Data Mining

Homework 1

Description

Homework 1
(Association Rule Mining)

Given the following database, use the Apriori algorithm to find all association rules

with minimum support = 50% and

minimum confidence = 70%.

TID	Items
1001	ink, pen,cheese,bag
1002	milk,pen,juice,cheese
1003	milk,juice
1004	juice,milk,cheese

Data Mining

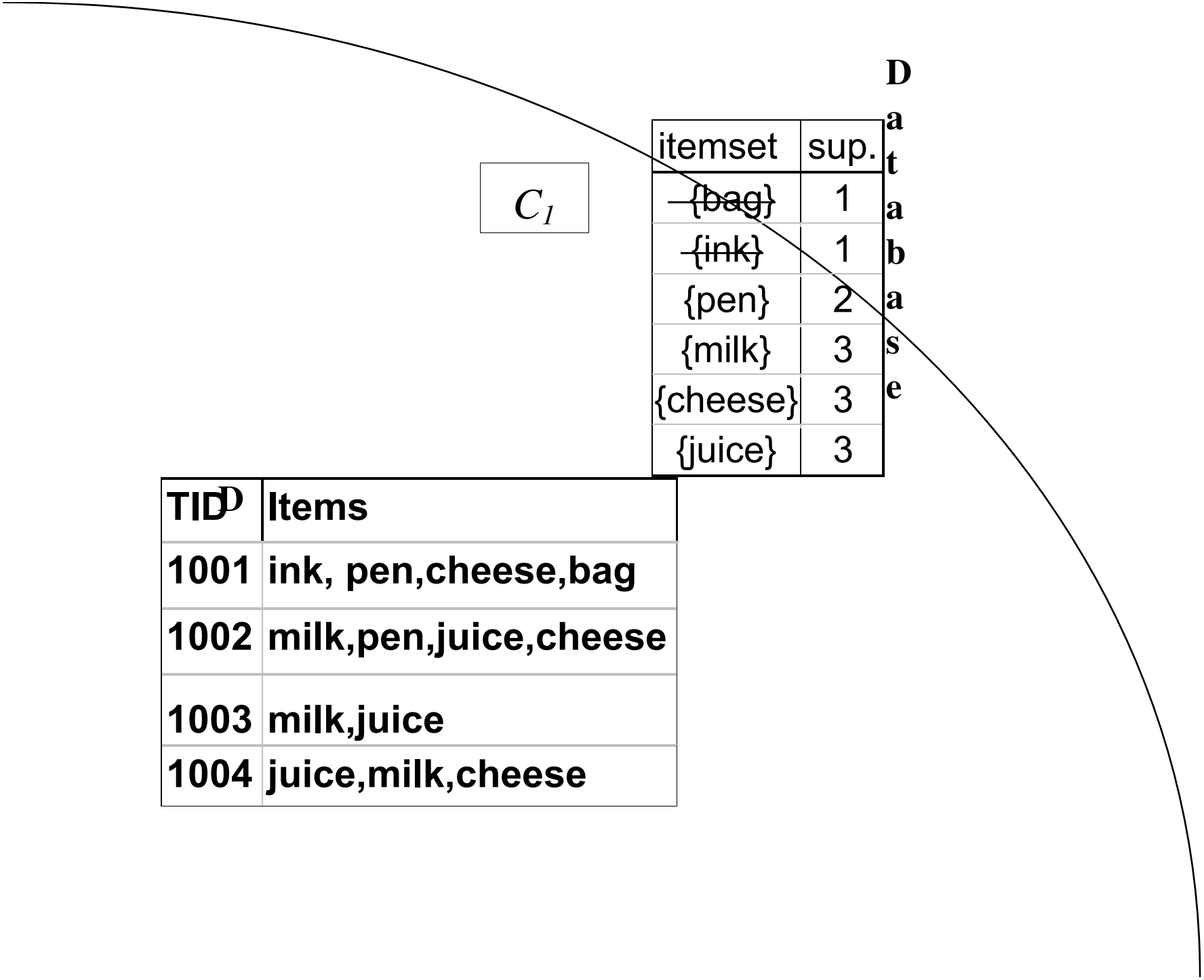
Homework 1 Solution

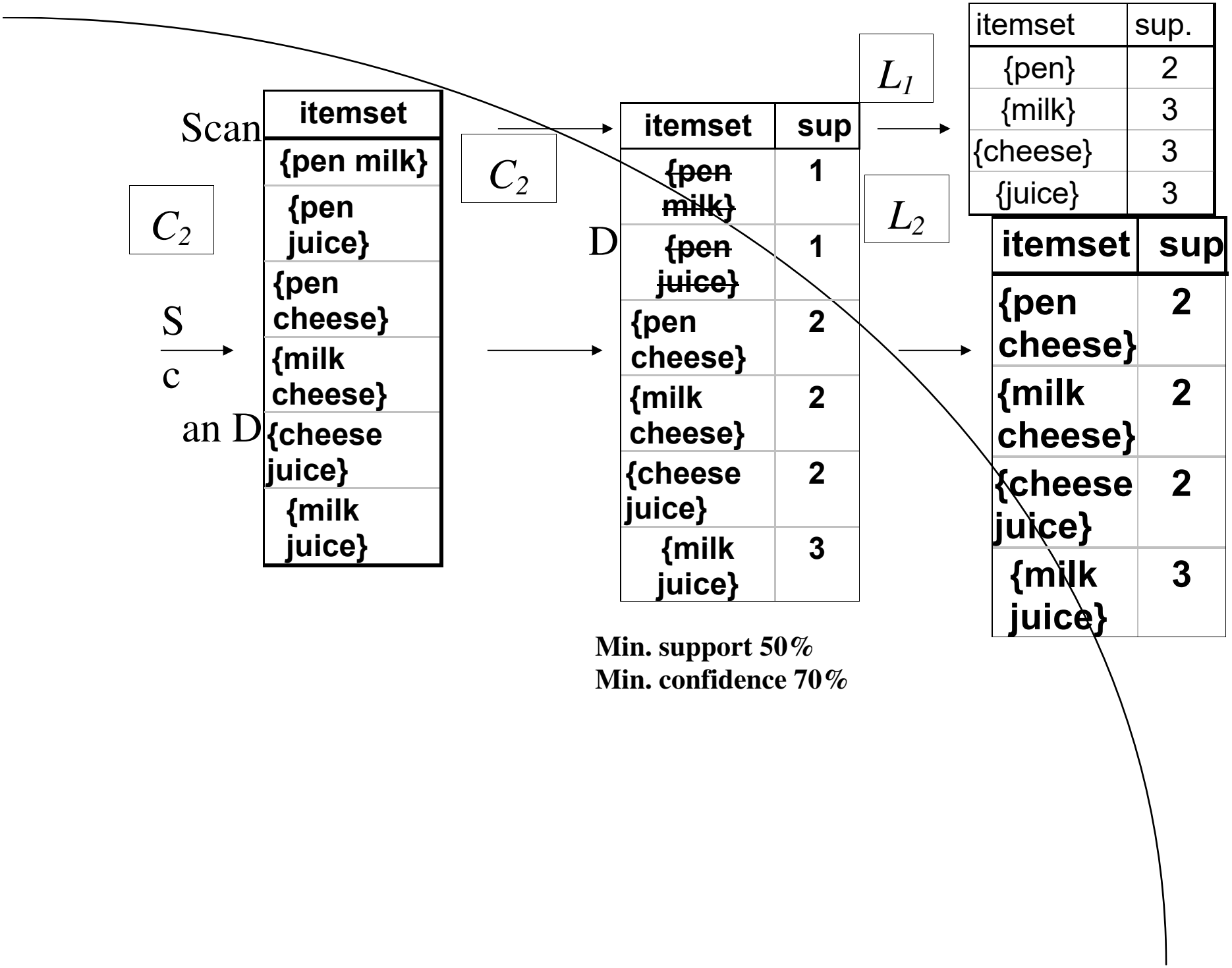
Homework 1 Solution
(Association Rule Mining)

Given the following database, find all association rules with minimum support = 50% and minimum confidence = 70%.

TID	Items
1001	ink, pen,cheese,bag
1002	milk,pen,juice,cheese
1003	milk,juice
1004	juice,milk,cheese

Min. support 50%
Min. confidence 70%


D
a
t
a
b
a
s
e
C_1
itemset
sup.
{bag}
1
{ink}
1
{pen}
2
{milk}
3
{cheese}
3
{juice}
3
TID
Items
1001
ink, pen,cheese,bag
1002
milk,pen,juice,cheese
1003
milk,juice
1004
juice,milk,cheese


Scan D
C_2
itemset
{pen milk}
{pen juice}
{pen cheese}
{milk cheese}
{cheese juice}
{milk juice}
C_2
itemset sup
{pen milk} 1
{pen juice} 1
{pen cheese} 2
{milk cheese} 2
{cheese juice} 2
{milk juice} 3
L_1
itemset sup.
{pen} 2
{milk} 3
{cheese} 3
{juice} 3
L_2
itemset sup
{pen cheese} 2
{milk cheese} 2
{cheese juice} 2
{milk juice} 3
Min. support 50%
Min. confidence 70%

C_3Sca n D

itemset
{milk cheese juice}

L_3

itemset	sup
{milk cheese juice}	2

Min. support 50%
Min. confidence 70%

Only the following association rules have confidence >= 70%:

{pen} =>{cheese}

{milk}=>{juice}

{juice}=>{milk}

{cheese juice}=>{milk}

{milk cheese}=>{juice}

Data Mining

Homework 2

Description

Homework 2
(Decision Tree Induction)

- Given is the training dataset on the following slide.
- Show the decision tree learned from the training dataset and all classification rules extracted from the tree.

Training Dataset - **“subscribe car magazine”**

Income	Marital_Status	Living_Area	Car_Type	Subscribe
Medium	Single	NJ	Sports	Yes
High	Married	NJ	Sedan	No
Low	Single	NY	Sedan	No
Medium	Single	CT	Sedan	Yes
Medium	Married	NJ	Van	No
Low	Married	NJ	Van	Yes
Medium	Single	NY	Sports	Yes
High	Married	CT	Van	No
Medium	Married	CT	Van	No
Low	Single	NJ	Sedan	Yes
Medium	Married	NJ	Sedan	No
High	Single	NY	Sedan	No
High	Single	NY	Sports	Yes
Medium	Single	NJ	Sedan	No
Low	Single	NY	Sports	Yes

Data Mining

Homework 2 Solution

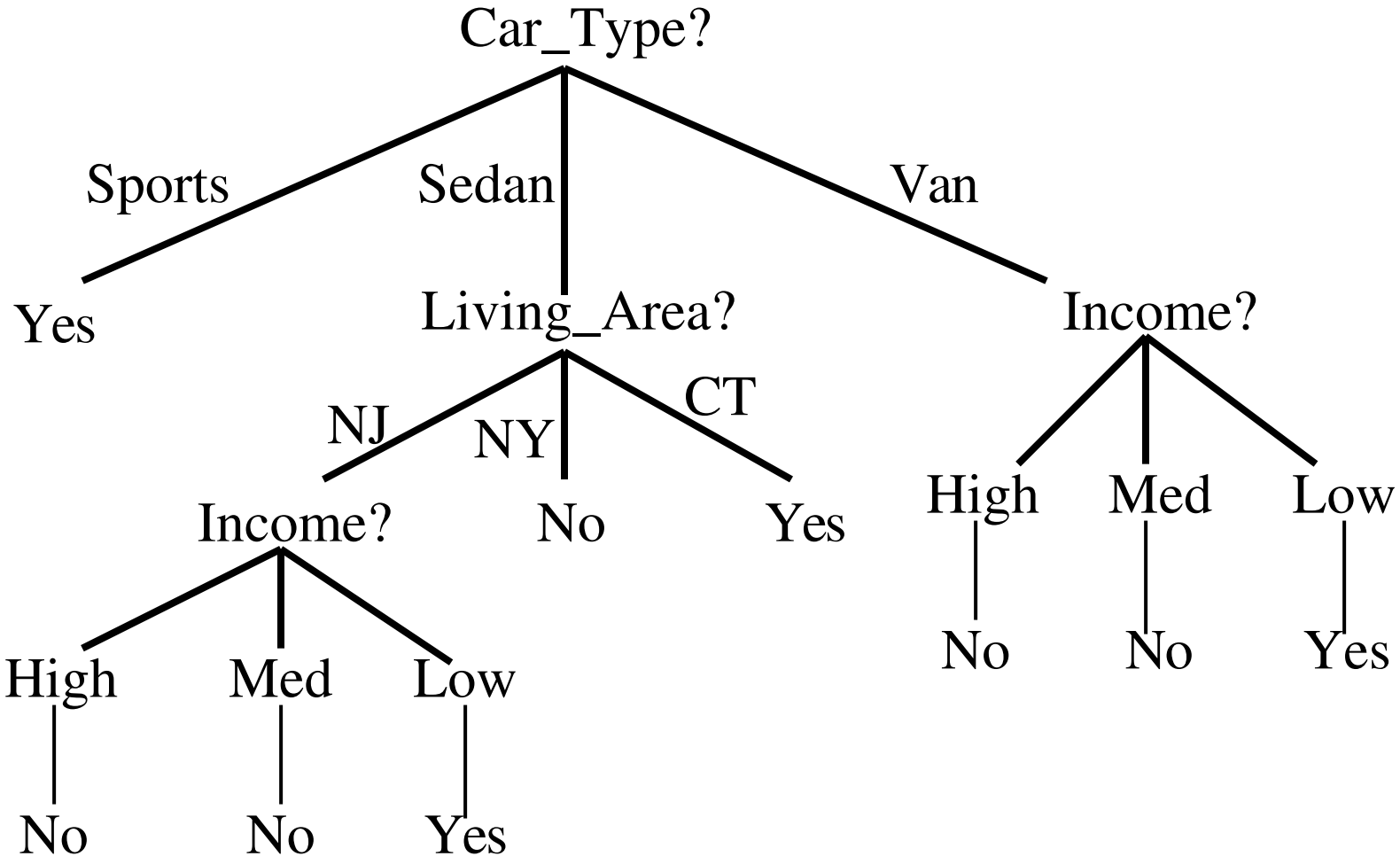
Training Dataset

“subscribe car magazine”

Income	Marital_Status	Living_Area	Car_Type	Subscribe
Medium	Single	NJ	Sports	Yes
High	Married	NJ	Sedan	No
Low	Single	NY	Sedan	No
Medium	Single	CT	Sedan	Yes
Medium	Married	NJ	Van	No
Low	Married	NJ	Van	Yes
Medium	Single	NY	Sports	Yes
High	Married	CT	Van	No
Medium	Married	CT	Van	No
Low	Single	NJ	Sedan	Yes
Medium	Married	NJ	Sedan	No
High	Single	NY	Sedan	No
High	Single	NY	Sports	Yes
Medium	Single	NJ	Sedan	No
Low	Single	NY	Sports	Yes

Output: A Decision Tree for

"subscribe car magazine"



Attribute Selection by Information Gain Computation

- Class P: change_plan = "yes"
- Class N: change_plan = "no"
- Compute the expected information needed to classify a given sample:

$$I(p,n) = I(7,8) = -0.997 \quad — \quad — \quad —$$

$$- \log \frac{7}{15} \quad \frac{2}{15}$$

$$- \log \frac{8}{15} \quad \frac{2}{15}$$

Information Gain Computation
(1) Income

- Compute the entropy and information gain

Income	p_i	n_i	$I(p_i, n_i)$
High	1	3	0.811
Medium	3	4	0.985
Low	3	1	0.811

$$E(Income) = \frac{4}{15} I(1,3) + \frac{7}{15} I(3,4) + \frac{4}{15} I(3,1) = 0.892$$

$$Gain(Income) = I(p,n) - E(Income)$$
$$= 0.997 - 0.892 = 0.105$$

Information Gain Computation
(2) Marital Status

- Compute the entropy and information gain

M_Status	p_i	n_i	$I(p_i, n_i)$
Single	6	3	0.917
Married	1	5	0.65

$$E(M\_Status) = \frac{9}{15} I(6,3) + \frac{6}{15} I(1,5) = 0.810$$

$$Gain(M\_Status) = I(p,n) - E(M\_Status)$$
$$= 0.997 - 0.810 = 0.187$$

Information Gain Computation
(3) Living Area

- Compute the entropy and information gain

L_Area	p_i	n_i	$I(p_i, n_i)$
NJ	3	4	0.985
NY	3	2	0.971
CT	1	2	0.919

$$E(L\_Area) = \frac{7}{15} I(3,4) + \frac{5}{15} I(3,2) + \frac{3}{15} I(1,2) = 0.967$$

$$Gain(L\_Area) = I(p,n) - E(L\_Area)$$
$$= 0.997 - 0.967 = 0.030$$

Information Gain Computation
(4) Car Type

- Compute the entropy and information gain

C_Type	p_i	n_i	$I(p_i, n_i)$
Sports	4	0	0
Sedan	2	5	0.864
Van	1	3	0.811

$$E(C\_Type) = \frac{4}{15} I(4,0) + \frac{7}{15} I(2,5) + \frac{4}{15} I(1,3) = 0.620$$

$$Gain(C\_Type) = I(p,n) - E(C\_Type)$$
$$= 0.997 - 0.620 = 0.377$$

Information Gain Computation

- Select Highest Gain

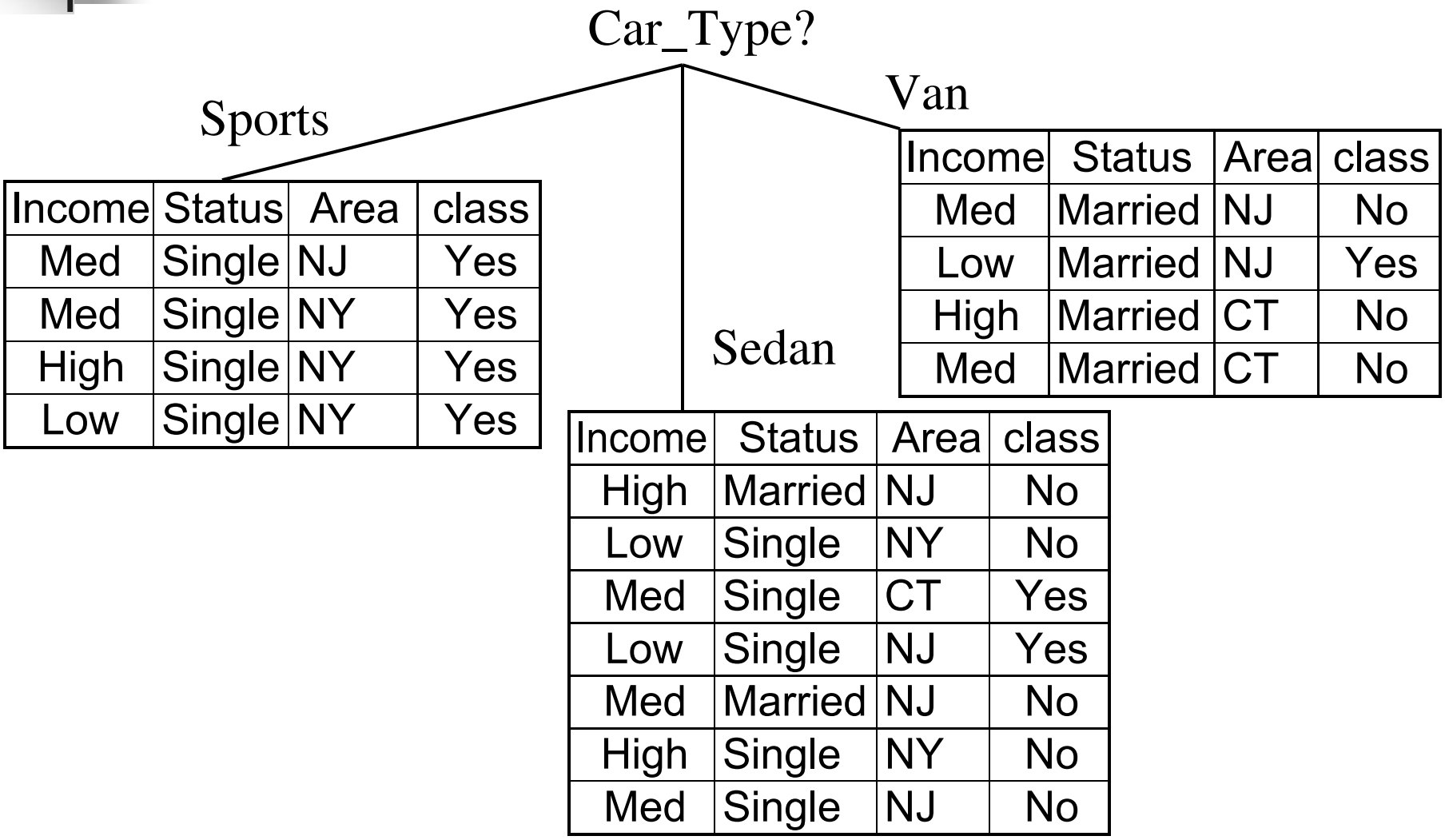
$$Gain\ (Income\ ) = 0.105$$

$$Gain\ (Marital\_Status\ ) = 0.187$$

$$Gain\ (Living\_Area\ ) = 0.030$$

$$Gain\ (Car\_Type\ ) = 0.377$$

Partitioned Training Data Set by "Car Type"

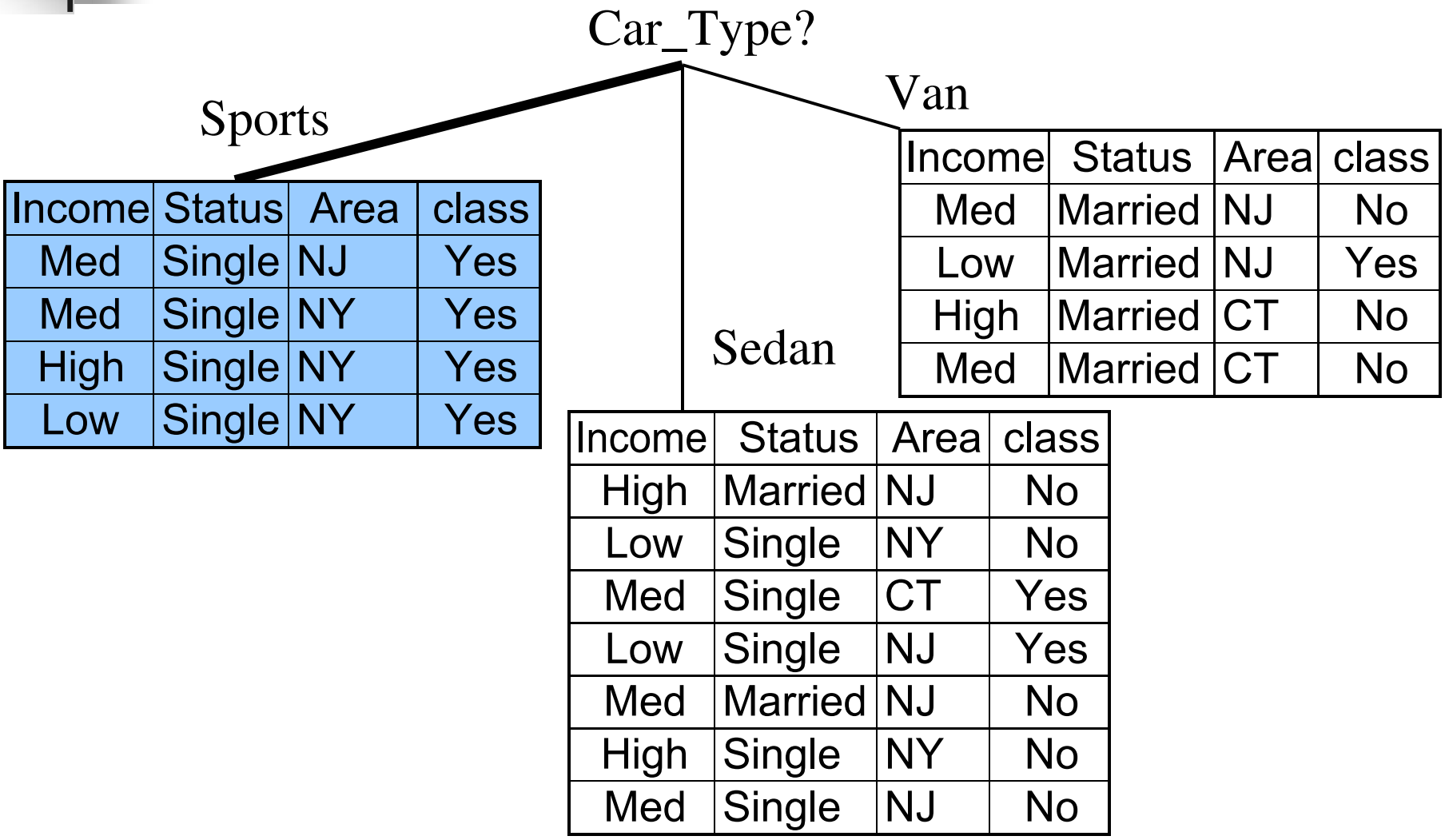


Income	Status	Area	class
Med	Single	NJ	Yes
Med	Single	NY	Yes
High	Single	NY	Yes
Low	Single	NY	Yes

Income	Status	Area	class
High	Married	NJ	No
Low	Single	NY	No
Med	Single	CT	Yes
Low	Single	NJ	Yes
Med	Married	NJ	No
High	Single	NY	No
Med	Single	NJ	No

Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

Partitioned Training Data Set by "Car Type"

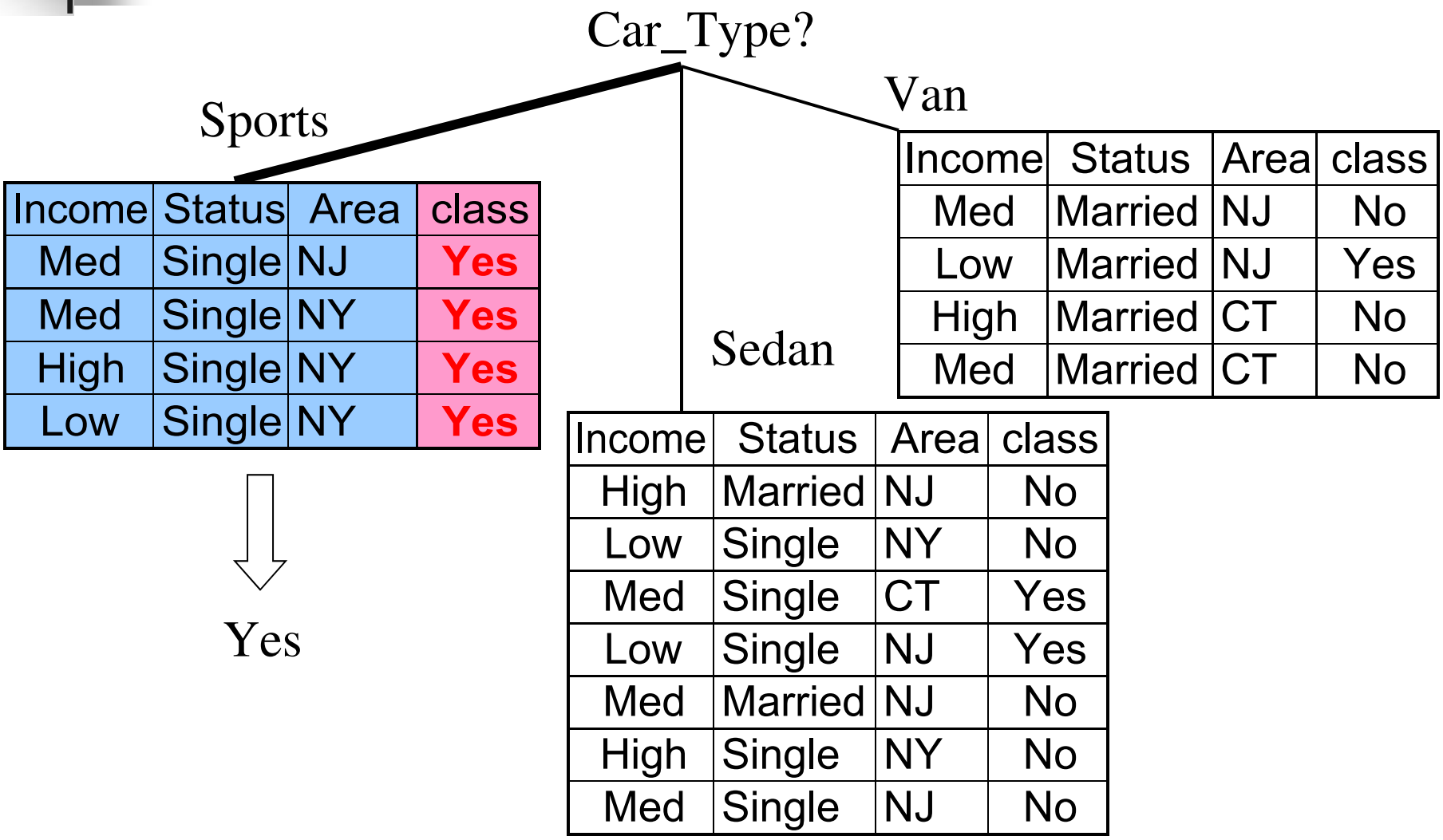


Income	Status	Area	class
Med	Single	NJ	Yes
Med	Single	NY	Yes
High	Single	NY	Yes
Low	Single	NY	Yes

Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

Income	Status	Area	class
High	Married	NJ	No
Low	Single	NY	No
Med	Single	CT	Yes
Low	Single	NJ	Yes
Med	Married	NJ	No
High	Single	NY	No
Med	Single	NJ	No

Partitioned Training Data Set by "Car Type"

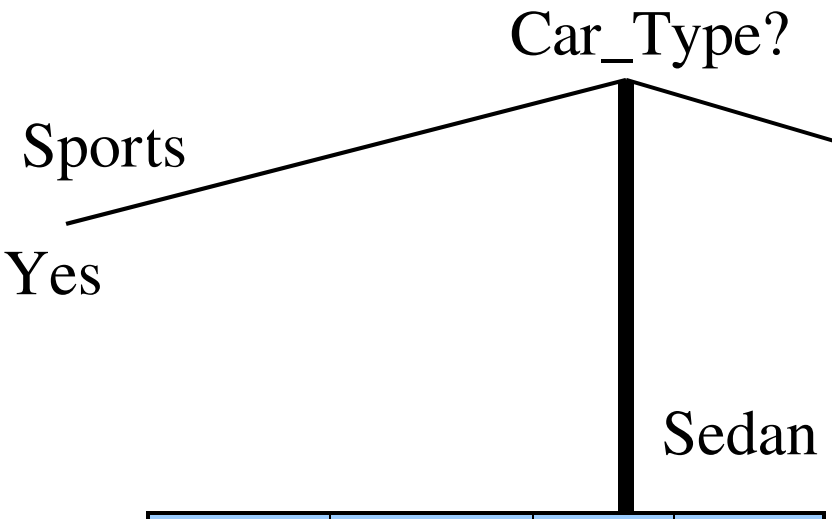


Income	Status	Area	class
Med	Single	NJ	**Yes**
Med	Single	NY	**Yes**
High	Single	NY	**Yes**
Low	Single	NY	**Yes**

Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

Income	Status	Area	class
High	Married	NJ	No
Low	Single	NY	No
Med	Single	CT	Yes
Low	Single	NJ	Yes
Med	Married	NJ	No
High	Single	NY	No
Med	Single	NJ	No

Partitioned Training Data Set and Partial Decision Tree



Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

Income	Status	Area	class
High	Married	NJ	No
Low	Single	NY	No
Med	Single	CT	Yes
Low	Single	NJ	Yes
Med	Married	NJ	No
High	Single	NY	No
Med	Single	NJ	No

Information Gain Computation for Sedan (1) Income

$$I(p,n) = I(2,5) = -\frac{2}{7}\log_2\frac{2}{7} - \frac{5}{7}\log_2\frac{5}{7} = 0.864$$

- Compute the entropy and information gain

Income	p_i	n_i	$I(p_i, n_i)$
High	0	2	0
Med	1	2	0.919
Low	1	1	1

$$E(Income) = \frac{2}{7}I(0,2) + \frac{3}{7}I(1,2) + \frac{2}{7}I(1,1) = 0.680$$

$$Gain(Income) = I(p,n) - E(Income)$$
$$= 0.864 - 0.680 = 0.184$$

Information Gain Computation for Sedan (2) Marital Status

- Compute the entropy and information gain

M_Status	p_i	n_i	$I(p_i, n_i)$
Single	2	3	0.971
Married	0	2	0

$$E(M\_Status) = \frac{5}{7} I(2,3) + \frac{2}{7} I(0,2) = 0.693$$

$$Gain(M\_Status) = I(p,n) - E(M\_Status)$$
$$= 0.864 - 0.693 = 0.171$$

Information Gain Computation for Sedan (3) Li ving Area

- Comp te the entropy and information gain

L_Area	p_i	n_i	$I(p_i, n_i)$
NJ	1	3	0.811
NY	0	2	0
CT	1	0	0

$$E(L\_Area) = \frac{4}{7} I(1,3) + \frac{2}{7} I(0,2) + \frac{1}{7} I(1,0) = 0.463$$

$$Gain(L\_Area) = I(p,n) - E(L\_Area)$$
$$= 0.864 - 0.463 = 0.401$$

Information Gain Computation for Sedan

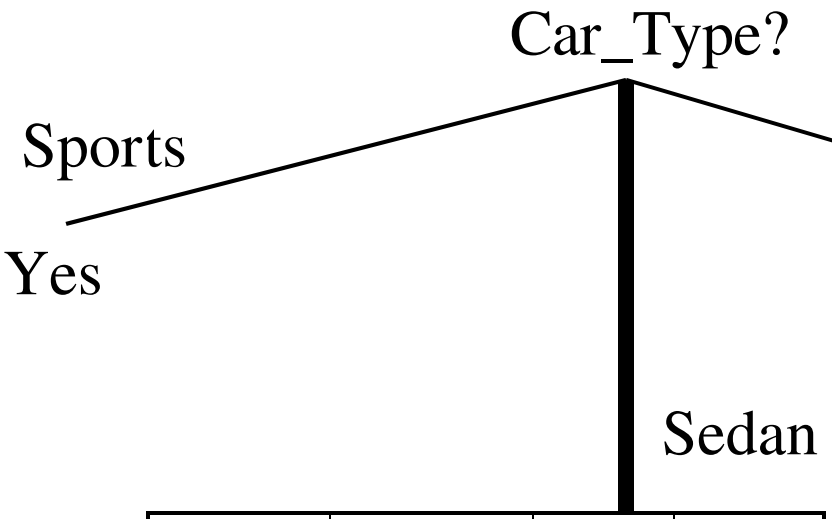
- Select Highest Gain

$$Gain\ (Income\ ) = 0.184$$

$$Gain\ (Marital\ \_\ Status\ ) = 0.171$$

$$Gain\ (Living\ \_\ Area\ ) = 0.401$$

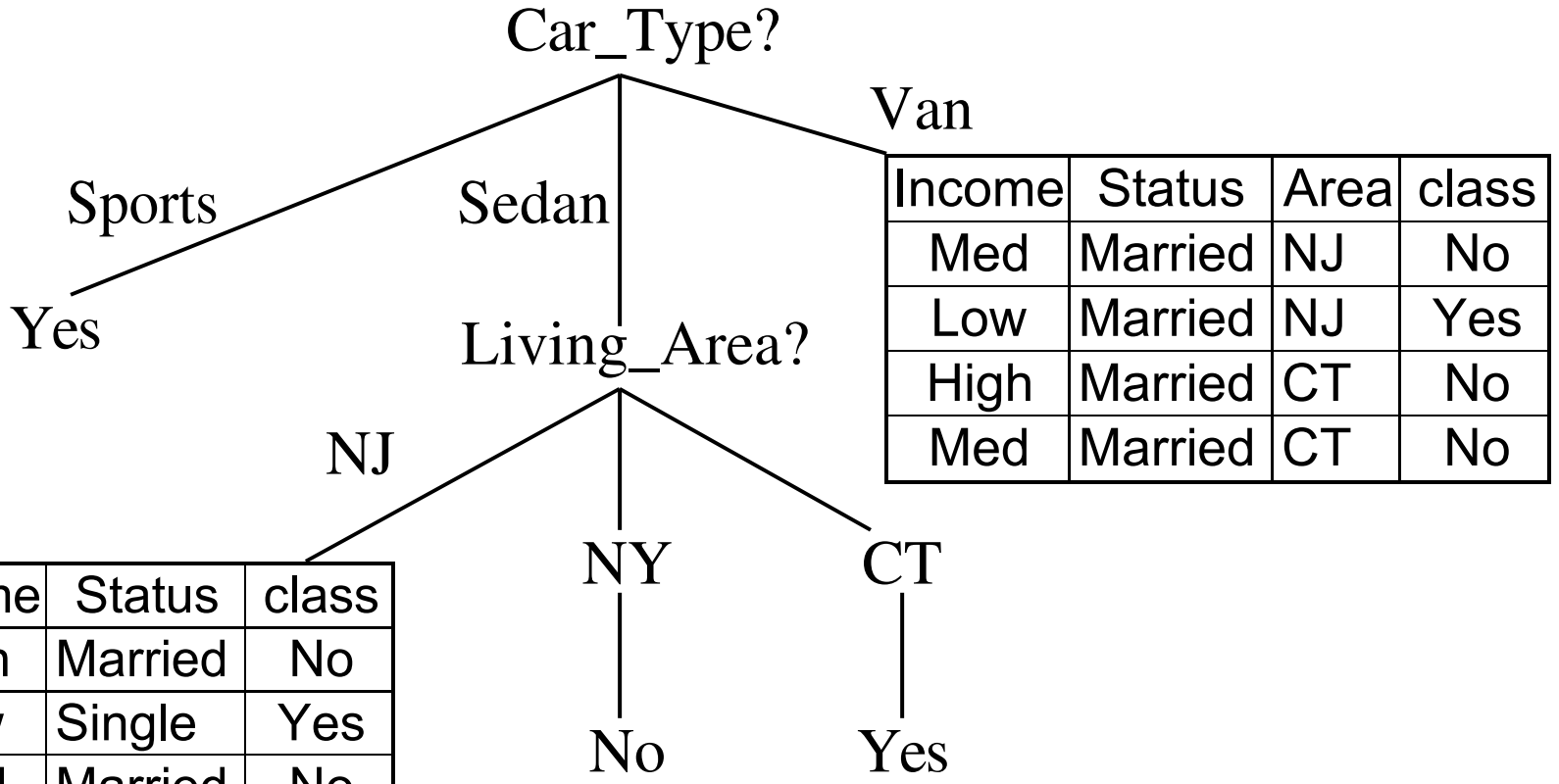
Partitioned Training Data Set and Partial Decision Tree



Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

Income	Status	Area	class
High	Married	**NJ**	**No**
Low	Single	**NY**	**No**
Med	Single	**CT**	**Yes**
Low	Single	**NJ**	**Yes**
Med	Married	**NJ**	**No**
High	Single	**NY**	**No**
Med	Single	**NJ**	**No**

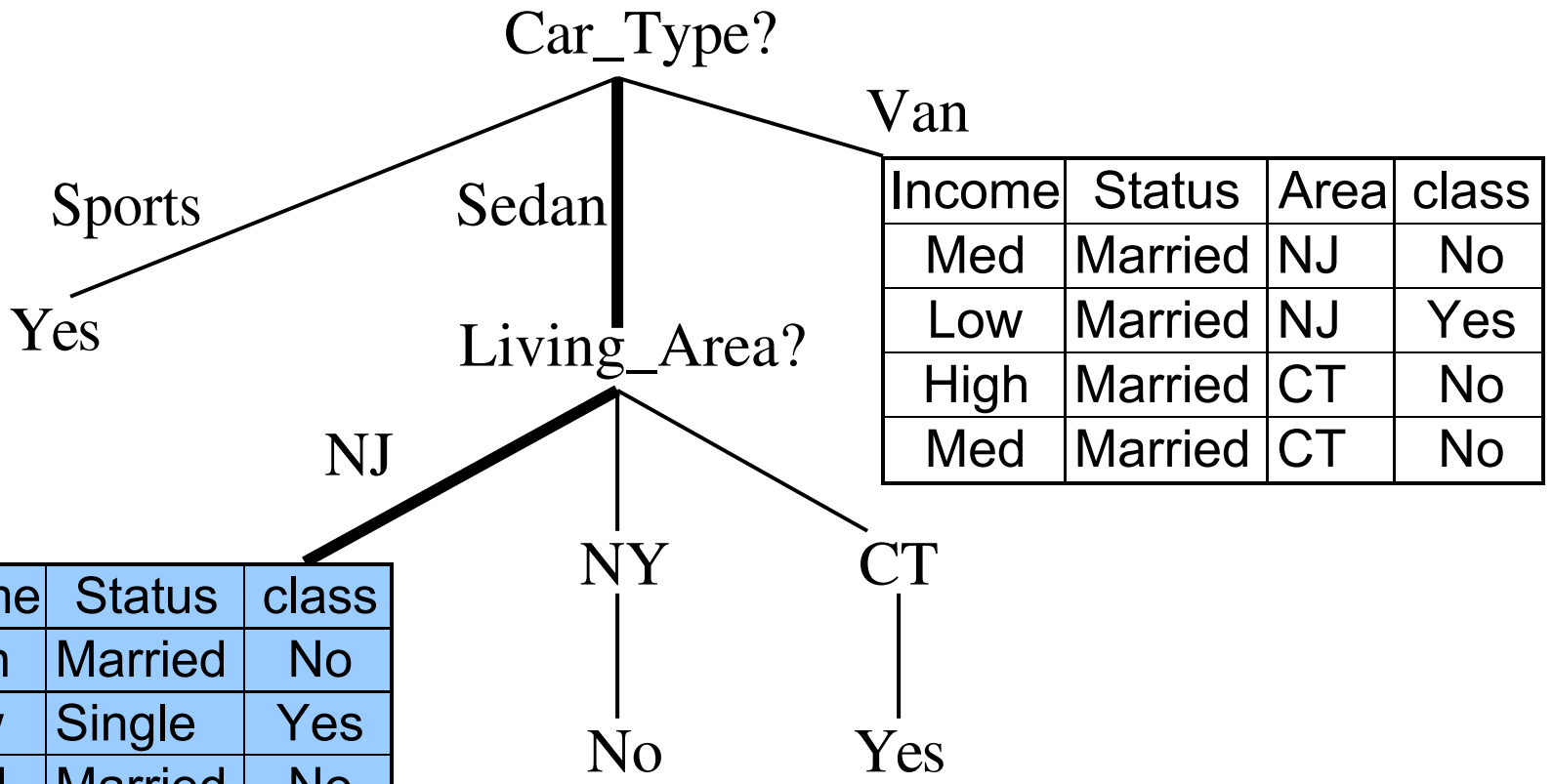
Partitioned Training Data Set and Partial Decision Tree



Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

Income	Status	class
High	Married	No
Low	Single	Yes
Med	Married	No
Med	Single	No

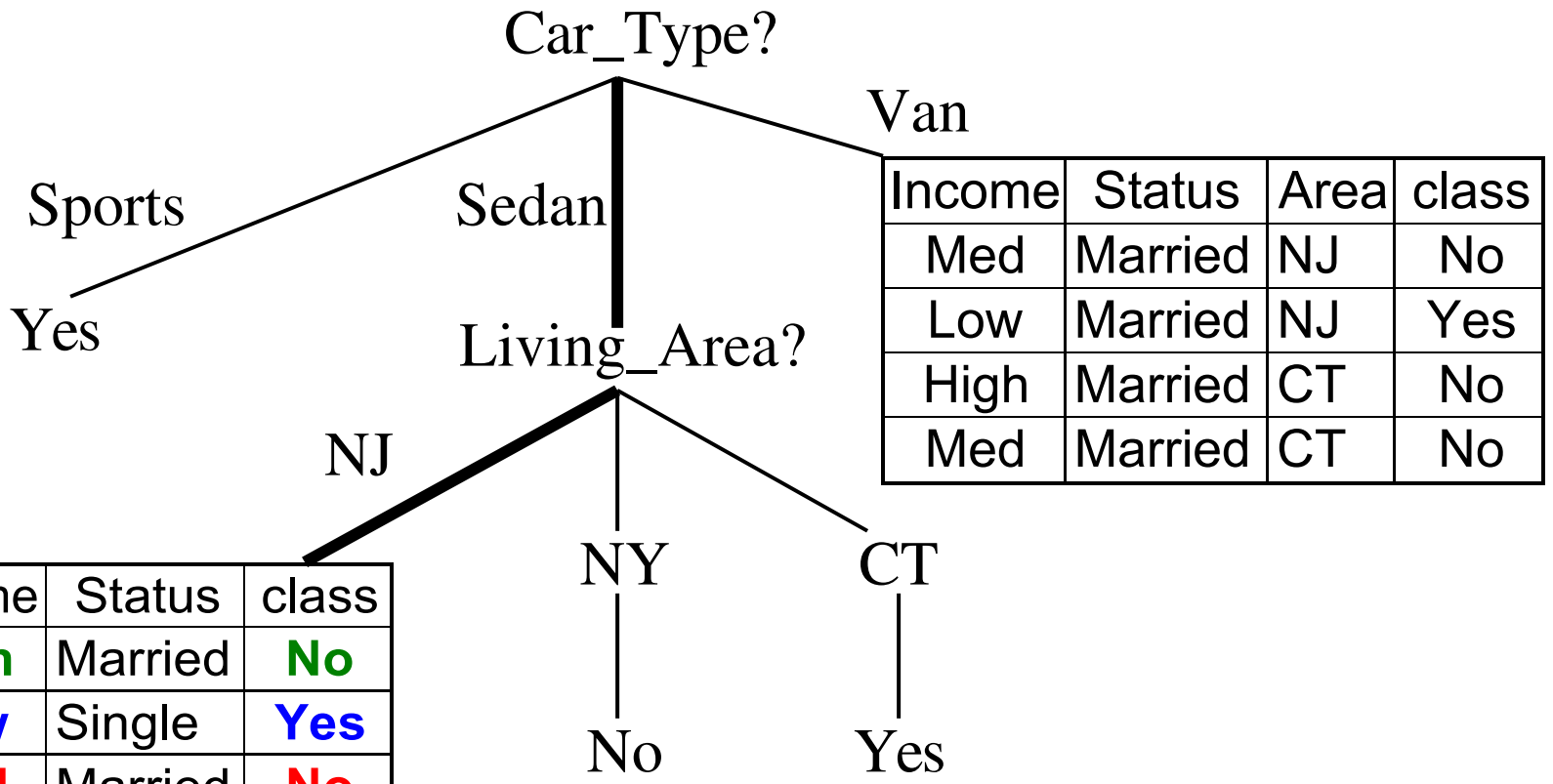
Partitioned Training Data Set and Partial Decision Tree



Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

Income	Status	class
High	Married	No
Low	Single	Yes
Med	Married	No
Med	Single	No

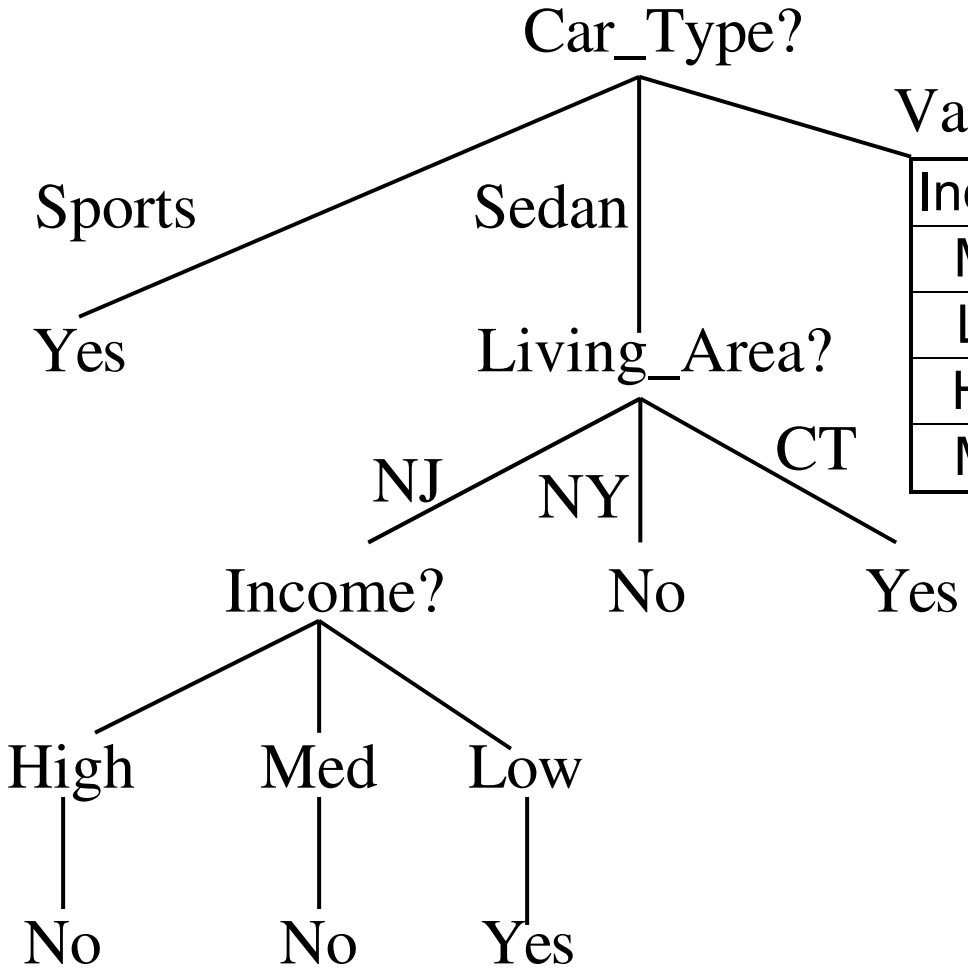
Partitioned Training Data Set and Partial Decision Tree



Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

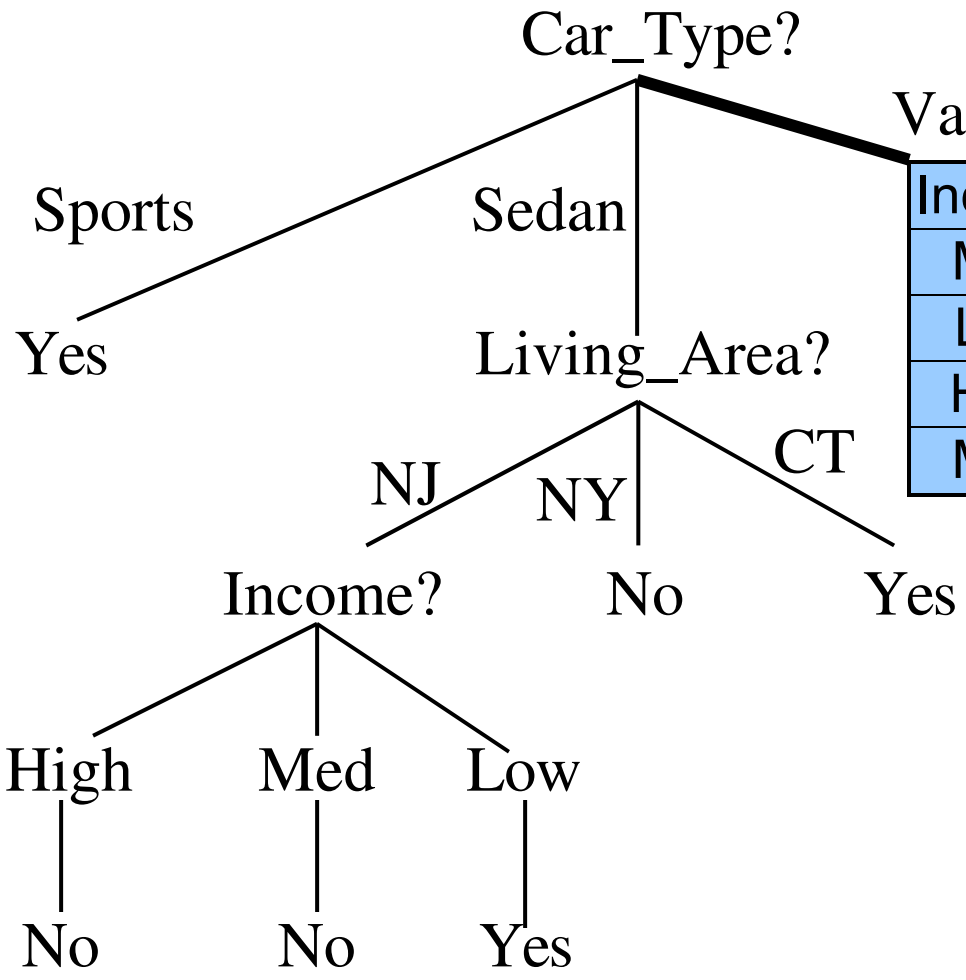
Income	Status	class
High	Married	**No**
Low	Single	**Yes**
Med	Married	**No**
Med	Single	**No**

Partitioned Training Data Set and Partial Decision Tree



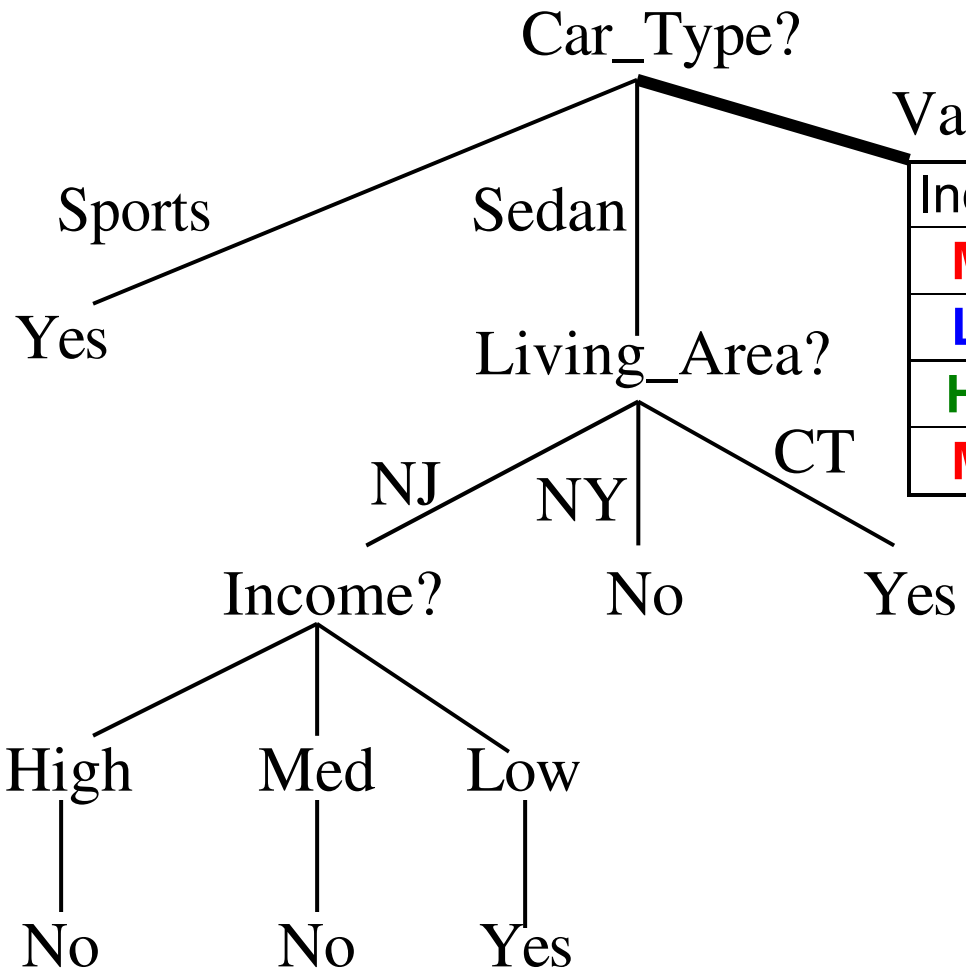
Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

Partitioned Training Data Set and Partial Decision Tree



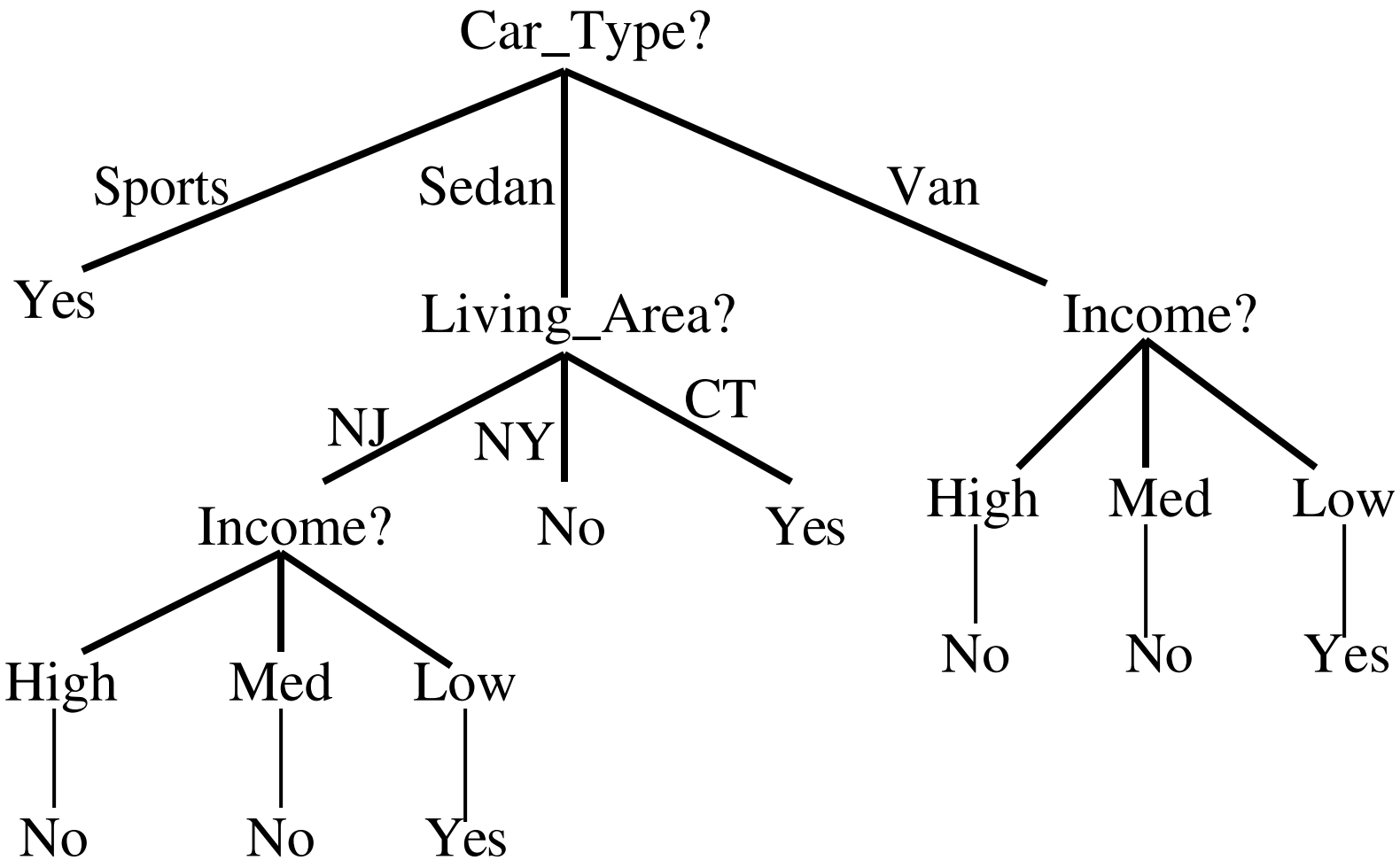
Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

Partitioned Training Data Set and Partial Decision Tree



Income	Status	Area	class
Med	Married	NJ	No
Low	Married	NJ	Yes
High	Married	CT	No
Med	Married	CT	No

Final Decision Tree



Extracting Classification Rules from Tree

IF car_type = "Sports"
THEN subscribe_car_magazine = "Yes"

IF car_type = "Sedan" AND area = "NJ" AND income="High"
THEN subscribe_*car_magazine* = "No"

IF car_type = "Sedan" AND area = "NJ" AND income="Medium"
THEN subscribe_car_magazine = "No"

IF car_type = "Sedan" AND area = "NJ" AND income="Low"
THEN subscribe_car_magazine = "Yes"

IF car_type = "Sedan" AND area = "NY"
THEN subscribe_*car_magazine* = "No"

IF car_type = "Sedan" AND area = "CT"
THEN subscribe_*car_magazine* = "Yes"

Extracting Classification Rules from Tree (cont.)

IF car_type = "Van" AND income="High"
THEN subscribe_car_magazine = "No"

IF car_type = "Van" AND income="Medium"
THEN subscribe_car_magazine = "No"

IF car_type = "Van" AND income="Low"
THEN subscribe_car_magazine = "Yes"

Data Mining

Homework 3

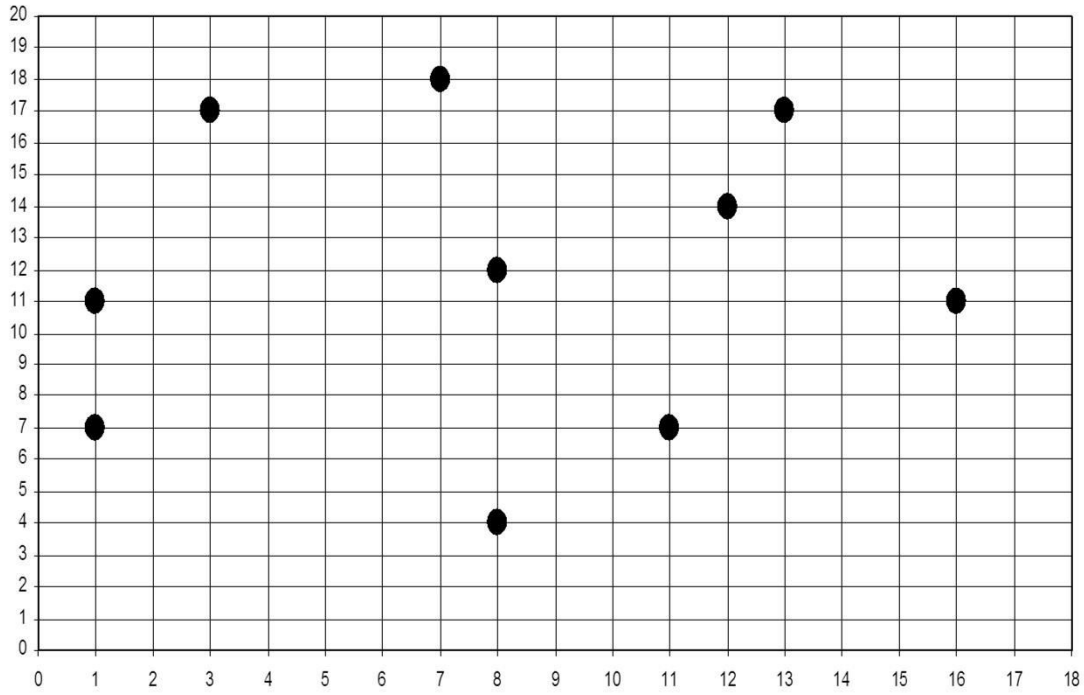
Description

Homework 3

(The Hierarchical Agglomerative Clustering Method)

- Given are ten 2-dimensional points as shown inthe figure on the next slide. The coordinates of the points can be seen from the figure.
- Divide the points into 2 clusters using the hierarchical agglomerative clustering method.

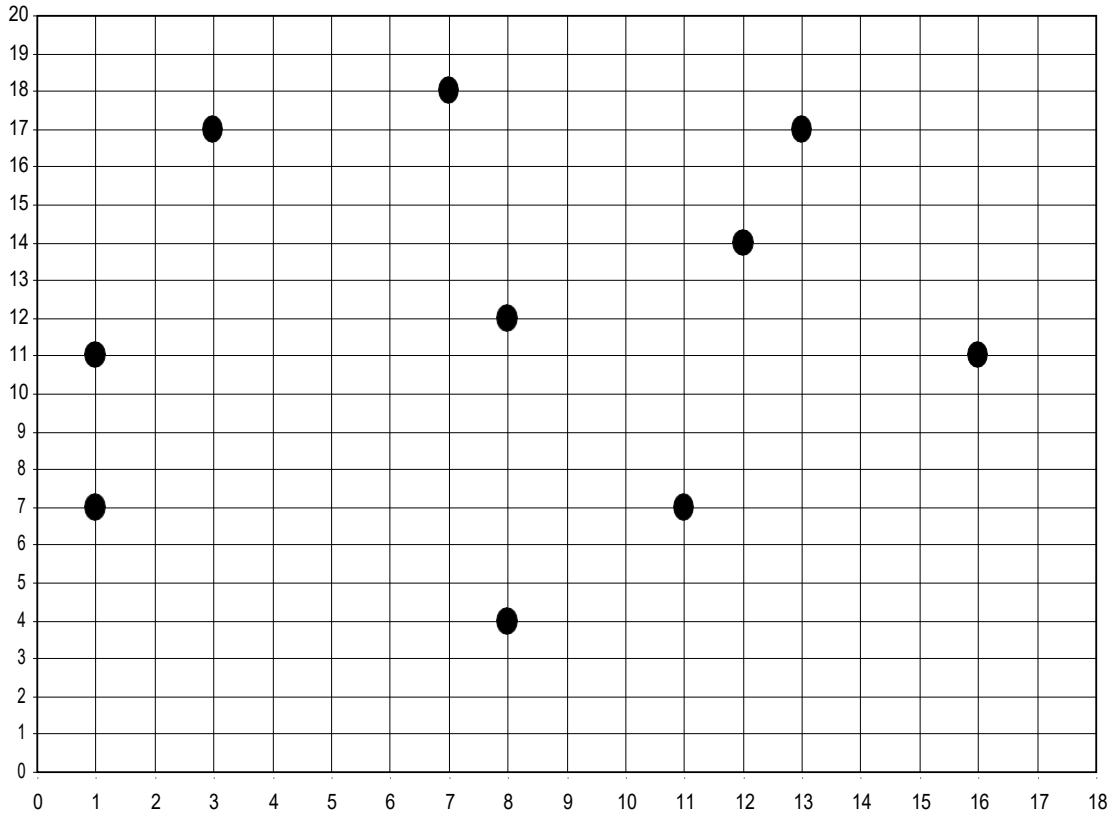
Hierarchical Agglomerative Clustering



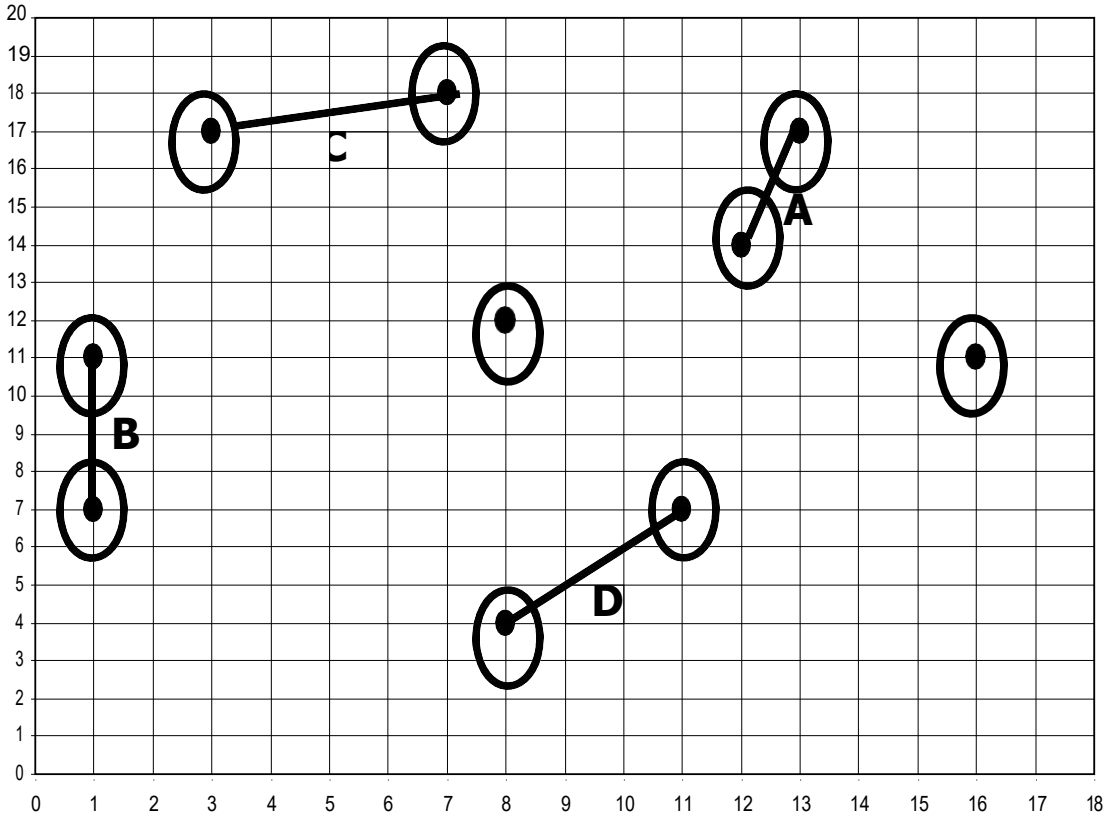
Data Mining

Homework 3 Solution

Agglomerative Clustering



Agglomerative Clustering



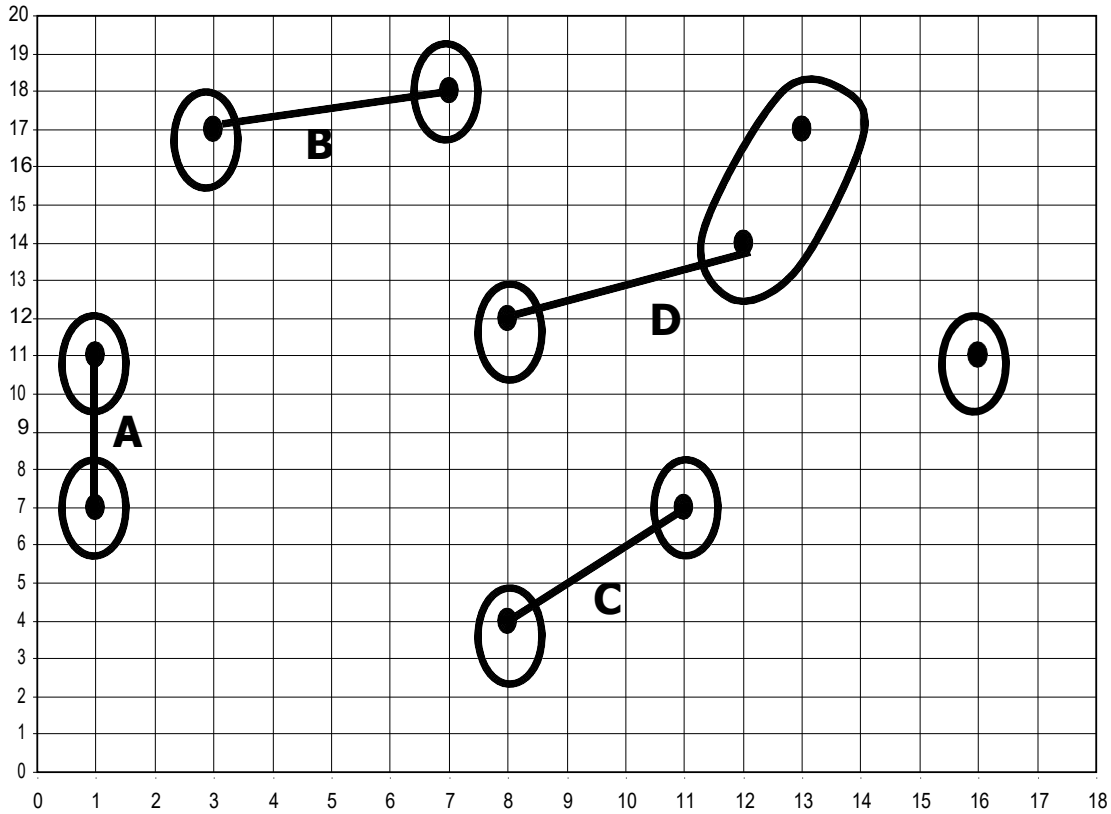
$$A = \sqrt{3^2 + 1^2} = 3.16$$

$$B = 4$$

$$C = \sqrt{4^2 + 1^2} = 4.12$$

$$D = \sqrt{3^2 + 3^2} = 4.24$$

Agglomerative Clustering



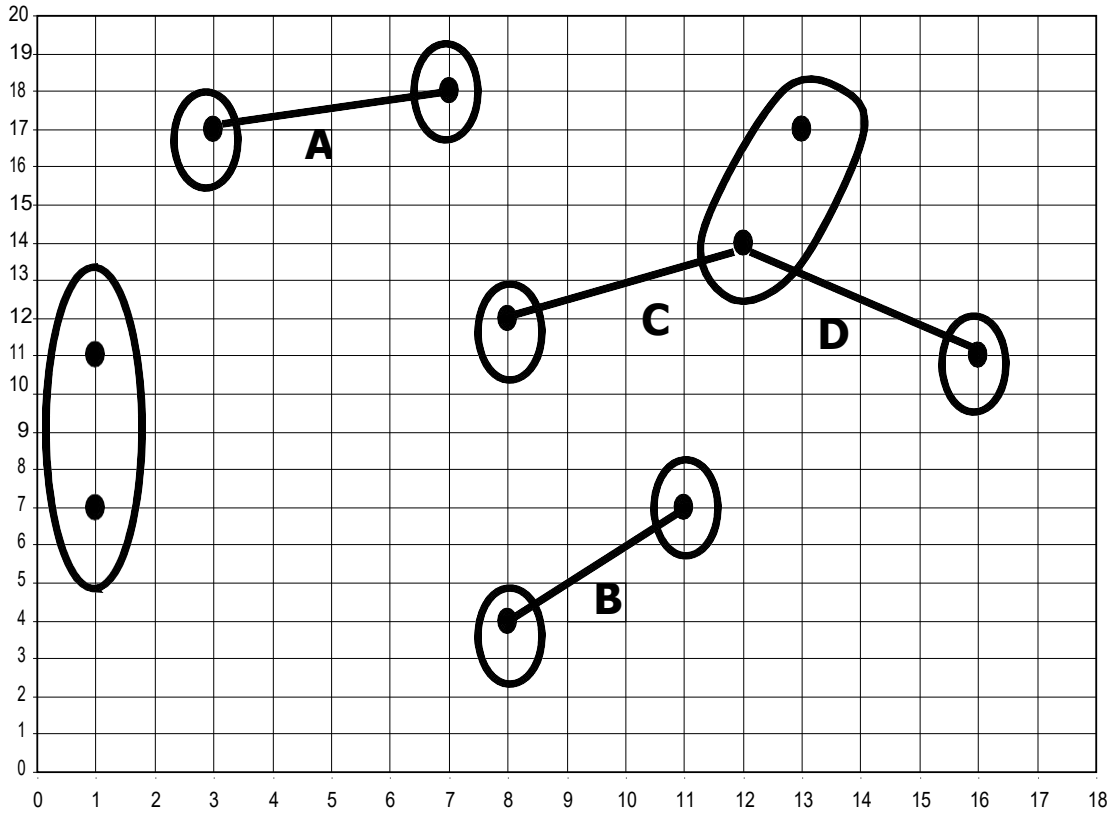
$$A = 4$$

$$B = \sqrt{4^2 + 1^2} = 4.12$$

$$C = \sqrt{3^2 + 3^2} = 4.24$$

$$D = \sqrt{4^2 + 2^2} = 4.47$$

Agglomerative Clustering



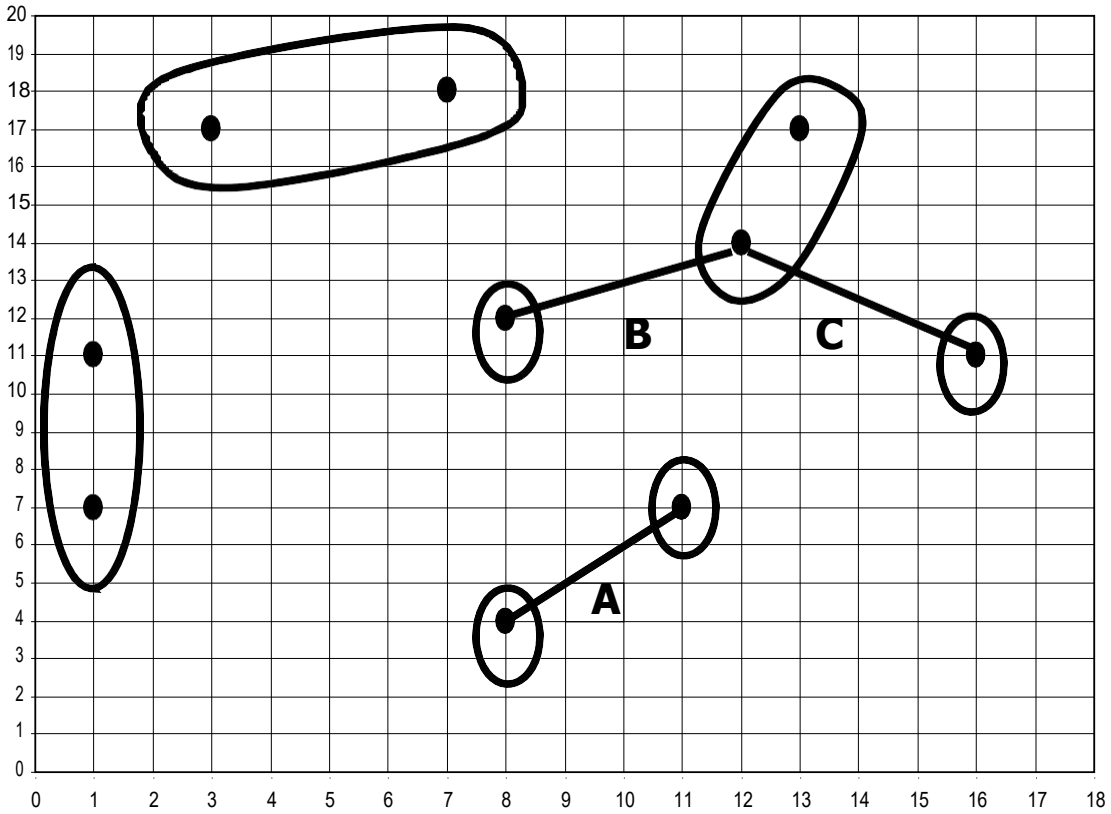
$$A = \sqrt{4^2 + 1^2} = 4.12$$

$$B = \sqrt{3^2 + 3^2} = 4.24$$

$$C = \sqrt{4^2 + 2^2} = 4.47$$

$$D = \sqrt{4^2 + 3^2} = 5$$

Agglomerative Clustering

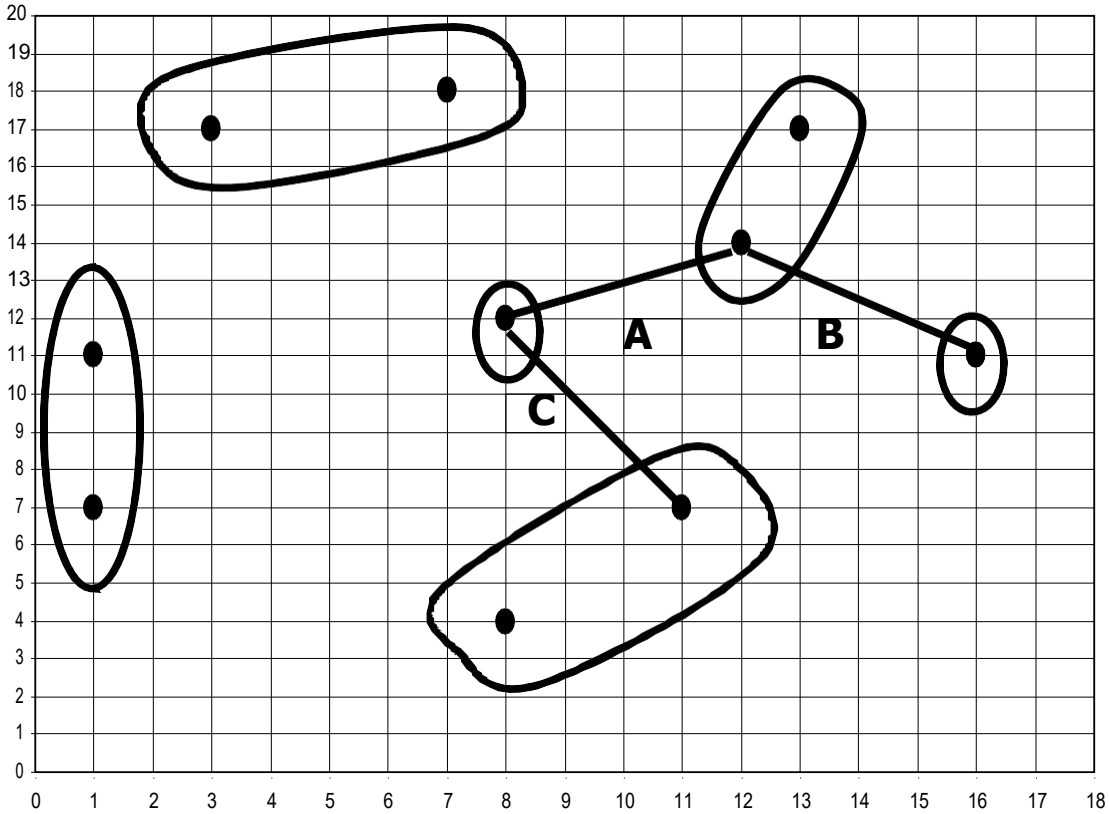


$$A = \sqrt{3^2 + 3^2} = 4.24$$

$$B = \sqrt{4^2 + 2^2} = 4.47$$

$$C = \sqrt{4^2 + 3^2} = 5$$

Agglomerative Clustering

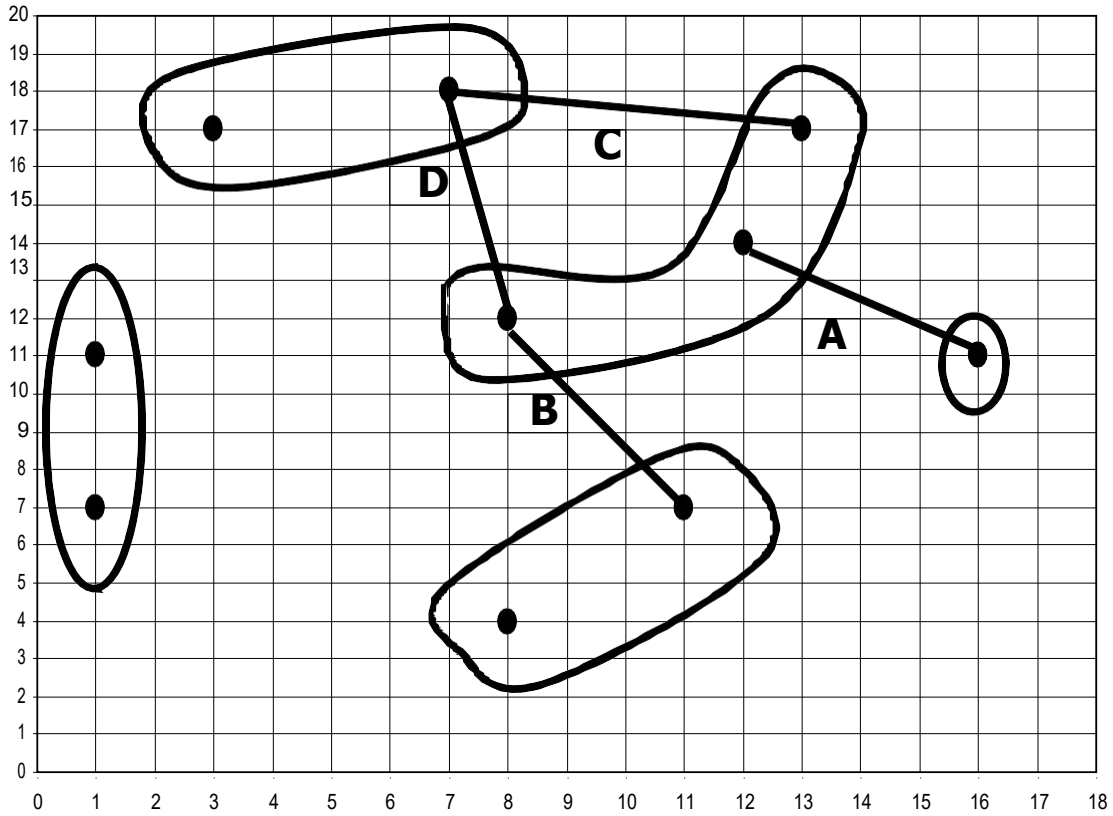


$$A = \sqrt{4^2 + 2^2} = 4.47$$

$$B = \sqrt{4^2 + 3^2} = 5$$

$$C = \sqrt{5^2 + 3^2} = 5.83$$

Agglomerative Clustering



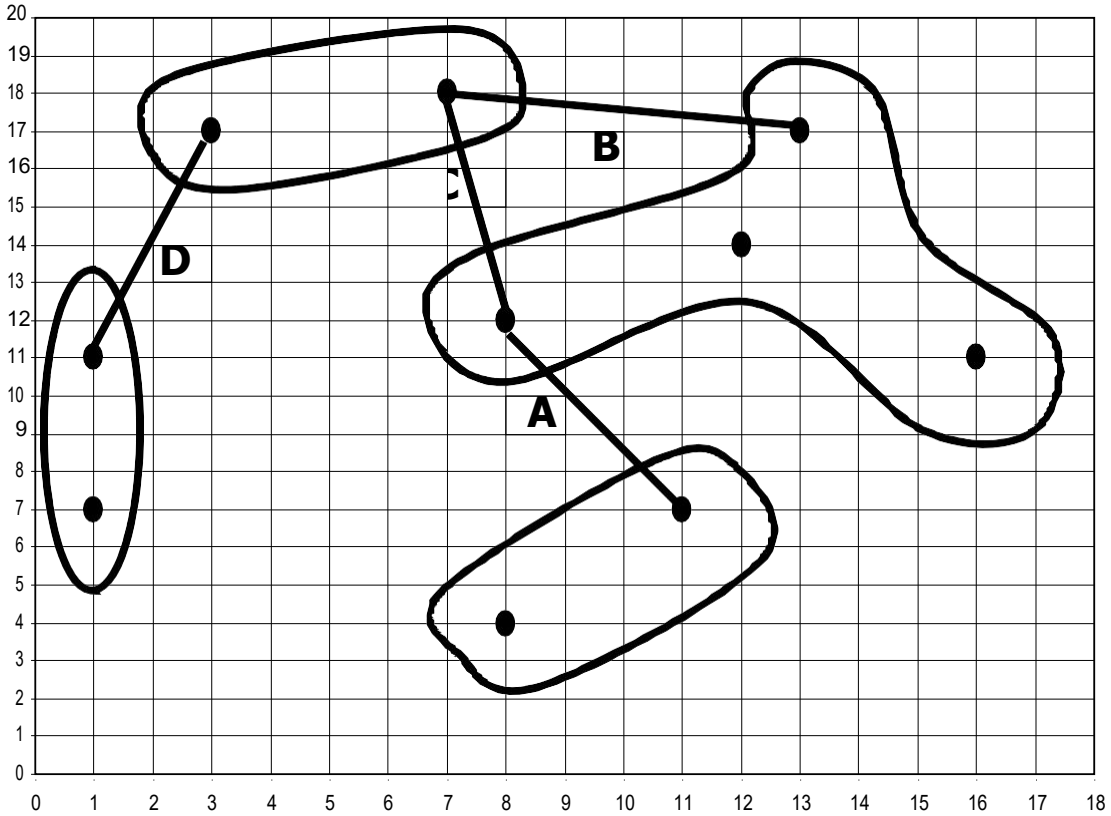
$$A = \sqrt{4^2 + 3^2} = 5$$

$$B = \sqrt{5^2 + 3^2} = 5.83$$

$$C = \sqrt{6^2 + 1^2} = 6.08$$

$$D = \sqrt{6^2 + 1^2} = 6.08$$

Agglomerative Clustering



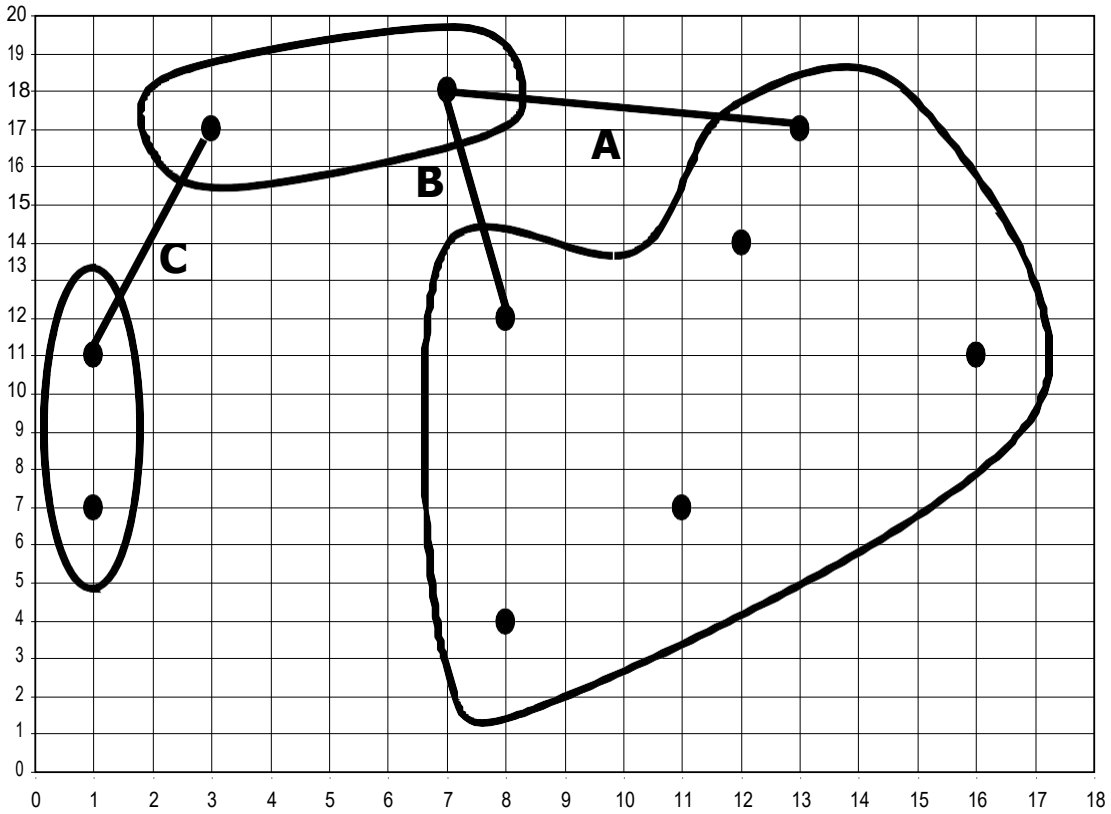
$$A = \sqrt{5^2 + 3^2} = 5.83$$

$$B = \sqrt{6^2 + 1^2} = 6.08$$

$$C = \sqrt{6^2 + 1^2} = 6.08$$

$$D = \sqrt{6^2 + 2^2} = 6.32$$

Agglomerative Clustering

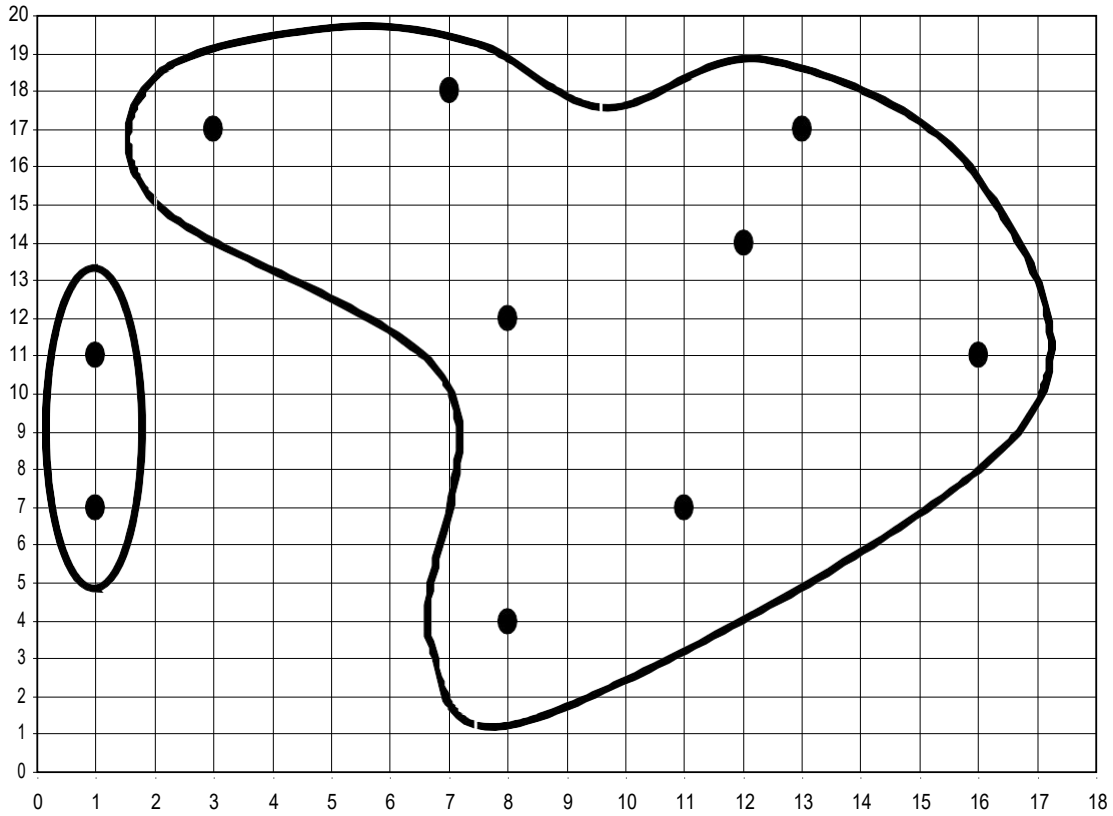


$$A = \sqrt{6^2 + 1^2} = 6.08$$

$$B = \sqrt{6^2 + 1^2} = 6.08$$

$$C = \sqrt{6^2 + 2^2} = 6.32$$

Agglomerative Clustering



Data Mining

Homework 4

Description

Homework 4

(Text Mining)

Given the database of documents shown in the next slide,

find all association rules

with minimum support = 2 and

minimum confidence = 80%.

Database D

DID	Keywords
D1	Behavior,Child,Therapy
D2	Dream,Psychology
D3	Child,Psychology,Therapy
D4	Behavior,Dream,Psychology
D5	Child,Dream,Psychology,Therapy
D6	Dream,Therapy
D7	Behavior,Child,Psychology

Data Mining

Homework 4 Solution

Text Mining Algorithm

Database D

DID	Keyword
D1	Behavior, Child, Therapy
D2	Dream,Psychology
D3	Child,Psychology,Therapy
D4	Behavior,Dream,Psychology
D5	Child,Dream,Psychology,Therapy
D6	Dream,Therapy
D7	Behavior,Child,Psychology

Scan D

C_1, L_1

Keywordset	Sup
{Behavior}	3
{Child}	4
{Dream}	4
{Psychology}	5
{Therapy}	4

Text Mining Algorithm

C_2

Keywordset
{Behavior,Child}
{Behavior,Dream}
{Behavior,Psychology}
{Behavior,Therapy}
{Child,Dream}
{Child,Psychology}
{Child,Therapy}
{Dream,Psychology}
{Dream,Therapy}
{Psychology,Therapy}

Scan D

C_2

Keywordset	Sup
{Behavior,Child}	2
{Behavior,Psychology}	2
{Child,Psychology}	3
{Child,Therapy}	3
{Dream,Psychology}	3
{Dream,Therapy}	2
{Psychology,Therapy}	2

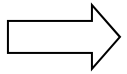
Text Mining Algorithm

L_2

Keywordset	Sup
{Behavior,Child}	2
{Behavior,Psychology}	2
{Child,Psychology}	3
{Child,Therapy}	3
{Dream,Psychology}	3
{Dream,Therapy}	2
{Psychology,Therapy}	2

C_3

Keywordset
{Behavior,Child,Psychology}
{Child,Psychology,Therapy}
{Dream,Psychology,Therapy}



Text Mining Algorithm

C_3

Keywordset	Sup
{Child,Psychology,Therapy}	2

L_3

Keywordset	Sup
{Child,Psychology,Therapy}	2

Association Rules

- Psychology, Therapy ⓪ Child (Support =2, confidence = 100%)

Data Mining

Homework 5

Description

Homework 5

(Connectivity Analysis)

Given is the table of linked web pages shown in the next slide, which forms a neighborhood graph.

(a) Find the authority in the neighborhood graph using six Iterations of the HITS algorithm.

(b) Find the hub in the neighborhood graph using six iterations of the HITS algorithm.

To answer the above questions, you need to calculate and show $\alpha(\rho)$ and $\lambda(\rho)$ for each web page ρ after the 6^{th} iteration of the HITS algorithm.

There is no need to consider normalization or convergence problems in this homework.

Web page	Linked to the web pages
A	C, E, F, G
B	C, F
C	A, D, F, G
D	B, C
E	A, D, G
F	C, E, G
G	A, B, C, F

Data Mining

Homework 5 Solution

Key for Homework 5

Site	Linked to the sites
A	C E F G
B	C F
C	A D F G
D	B C
E	A D G
F	C E G
G	A B C F

Initialization

ρ	α(ρ)	λ(ρ)
A	1	1
B	1	1
C	1	1
D	1	1
E	1	1
F	1	1
G	1	1

First Iteration

$\alpha(A) = \lambda(C)+\lambda(E)+\lambda(G)$
$= 1+1+1 = 3$

$\alpha(B) = \lambda(D)+\lambda(G)$
$= 1+1 = 2$

$\alpha(C) = \lambda(A)+\lambda(B)+\lambda(D)+\lambda(F)+\lambda(G)$
$= 1+1+1+1+1 = 5$

$\alpha(D) = \lambda(C)+\lambda(E)$
$= 1+1 = 2$

$\alpha(E) = \lambda(A)+ \lambda(F)$
$= 1 +1 =2$

$\alpha(F) = \lambda(A)+\lambda(B)+\lambda(C)+\lambda(G)$
$= 1+1+1+1 = 4$

$\alpha(G) = \lambda(A)+\lambda(C)+\lambda(E)+\lambda(F)$
$= 1+1+1+1 = 4$

$\lambda(A) = \alpha(C)+\alpha(E)+\alpha(F)+\alpha(G)$
$= 1+1+1+1 = 4$

$\lambda(B) = \alpha(C)+\alpha(F)$
$= 1+1 = 2$

$\lambda(C) = \alpha(A)+\alpha(D)+\alpha(F)+\alpha(G)$
$= 1+1+1+1 = 4$

$\lambda(D) = \alpha(B)+\alpha(C)$
$= 1+1 = 2$

$\lambda(E) = \alpha(A)+\alpha(D)+\alpha(G)$
$= 1+1+1 = 3$

$\lambda(F) = \alpha(C)+\alpha(E)+\alpha(G)$
$= 1+1+1 = 3$

$\lambda(G) = \alpha(A)+\alpha(B)+\alpha(C)+\alpha(F)$
$= 1+1+1+1 = 4$

Second Iteration

$\alpha(A) = \lambda(C)+\lambda(E)+\lambda(G)$
$= 4+3+4 = 11$

$\alpha(B) = \lambda(D)+\lambda(G)$
$= 2+4 = 6$

$\alpha(C) = \lambda(A)+\lambda(B)+\lambda(D)+\lambda(F)+\lambda(G)$
$= 4+2+2+3+4 = 15$

$\alpha(D) = \lambda(C)+\lambda(E)$
$= 4+3 = 7$

$\alpha(E) = \lambda(A) + \lambda(F)$
$= 4+3 = 7$

$\alpha(F) = \lambda(A)+\lambda(B)+\lambda(C)+\lambda(G)$
$= 4+2+4+4 = 14$

$\alpha(G) = \lambda(A)+\lambda(C)+\lambda(E)+\lambda(F)$
$= 4+4+3+3 = 14$

$\lambda(A) = \alpha(C)+\alpha(E)+\alpha(F)+\alpha(G)$
$= 5+2+4+4 = 15$

$\lambda(B) = \alpha(C)+\alpha(F)$
$= 5+4 = 9$

$\lambda(C) = \alpha(A)+\alpha(D)+\alpha(F)+\alpha(G)$
$= 3+2+4+4 = 13$

$\lambda(D) = \alpha(B)+\alpha(C)$
$= 2+5 = 7$

$\lambda(E) = \alpha(A)+\alpha(D)+\alpha(G)$
$= 3+2+4 = 9$

$\lambda(F) = \alpha(C)+\alpha(E)+\alpha(G)$
$= 5+2+4 = 11$

$\lambda(G) = \alpha(A)+\alpha(B)+\alpha(C)+\alpha(F)$
$= 3+2+5+4 = 14$

Third Iteration

$\alpha(A) = \lambda(C)+\lambda(E)+\lambda(G)$
$= 13+9+14 = 36$

$\alpha(B) = \lambda(D)+\lambda(G)$
$= 7+14 = 21$

$\alpha(C) = \lambda(A)+\lambda(B)+\lambda(D)+\lambda(F)+\lambda(G)$
$= 15+9+7+11+14 = 56$

$\alpha(D) = \lambda(C)+\lambda(E)$
$= 13+9 = 22$

$\alpha(E) = \lambda(A) + \lambda(F)$
$= 15+11 = 26$

$\alpha(F) = \lambda(A)+\lambda(B)+\lambda(C)+\lambda(G)$
$= 15+9+13+14 = 51$

$\alpha(G) = \lambda(A)+\lambda(C)+\lambda(E)+\lambda(F)$
$= 15+13+9+11 = 48$

$\lambda(A) = \alpha(C)+\alpha(E)+\alpha(F)+\alpha(G)$
$= 15+7+14+14 = 50$

$\lambda(B) = \alpha(C)+\alpha(F)$
$= 15+14 = 29$

$\lambda(C) = \alpha(A)+\alpha(D)+\alpha(F)+\alpha(G)$
$= 11+7+14+14 = 46$

$\lambda(D) = \alpha(B)+\alpha(C)$
$= 6+15 = 21$

$\lambda(E) = \alpha(A)+\alpha(D)+\alpha(G)$
$= 11+7+14 = 32$

$\lambda(F) = \alpha(C)+\alpha(E)+\alpha(G)$
$= 15+7+14 = 36$

$\lambda(G) = \alpha(A)+\alpha(B)+\alpha(C)+\alpha(F)$
$= 11+6+15+14 = 46$

Fourth Iteration

$\alpha(A) = \lambda(C)+\lambda(E)+\lambda(G)$
$= 46+32+46 = 124$

$\alpha(B) = \lambda(D)+\lambda(G)$
$= 21+46 = 67$

$\alpha(C) = \lambda(A)+\lambda(B)+\lambda(D)+\lambda(F)+\lambda(G)$
$= 50+29+21+36+46 = 182$

$\alpha(D) = \lambda(C)+\lambda(E)$
$= 46+32 = 78$

$\alpha(E) = \lambda(A) + \lambda(F)$
$= 50 + 36 = 86$

$\alpha(F) = \lambda(A)+\lambda(B)+\lambda(C)+\lambda(G)$
$= 50+29+46+46 = 171$

$\alpha(G) = \lambda(A)+\lambda(C)+\lambda(E)+\lambda(F)$
$= 50+46+32+36 = 164$

$\lambda(A) = \alpha(C)+\alpha(E)+\alpha(F)+\alpha(G)$
$= 56+26+51+48 = 181$

$\lambda(B) = \alpha(C)+\alpha(F)$
$= 56+51 = 107$

$\lambda(C) = \alpha(A)+\alpha(D)+\alpha(F)+\alpha(G)$
$= 36+22+51+48 = 157$

$\lambda(D) = \alpha(B)+\alpha(C)$
$= 21+56 = 77$

$\lambda(E) = \alpha(A)+\alpha(D)+\alpha(G)$
$= 36+22+48 = 106$

$\lambda(F) = \alpha(C)+\alpha(E)+\alpha(G)$
$= 56+26+48 = 130$

$\lambda(G) = \alpha(A)+\alpha(B)+\alpha(C)+\alpha(F)$
$= 36+21+56+51 = 164$

Fifth Iteration

$\alpha(A) = \lambda(C)+\lambda(E)+\lambda(G)$
$= 157+106+164 = 427$

$\alpha(B) = \lambda(D)+\lambda(G)$
$= 77+164 = 241$

$\alpha(C) = \lambda(A)+\lambda(B)+\lambda(D)+\lambda(F)+\lambda(G)$
$= 181+107+77+130+164$
$= 659$

$\alpha(D) = \lambda(C)+\lambda(E)$
$= 157+106 = 263$

$\alpha(E) = \lambda(A) + \lambda(F)$
$= 181+130=311$

$\alpha(F) = \lambda(A)+\lambda(B)+\lambda(C)+\lambda(G)$
$= 181+107+157+164 = 609$

$\alpha(G) = \lambda(A)+\lambda(C)+\lambda(E)+\lambda(F)$
$= 181+157+106+130 = 574$

$\lambda(A) = \alpha(C)+\alpha(E)+\alpha(F)+\alpha(G)$
$= 182+86+171+164 = 603$

$\lambda(B) = \alpha(C)+\alpha(F)$
$= 182+171 = 353$

$\lambda(C) = \alpha(A)+\alpha(D)+\alpha(F)+\alpha(G)$
$= 124+78+171+164 = 537$

$\lambda(D) = \alpha(B)+\alpha(C)$
$= 67+182 = 249$

$\lambda(E) = \alpha(A)+\alpha(D)+\alpha(G)$
$= 124+78+164 = 366$

$\lambda(F) = \alpha(C)+\alpha(E)+\alpha(G)$
$= 182+86+164 = 432$

$\lambda(G) = \alpha(A)+\alpha(B)+\alpha(C)+\alpha(F)$
$= 124+67+182+171 = 544$

Sixth Iteration

$\alpha(A) = \lambda(C)+\lambda(E)+\lambda(G)$
$= 537+366+544 = 1447$

$\alpha(B) = \lambda(D)+\lambda(G)$
$= 249+544 = 793$

$\alpha(C) = \lambda(A)+\lambda(B)+\lambda(D)+\lambda(F)+\lambda(G)$
$= 603+353+249+432+544$
$= 2181$

$\alpha(D) = \lambda(C)+\lambda(E)$
$= 537+366 = 903$

$\alpha(E) = \lambda(A) + \lambda(F)$
$= 603+432= 1035$

$\alpha(F) = \lambda(A)+\lambda(B)+\lambda(C)+\lambda(G)$
$= 603+353+537+544 = 2037$

$\alpha(G) = \lambda(A)+\lambda(C)+\lambda(E)+\lambda(F)$
$= 603+537+366+432 = 1938$

$\lambda(A) = \alpha(C)+\alpha(E)+\alpha(F)+\alpha(G)$
$= 659+311+609+574 = 2153$

$\lambda(B) = \alpha(C)+\alpha(F)$
$= 659+609 = 1268$

$\lambda(C) = \alpha(A)+\alpha(D)+\alpha(F)+\alpha(G)$
$= 427+263+609+574 = 1873$

$\lambda(D) = \alpha(B)+\alpha(C)$
$= 241+659 = 900$

$\lambda(E) = \alpha(A)+\alpha(D)+\alpha(G)$
$= 427+263+574 = 1264$

$\lambda(F) = \alpha(C)+\alpha(E)+\alpha(G)$
$= 659+311+574 = 1544$

$\lambda(G) = \alpha(A)+\alpha(B)+\alpha(C)+\alpha(F)$
$= 427+241+659+609 = 1936$

After Six Iterations

ρ	α(ρ)	λ(ρ)
A	1447	2153
B	793	1268
C	2181	1873
D	903	900
E	1035	1264
F	2037	1544
G	1938	1936