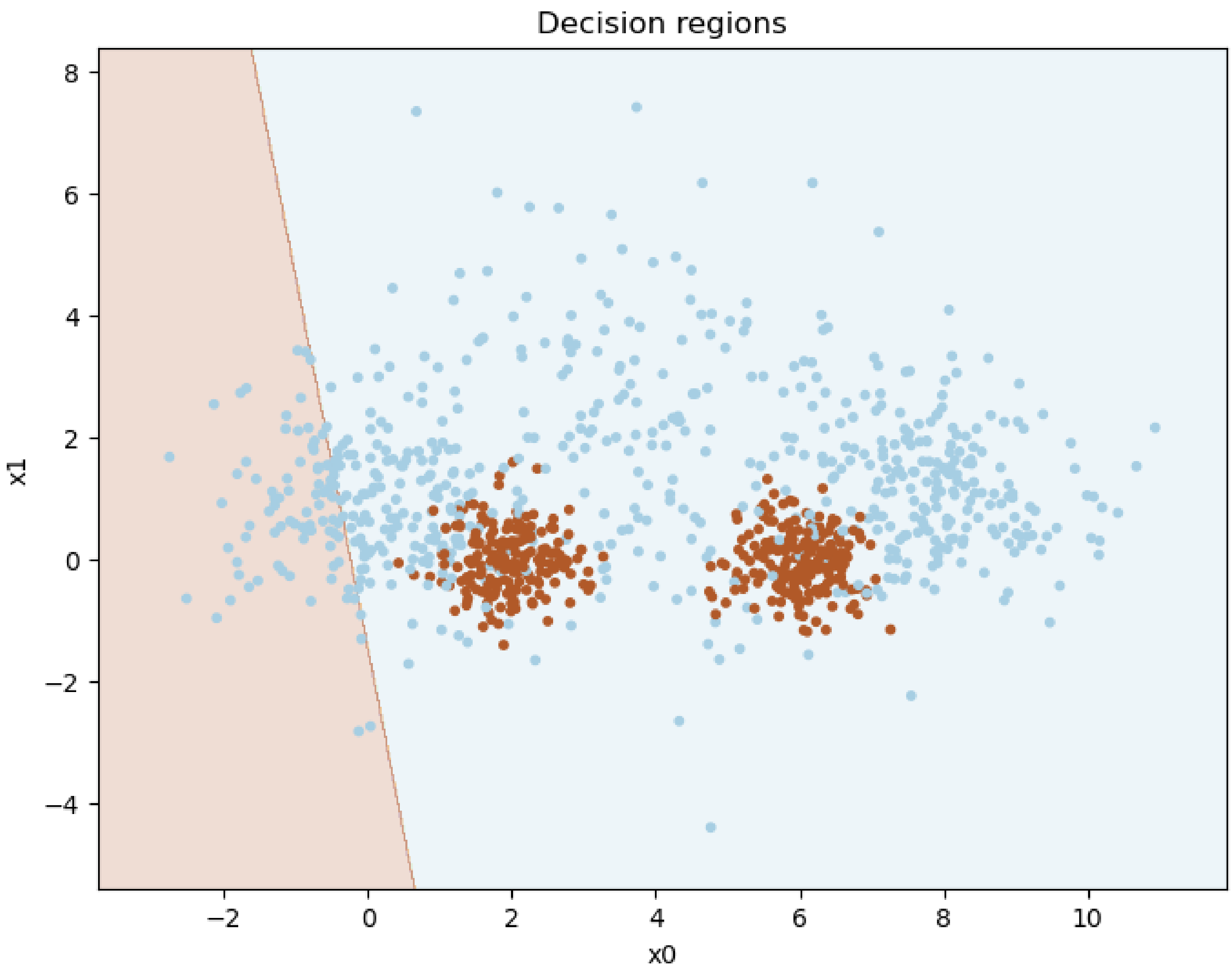
The following is a small procedure which plots the data set together with the decision boundaries. You may modify the colors and the rest of the graphics as you like. The procedure will also work for multi-class classifiers

In [ ]:
```python
def plot_decision_regions(X, t, clf=[], size=(8,6)):
    """Plot the data set (X,t) together with the decision boundary of the classifier clf"""
    # The region of the plane to consider determined by X
    x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1

    # Make a make of the whole region
    h = 0.02  # step size in the mesh
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))
```

```
    Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
    # Classify each meshpoint.
    Z = Z.reshape(xx.shape)

    plt.figure(figsize=size) # You may adjust this

    # Put the result into a color plot
    plt.contourf(xx, yy, Z, alpha=0.2, cmap = 'Paired')

    plt.scatter(X[:,0], X[:,1], c=t, s=10.0, cmap='Paired')

    plt.xlim(xx.min(), xx.max())
    plt.ylim(yy.min(), yy.max())
    plt.title("Decision regions")
    plt.xlabel("x0")
    plt.ylabel("x1")

#    plt.show()
```

In [ ]:

```
plot_decision_regions(X_train, t2_train, cl)
```



Task: Tuning

The result is far from impressive. Remember that a classifier which always chooses the majority class will have an accuracy of 0.6 on this data set.

Your task is to try various settings for the two training hyper-parameters, *eta* and *epochs*, to get the best accuracy on the validation set.

Report how the accuracy vary with the hyper-parameter settings. It it not sufficient to give the final hyperparemters. You must also show how you found them and results for alternative values you tried out.

When you are satisfied with the result, you may plot the decision boundaries, as above.

In [ ]:

```
# Finding best hyper-parameters
def test_hyperpars(X_train, t_train, X_val, t_val, eta_values, epoch_values, cl = cl):
    # Testing various eta and epochs to increase accuracy

    for et in eta_values:
        for ep in epoch_values:
            cl.fit(X_train, t_train, X_val, t_val, eta = et, epochs = ep)
            print(f"eta: {et:5}    epochs: {ep:5}    accuracy: {cl.accuracy(cl.predict(X_val), t_val):8.3f}"
```

In [ ]:

```
eta_values = [0.3, 0.1, 0.05, 0.01, 0.005, 0.001]
epoch_values = [10, 50, 100, 500, 1000, 5000]

test_hyperpars(X_train, t2_train, X_val, t2_val, eta_values, epoch_values)
```

```
eta:    0.3    epochs:    10    accuracy:    0.516
eta:    0.3    epochs:    50    accuracy:    0.516
eta:    0.3    epochs:   100    accuracy:    0.516
eta:    0.3    epochs:   500    accuracy:    0.576
```

```
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/1619145062.py:5: RuntimeWarning: overflow e
ncountered in double_scalars
  return sum((x - y)**2)/x.shape[0]
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/1619145062.py:5: RuntimeWarning: overflow e
ncountered in square
  return sum((x - y)**2)/x.shape[0]
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/2818609812.py:28: RuntimeWarning: overflow
encountered in matmul
  loss.append(self.MSE(t_train, X_train @ weights))
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/2818609812.py:48: RuntimeWarning: overflow
encountered in matmul
  ys = X @ self.weights
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/2818609812.py:26: RuntimeWarning: overflow
encountered in matmul
  weights -= eta / N * X_train.T @ (X_train @ weights - t_train)
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/2818609812.py:26: RuntimeWarning: invalid v
alue encountered in matmul
  weights -= eta / N * X_train.T @ (X_train @ weights - t_train)
eta:   0.3    epochs:  1000    accuracy:    0.576
eta:   0.3    epochs:  5000    accuracy:    0.576
eta:   0.1    epochs:    10    accuracy:    0.522
eta:   0.1    epochs:    50    accuracy:    0.516
eta:   0.1    epochs:   100    accuracy:    0.516
eta:   0.1    epochs:   500    accuracy:    0.516
eta:   0.1    epochs:  1000    accuracy:    0.516
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/2818609812.py:28: RuntimeWarning: invalid v
alue encountered in matmul
  loss.append(self.MSE(t_train, X_train @ weights))
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/2818609812.py:48: RuntimeWarning: invalid v
alue encountered in matmul
  ys = X @ self.weights
```

```
eta:   0.1    epochs:  5000    accuracy:    0.576
eta:  0.05    epochs:    10    accuracy:    0.506
eta:  0.05    epochs:    50    accuracy:    0.590
eta:  0.05    epochs:   100    accuracy:    0.658
eta:  0.05    epochs:   500    accuracy:    0.704
eta:  0.05    epochs:  1000    accuracy:    0.704
eta:  0.05    epochs:  5000    accuracy:    0.704
eta:  0.01    epochs:    10    accuracy:    0.502
eta:  0.01    epochs:    50    accuracy:    0.504
eta:  0.01    epochs:   100    accuracy:    0.560
eta:  0.01    epochs:   500    accuracy:    0.658
eta:  0.01    epochs:  1000    accuracy:    0.686
eta:  0.01    epochs:  5000    accuracy:    0.704
eta: 0.005    epochs:    10    accuracy:    0.576
eta: 0.005    epochs:    50    accuracy:    0.478
eta: 0.005    epochs:   100    accuracy:    0.504
eta: 0.005    epochs:   500    accuracy:    0.588
eta: 0.005    epochs:  1000    accuracy:    0.658
eta: 0.005    epochs:  5000    accuracy:    0.704
eta: 0.001    epochs:    10    accuracy:    0.576
eta: 0.001    epochs:    50    accuracy:    0.576
eta: 0.001    epochs:   100    accuracy:    0.522
eta: 0.001    epochs:   500    accuracy:    0.504
eta: 0.001    epochs:  1000    accuracy:    0.560
eta: 0.001    epochs:  5000    accuracy:    0.658
```

The effect of the hyper-parameter settings on the accuracy is printed above.

A learning rate of 0.3 and 0.1 is too high and diverges. Lower epoch values result in lower accuracy independant of the learning rate.

An eta of 0.3 and 0.1 and few epochs yields the lowest accuracies of 0.516.

Optimizing test intervals: The higest accuracy with the least amount of epochs is: eta: 0.05 epochs: 500 accuracy: 0.704

Testing in the intervals around these values.

In [ ]:
```
eta_values = [0.1, 0.04, 0.01, 0.009, 0.008, 0.005]
epoch_values = [100, 250, 350, 500, 1000, 5000]
```

```python
test_hyperpars(X_train, t2_train, X_val, t2_val, eta_values, epoch_values)
```

```
eta:   0.1    epochs:   100    accuracy:    0.516
eta:   0.1    epochs:   250    accuracy:    0.516
eta:   0.1    epochs:   350    accuracy:    0.516
eta:   0.1    epochs:   500    accuracy:    0.516
eta:   0.1    epochs:  1000    accuracy:    0.516
```

```
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/1619145062.py:5: RuntimeWarning: overflow e
ncountered in double_scalars
  return sum((x - y)**2)/x.shape[0]
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/1619145062.py:5: RuntimeWarning: overflow e
ncountered in square
  return sum((x - y)**2)/x.shape[0]
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/1541892746.py:28: RuntimeWarning: overflow
encountered in matmul
  loss.append(self.MSE(t_train, X_train @ weights))
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/1541892746.py:48: RuntimeWarning: overflow
encountered in matmul
  ys = X @ self.weights
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/1541892746.py:26: RuntimeWarning: overflow
encountered in matmul
  weights -= eta / N * X_train.T @ (X_train @ weights - t_train)
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/1541892746.py:26: RuntimeWarning: invalid v
alue encountered in matmul
  weights -= eta / N * X_train.T @ (X_train @ weights - t_train)
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/1541892746.py:28: RuntimeWarning: invalid v
alue encountered in matmul
  loss.append(self.MSE(t_train, X_train @ weights))
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_77971/1541892746.py:48: RuntimeWarning: invalid v
alue encountered in matmul
  ys = X @ self.weights
```

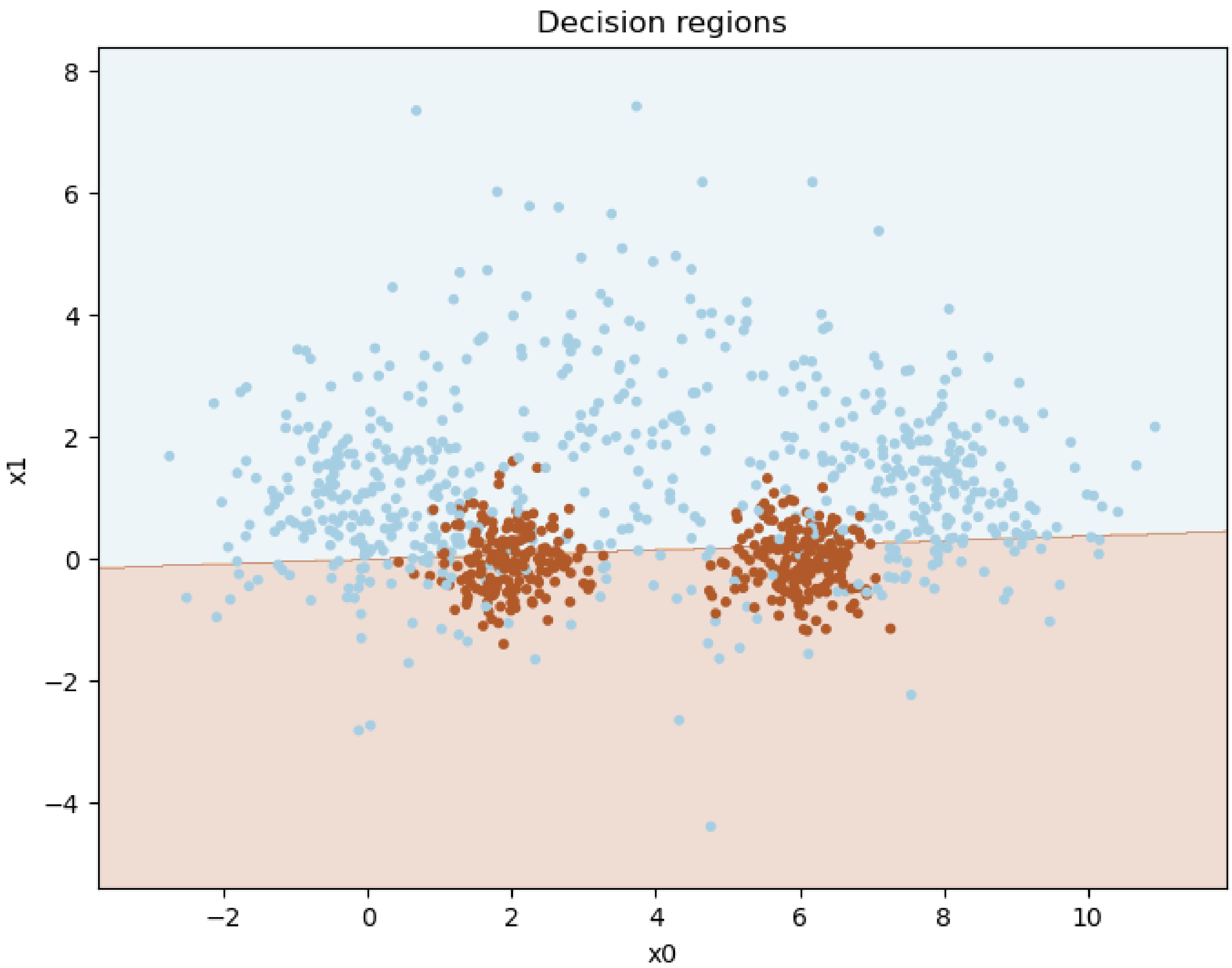
```
eta:   0.1    epochs:  5000    accuracy:    0.576
eta:  0.04    epochs:   100    accuracy:    0.634
eta:  0.04    epochs:   250    accuracy:    0.686
eta:  0.04    epochs:   350    accuracy:    0.704
eta:  0.04    epochs:   500    accuracy:    0.704
eta:  0.04    epochs:  1000    accuracy:    0.704
eta:  0.04    epochs:  5000    accuracy:    0.704
eta:  0.01    epochs:   100    accuracy:    0.560
eta:  0.01    epochs:   250    accuracy:    0.588
eta:  0.01    epochs:   350    accuracy:    0.622
eta:  0.01    epochs:   500    accuracy:    0.658
eta:  0.01    epochs:  1000    accuracy:    0.686
eta:  0.01    epochs:  5000    accuracy:    0.704
eta: 0.009    epochs:   100    accuracy:    0.544
eta: 0.009    epochs:   250    accuracy:    0.588
eta: 0.009    epochs:   350    accuracy:    0.620
eta: 0.009    epochs:   500    accuracy:    0.650
eta: 0.009    epochs:  1000    accuracy:    0.692
eta: 0.009    epochs:  5000    accuracy:    0.704
eta: 0.008    epochs:   100    accuracy:    0.530
eta: 0.008    epochs:   250    accuracy:    0.586
eta: 0.008    epochs:   350    accuracy:    0.602
eta: 0.008    epochs:   500    accuracy:    0.634
eta: 0.008    epochs:  1000    accuracy:    0.686
eta: 0.008    epochs:  5000    accuracy:    0.704
eta: 0.005    epochs:   100    accuracy:    0.504
eta: 0.005    epochs:   250    accuracy:    0.560
eta: 0.005    epochs:   350    accuracy:    0.586
eta: 0.005    epochs:   500    accuracy:    0.588
eta: 0.005    epochs:  1000    accuracy:    0.658
eta: 0.005    epochs:  5000    accuracy:    0.704
```

The optimal accuracy using a low enough learning rate and the lowest amount of epochs in this test:

eta: 0.04 epochs: 350 accuracy: 0.704

In [ ]:

```
best_eta = 0.04
best_epochs = 350
cl.fit(X_train, t2_train, X_val, t2_val, eta = best_eta, epochs = best_epochs)
plot_decision_regions(X_train, t2_train, cl)
```



Task: Loss

The linear regression classifier is trained with mean squared error loss. So far, we have not calculated the loss explicitly in the code. Extend the code to calculate the loss on the training set for each epoch and to store the losses such that the losses can be inspected after training.

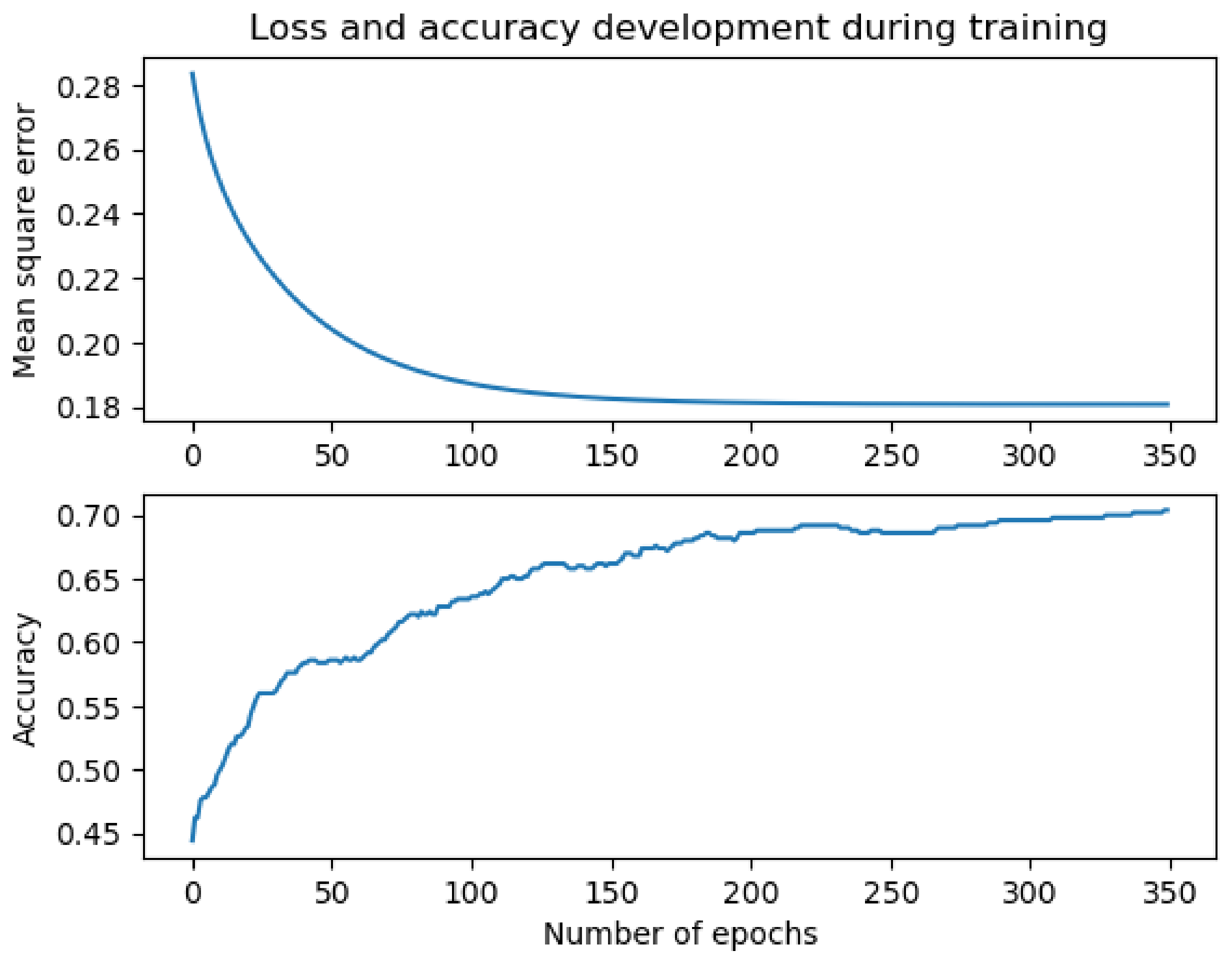
Also extend the classifier to calculate the accuracy on the training data after each epoch.

Train a classifier with your best settings from last point. After training, plot the loss as a function of the number of epochs. Then plot the accuracy as a function of the number of epochs.

Comment on what you see: Are the function monotone? Is this as expected?

In [ ]:

```
# Training with best hyper-parameter settings
cl.fit(X_train, t2_train, X_val, t2_val, eta = best_eta, epochs = best_epochs)

losses = cl.mse_loss()
accuracies = cl.accuracy_func()


fig, ax = plt.subplots(2)
ax[0].plot(range(best_epochs), losses)
ax[0].set_ylabel('Mean square error')
ax[1].plot(range(best_epochs), accuracies)
ax[1].set_ylabel('Accuracy')
ax[1].set_xlabel('Number of epochs')
ax[0].set_title('Loss and accuracy development during training')
plt.show()
```



Comment:

The functions are monotone which is to be expected. With a learning rate that is too high the function will generally not be monotone.

Task: Scaling

we have seen in the lectures that scaling the data may improve training speed.

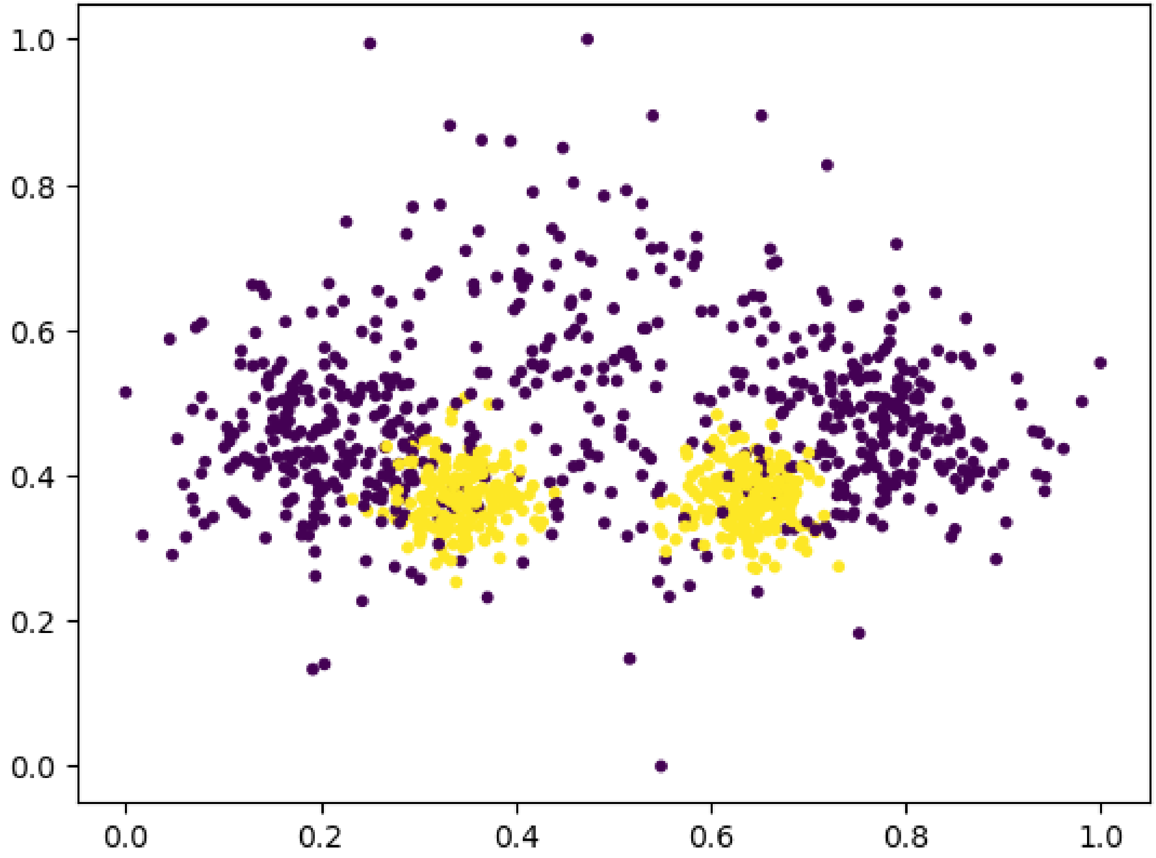
- Implement a scaler, either standard scaler (normalizer) or max-min scaler
- Scale the data
- Train the model on the scaled data
- Experiment with hyper-parameter settings and see whether you can speed up the training.
- Report final hyper-meter settings and show how you found them.
- Plot the loss curve and the accuracy curve for the classifier trained on scaled data with the best settings you found.

In [ ]:
```
# Implementing scaler (normalizer)

def scale(data):
    mean = np.mean(data, axis = 0)
    std = np.std(data, axis = 0)

    scaled = (data - mean)/std

    return scaled
```

In [ ]:
```
# Scaler from w8 (minmax scaler)

class MMScaler():

    def fit(self, X_train):
        self.maxes = np.max(X_train, axis=0)
        self.mins = np.min(X_train, axis=0)

    def transform(self, X):
        return (X - self.mins)/(self.maxes - self.mins)
```

In [ ]:
```
# Scaling data
# scaled_X_train = scale(X_train)
# scaled_X_val = scale(X_val)

sc = MMScaler()
sc.fit(X_train)
scaled_X_train = sc.transform(X_train)

sc.fit(X_val)
```

```python
scaled_X_val = sc.transform(X_val)


# Plotting the scaled data
plt.scatter(scaled_X_train[:, 0], scaled_X_train[:, 1], c = t2_train, s = 10.0)
plt.show()


# from sklearn.preprocessing import StandardScaler

# scaler = StandardScaler()

# scaler.fit(X_train)
# s_X_train = scaler.transform(X_train)

# scaler.fit(t2_train.reshape(1, -1))
# s_t2_train = scaler.transform(t2_train.reshape(1, -1))

# scaler.fit(X_val)
# s_X_val = scaler.transform(X_val)

# scaler.fit(t2_val.reshape(1, -1))
# s_t2_val = scaler.transform(t2_val.reshape(1, -1))

# plt.scatter(scaled_X_train[:, 0], scaled_X_train[:, 1], c = scaled_t2_train, s = 10.0)
# plt.show()

# print(1, scaled_t2_train[-1])
# print(2, s_t2_train[-1])
```



Experimenting with hyper-parameter settings.

In [ ]:
```
cl_s = NumpyLinRegClass()
cl_s.fit(scaled_X_train, t2_train, scaled_X_val, t2_val)
print(accuracy_(cl_s.predict(scaled_X_val), t2_val))
print(cl_s.accuracy(cl_s.predict(scaled_X_val), t2_val))
```

```
0.576
0.576
```

The accuracy has improved by just using the scaled data.

In [ ]:

```
eta_values = [0.5, 0.4, 0.3, 0.2, 0.1, 0.05]
epoch_values = [10, 30, 100, 200, 1000, 5000]


test_hyperpars(scaled_X_train, t2_train, scaled_X_val, t2_val, eta_values, epoch_values)
```

```
eta:   0.5    epochs:    10    accuracy:    0.576
eta:   0.5    epochs:    30    accuracy:    0.566
eta:   0.5    epochs:   100    accuracy:    0.770
eta:   0.5    epochs:   200    accuracy:    0.742
eta:   0.5    epochs:  1000    accuracy:    0.690
eta:   0.5    epochs:  5000    accuracy:    0.690
eta:   0.4    epochs:    10    accuracy:    0.576
eta:   0.4    epochs:    30    accuracy:    0.576
eta:   0.4    epochs:   100    accuracy:    0.738
eta:   0.4    epochs:   200    accuracy:    0.752
eta:   0.4    epochs:  1000    accuracy:    0.690
eta:   0.4    epochs:  5000    accuracy:    0.690
eta:   0.3    epochs:    10    accuracy:    0.576
eta:   0.3    epochs:    30    accuracy:    0.576
eta:   0.3    epochs:   100    accuracy:    0.652
eta:   0.3    epochs:   200    accuracy:    0.768
eta:   0.3    epochs:  1000    accuracy:    0.690
eta:   0.3    epochs:  5000    accuracy:    0.690
eta:   0.2    epochs:    10    accuracy:    0.576
eta:   0.2    epochs:    30    accuracy:    0.576
eta:   0.2    epochs:   100    accuracy:    0.560
eta:   0.2    epochs:   200    accuracy:    0.738
eta:   0.2    epochs:  1000    accuracy:    0.706
eta:   0.2    epochs:  5000    accuracy:    0.690
eta:   0.1    epochs:    10    accuracy:    0.576
eta:   0.1    epochs:    30    accuracy:    0.576
eta:   0.1    epochs:   100    accuracy:    0.576
eta:   0.1    epochs:   200    accuracy:    0.560
eta:   0.1    epochs:  1000    accuracy:    0.742
eta:   0.1    epochs:  5000    accuracy:    0.690
eta:  0.05    epochs:    10    accuracy:    0.576
eta:  0.05    epochs:    30    accuracy:    0.576
eta:  0.05    epochs:   100    accuracy:    0.576
eta:  0.05    epochs:   200    accuracy:    0.576
eta:  0.05    epochs:  1000    accuracy:    0.770
eta:  0.05    epochs:  5000    accuracy:    0.692
```

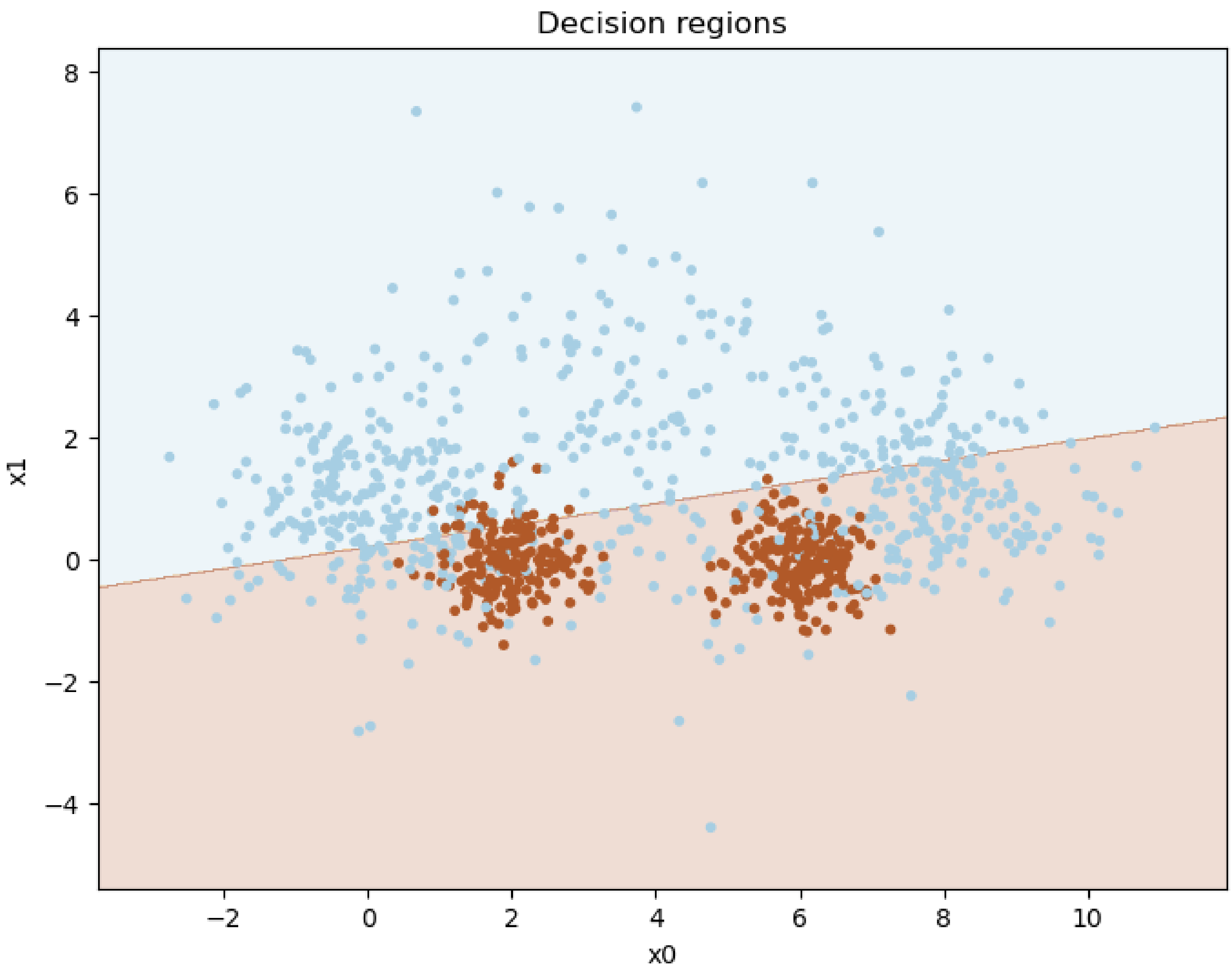
The accuracy improves comparing to the non-scaled data, with the highest accuracy in the test above being 0.770

I am choosing a low enough eta to ensure a monotone loss function, with the lowest number of epochs:

eta: 0.5 epochs: 100 accuracy: 0.770

In [ ]:
```
# Training with best hyper-parameter settings for scaled data
best_scaled_eta = 0.5
best_scaled_epochs = 100

cl = NumpyLinRegClass()
cl.fit(scaled_X_train, t2_train, scaled_X_val, t2_val, eta = best_scaled_eta, epochs = best_scaled_epochs)
print('Accuracy using scaled data: ',cl.accuracy_func()[-1])
print('Optimal eta and epochs values:', '\neta: ', best_scaled_eta, '\nepochs: ', best_scaled_epochs)

plot_decision_regions(X_train, t2_train, cl)
```

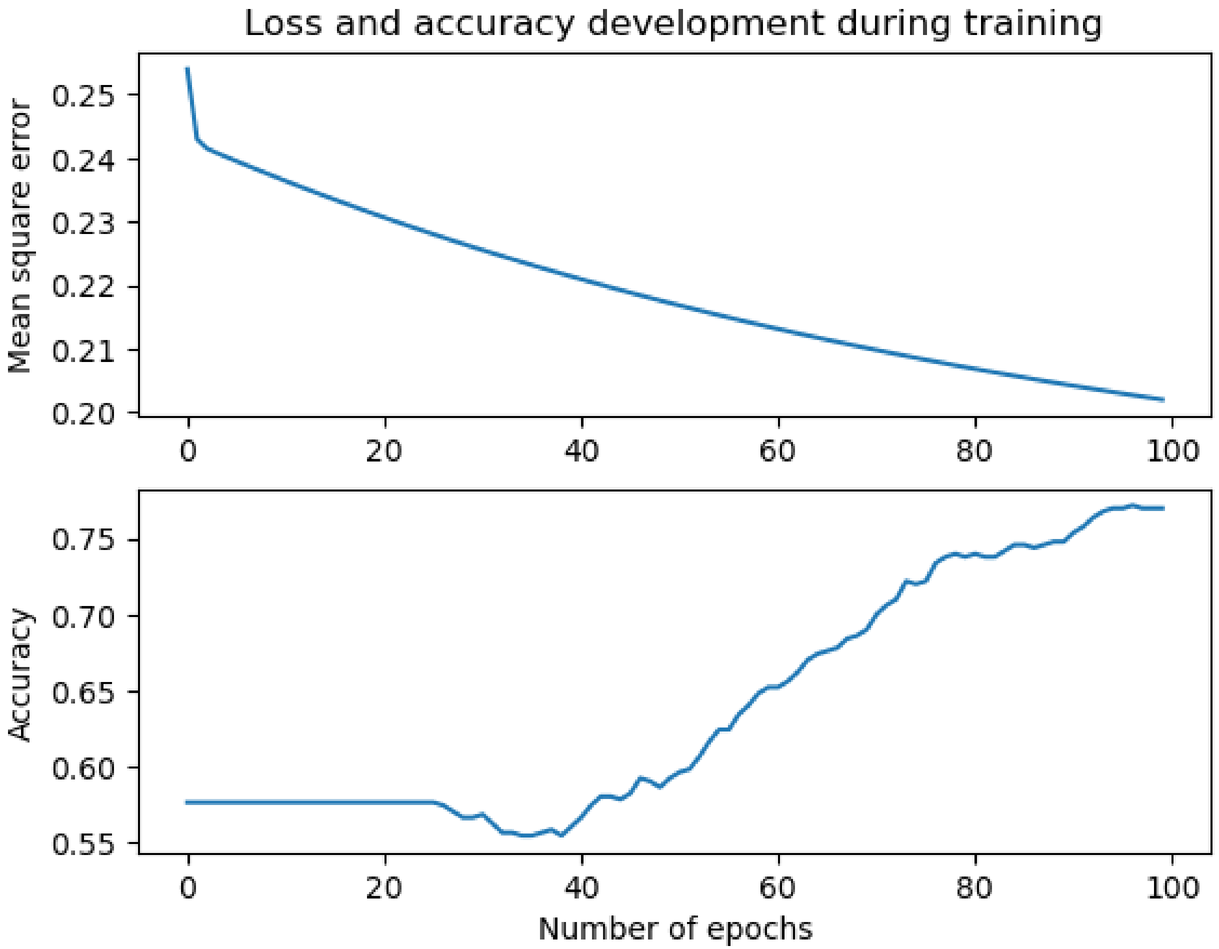
```
Accuracy using scaled data:  0.77
Optimal eta and epochs values:
eta:  0.5
epochs:  100
```



This result is better than with the unscaled data. The accuracy is high even for a lower number of epochs.

In [ ]:
```
# plotting the loss and the accuracy curve for the classifier trained
# on scaled data with the best settings found.
losses = cl.mse_loss()
```

```
accuracies = cl.accuracy_func()

fig, ax = plt.subplots(2)
ax[0].plot(range(best_scaled_epochs), losses)
ax[0].set_ylabel('Mean square error')
ax[1].plot(range(best_scaled_epochs), accuracies)
ax[1].set_ylabel('Accuracy')
ax[1].set_xlabel('Number of epochs')
ax[0].set_title('Loss and accuracy development during training')
plt.show()
```



The loss function is not as monotone as expected for the first few values, which might be an indexing error that I have not been able to identify. The accuracy function is not monotone. There might be an error in calculating the accuracy.

Logistic regression

a) You should now implement a logistic regression classifier similarly to the classifier based on linear regression. You may use code from the solution to weekly exercise set week07.

b) In addition to the method `predict` which predicts a class for the data, include a method `predict_probability` which predicts the probability of the data belonging to the positive class.

c) As with the classifier based on linear regression, we want to calculate loss and accuracy after each epoch. The prefered loss for logistic regression is binary cross-entropy. You could have used mean squared error. The most important is that your implementation of the loss corresponds to your implementation of the gradient descent.

d) In addition, extend the fit-method with optional arguments for a validation set (X_val, t_val). If a validation set is included in the call to fit, calculate the loss and the accuracy for the validation set after each epoch.

e) The training runs for a number of epochs. We cannot know beforehand for how many epochs it is reasonable to run the training. One possibility is to run the training until the learning does not improve much. Extend the fit-method with two keyword arguments, `tol` and `n_epochs_no_update` and stop training when the loss has not improved with more than `tol` after running `n_epochs_no_update` epochs. A possible default value for `n_epochs_no_update` is 5. Also, add an attribute to the classifier which tells us after fitting how many epochs were ran.

f) Train classifiers with various learning rates, and with varying values for `tol` for finding optimal values. Also consider the effect of scaling the data.

g) After a succesful training, plot both training loss and validation loss as functions of the number of epochs in one figure, and both accuracies as functions of the number of epochs in another figure. Comment on what you see.

In [ ]:

```python
# Week 7 solution + my own additions:

class NumpyLogReg(NumpyClassifier):

    def __init__(self, bias = 0):
        self.bias = bias

    def fit(self, X_train, t_train, X_val, t_val, tol, eta = 0.1, n_epochs_no_update = 5):
        """X_train is a Nxm matrix, N data points, m features
        t_train is avector of length N,
        the targets values for the training data"""

        X_train_no_bias = np.copy(X_train)

        (N, m) = X_train.shape
        X_train = add_bias(X_train, 0)

        self.weights = weights = np.zeros(m+1)
        self.loss = loss = []
        self.val_loss = val_loss = []
        self.lr_accuracy = lr_accuracy = []

        i = 0
        test_loss = tol - 1
        for _ in range(200):
            i += 1
            if i > 15 and test_loss < tol:
                # print(f'{n_epochs_no_update} epochs were ran')
                # print('Accuracy: ', e_accuracy[-1])
                self.final_epochs = n_epochs_no_update
                break

            for e in range(n_epochs_no_update):
                weights -= eta / N *  X_train.T @ (self.forward(X_train) - t_train)
                loss.append(self.BCE(X_train, t_train, self.predict(X_train_no_bias)))
                val_loss.append(self.BCE(X_train, t_train, self.predict(X_val)))
                lr_accuracy.append(np.mean(self.predict(X_val) == t_val))
            test_loss = np.abs(loss[-1] - loss[(n_epochs_no_update - 5)])
            n_epochs_no_update += 1
```

```python
    def get_number_of_epochs(self):
        return self.final_epochs

    def forward(self, X):
        return self.logistic(X @ self.weights)

    def predict(self, x, threshold=0.5):
        """X is a Kxm matrix for some K>=1
        predict the value for each point in X"""
        z = add_bias(x, 0)
        return (self.forward(z) > threshold).astype('int')

    def predict_probability(self, x):
        """Predicts probability of the data
        belonging to the positive class"""
        s = add_bias(x, 0)
        return self.logistic(s @ self.weights)

    def bce_loss(self):
        return self.loss[-self.final_epochs:]

    def validation_loss(self):
        return self.val_loss[-self.final_epochs:]

    def accuracy_lr(self):
        return self.lr_accuracy[-self.final_epochs:]
```

In [ ]:

```python
lr_cl = NumpyLogReg()
lr_cl.fit(X_train, t2_train, X_val, t2_val, 0.5)

print(lr_cl.accuracy_lr()[-1], lr_cl.get_number_of_epochs())
```

```
0.706 202
```

In 202 epochs the logistic regression model found an accuracy of 0.706 with an eta of 0.1 and a tolerance of 0.5. This is the best value yet for the first test of any model.

In [ ]:

```
# Finding best hyper-parameters
def test_eta_tol(X_train, t_train, X_val, t_val, eta_values, tol_values):
    # Testing various eta and tolerances to increase accuracy

    # varying eta and tol and printing accuracy

    for et in eta_values:
        for tol in tol_values:
            lr_cl.fit(X_train, t_train, X_val, t_val, tol, eta = et)
            print(f"eta: {et:7}    epochs: {lr_cl.get_number_of_epochs():4}    tol: {tol:5}    accuracy: {lr
```

In [ ]:

```
eta_values = [3, 2, 1, 0.1, 0.01, 0.001]
tol_values = [0.001, 0.05, 0.1, 0.5, 1, 1.5]

test_eta_tol(X_train, t2_train, X_val, t2_val, eta_values, tol_values)
```

```
eta:       3    epochs:  202    tol: 0.001    accuracy:    0.712
eta:       3    epochs:  173    tol:  0.05    accuracy:    0.712
eta:       3    epochs:  149    tol:   0.1    accuracy:    0.712
eta:       3    epochs:   85    tol:   0.5    accuracy:    0.712
eta:       3    epochs:   69    tol:     1    accuracy:    0.712
eta:       3    epochs:   65    tol:   1.5    accuracy:    0.712
eta:       2    epochs:   65    tol: 0.001    accuracy:    0.712
eta:       2    epochs:  161    tol:  0.05    accuracy:    0.712
eta:       2    epochs:  137    tol:   0.1    accuracy:    0.712
eta:       2    epochs:   38    tol:   0.5    accuracy:    0.574
eta:       2    epochs:   38    tol:     1    accuracy:    0.574
eta:       2    epochs:   34    tol:   1.5    accuracy:    0.574
eta:       1    epochs:  178    tol: 0.001    accuracy:    0.588
eta:       1    epochs:  102    tol:  0.05    accuracy:    0.588
eta:       1    epochs:   90    tol:   0.1    accuracy:    0.588
eta:       1    epochs:   21    tol:   0.5    accuracy:    0.716
eta:       1    epochs:   21    tol:     1    accuracy:    0.716
eta:       1    epochs:   21    tol:   1.5    accuracy:    0.716
eta:     0.1    epochs:   21    tol: 0.001    accuracy:    0.706
eta:     0.1    epochs:   21    tol:  0.05    accuracy:    0.706
eta:     0.1    epochs:   21    tol:   0.1    accuracy:    0.706
eta:     0.1    epochs:  202    tol:   0.5    accuracy:    0.706
eta:     0.1    epochs:  131    tol:     1    accuracy:    0.706
eta:     0.1    epochs:   92    tol:   1.5    accuracy:    0.706
eta:    0.01    epochs:   92    tol: 0.001    accuracy:    0.706
eta:    0.01    epochs:   92    tol:  0.05    accuracy:    0.706
eta:    0.01    epochs:   92    tol:   0.1    accuracy:    0.706
eta:    0.01    epochs:   92    tol:   0.5    accuracy:    0.706
eta:    0.01    epochs:   92    tol:     1    accuracy:    0.706
eta:    0.01    epochs:   92    tol:   1.5    accuracy:    0.706
eta:   0.001    epochs:   92    tol: 0.001    accuracy:    0.704
eta:   0.001    epochs:   92    tol:  0.05    accuracy:    0.704
eta:   0.001    epochs:   92    tol:   0.1    accuracy:    0.704
eta:   0.001    epochs:   92    tol:   0.5    accuracy:    0.704
eta:   0.001    epochs:   92    tol:     1    accuracy:    0.704
eta:   0.001    epochs:   20    tol:   1.5    accuracy:    0.542
```

There seems to be a maximum accuracy of 0.716 when training the unscaled data by logistic regression. I expected to find a higher accuracy for this model.

choosing hyper-parameters for a faster training (few epochs) and the highest accuracy

eta: 1 epochs: 21 tol: 0.5 accuracy: 0.716

In [ ]:
```
best_tol = 0.5
best_eta = 1

lr_cl.fit(X_train, t2_train, X_val, t2_val, best_tol, eta = best_eta)

print(lr_cl.get_number_of_epochs(), lr_cl.accuracy_lr()[-1])
```

```
21 0.716
```

A high accuracy is found in few epochs, but this accuracy value could be higher considering the much higher accuracy found in linear regression on scaled data.

In [ ]:
```
# Same test on scaled data:
eta_values = [1, 0.1, 0.01]
tol_values = [0.05, 0.1, 0.5]

test_eta_tol(scaled_X_train, t2_train, scaled_X_val, t2_val, eta_values, tol_values)
```

```
eta:       1    epochs:  175    tol:  0.05    accuracy:    0.658
eta:       1    epochs:   24    tol:   0.1    accuracy:    0.672
eta:       1    epochs:   20    tol:   0.5    accuracy:    0.654
eta:     0.1    epochs:   20    tol:  0.05    accuracy:    0.658
eta:     0.1    epochs:  284    tol:   0.1    accuracy:    0.658
eta:     0.1    epochs:   20    tol:   0.5    accuracy:    0.576
eta:    0.01    epochs:   20    tol:  0.05    accuracy:    0.658
eta:    0.01    epochs:   20    tol:   0.1    accuracy:    0.576
eta:    0.01    epochs:   20    tol:   0.5    accuracy:    0.576
```

Neither of these accuracies are good compared to earlier values. The logistic regression model seems to do a lot better on unscaled data.

Choosing the hyper-parameters yielding the fewest epochs and highest accuracy.

eta: 1 epochs: 24 tol: 0.1 accuracy: 0.672

In [ ]:
```
best_tol = 0.1
best_eta = 1

lr_cl.fit(scaled_X_train, t2_train, scaled_X_val, t2_val, best_tol, eta = best_eta)

print(lr_cl.get_number_of_epochs(), lr_cl.accuracy_lr()[-1])
```

```
24 0.672
```

The best accuracy in the logistic regression model is 0.672, found in 24 epochs. The logistic regression model performs better, at least on this data, with unscaled data.

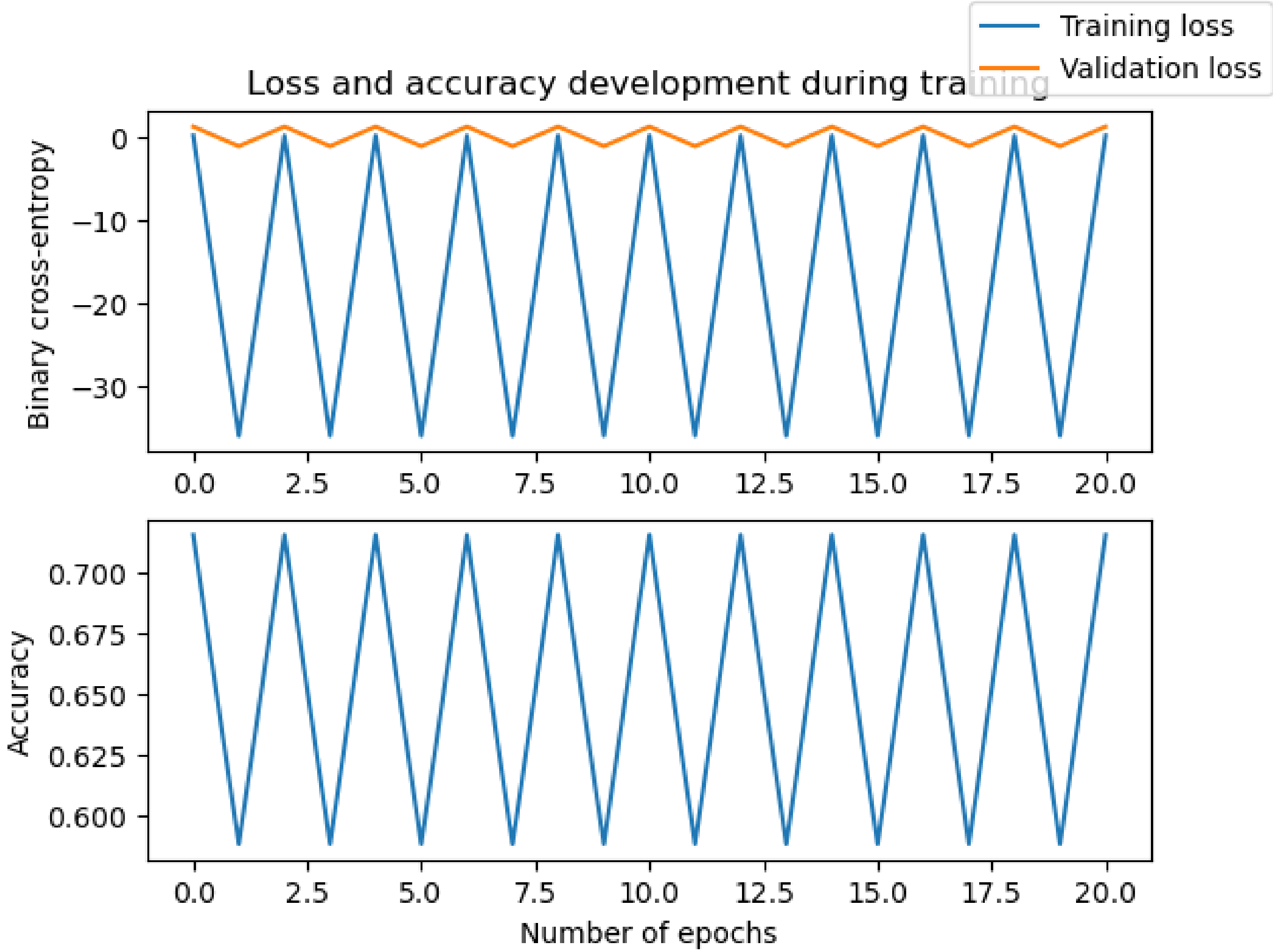
In [ ]:
```
# Final training using the best data (not scaled)

best_tol = 0.5
best_eta = 1

lr_cl.fit(X_train, t2_train, X_val, t2_val, best_tol, best_eta)

print(lr_cl.accuracy_lr()[-1], lr_cl.get_number_of_epochs())
```

```
0.716 21
```

In [ ]:
```
# Plotting training loss and validation loss vs. epochs
# Plotting accuracies vs. epochs

epochs = lr_cl.get_number_of_epochs()
losses = lr_cl.bce_loss()
print(losses)
val_losses = lr_cl.validation_loss()
accuracies = lr_cl.accuracy_lr()

fig, ax = plt.subplots(2)
ax[0].plot(range(epochs), losses, label = 'Training loss')
ax[0].plot(range(epochs), val_losses, label = 'Validation loss')
ax[0].set_ylabel('Binary cross-entropy')
ax[1].plot(range(epochs), accuracies)
ax[1].set_ylabel('Accuracy')
ax[1].set_xlabel('Number of epochs')
ax[0].set_title('Loss and accuracy development during training')
```

```
fig.legend()
plt.show()
```

```
[0.20243626179544197, -35.85568652275703, 0.202331436980664, -35.85562755044879, 0.20223704688233943, -35.85557444898159, 0.2021520531121397, -35.855526634039975, 0.2020755205525059, -35.85548357943804, 0.20200660709788104, -35.855444811340824, 0.20194455441278858, -35.855409903060675, 0.2018886796062587, -35.85537847036892, 0.20183836773249453, -35.85535016727345, 0.20179306503555264, -35.85532468221647, 0.20175227286487563]
```



This plot looks completely wrong.

Multi-class classifiers

We turn to the task of classifying when there are more than two classes, and the task is to ascribe one class to each input. We will now use the set (X, t_multi).

"One-vs-rest" with logistic regression

We saw in the lecture how a logistic regression classifier can be turned into a multi-class classifier using the one-vs-rest approach. We train one logistic regression classifier for each class. To predict the class of an item, we run all the binary classifiers and collect the probability score from each of them. We assign the class which ascribes the highest probability.

Build such a classifier. Train the resulting classifier on (X_train, t_multi_train), test it on (X_val, t_multi_val), tune the hyper-parameters and report the accuracy.

Also plot the decision boundaries for your best classifier similarly to the plots for the binary case.

In [ ]:
```
class OneVsRest():

    def OvRfit(self, X_train, t_multi_train, X_val, t_multi_val, tol, eta, C = 5):
        # C - number of classes
        self.C = C
        self.t_train_hot = t_train_hot = self.oneHot(t_multi_train)
        self.t_val_hot = t_val_hot = self.oneHot(t_multi_val)

        lc = NumpyLogReg()
        self.list_of_logclass = list_of_logclass = np.zeros(C)
        self.predictions = predictions = np.zeros(C)

        for i in range(C):
            list_of_logclass[i] = lc.fit(X_train, t_train_hot[:, i], X_val, t_val_hot[:, i], tol, eta)
            predictions[i] = lc.predict(X_val)

    def oneHot(self, X):
        x = np.zeros((X.size, X.max() + 1))
```

```
        x[np.arange(X.size), X] = 1
        return x

    def predict(self, X):
        # (n) shaped array of predictions, NOT onehot encoded
        return x

    def predict_probability(self, X):
        # (n x C) table of n inputs and C classes
        x = np.zeros(X.size) # X.size = n?
        for i in range(self.C):
            x[i] = self.list_of_logclass[i].forward(X)


    def get_predictions(self):
        return self.predictions
```

Part II Multi-layer neural networks

A first non-linear classifier

The following code it a simple implementation of a multi-layer perceptron. It is quite restricted. There is only one hidden layer. It can only handle binary classification. In addition, it uses a simple final layer similar to the linear regression classifier above. One way to look at it is what happens when we add a hidden layer to the linear regression classifier.

It can be used to make a non-linear classifier for the set (X, t2). Experiment with settings for learning rate and epochs and see how good results you can get. Report results for variouse settings. Be prepared to train for a looooong time. Plot the training set together with the decision regions as in part I.

In [ ]:
```
class MLPBinaryLinRegClass(NumpyClassifier):
    """A multi-layer neural network with one hidden layer"""
```

```
    def __init__(self, bias=-1, dim_hidden = 6):
        """Intialize the hyperparameters"""
        self.bias = bias
        self.dim_hidden = dim_hidden

        def logistic(x):
            return 1/(1+np.exp(-x))
        self.activ = logistic

        def logistic_diff(y):
            return y * (1 - y)
        self.activ_diff = logistic_diff

    def fit(self, X_train, t_train, eta=0.001, epochs = 100):
        """Intialize the weights. Train *epochs* many epochs.

        X_train is a Nxm matrix, N data points, m features
        t_train is a vector of length N of targets values for the training data,
        where the values are 0 or 1.
        """
        self.eta = eta

        T_train = t_train.reshape(-1,1)

        dim_in = X_train.shape[1]
        dim_out = T_train.shape[1]

        # Itilaize the wights
        self.weights1 = (np.random.rand(
            dim_in + 1,
            self.dim_hidden) * 2 - 1)/np.sqrt(dim_in)
        self.weights2 = (np.random.rand(
            self.dim_hidden+1,
            dim_out) * 2 - 1)/np.sqrt(self.dim_hidden)
        X_train_bias = add_bias(X_train, self.bias)

        for e in range(epochs):
            # One epoch
            hidden_outs, outputs = self.forward(X_train_bias)
            # The forward step
```

```python
            out_deltas = (outputs - T_train)
            # The delta term on the output node
            hiddenout_diffs = out_deltas @ self.weights2.T
            # The delta terms at the output of the jidden layer
            hiddenact_deltas = (hiddenout_diffs[:, 1:] *
                                self.activ_diff(hidden_outs[:, 1:]))
            # The deltas at the input to the hidden layer
            self.weights2 -= self.eta * hidden_outs.T @ out_deltas
            self.weights1 -= self.eta * X_train_bias.T @ hiddenact_deltas
            # Update the weights

    def forward(self, X):
        """Perform one forward step.
        Return a pair consisting of the outputs of the hidden_layer
        and the outputs on the final layer"""
        hidden_activations = self.activ(X @ self.weights1)
        hidden_outs = add_bias(hidden_activations, self.bias)
        outputs = hidden_outs @ self.weights2
        return hidden_outs, outputs

    def predict(self, X):
        """Predict the class for the mebers of X"""
        Z = add_bias(X, self.bias)
        forw = self.forward(Z)[1]
        score= forw[:, 0]
        return (score > 0.5)
```

In [ ]:
```python
def fit_accuracy(predicted, gold):
    return np.mean(predicted == gold)
```

In [ ]:
```python
mlp = MLPBinaryLinRegClass()
mlp.fit(X_train, t2_train)

print(fit_accuracy(mlp.predict(X_val), t2_val))
```

```
0.77
```

In [ ]:
```python
for epochs in [10, 20, 50, 100, 200, 500, 1000, 10000]:
    mlp = MLPBinaryLinRegClass()
    mlp.fit(X_train, t2_train, epochs=epochs)
```

```
    accuracy_mlp = fit_accuracy(mlp.predict(X_val), t2_val)
    print("epochs: {:8}  accuracy: {:10.3f}".format(epochs, accuracy_mlp))
```

```
epochs:       10  accuracy:      0.538
epochs:       20  accuracy:      0.576
epochs:       50  accuracy:      0.828
epochs:      100  accuracy:      0.648
epochs:      200  accuracy:      0.830
epochs:      500  accuracy:      0.792
epochs:     1000  accuracy:      0.838
epochs:    10000  accuracy:      0.858
```

In [ ]:

```
for eta in [10, 1, 0.1, 0.01, 0.001, 0.0001, 0.00001, 0.000001]:
    mlp = MLPBinaryLinRegClass()
    mlp.fit(X_train, t2_train, eta = eta)
    accuracy_mlp = fit_accuracy(mlp.predict(X_val), t2_val)
    print("eta: {:8}  accuracy: {:10.3f}".format(eta, accuracy_mlp))
```

```
eta:       10  accuracy:      0.576
eta:        1  accuracy:      0.576
eta:      0.1  accuracy:      0.576
eta:     0.01  accuracy:      0.424
eta:    0.001  accuracy:      0.782
eta:   0.0001  accuracy:      0.590
eta:    1e-05  accuracy:      0.548
eta:    1e-06  accuracy:      0.510
```

```
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_43913/3619876166.py:10: RuntimeWarning: overflow
encountered in exp
  return 1/(1+np.exp(-x))
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_43913/3619876166.py:46: RuntimeWarning: overflow
encountered in matmul
  hiddenout_diffs = out_deltas @ self.weights2.T
/var/folders/qp/v9d822514d39n5b42h3_wy100000gn/T/ipykernel_43913/3619876166.py:48: RuntimeWarning: invalid v
alue encountered in multiply
  hiddenact_deltas = (hiddenout_diffs[:, 1:] *
```

From the results above I am choosing to move forward in testing with an eta of 0.001

In [ ]:

```
n = 10
epochss = np.linspace(10, 100000, n, dtype = 'int')
```

```python
for epochs in epochss:
    mlp = MLPBinaryLinRegClass()
    mlp.fit(X_train, t2_train, eta = 0.001, epochs = epochs)
    accuracy_mlp = fit_accuracy(mlp.predict(X_val), t2_val)
    print("epochs: {:8}  accuracy: {:10.5f}".format(epochs, accuracy_mlp))
```

```
epochs:       10  accuracy:    0.54800
epochs:    11120  accuracy:    0.85400
epochs:    22230  accuracy:    0.88800
epochs:    33340  accuracy:    0.89400
epochs:    44450  accuracy:    0.84400
epochs:    55560  accuracy:    0.85200
epochs:    66670  accuracy:    0.85000
epochs:    77780  accuracy:    0.84200
epochs:    88890  accuracy:    0.82400
epochs:   100000  accuracy:    0.87000
```

In [ ]:

```python
n = 7
epochss = np.linspace(100000, 190000, n, dtype = 'int')

for epochs in epochss:
    mlp = MLPBinaryLinRegClass()
    mlp.fit(X_train, t2_train, eta = 0.001, epochs = epochs)
    accuracy_mlp = fit_accuracy(mlp.predict(X_val), t2_val)
    print("epochs: {:8}  accuracy: {:10.5f}".format(epochs, accuracy_mlp))
```

```
epochs:   100000  accuracy:    0.84000
epochs:   115000  accuracy:    0.88800
epochs:   130000  accuracy:    0.86200
epochs:   145000  accuracy:    0.86200
epochs:   160000  accuracy:    0.90200
epochs:   175000  accuracy:    0.85800
epochs:   190000  accuracy:    0.83800
```

In [ ]:

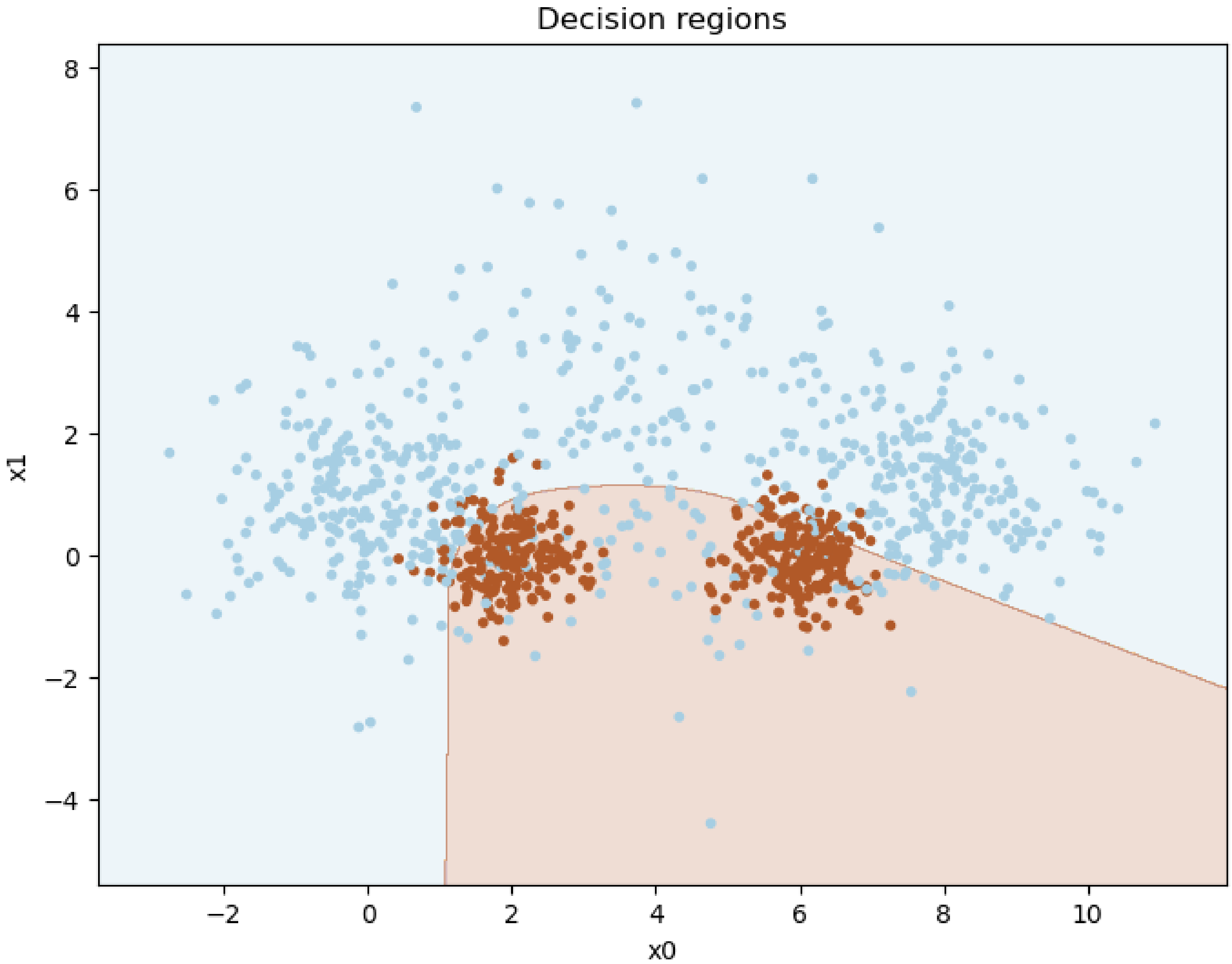
```python
mlp = MLPBinaryLinRegClass()
mlp.fit(X_train, t2_train, eta = 0.001, epochs = 500000)
accuracy_mlp = fit_accuracy(mlp.predict(X_val), t2_val)
print(accuracy_mlp)
```

```
0.88
```

In [ ]:
```
# As the accuracy is around 0.85 for epochs from 11 000 to 500 000,
# I will select the number of epochs to be 30 000

mlp = MLPBinaryLinRegClass()
mlp.fit(X_train, t2_train, eta = 0.001, epochs = 30000)
accuracy_mlp = fit_accuracy(mlp.predict(X_val), t2_val)
print(accuracy_mlp)
```

```
0.84
```

In [ ]:
```
# plotting decision regions

plot_decision_regions(X_train, t2_train, mlp)
```



Improving the classifier

You should now make changes to the classifier similarly to what you did with the logistic regression classifier in part 1.

a) In addition to the method predict, which predicts a class for the data, include a method predict_probability which predict the probability of the data belonging to the positive class. The training should be based on this value as with logistic regression.

b) Calculate the loss and the accuracy after each epoch and store them for inspection after training.

c) In addition, extend the fit-method with optional arguments for a validation set (X_val, t_val). If a validation set is included in the call to fit, calculate the loss and the accuracy for the validation set after each epoch.

d) The training runs for a number of epochs. We cannot know beforehand for how many epochs it is reasonable to run the training. One possibility is to run the training until the learning does not improve much. Extend the fit method with two keyword arguments, `tol` and `n_epochs_no_update` and stop training when the loss has not improved with more than tol after `n_epochs_no_update`. A possible default value for `n_epochs_no_update` is 5. Also, add an attribute to the classifier which tells us after fitting how many epochs were ran.

e) Tune the hyper-parameters: `eta`, `tol` and `dim-hidden`. Also consider the effect of scaling the data.

f) After a succesful training with a best setting for the hyper-parameters, plot both training loss and validation loss as functions of the number of epochs in one figure, and both accuracies as functions of the number of epochs in another figure. Comment on what you see.

g) The algorithm contains an element of non-determinism. Hence, train the classifier 10 times with the optimal hyper-parameters and report the mean and standard deviation of the accuracies over the 10 runs.

Part III: Final testing

We can now perform a final testing on the held-out test set.

Binary task (X, t2)

Consider the linear regression classifier, the logistic regression classifier and the multi-layer network with the best settings you found. Train each of them on the training set and calculate accuracy on the held-out test set, but also on the validation set and the training set. Report in a 3 by 3 table.

Comment on what you see. How do the three different algorithms compare? Also, compare the results between the different data sets. In cases like these, one might expect slightly inferior results on the held-out test data compared to the validation data. Is that the case here?

Also report precision and recall for class 1.