Assignment 2 Sonya Stuhec-Leonard

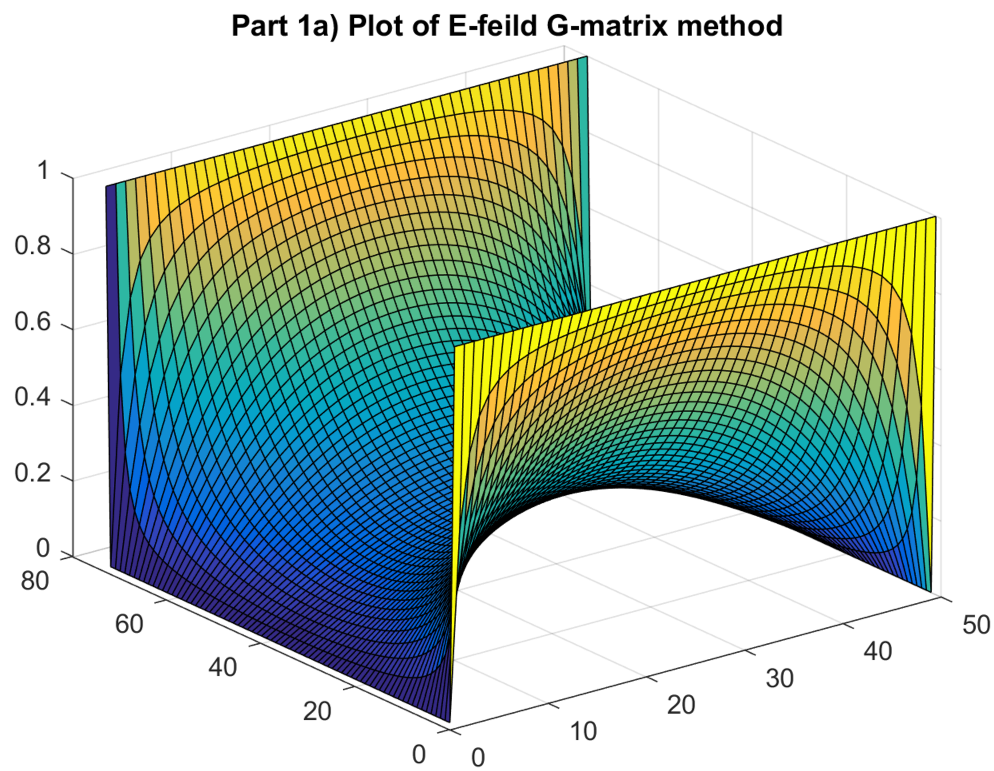
Table of Contents

Part 1: solve by Finite difference a box of L X W

```
clear
close

%box dimensions
W = 50;
L = W*3/2;

%Boundary condition values
V0 = 3;

%Create conductivity map of area LXW
Map = zeros(L, W);
%background conductivity
background = 1;
%conductivijty object
object = 0.1;
%wijdth object
wobj = L/4;
%height object
hobj = W/5;
%object positions (manually selected)
PosObj = [W/2, hobj/2; W/2, L-hobj/2];
NumObj = 2;
%Create G matrjx
G = sparse(L*W);
%create conductivity map in n space
% map_n = zeros(L*W);
%B matrix defines boundary conditions
B = zeros(1, L*W);

%populate G-matrijx
for i = 1:L
    for j = 1:W
        %numbering scheme for G
        n = j+(i-1)*W;

        if i==1
```

```
            %at x=0 Bc = V0, and djagonal of BC = 1
            G(n, n) = V0;
            B(n) = V0;

        elseif i==L
            %at x=L BC=0, and djagonal of BC = 1
            G(n, n) = V0;
            B(n) = V0;

        elseif j==1

            G(n, n) = 1;
            B(n) = 0;

        elseif j==W
            G(n, n) = 1;
            B(n) = 0;

        else
            %solve FD equations or put 1 an d4 in the row coresponding
 to
            %the j, i posjtjon
            L_Minus1 = j+(i-2)*W;
            L_Plus1 = j+ (i)*W;
            W_Minus1 = j-1+(i-1)*W;
            W_Plus1 = j+1+(i-1)*W;

            G(n, n)= -4;
            G(n, L_Minus1) = 1;
            G(n, L_Plus1) = 1;
            G(n, W_Minus1) = 1;
            G(n, W_Plus1) = 1;

        end
    end
end

% %useful for quick verification of G
% figure
% spy(G)
% title('Spying on G matrix')

V = (G)\B'; %inverse(G)*B

% Define Eigen vector of nx by ny plane

for i=1:L
    for j=1:W
        n=j+(i-1)*W;
        V_vec(i, j) = V(n);
    end
end

figure
```

```
surf(V_vec)
title('Part 1a) Plot of E-feild G-matrix method')
```

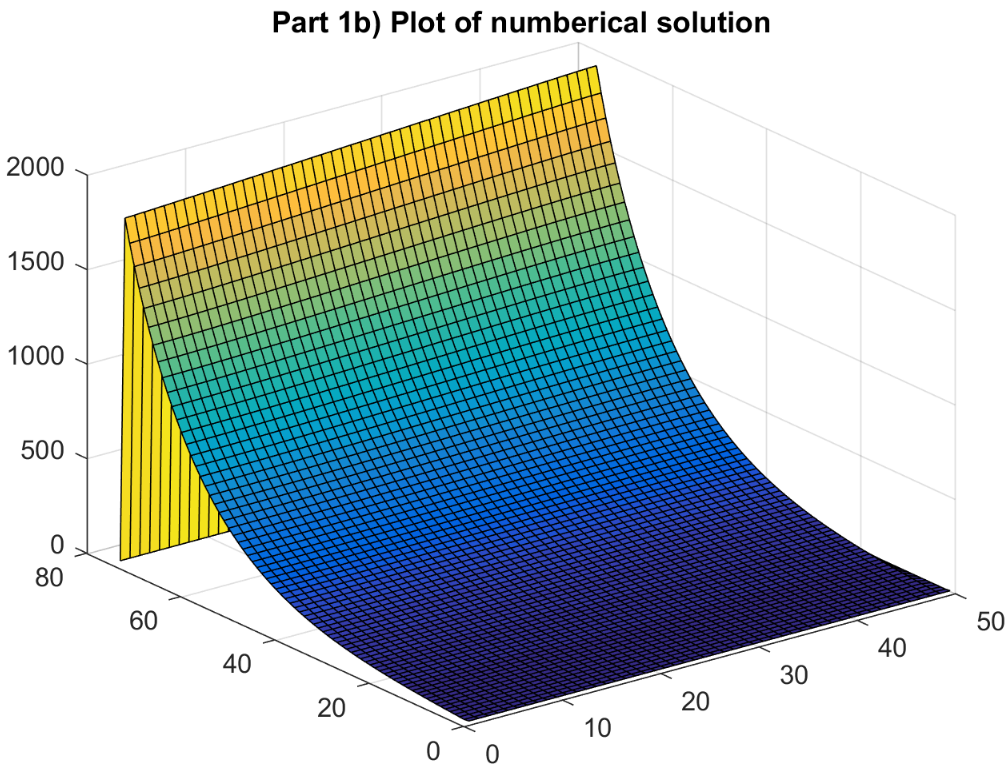


1b) anlytical serise solution for comparison

```
% reset value of n
n=0;
sumation=0;
a=W;
b=L;
%meshing vairable
nMax=100;

V_numberical = zeros(L, W);

for x = 1:L
    for y = 1:W

        if x==1 || x==L
            %at x=0 Bc = V0, and diagonal of BC = 1
            V_numberical(x, y) = V0;

        elseif y==0 || y==W
            V_numberical(x, y) = 0;

        else
```

```
            for n=1:2:nMax
                sumation=sumation+(1/n*cosh(n*pi*x/a)/cosh(n*pi*b/
a)*sin(n*pi*y/a));
            end
            V_numberical(x, y) = 4*V0/pi*sumation;
        end
    end
end

figure
surf(V_numberical)
title('Part 1b) Plot of numberical solution')

% Hello Aaron,
% This paragraph of comments is my discusson of numberical vs
 analytical
% methods. I thought it might be more efficent to put it here rather
 than
% in a seperate document.

% This solutions should approach the anaytica solutions withign a
 margin of
% error since the sum cannot be taken to infinity. The first few terms
% should be the largest contributing factors to the soultions, with
 ach
% additional term improving the acuracy by smaller and smaller
 amounts.

% You should stop an analytical seriser when you are within a
 reasonable
% margin of error of the expected or analytical solution. The precise
% number of iterations to go throuh will depend on that application,
 as
% the definitionfor a reasonable margin of error.

% Numberical solution can be much faster and tends to be more
 intuative
% forward however its cons include lack of acuracey, difficult for
 complex
% problmes and gemetries also, fine meshing improves accuracy but also
% makes the simulations slow.

% An analytical method is more acturate, applicable to complex systems
% and faster to simulate. However, they are limited in terms of
% application becase they usally solve for a simplified version of a
% system.

% My simulation is not correct here. After discussing with other
 students
% the code iteslf seems to be fine, and it is a matter of debugging,
% however my attempts have not found the problem and I hope the
 description
% of my understanding is sufficent, despite not have a correct graph
 to
```

```
% show.
```



Part 2a)Current flow with bottle neck

```
%populate conductivity map
for i=1:L
    for j=1:W
        Map(i, j) = background;
        %set conductivity of objects
        for obj = 1:NumObj
            dx = PosObj(obj,1)-j;
            dy = PosObj(obj,2)-i;
            if (dx)^2 <= wobj && (dy-1)^2 <= hobj
                Map(i, j) = object;
            elseif(dx)^2 <= wobj && (dy+1)^2 <= hobj
                Map(i, j) = object;
            end
        end
    end
end

%plot conductivity
figure
surf(Map)
title('Part 2 Conductivity map')
colorbar
```

```
% put conductivity map into n space
for i=1:L
    for j=1:W
        n=j+(i-1)*W;
        map_n(n) = Map(i, j);
    end
end

%conductivity acounted for G matrjx
G2 = G.*map_n;

V2 = zeros(L, W);

V2 = G2\B'; %inverse(G)*B

% Define Ejgen vector of nx by ny plane

for i=1:L
    for j=1:W
        n=j+(i-1)*W;
        %changing fjrst col of E (ejgen vetor) to the nx X ny matrjx
 for
        %plotting. Chaningn whjch col of E you convert wjll change
 withc
        %ejgen vector you look at
        V_vec2(i, j) = V2(n);
    end
end

figure
surf(V_vec2)
title('Plot of e-feild in x-y plane')


[Ex, Ey] = gradient(V_vec);

% [ExA, EyA] = gradient(V_numberical);

%plot of the Ex and Ey fejld
figure
quiver (Ex, Ey)
title('Part 2 E-feild')

Y0 = zeros(L, W);
X0 = zeros(L, W);

% %Ex plot
% figure
% quiver (Ex, Y0)
% title('Part 2 E_x-feild')
%
% %Ey plot
% figure
```

```
% quiver (X0, Ey)
% title('Part 2 E_y-feild')

%initalize current moatrix
J = zeros(L, W);

for i=1:L
    for j=1:W
        J(:, j) = Map(:, j).*-Ex(:, j);
        J(i, :) = Map(i, :).*-Ey(i, :);
    end
end

%Ey plot
figure
surf (J)
title('Part 2 Electric feild density')

%output of current on the x, y plane
fprintf('output of current on the x, y plane for upper contact
 region')
J(W/2-wobj-1:W/2+obj-1, hobj-1:W-1)

output of current on the x, y plane for upper contact regionWarning:
 Integer operands are required for
colon operator when used as index

ans =

  Columns 1 through 7

   -0.0306   -0.0249   -0.0205   -0.0170   -0.0142   -0.0119   -0.0100
   -0.0340   -0.0282   -0.0235   -0.0198   -0.0167   -0.0141   -0.0119
   -0.0360   -0.0304   -0.0257   -0.0218   -0.0186   -0.0158   -0.0135
   -0.0368   -0.0316   -0.0271   -0.0233   -0.0200   -0.0172   -0.0148
   -0.0368   -0.0320   -0.0278   -0.0242   -0.0210   -0.0182   -0.0157
   -0.0361   -0.0319   -0.0280   -0.0246   -0.0215   -0.0188   -0.0164
   -0.0351   -0.0313   -0.0279   -0.0247   -0.0218   -0.0192   -0.0168
   -0.0338   -0.0305   -0.0274   -0.0245   -0.0218   -0.0193   -0.0170
   -0.0324   -0.0295   -0.0267   -0.0240   -0.0215   -0.0192   -0.0170
   -0.0310   -0.0284   -0.0259   -0.0234   -0.0211   -0.0189   -0.0168
   -0.0295   -0.0272   -0.0250   -0.0228   -0.0206   -0.0186   -0.0166
   -0.0280   -0.0260   -0.0240   -0.0220   -0.0200   -0.0181   -0.0163
   -0.0266   -0.0248   -0.0230   -0.0212   -0.0194   -0.0176   -0.0159
   -0.0253   -0.0237   -0.0220   -0.0204   -0.0187   -0.0171   -0.0154
   -0.0240   -0.0226   -0.0211   -0.0196   -0.0180   -0.0165   -0.0150
   -0.0228   -0.0215   -0.0202   -0.0188   -0.0173   -0.0159   -0.0145
   -0.0217   -0.0205   -0.0193   -0.0180   -0.0167   -0.0153   -0.0140
   -0.0206   -0.0196   -0.0184   -0.0172   -0.0160   -0.0148   -0.0135
   -0.0197   -0.0187   -0.0176   -0.0165   -0.0154   -0.0142   -0.0130
   -0.0188   -0.0179   -0.0169   -0.0159   -0.0148   -0.0137   -0.0126
   -0.0180   -0.0171   -0.0162   -0.0153   -0.0143   -0.0132   -0.0121

  Columns 8 through 14
```

```
  -0.0085   -0.0071   -0.0059   -0.0049   -0.0040   -0.0003   -0.0002
  -0.0101   -0.0085   -0.0072   -0.0059   -0.0048   -0.0004   -0.0003
  -0.0115   -0.0097   -0.0082   -0.0068   -0.0056   -0.0004   -0.0003
  -0.0126   -0.0108   -0.0091   -0.0076   -0.0062   -0.0005   -0.0004
  -0.0135   -0.0116   -0.0098   -0.0082   -0.0068   -0.0005   -0.0004
  -0.0142   -0.0122   -0.0104   -0.0087   -0.0072   -0.0057   -0.0044
  -0.0146   -0.0126   -0.0108   -0.0091   -0.0075   -0.0060   -0.0046
  -0.0148   -0.0129   -0.0110   -0.0093   -0.0077   -0.0062   -0.0048
  -0.0149   -0.0130   -0.0112   -0.0095   -0.0079   -0.0064   -0.0049
  -0.0149   -0.0130   -0.0112   -0.0096   -0.0080   -0.0064   -0.0050
  -0.0147   -0.0129   -0.0112   -0.0096   -0.0080   -0.0065   -0.0050
  -0.0145   -0.0128   -0.0111   -0.0095   -0.0079   -0.0064   -0.0050
  -0.0142   -0.0125   -0.0109   -0.0094   -0.0079   -0.0064   -0.0049
  -0.0138   -0.0122   -0.0107   -0.0092   -0.0077   -0.0063   -0.0049
  -0.0134   -0.0119   -0.0105   -0.0090   -0.0076   -0.0062   -0.0048
  -0.0130   -0.0116   -0.0102   -0.0088   -0.0074   -0.0060   -0.0047
  -0.0126   -0.0113   -0.0099   -0.0086   -0.0072   -0.0059   -0.0046
  -0.0122   -0.0109   -0.0096   -0.0083   -0.0070   -0.0058   -0.0045
  -0.0118   -0.0106   -0.0093   -0.0081   -0.0068   -0.0056   -0.0044
  -0.0114   -0.0102   -0.0090   -0.0078   -0.0066   -0.0054   -0.0042
  -0.0110   -0.0099   -0.0088   -0.0076   -0.0065   -0.0053   -0.0041

 Columns 15 through 21

  -0.0002   -0.0001   -0.0000    0.0000    0.0001    0.0002    0.0002
  -0.0002   -0.0001   -0.0000    0.0000    0.0001    0.0002    0.0003
  -0.0002   -0.0001   -0.0000    0.0000    0.0001    0.0002    0.0003
  -0.0003   -0.0002   -0.0001    0.0001    0.0002    0.0003    0.0004
  -0.0003   -0.0002   -0.0001    0.0001    0.0002    0.0003    0.0004
  -0.0031   -0.0018   -0.0006    0.0006    0.0018    0.0031    0.0044
  -0.0033   -0.0019   -0.0006    0.0006    0.0019    0.0033    0.0046
  -0.0034   -0.0020   -0.0007    0.0007    0.0020    0.0034    0.0048
  -0.0035   -0.0021   -0.0007    0.0007    0.0021    0.0035    0.0049
  -0.0035   -0.0021   -0.0007    0.0007    0.0021    0.0035    0.0050
  -0.0035   -0.0021   -0.0007    0.0007    0.0021    0.0035    0.0050
  -0.0035   -0.0021   -0.0007    0.0007    0.0021    0.0035    0.0050
  -0.0035   -0.0021   -0.0007    0.0007    0.0021    0.0035    0.0049
  -0.0035   -0.0021   -0.0007    0.0007    0.0021    0.0035    0.0049
  -0.0034   -0.0020   -0.0007    0.0007    0.0020    0.0034    0.0048
  -0.0033   -0.0020   -0.0007    0.0007    0.0020    0.0033    0.0047
  -0.0033   -0.0020   -0.0007    0.0007    0.0020    0.0033    0.0046
  -0.0032   -0.0019   -0.0006    0.0006    0.0019    0.0032    0.0045
  -0.0031   -0.0019   -0.0006    0.0006    0.0019    0.0031    0.0044
  -0.0030   -0.0018   -0.0006    0.0006    0.0018    0.0030    0.0042
  -0.0029   -0.0018   -0.0006    0.0006    0.0018    0.0029    0.0041

 Columns 22 through 28

   0.0032    0.0040    0.0049    0.0059    0.0071    0.0085    0.0100
   0.0038    0.0048    0.0059    0.0072    0.0085    0.0101    0.0119
   0.0044    0.0056    0.0068    0.0082    0.0097    0.0115    0.0135
   0.0050    0.0062    0.0076    0.0091    0.0108    0.0126    0.0148
   0.0054    0.0068    0.0082    0.0098    0.0116    0.0135    0.0157
```

0.0057	0.0072	0.0087	0.0104	0.0122	0.0142	0.0164
0.0060	0.0075	0.0091	0.0108	0.0126	0.0146	0.0168
0.0062	0.0077	0.0093	0.0110	0.0129	0.0148	0.0170
0.0064	0.0079	0.0095	0.0112	0.0130	0.0149	0.0170
0.0064	0.0080	0.0096	0.0112	0.0130	0.0149	0.0168
0.0065	0.0080	0.0096	0.0112	0.0129	0.0147	0.0166
0.0064	0.0079	0.0095	0.0111	0.0128	0.0145	0.0163
0.0064	0.0079	0.0094	0.0109	0.0125	0.0142	0.0159
0.0063	0.0077	0.0092	0.0107	0.0122	0.0138	0.0154
0.0062	0.0076	0.0090	0.0105	0.0119	0.0134	0.0150
0.0060	0.0074	0.0088	0.0102	0.0116	0.0130	0.0145
0.0059	0.0072	0.0086	0.0099	0.0113	0.0126	0.0140
0.0058	0.0070	0.0083	0.0096	0.0109	0.0122	0.0135
0.0056	0.0068	0.0081	0.0093	0.0106	0.0118	0.0130
0.0054	0.0066	0.0078	0.0090	0.0102	0.0114	0.0126
0.0053	0.0065	0.0076	0.0088	0.0099	0.0110	0.0121

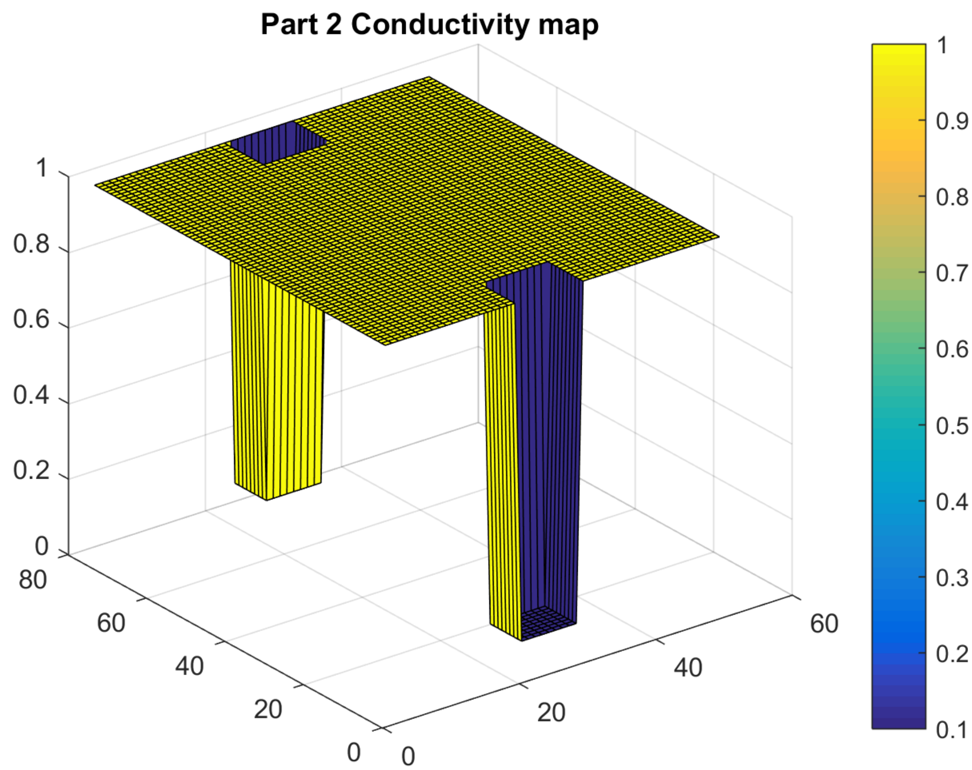
Columns 29 through 35

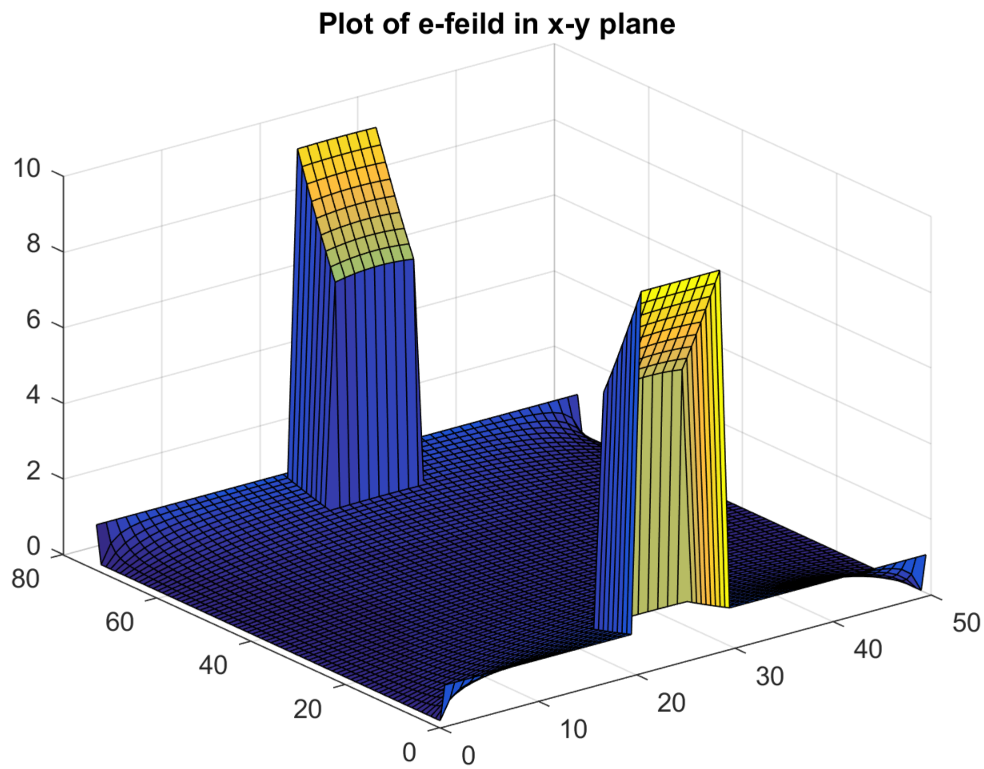
0.0119	0.0142	0.0170	0.0205	0.0249	0.0306	0.0381
0.0141	0.0167	0.0198	0.0235	0.0282	0.0340	0.0414
0.0158	0.0186	0.0218	0.0257	0.0304	0.0360	0.0428
0.0172	0.0200	0.0233	0.0271	0.0316	0.0368	0.0429
0.0182	0.0210	0.0242	0.0278	0.0320	0.0368	0.0421
0.0188	0.0215	0.0246	0.0280	0.0319	0.0361	0.0408
0.0192	0.0218	0.0247	0.0279	0.0313	0.0351	0.0391
0.0193	0.0218	0.0245	0.0274	0.0305	0.0338	0.0373
0.0192	0.0215	0.0240	0.0267	0.0295	0.0324	0.0354
0.0189	0.0211	0.0234	0.0259	0.0284	0.0310	0.0335
0.0186	0.0206	0.0228	0.0250	0.0272	0.0295	0.0317
0.0181	0.0200	0.0220	0.0240	0.0260	0.0280	0.0300
0.0176	0.0194	0.0212	0.0230	0.0248	0.0266	0.0284
0.0171	0.0187	0.0204	0.0220	0.0237	0.0253	0.0268
0.0165	0.0180	0.0196	0.0211	0.0226	0.0240	0.0254
0.0159	0.0173	0.0188	0.0202	0.0215	0.0228	0.0240
0.0153	0.0167	0.0180	0.0193	0.0205	0.0217	0.0228
0.0148	0.0160	0.0172	0.0184	0.0196	0.0206	0.0216
0.0142	0.0154	0.0165	0.0176	0.0187	0.0197	0.0206
0.0137	0.0148	0.0159	0.0169	0.0179	0.0188	0.0196
0.0132	0.0143	0.0153	0.0162	0.0171	0.0180	0.0188

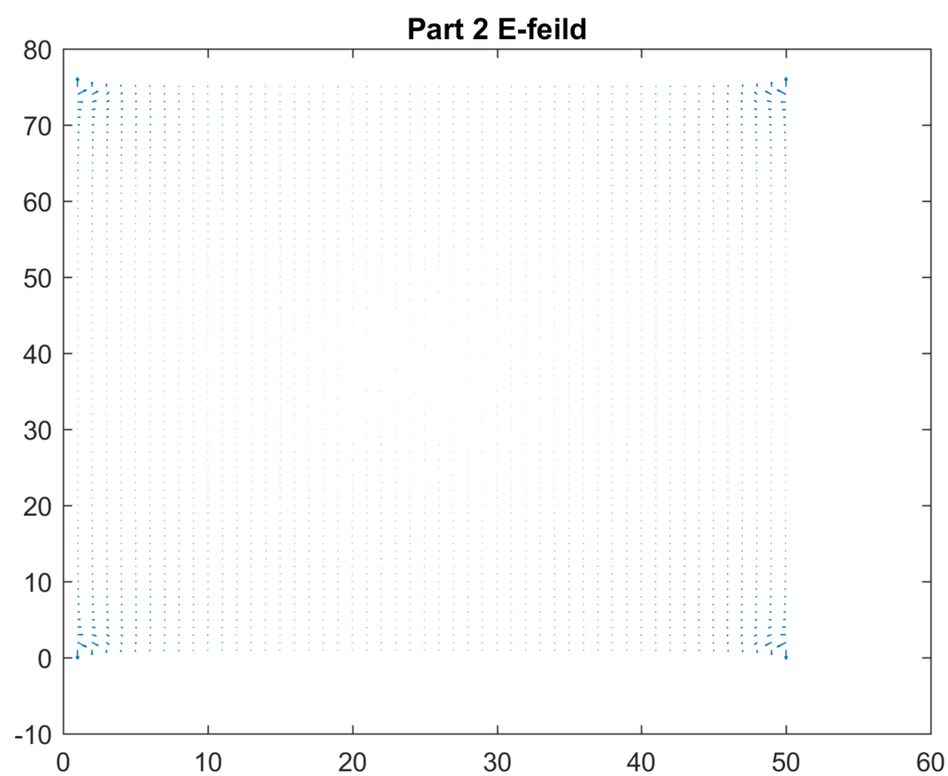
Columns 36 through 41

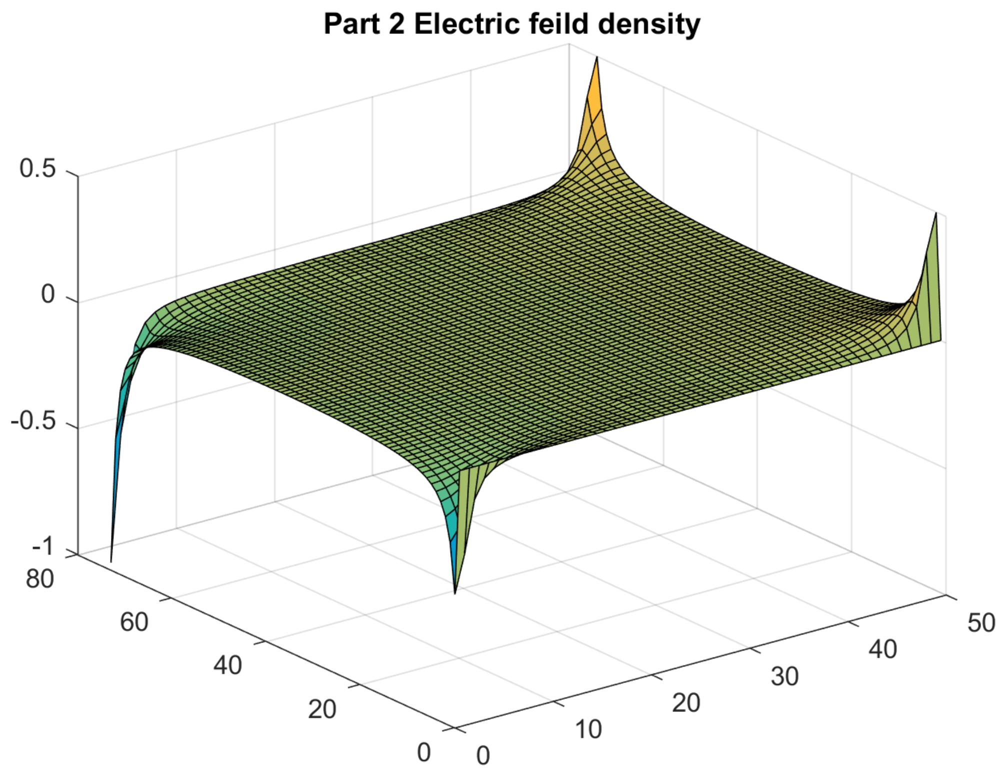
0.0480	0.0610	0.0782	0.0999	0.1247	0.1474
0.0505	0.0619	0.0756	0.0913	0.1073	0.1203
0.0509	0.0604	0.0711	0.0824	0.0931	0.1010
0.0498	0.0576	0.0659	0.0742	0.0815	0.0867
0.0480	0.0543	0.0607	0.0669	0.0721	0.0757
0.0457	0.0508	0.0559	0.0606	0.0644	0.0669
0.0433	0.0474	0.0515	0.0551	0.0579	0.0598
0.0408	0.0442	0.0475	0.0503	0.0525	0.0539
0.0384	0.0412	0.0439	0.0461	0.0479	0.0490
0.0361	0.0385	0.0406	0.0425	0.0439	0.0447
0.0339	0.0359	0.0377	0.0392	0.0404	0.0411

0.0319	0.0336	0.0351	0.0364	0.0373	0.0379
0.0300	0.0315	0.0328	0.0338	0.0346	0.0351
0.0282	0.0295	0.0306	0.0316	0.0322	0.0327
0.0266	0.0278	0.0287	0.0295	0.0301	0.0305
0.0252	0.0262	0.0270	0.0277	0.0282	0.0285
0.0238	0.0247	0.0255	0.0261	0.0265	0.0268
0.0226	0.0234	0.0240	0.0246	0.0250	0.0252
0.0214	0.0221	0.0228	0.0232	0.0236	0.0238
0.0204	0.0210	0.0216	0.0220	0.0224	0.0226
0.0194	0.0201	0.0206	0.0210	0.0213	0.0214




Plot of e-feild in x-y plane
10
8
6
4
2
0
80
60
40
20
0
0
10
20
30
40
50


Part 2 E-feild
80
70
60
50
40
30
20
10
0
-10
0
10
20
30
40
50
60



Part 2b) effect of different parameters on current

```
%variation of current with mesh size
%anaytical solution altered since there is no change in for G method
m = [3, 25, 50, 100];
J1_mesh = Assignment2ParameterVariable_numberical( m(1), object,
 hobj );
J2_mesh = Assignment2ParameterVariable_numberical( m, object, hobj );
J3_mesh = Assignment2ParameterVariable_numberical( 50, object, hobj );
J4_mesh = Assignment2ParameterVariable_numberical( 100, object,
 hobj );

J_mesh = [mean(mean(J1_mesh)), mean(mean(J2_mesh)),
 mean(mean(J3_mesh)), mean(mean(J4_mesh))];

% P_mesh = polyfit(m,J_mesh,1);
% yfit_mesh = P_mesh(1).*m+P_mesh(2);

figure
plot(m, J_mesh, '.r', 'MarkerSize', 20)
% hold on
% plot(m,yfit_mesh,'r-.');
title('Effect on increaing meshing size')
```

```
xlabel('Mesh size')
ylabel('Average current')

%variation in current with bottle neck size
h = [W/4, W/6, W/8, W/10];
J1_neck = Assignment2ParameterVariable( object, h(1) );
J2_neck = Assignment2ParameterVariable( object, h(2) );
J3_neck = Assignment2ParameterVariable( object, h(3) );
J4_neck = Assignment2ParameterVariable( object, h(4) );

J_neck = [mean(mean(J1_neck)), mean(mean(J2_neck)),
 mean(mean(J3_neck)), mean(mean(J4_neck))];
BN = abs(h.*2-W); %bottle neak is the width -2*height of each object

% P_neck = polyfit(BN ,J_neck,1);
% yfit_neck = P_neck(1).*BN+P_neck(2);

figure
plot(BN, J_neck, '.c', 'MarkerSize', 20)
% hold on
% plot(BN,yfit_neck,'r-.');
title('Effect on increaing bottle neck size')
xlabel('Bottle neack seperation')
ylabel('Average current')


% %variation in current with conductivity
c = [0.01, 0.1, 1, 10];
J1_cond = Assignment2ParameterVariable( c(1), hobj );
J2_cond = Assignment2ParameterVariable( c(2), hobj );
J3_cond = Assignment2ParameterVariable( c(3), hobj );
J4_cond = Assignment2ParameterVariable( c(4), hobj );

J_cond = [mean(mean(J1_cond)), mean(mean(J2_cond)),
 mean(mean(J3_cond)), mean(mean(J4_cond))];

P_cond =polyfit(c ,J_cond,1);
yfit_cond = P_cond(1).*c+P_cond(2);

figure
plot(c, J_cond, '.b', 'MarkerSize', 20)
hold on
plot(c,yfit_cond,'r-.');
title('Effect on increaing conductivity of squares')
xlabel('Conductity of boxes')
ylabel('Average current')
```

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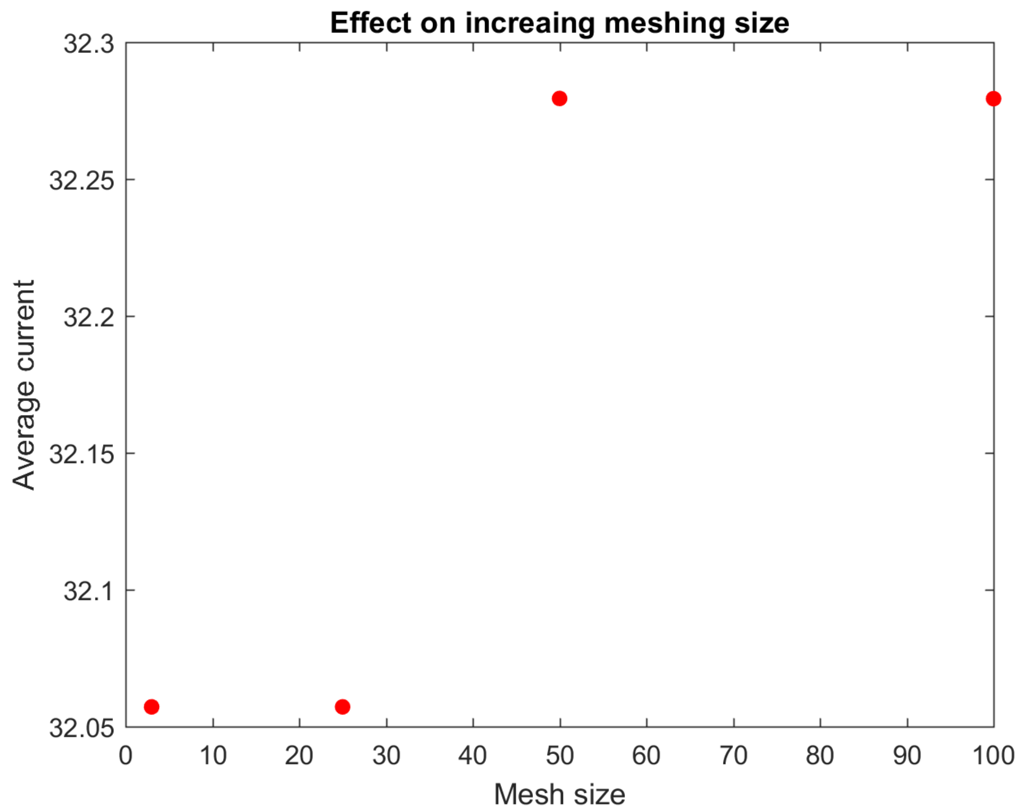
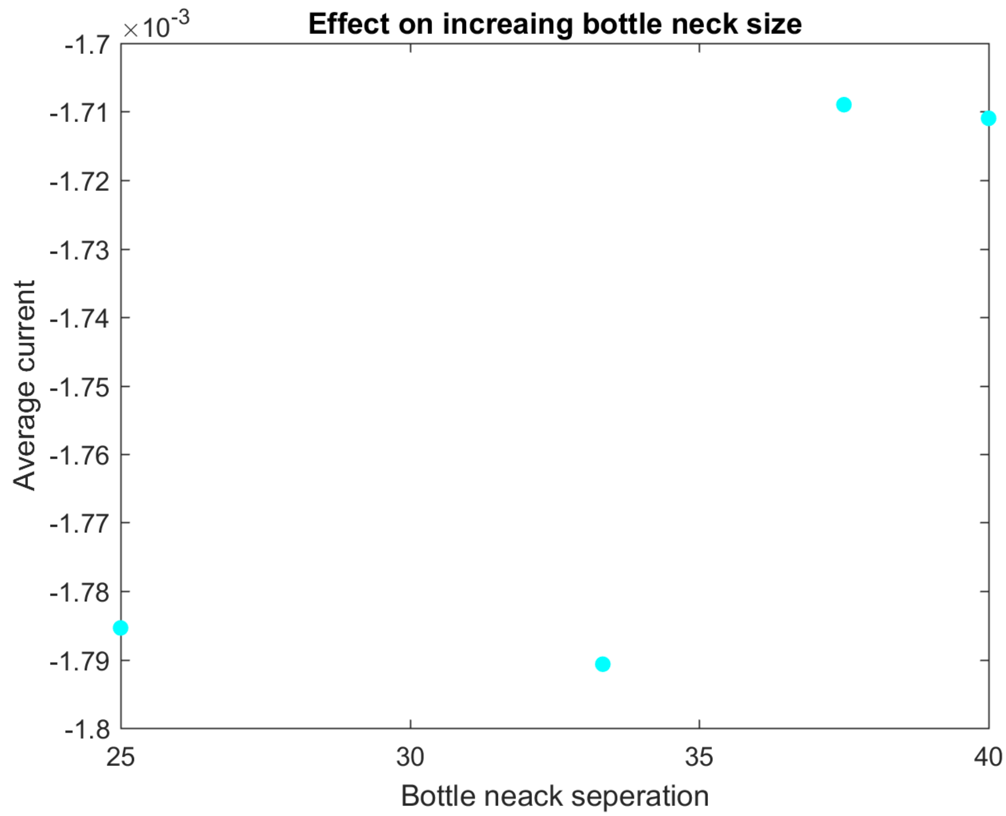
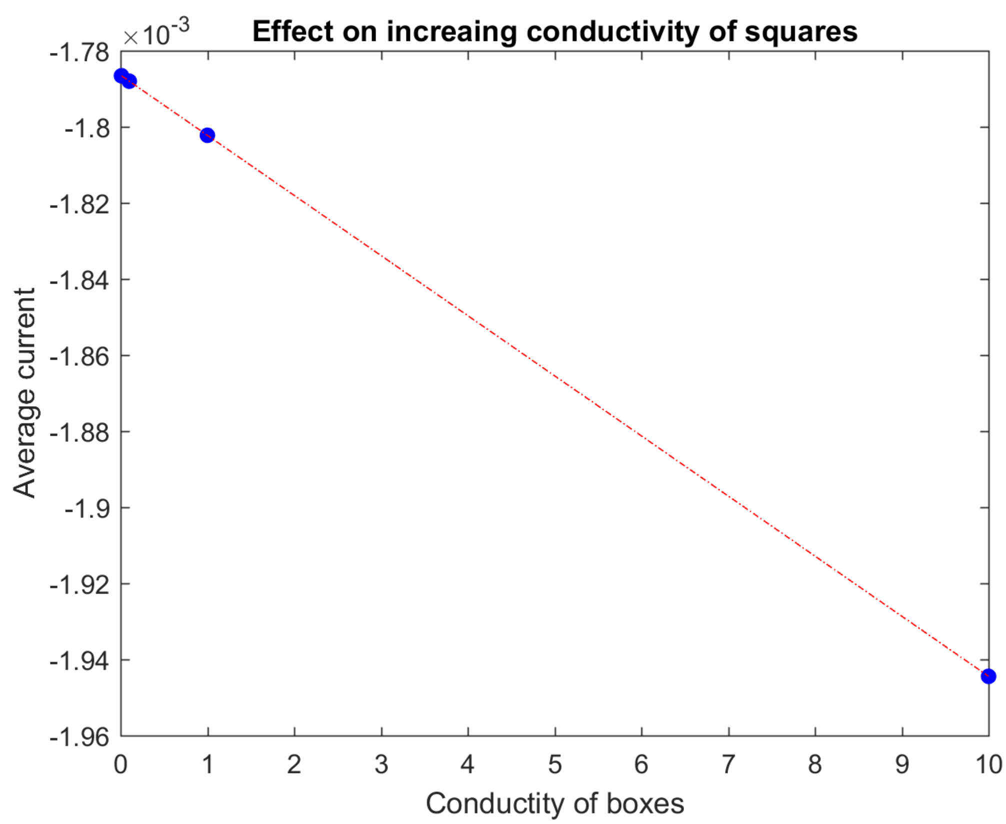



Effect on increaing bottle neck size
×10^-3
-1.7
-1.71
-1.72
-1.73
-1.74
-1.75
-1.76
-1.77
-1.78
-1.79
-1.8
Average current
25
30
35
40
Bottle neack seperation


Effect on increaing conductivity of squares
×10^-3
-1.78
-1.8
-1.82
-1.84
-1.86
-1.88
-1.9
-1.92
-1.94
-1.96
Average current
0
1
2
3
4
5
6
7
8
9
10
Conductity of boxes

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