Name - Sarang Sampatrao Gaikwad

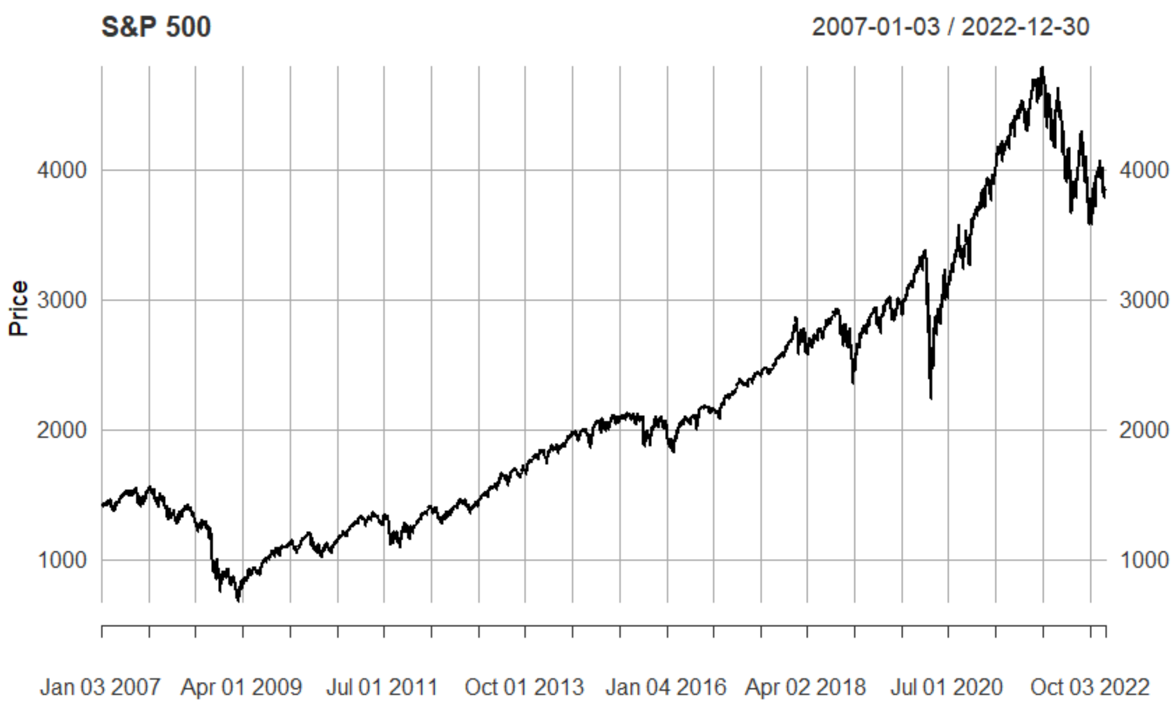
PROJECT - 3

install.packages('quantmod')

library('quantmod')

getSymbols("^GSPC",from="2007-01-03",to="2023-1-03")

plot(GSPC$GSPC.Adjusted,xlab = 'Time',ylab = 'Price', main='S&P 500')

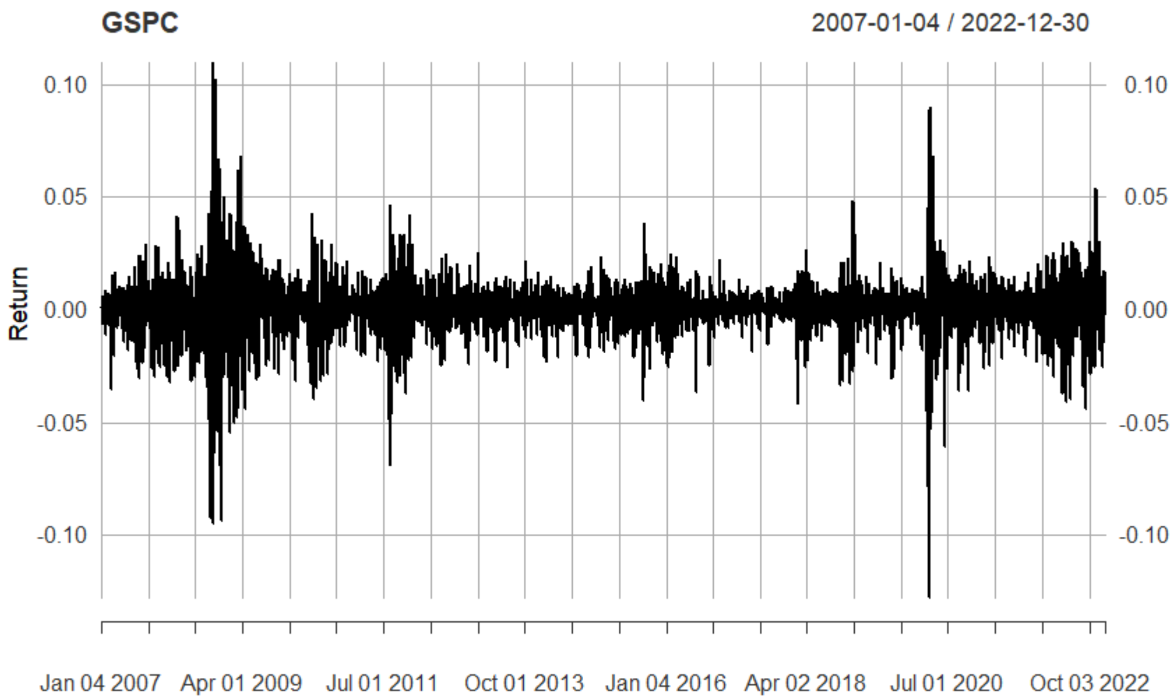


#The depicted graph portrays a noticeable upward trend in price data, initially characterized by a gradually decreasing rate of ascent. However, towards April 2009, there was a sudden fall of price data. #Subsequently, there is a return to upward movement, punctuated by a sharp price decline on Jan 04, 2016. Following this downturn, the data begins to trend upward again after the sudden fall in price data Jan 2016. There is a significant drop in price below $2300, followed by a subsequent upward movement. #Towards the end of 2020, a consistent price increase is evident, indicating a robust economy just before the onset of COVID-19.

```
1]  "GSPC"
> plot(GSPC$GSPC.Adjusted,xlab = 'Time',ylab = 'Price', main='S&P 500')
> GSPC.rtn <- diff(log(GSPC$GSPC.Adjusted))
> GSPC.rtn <- GSPC.rtn[2:length(GSPC.rtn),]
> dim(GSPC.rtn)
```

```
[1] 4027    1
> t <- seq(1:3503)
> head(GSPC.rtn)
           GSPC.Adjusted
2007-01-04  0.0012275325
2007-01-05 -0.0061031678
2007-01-08  0.0022178571
2007-01-09 -0.0005168095
2007-01-10  0.0019384718
2007-01-11  0.0063198616
```

```
> y<-GSPC.rtn
> plot(y,xlab = 'Time',ylab = 'Return', main='GSPC')

>
```



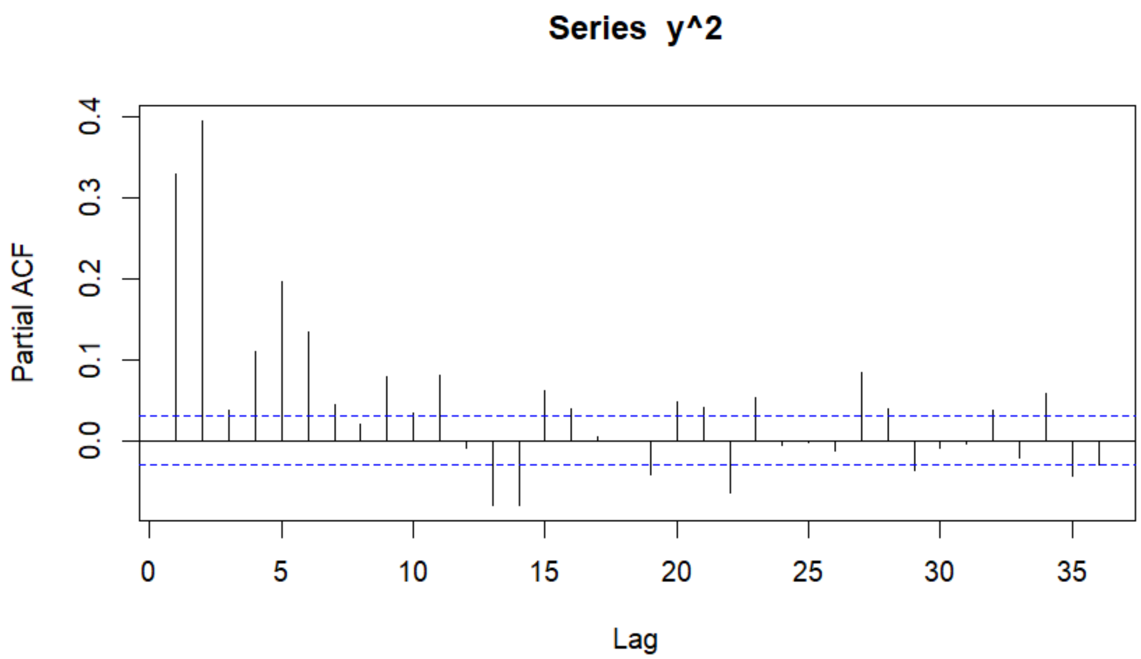
After taking the logged difference of the price data for S&P 500 we can see # that the upward trend has been removed from the data. Now we can proceed to # perform a t-test on the data to confirm that the mean is 0#

```
> t.test(y)

        One Sample t-test

data:  y
t = 1.2058, df = 4026, p-value = 0.228
```

```
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -0.0001549715  0.0006501702
sample estimates:
   mean of x 
0.0002475993 
```

Since the p-value for the t-test is 0.228, higher than 0.05 we can say we can # reject the null hypothesis and confirm the mean is zero. We then proceed to # perform the Arch and Box tests to look for arch effects#

```
> pacf(y^2)
```




```
> library('FinTS')

> #source('archTest.R' )
> #archTest(y^2,12)
> ArchTest(y^2,lags=12,demean=T)

	ARCH LM-test; Null hypothesis: no ARCH effects

data:  y^2
Chi-squared = 690.83, df = 12, p-value < 2.2e-16

> Box.test(y^2,lag=12,type='Ljung')

	Box-Ljung test

data:  y^2
X-squared = 4773.2, df = 12, p-value < 2.2e-16
```

Since the p-values are lower than 0.05 for both the Arch and Box-Ljung we can # confirm that there are no arch effects and we can proceed to use the ARCH and # GARCH models#

```
?fGarch
> ?garchFit
> m1<-garchFit (~1+garch(10,0),data=y,trace=F) #Fit an ARCH(10) model
> #Use subcommand " trace = F" to reduce the output .
> summary (m1)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~1 + garch(10, 0), data = y, trace = F)

Mean and Variance Equation:
 data ~ 1 + garch(10, 0)
<environment: 0x0000029f22588930>
 [data = y]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1      alpha2      alpha3      alpha4
alpha5
6.9784e-04  2.0429e-05  9.9443e-02  1.5930e-01  1.1480e-01  1.4673e-01  6.
4398e-02
    alpha6      alpha7      alpha8      alpha9     alpha10
6.9776e-02  6.0624e-02  6.3518e-02  5.8716e-02  5.0160e-02

Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      6.978e-04   1.238e-04    5.637 1.73e-08 ***
omega   2.043e-05   1.658e-06   12.324  < 2e-16 ***
alpha1  9.944e-02   1.855e-02    5.361 8.28e-08 ***
alpha2  1.593e-01   2.131e-02    7.475 7.73e-14 ***
alpha3  1.148e-01   2.097e-02    5.476 4.36e-08 ***
alpha4  1.467e-01   2.240e-02    6.551 5.72e-11 ***
alpha5  6.440e-02   1.682e-02    3.829 0.000129 ***
alpha6  6.978e-02   1.809e-02    3.858 0.000114 ***
alpha7  6.062e-02   1.662e-02    3.647 0.000266 ***
alpha8  6.352e-02   1.750e-02    3.629 0.000285 ***
alpha9  5.872e-02   1.798e-02    3.265 0.001095 **
alpha10 5.016e-02   1.601e-02    3.133 0.001732 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 12939.72    normalized:  3.213241

Description:
 Wed Apr 10 14:24:44 2024 by user: 91900


Standardised Residuals Tests:
                                 Statistic    p-Value
 Jarque-Bera Test   R    Chi^2  949.8842355 0.00000000
 Shapiro-Wilk Test  R    W        0.9737147 0.00000000
 Ljung-Box Test     R    Q(10)   18.1484716 0.05251019
 Ljung-Box Test     R    Q(15)   25.4763917 0.04390005
```

```
 Ljung-Box Test     R    Q(20)   28.1691648 0.10545498
 Ljung-Box Test     R^2  Q(10)    5.2934902 0.87073025
 Ljung-Box Test     R^2  Q(15)    6.2377978 0.97547044
 Ljung-Box Test     R^2  Q(20)    6.5347173 0.99795515
 LM Arch Test       R    TR^2     6.0635145 0.91284650

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC 
-6.420522 -6.401746 -6.420539 -6.413869 
```

m2<-garchFit (~1+garch(2,2) ,data=y,trace=F)

summary(m2)

```
Title:
 GARCH Modelling 

Call:
 garchFit(formula = ~1 + garch(2, 2), data = y, trace = F) 

Mean and Variance Equation:
 data ~ 1 + garch(2, 2)
<environment: 0x0000029f2830cad0>
 [data = y]

Conditional Distribution:
 norm 

Coefficient(s):
        mu       omega      alpha1      alpha2       beta1       beta2  
7.0858e-04  5.5574e-06  1.1082e-01  1.5372e-01  1.3986e-01  5.6165e-01  

Std. Errors:
 based on Hessian 

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)    
mu     7.086e-04   1.245e-04    5.691 1.26e-08 ***
omega  5.557e-06   7.513e-07    7.397 1.39e-13 ***
alpha1 1.108e-01   1.761e-02    6.292 3.13e-10 ***
alpha2 1.537e-01   2.069e-02    7.431 1.08e-13 ***
beta1  1.399e-01   1.299e-01    1.077    0.282    
beta2  5.616e-01   1.144e-01    4.910 9.09e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 12944.99    normalized:  3.21455 

Description:
 Wed Apr 10 14:25:44 2024 by user: 91900 


Standardised Residuals Tests:
                                   Statistic    p-Value
 Jarque-Bera Test   R    Chi^2  1043.1122687 0.00000000
 Shapiro-Wilk Test  R    W         0.9726086 0.00000000
 Ljung-Box Test     R    Q(10)    19.2677308 0.03699095
 Ljung-Box Test     R    Q(15)    26.2627134 0.03535033
 Ljung-Box Test     R    Q(20)    28.8234291 0.09129474
 Ljung-Box Test     R^2  Q(10)     7.2349071 0.70309776
 Ljung-Box Test     R^2  Q(15)     8.9860428 0.87824610
 Ljung-Box Test     R^2  Q(20)    10.1195510 0.96595196
 LM Arch Test       R    TR^2      8.1018990 0.77711747
```

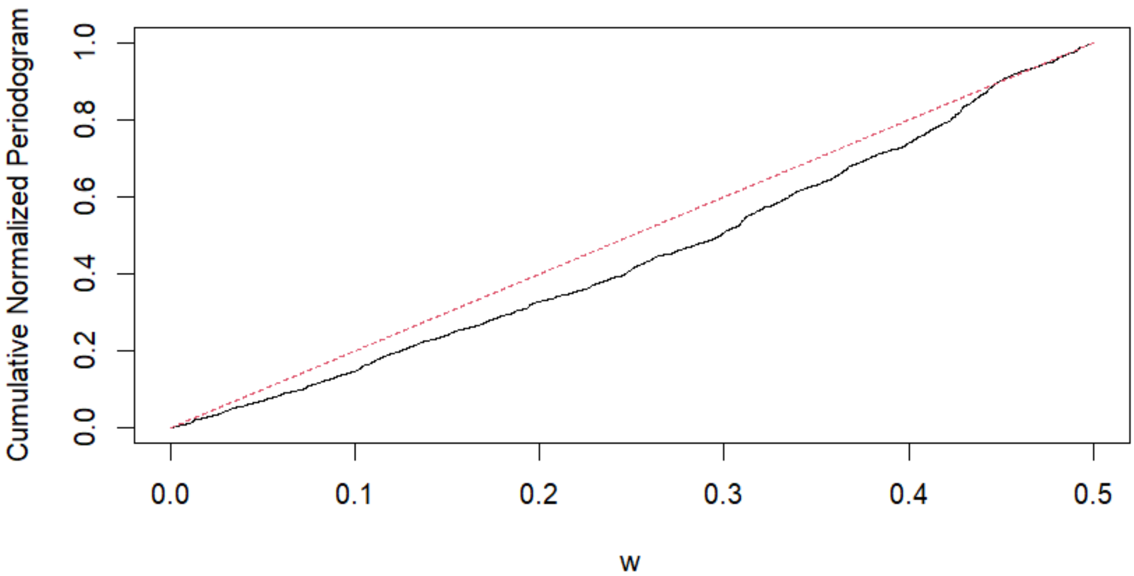
```
Information Criterion Statistics:
      AIC       BIC       SIC      HQIC 
-6.426121 -6.416733 -6.426125 -6.422794
```

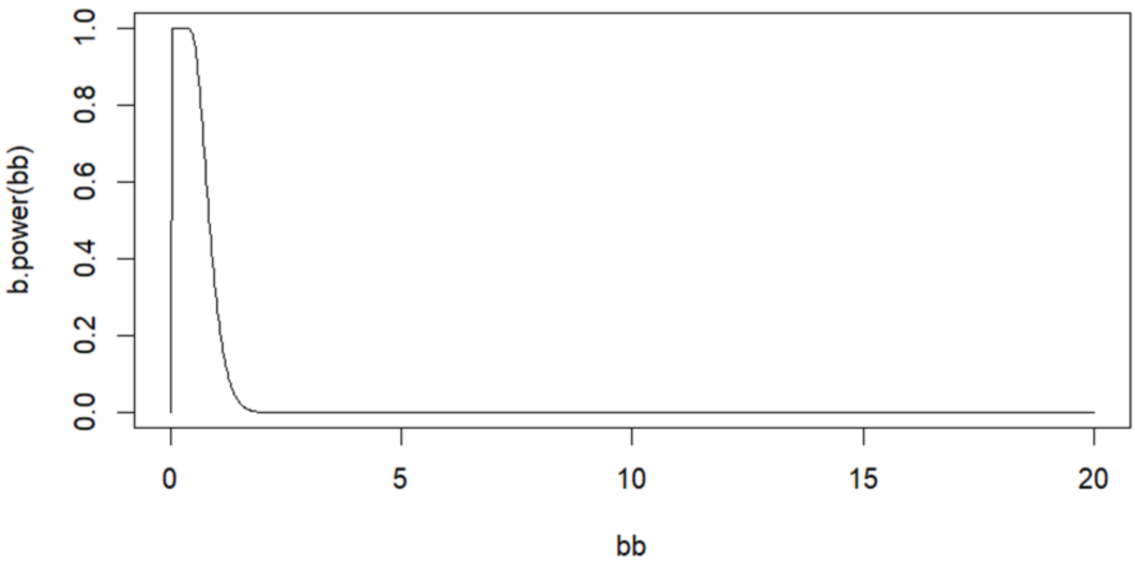
m3<-garchFit (~1+garch(1,4) ,data=y,trace=F)

summary(m3)

```
Title:
 GARCH Modelling 

Call:
 garchFit(formula = ~1 + garch(1, 4), data = y, trace = F) 

Mean and Variance Equation:
 data ~ 1 + garch(1, 4)
<environment: 0x0000029f2985fec0>
 [data = y]

Conditional Distribution:
 norm 

Coefficient(s):
        mu       omega      alpha1       beta1       beta2       beta3       
beta4  
7.0458e-04  3.0272e-06  1.4946e-01  8.3265e-01  1.0000e-08  1.0000e-08  1.
0000e-08  

Std. Errors:
 based on Hessian 

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)    
mu     7.046e-04   1.250e-04    5.637 1.73e-08 ***
omega  3.027e-06   5.987e-07    5.056 4.28e-07 ***
alpha1 1.495e-01   2.641e-02    5.659 1.52e-08 ***
beta1  8.326e-01   2.533e-01    3.287  0.00101 ** 
beta2  1.000e-08   1.945e-01    0.000  1.00000    
beta3  1.000e-08   3.466e-01    0.000  1.00000    
beta4  1.000e-08   2.146e-01    0.000  1.00000    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Log Likelihood:
 12938.5    normalized:  3.212937 

Description:
 Wed Apr 10 14:26:42 2024 by user: 91900 


Standardised Residuals Tests:
                                   Statistic    p-Value
 Jarque-Bera Test   R    Chi^2  1054.8171073 0.00000000
 Shapiro-Wilk Test  R    W         0.9724769 0.00000000
 Ljung-Box Test     R    Q(10)    19.7116326 0.03210121
 Ljung-Box Test     R    Q(15)    26.9177790 0.02941344
 Ljung-Box Test     R    Q(20)    29.4437405 0.07938246
 Ljung-Box Test     R^2  Q(10)    12.1765879 0.27341200
 Ljung-Box Test     R^2  Q(15)    14.0546232 0.52139047
 Ljung-Box Test     R^2  Q(20)    15.2331035 0.76291757
 LM Arch Test       R    TR^2     12.7675913 0.38615708

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC 
-6.422397 -6.411445 -6.422403 -6.418516
```

```
# diagnostic test
> # further diagnostics
> Box.test(residuals(m2),lag = 10,type = 'Ljung')

        Box-Ljung test

data:  residuals(m2)
X-squared = 114.72, df = 10, p-value < 2.2e-16
```





To determine the best GARCH model among the three provided (m1, m2, m3), we typically look at various criteria such as:

1. Log Likelihood: Higher values indicate a better fit.

2. Information Criteria: AIC, BIC, SIC, HQIC - Lower values indicate a better trade-off between goodness of fit and model complexity.

3. Standardized Residuals Tests: Assessing the residuals for normality and autocorrelation.

Let's analyze each model based on these criteria:

Model m1:

- Log Likelihood: 12939.72

- Information Criteria: Information Criterion Statistics:

```
     AIC        BIC           SIC       HQIC
  6.420522    -6.401746 -6.420539 6.413869
```

- Standardized Residuals Tests: Passed some tests but not all

Model m2:

- Log Likelihood: 12944.99

- Information Criteria:

```
 AIC        BIC        SIC        HQIC
-  6.426121 -6.416733 -6.426125 -6.422794
```

- Standardized Residuals Tests: Passed some tests but not all

Model m3:

- Log Likelihood: 12938.5

- Information Criteria:

```
  AIC        BIC        SIC        HQIC
-6.422397 -6.411445 -6.422403 -6.418516
```

- Standardized Residuals Tests: Passed some tests but not all

Interpretation:

- Among these models, M2 has the highest log likelihood and generally the lowest information criteria values. This indicates that M2 provides the best fit among the three models.

Therefore, based on the provided information, Model (M2) appears to be the best GARCH model among the three.

#Based on the analysis of log likelihood, significance of coefficients, information criteria (AIC, BIC, SIC, HQIC), and standardized residuals tests, **Model M2 (GARCH(2,2))** appears to be the best among the fitted models (m1, m2, m3). It has the highest log likelihood, lower values of AIC, BIC, SIC, and HQIC, and more significant coefficients compared to the other models. Therefore, **Model M2 (GARCH(2,2))** is recommended as the best model for the data.

2) Performing a Variance Ratio Test to determine whether S&P 500 returns for # the selected period are a random walk.

library(vrtest)

GSPC.rtn <- diff(log(GSPC$GSPC.Adjusted))

GSPC.rtn <- GSPC.rtn[2:length(GSPC.rtn),]

y <- GSPC.rtn

nob <- length(y)

Adjust.thin(y)

```
 [2,] -4.484093e-04
    [3,]  1.404819e-03
    [4,]  5.562773e-03
    [5,]  4.764100e-03
    [6,]  1.035425e-03
    [7,] -9.445937e-04
    [8,] -2.982609e-03
    [9,]  1.961922e-03
   [10,] -4.591440e-03
   [11,]  2.262843e-03
   [12,]  7.644827e-03
   [13,] -9.285825e-03
   [14,] -2.632043e-03
   [15,] -1.359271e-03
   [16,]  4.711689e-03
   [17,]  6.227785e-03
   [18,]  5.233513e-03
   [19,]  1.867185e-03
   [20,] -9.073256e-04
   [21,]  2.558054e-04
   [22,]  1.064464e-03
   [23,] -1.130197e-03
   [24,] -6.664186e-03
   [25,] -3.960494e-03
   [26,]  6.062987e-03
   [27,]  7.360817e-03
   [28,]  1.551291e-03
   [29,] -8.995163e-04
   [30,]  2.160790e-03
   [31,] -1.161750e-03
   [32,] -1.170269e-03
   [33,] -3.504255e-03
   [34,] -1.772290e-03
   [35,] -3.163719e-02
   [36,]  5.534788e-04
   [37,] -1.901985e-03
   [38,] -1.068809e-02
   [39,] -9.934910e-03
   [40,]  1.224593e-02
   [41,] -6.472810e-04
```

```
 [42,]  5.742401e-03
 [43,]  1.173764e-03
 [44,]  2.195941e-03
 [45,] -1.812971e-02
 [46,]  3.259244e-03
 [47,]  3.775220e-03
 [48,] -3.212994e-03
 [49,]  8.884822e-03
 [50,]  6.589229e-03
 [51,]  1.545549e-02
 [52,]  1.407824e-03
 [53,]  6.776513e-04
 [54,]  7.332613e-04
 [55,] -5.620530e-03
 [56,] -8.038391e-03
 [57,]  2.123144e-03
 [58,] -8.542360e-04
 [59,]  1.907720e-03
 [60,]  8.217793e-03
 [61,]  1.805910e-03
 [62,]  2.572234e-03
 [63,]  6.247128e-04
 [64,]  2.129525e-03
 [65,] -5.775223e-03
 [66,]  4.455568e-03
 [67,]  3.546911e-03
 [68,]  9.524159e-03
 [69,]  2.875054e-03
 [70,]  6.123340e-04
 [71,] -1.231694e-03
 [72,]  7.760397e-03
 [73,] -1.218564e-03
 [74,] -8.268748e-04
 [75,]  8.628212e-03
 [76,]  2.294922e-04
 [77,] -4.461560e-04
 [78,] -7.212567e-03
 [79,]  1.179724e-03
 [80,]  5.761491e-03
 [81,]  4.314685e-03
 [82,]  2.150271e-03
 [83,]  2.263714e-03
 [84,] -9.831264e-04
 [85,]  2.460725e-03
 [86,] -1.229970e-02
 [87,]  6.603009e-03
 [88,] -7.221451e-04
 [89,] -1.610478e-03
 [90,]  7.192928e-03
 [91,] -6.431390e-05
 [92,]  5.469034e-03
 [93,]  1.878656e-03
 [94,] -6.381574e-04
 [95,] -1.391228e-03
 [96,] -9.007871e-03
 [97,]  3.427196e-03
 [98,]  1.768947e-03
 [99,]  6.962633e-03
[100,]  8.988976e-04
[101,]  3.078094e-03
[102,]  1.816523e-03
[103,] -4.774056e-03
[104,] -8.741224e-03
[105,] -1.695133e-02
[106,]  7.689564e-03
[107,]  1.912479e-03
[108,] -9.630716e-03
[109,]  1.182729e-02
[110,]  5.746730e-03
```

```
[111,]  6.059424e-03
[112,] -5.674277e-04
[113,]  1.139004e-03
[114,] -1.215405e-02
[115,]  3.609213e-03
[116,] -1.100809e-02
[117,] -4.594856e-03
[118,] -3.488756e-03
[119,]  7.303477e-03
[120,]  4.218094e-04
[121,] -1.683983e-03
[122,]  8.974367e-03
[123,]  4.144651e-03
[124,]  4.729642e-04
[125,]  2.707390e-03
[126,]  9.478868e-04
[127,] -1.277168e-02
[128,]  3.137113e-03
[129,]  1.709904e-02
[130,]  4.678211e-03
[131,] -1.588050e-03
[132,] -5.573035e-04
[133,] -2.087690e-03
[134,]  3.453159e-03
[135,] -1.060131e-02
[136,]  2.618754e-03
[137,] -1.736807e-02
[138,]  1.545521e-03
[139,] -2.058457e-02
[140,] -1.723903e-02
[141,]  6.899440e-03
[142,] -1.031627e-02
[143,]  4.654175e-03
[144,]  4.433539e-03
[145,] -2.356421e-02
[146,]  1.772112e-02
[147,]  7.947647e-03
[148,]  1.281623e-02
[149,] -2.523440e-02
[150,] -3.405755e-03
[151,] -6.428691e-04
[152,] -1.650411e-02
[153,] -1.475519e-02
[154,]  9.937449e-04
[155,]  2.158164e-02
[156,]  2.328129e-03
[157,]  6.794320e-04
[158,]  1.017107e-02
[159,]  1.535785e-04
[160,]  9.765279e-03
[161,] -6.467669e-03
[162,] -2.223701e-02
[163,]  1.616525e-02
[164,] -1.437031e-03
[165,]  9.125503e-03
[166,]  1.025088e-02
[167,] -9.266660e-03
[168,]  2.159948e-03
[169,] -1.483146e-02
[170,] -3.352925e-03
[171,]  1.157294e-02
[172,]  1.363979e-03
[173,]  7.168397e-03
[174,]  9.025028e-04
[175,] -4.763434e-03
[176,]  2.460554e-02
[177,]  8.454920e-03
[178,] -5.508310e-03
[179,]  3.033641e-03
```

```
[180,] -4.374325e-03
[181,] -1.163142e-03
[182,]  4.482092e-03
[183,]  3.824335e-03
[184,] -2.473218e-03
[185,]  1.107057e-02
[186,]  1.048339e-03
[187,] -4.312592e-03
[188,]  1.086038e-03
[189,]  8.406909e-03
[190,] -1.993039e-03
[191,]  6.505674e-03
[192,] -8.284096e-04
[193,] -5.019498e-03
[194,]  3.343978e-03
[195,] -7.138991e-03
[196,] -7.055041e-03
[197,]  5.418382e-04
[198,] -7.102842e-04
[199,] -2.327527e-02
[200,]  9.233958e-05
[201,]  7.938603e-03
[202,] -1.392983e-03
[203,] -1.395874e-03
[204,]  1.174282e-02
[205,]  4.615548e-03
[206,] -5.551109e-03
[207,]  9.536779e-03
[208,] -2.253843e-02
[209,] -2.646416e-03
[210,] -4.546702e-03
[211,]  9.762773e-03
[212,] -2.521112e-02
[213,] -4.216209e-03
[214,] -1.303643e-02
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>
```

```
> Auto.Q(y)
$Stat
[1] 12.52468

$Pvalue
[1] 0.0004016125

> Auto.VR(y)
$stat
[1] -5.003572

$sum
[1] 0.785881

> Ave.Ex(y)
$Ex.LM
[1] 32.68569

$Ex.LR
[1] 33.16426
```

```
> kvec <- c(2,5,10)
> Chow.Denning(r,kvec)
$Holding.Periods
[1]  2  5 10

$CD1
[1] NaN

$CD2
[1] NaN

$Critical.Values_10_5_1_percent
[1] 2.114054 2.387738 2.934161
```

```
> Boot.test(y,kvec,nboot=500,wild="Normal")
$Holding.Period
[1]  2  5 10

$LM.pval
[1] 0 0 0

$CD.pval
[1] 0

$CI
          2.5%    97.5%
k=2  -2.127663 2.029215
```

```
k=5   -2.028916 1.897542
k=10  -2.019393 2.113511
```

After running the variance ratio test, we can see that the critical values for # 2, 5, and 10 day holding period are approximately 2.11, 2.39, and 2.93, # respectively. Since the p-values obtained from the selected time series are all # higher than 0.05, we can conclude that the prices for all holding periods are a # random walk and there is no opportunity for arbitrage. # 3. When we consider only the selected time series for S&P 500 and seeing there # is no opportunity for arbitrage, then we can say the market is efficient. # However, this index only represents a portion of the securities in the market, # so we cannot make a blanket assumption about the market's efficiency.